

Determination of Fracture Parameters of Effective Crack Model by Wedge-Splitting Test

A. Tevfik Bildik*, Ragıp İnce

Firat University Engineering Faculty Civil Engineering Department Elazığ
*tbildik@firat.edu.tr

(Received: 07.02.2017; Accepted: 17.04.2017)

Abstract

Although the cracked beams have been widely utilized in fracture mechanics of concrete, there have been some advantages of the cubical/cylindrical specimens such as compactness and lightness. In the present work, the wedge-split-tension tests on cubical specimens with different cement contents and water/cement ratios were initially performed for the effective crack model. Finally, some relationships based on regression between the fracture parameters and the strength properties of concrete were derived. The results of the split-tension cube tests look viable and very promising.

Keywords: Concrete; Effective Crack Model; Wedge-splitting Test.

Kama-Yarma Testi ile Efektif Çatlak Modelinin Kırılma Parametrelerinin Belirlenmesi

Özet

Betonun kırılma mekaniğinde genellikle çentikli kiriş numuneler kullanılmakla birlikte küp ve silindirik numunelerin de bazı avantajları vardır. Bu çalışmada, çimento miktarları ve su/çimento oranları farklı olan kama-yarma numuneleri efektif çatlak modeline göre teste tabi tutulmuştur. Sonuç olarak betonun kırılma parametreleri ile basınç ve yarma-çekme mukavemetleri arasında bazı regresyon bağıntıları türetilmiştir.

Anahtar Kelimeler: Beton; Efektif Çatlak Modeli; Kama-Yarma Testi.

1. Introduction

Fracture mechanics applications of cement-based materials were initiated by Kaplan [1]. However, in 1970s, experimental investigations on concrete fracture revealed that Linear Elastic Fracture Mechanics (LEFM) has been no longer valid for cementitious materials such as rock and concrete [2]. Because of the existence of a relatively large process zone in front of and around the tip of the main notches and this inelastic zone is ignored by LEFM, it is inapplicable for concrete. Therefore, several deterministic fracture-mechanics approaches have been developed to describe fracture-dominated failure of concrete structures [3-8].

These models can be categorized as cohesive crack models [3, 4] and effective crack models [5-8]. LEFM uses a single fracture parameter such as the critical strain energy release rate, but these models need at least two experimentally

determined fracture parameters to estimate failure of concrete/reinforced concrete structures.

The cracked beam tests have been widely utilized, because they were used in the first LEFM standard test for metals in order to estimate fracture parameters of quasi-brittle materials. Nevertheless, there are some important advantages of cubical/cylindrical specimens as follows [9-13]:

1) They are compact and lighter than notched beams. Therefore they are useful for investigating the size effect.

2) They can be easily cast at the construction site by using the same molds as for strength tests.

3) The self-weight of the specimens can be neglected in the computing of fracture parameters, contrary to cracked beams.

The tests on the cylinder and cube samples in fracture mechanics of concrete can be classified as split-tension tests and wedge-splitting tests.

The wedge-splitting tests have been performed on the cylinder/cube specimens with an edge crack. The wedge-splitting tests are also useful for the effective crack models although they were initially developed for the cohesive crack fracture models [9].

In the present work, the wedge-splitting tests (WST) on cube specimens with different cement contents and water/cement ratios were initially performed for the effective crack fracture model (ECM). Finally, some relationships based on regression between the strength properties and the fracture parameters of concrete were determined.

2. Effective Crack Model (ECM)

The effective notch length a_e in the effective crack model for the fracture of cementitious materials recommended by Nallathambi and Karihaloo [6] is computed from the secant stiffness of the real concrete body at the maximum load. The main idea behind this approach may be emphasized with Figure 1, in which the load and deflection curve of the cracked three-point beam up to maximum load is indicated. When the secant stiffness of the real body is equal to tangent stiffness of the body, of which the notch length is a_e , the fracture toughness achieves its critical value K_{Ic}^e . Consequently, according to the effective crack model, the fracture of cement-based materials is defined by two-parameter: the critical fracture toughness K_{Ic}^e and the effective notch length a_e .

Though the effective notch length is computed from the load and displacement curves by trial and error approach in practice, it can also be calculated by the following regression formula:

$$\frac{a_e}{d} = \gamma_1 \left(\frac{\sigma_u}{E_c} \right)^{\gamma_2} \left(\frac{a_0}{d} \right)^{\gamma_3} \left(1 + \frac{d_{max}}{d} \right)^{\gamma_4} \quad (1)$$

in which a_0 is the initial notch length, d is the specimen depth, d_{max} is the maximum aggregate size, the nominal strength $\sigma_u = 6M_u / (bd^2)$, M_u is the maximum moment and $\gamma_1 = 0.088$, $\gamma_2 = -0.208$, $\gamma_3 = 0.451$ and $\gamma_4 = 1.653$. When elasticity modulus of cement-based materials E_c is

estimated from the separate experiments, these constants are $\gamma_1 = 0.198$, $\gamma_2 = -0.131$, $\gamma_3 = 0.394$ and $\gamma_4 = 0.600$ [14]. Nevertheless, elasticity modulus of cementitious materials in Eq. (1) may be determined according to the following expression [15]:

$$E_c = 4730 \sqrt{f'_c} \quad (2)$$

where f'_c is the cylindrical strength for concrete. E_c and f'_c in Eq. (2) are in [MPa]. The critical fracture toughness K_{Ic}^e according to the effective crack model may be determined from Eq. (3):

$$K_{Ic}^e = \sigma_u \sqrt{a_e} Y(\alpha_e = a_e/d) \quad (3)$$

in which α is the relative notch length (a/d) and $Y(\alpha)$ is the function of geometry for computing the fracture toughness of the notched three-point beam and may be obtained from any fracture mechanics handbook [16].

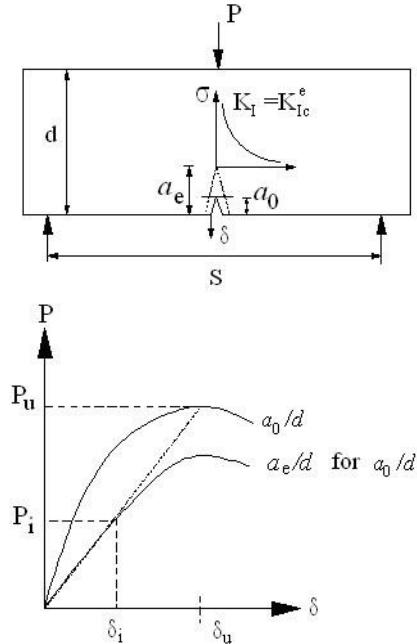


Figure 1. Determination of fracture parameters of concrete according to ECM

3. Wedge-Splitting Test

Although notched-beam specimens have been widely used in concrete fracture mechanics, compact tension (CT) and wedge-splitting (WS) specimens have some advantages over beams,

such as compactness and lightness (Fig. 2). CT specimens were initially used by Wittmann et al. [10] to determine the fracture energy and evaluate the strain-softening behavior of cement-based materials. Brühwiler and Wittmann [9] proposed a popular wedge-splitting test, which has been used in recent years in concrete fracture testing with test specimens.

A WS specimen can be considered a compact form of the three-point bending beam, as shown in Fig. 2b. WS specimens with grooves were developed for use as CT specimens, as shown in Fig. 2c. WS and CT testing can be conducted on both cylindrical and cubical specimens. The use of cylindrical test specimens, which may be obtained from existing concrete/reinforced concrete structures by coring, offers the great advantage of estimating the fracture properties of existing structures based on fracture mechanics. In WS testing, the load is applied to the specimen by means of a wedge and a loading device with roller bearings, as illustrated in Fig. 2d. The horizontal load P_H acts on the rollers because of the vertical load (P_V) on the wedge, as shown in Fig. 2e. Friction forces also occur between the rollers and the wedge. However, the friction forces can be ignored when the wedge angle $\theta=15^\circ$. The horizontal load can be calculated as follows:

$$P_H = \frac{P_V}{2 \tan \theta} \quad (4)$$

For CT and WS test samples, the fracture toughness can be computed as follows:

$$K_I = \frac{P_H}{bd} \sqrt{d} Y(\alpha) \quad (5)$$

where the dimensionless function $Y(\alpha)$ is given by the following equation [16]:

$$Y(\alpha) = \frac{(2 + \alpha)(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4)}{(1 - \alpha)^{3/2}} \quad (6)$$

The accuracy of Eq. (6) is $\pm 0.5\%$ for $\alpha > 0.2$.

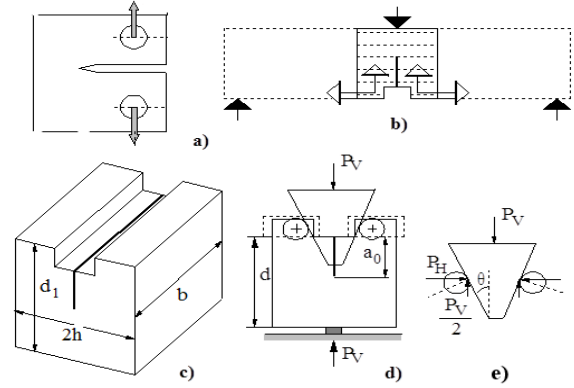


Figure 2. Wedge-splitting test

a) Standard CT specimen b) WS specimen as “compact” three-point bending specimen [9] c) Specimen configuration d) Loading e) Wedge forces

4. Experimental Studies

Cubical wedge-splitting (WS) specimens with 150 mm were used in this study (Figure 2). The maximum aggregate diameter of 16mm was used. The cement contents varied from 250 kg/m³ to 490 kg/m³ whereas the water-cement ratios (w/c) varied from 0.44 to 0.81. The batches were designed for two slump values=6±1 and 12±2 cm. Eight series specimens (48 cube specimens), namely A, B, C, D, E, F, G and H, were tested concerning the above variables in different combinations. The test specimens in each batches were cast from the same mix of concrete. The cracks were precast in all test samples. The test specimens in all series were of the same size, but they had different notch lengths for each specimen geometry. Table 1 summarizes the cement content, the initial crack length, a_0 , the concrete compressive strength, f'_c , and the ultimate vertical load, P_V , of the test specimens.

Three identical cube specimens with 150 mm were also cast from each mix of concrete in order to estimate the compressive strength of concrete. All test specimens and identical test cubes were removed from the mold after 1 day and then were cured at 20 °C in water until testing at 28-day. The compression tests and the split-tension tests were made by using a digital compression machine with a capacity of 100 kN. Typically, approximately 3 min (± 30 sec) elapsed before the peak load capacity for each test specimen was reached. Identical cubes were tested at an age similar to the other specimens.

5. Analysis of Test Results

In this study, WS specimens were analyzed according to ECM. For this, equations 1-6 were utilized. The nominal strength in Eq. (1) may be computed for WS specimens according to the principles of the classical strength of materials as follows:

$$\sigma_u = \frac{P_H}{bd} + \frac{6P_H d/2}{bd^2} = \frac{4P_H}{bd} \quad (7)$$

The computed fracture parameters of concrete based on ECM: the effective fracture toughness and the relative effective notch length values were also given in the last two columns in table 2. It is well known that there is a very strong correlation between f'_c and water-cement ratio. f'_c decreases with increasing w/c. The following formula by Abrams may be utilized to stabilize the effects of the factors related to curing conditions, concrete age and cement properties, which directly influence the internal structure of cement-based materials:

$$f_c = \frac{K_1}{K_2^{w/c}} \quad (8)$$

where K_1 and K_2 are the empirical constant and the constants which depends on the cement properties, respectively [17]. These constants may be computed as $K_1=A$ and $K_2=e^{-C}$ from the regression based on exponential function performed on $Y=AX+C$ with $Y=f'_c$, $X=w/c$. Fig. 3 shows the two relation-ships $K_{lc}^e - f'_c$ and $K_{lc}^e - w/c$. The two empirical formulas were derived in this figure. The results indicate that K_{lc}^e increases with increasing f'_c while K_{lc}^e decreases with increasing w/c.

Tests have revealed that fracture parameters of cement-based materials are generally influenced by the four material parameters; namely compressive strength, aggregate type, maximum aggregate diameter and water/cement ratio (w/c) [11-13, 18-20]. It is noted that fracture parameters of concrete can also be affected by other material parameters such as cement type, aggregate/sand ratio, curing conditions and porosity etc. Therefore, these empirical formulas are approximate and they should only be utilized for preliminary design

and for the bodies of low fracture sensitivity although their correlation coefficients are very high $r>0.900$.

Table 1. Experimental results and analysis results

Series-Slump	Cement kg/m ³	w/c	f'_c MPa	a_0 mm	P_v kN	K_{lc}^e MPa√m	a_e/d
A-12	250	0.81	19.24	50	2.59	0.883	0.523
				50	2.71	0.914	0.520
				80	1.23	0.843	0.695
				80	1.21	0.836	0.696
				95	0.71	0.948	0.799
				95	0.70	0.945	0.800
B-6	250	0.76	22.45	50	2.78	0.949	0.524
				50	2.93	0.988	0.520
				80	1.42	0.944	0.689
				80	1.36	0.922	0.692
				95	0.89	1.056	0.783
				95	0.87	1.051	0.786
C-12	330	0.58	35.56	50	3.37	1.160	0.527
				50	3.65	1.234	0.521
				80	1.68	1.149	0.694
				80	1.68	1.149	0.694
				95	1.01	1.299	0.794
				95	0.90	1.275	0.806
D-6	325	0.56	36.97	50	3.45	1.186	0.526
				50	3.35	1.159	0.528
				80	1.72	1.174	0.694
				80	1.71	1.171	0.694
				95	0.97	1.311	0.800
				95	0.78	1.281	0.823
E-12	410	0.54	44.07	50	3.71	1.280	0.527
				50	3.71	1.280	0.527
				80	1.78	1.247	0.699
				80	1.63	1.193	0.707
				95	0.96	1.412	0.811
				95	0.97	1.413	0.809
F-6	400	0.53	46.63	50	3.60	1.258	0.531
				50	3.64	1.269	0.531
				80	1.82	1.279	0.699
				80	1.83	1.282	0.699
				95	1.00	1.454	0.809
				95	0.99	1.452	0.810
G-12	490	0.45	51.35	50	3.99	1.378	0.528
				50	4.18	1.428	0.524
				80	1.86	1.324	0.702
				80	1.78	1.295	0.706
				95	1.03	1.523	0.811
				95	0.95	1.513	0.820
H-6	480	0.44	54.07	50	3.87	1.353	0.531
				50	4.11	1.418	0.527
				80	1.79	1.316	0.708
				80	1.93	1.366	0.701
				95	1.06	1.563	0.811
				95	0.95	1.550	0.823

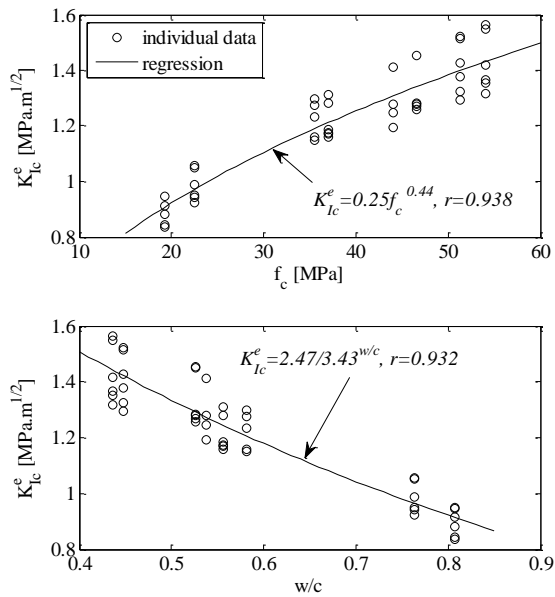


Figure 3. Variation of K_{Ic}^e with f_c and w/c

6. Conclusions

In recent years, splitting specimens such as compact-tension, wedge-splitting and cylindrical/cubical splitting tension test specimens have been commonly preferred over beams for use in concrete fracture testing. Wedge-splitting test results were used for the first time in this study to determine the fracture parameters of cement-based materials via the effective crack model. The following conclusions should be drawn from the results of this study:

- 1) Notched compact tension specimens and beams have been used with the effective crack model. The results of this study indicate that the effective crack model can be successfully applied to wedge-splitting specimens.
- 2) Many structural laboratories do not have sophisticated testing equipment such as closed-loop testing systems and displacement-controlled testing machines. The effective crack model offers the great advantage of requiring measurement of only the maximum load applied to specimens to determine the values of the fracture parameters of concrete.
- 3) The fracture parameters of cementitious materials required for the effective crack model were investigated in this study. Nevertheless, the results obtained may easily be transformed to other fracture models, such as the size effect

fracture model, the two-parameter model and the double-K model by the related LFM formulas.

7. References

1. Kaplan, M.F. (1961). Crack propagation and the fracture of concrete. *ACI J.*, **58(11)**: 591-610.
2. Kesler, C.E., Naus, D.J. and Lott, L.L. (1972). Fracture mechanics-its applicability to concrete. *The Soc. of Mater. Sci.*, **4**: 113-124.
3. Hillerborg, A., Modeer, M. and Petersson, P.E. (1976). Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cem. Conc. Res.*, **6**: 773-782.
4. Bazant, Z.P. and Oh, B.H. (1983). Crack band theory for fracture concrete. *Mater. and Struct. (RILEM)*, **16(93)**: 155-157.
5. Jenq, Y.S. and Shah, S.P. (1985). Two-parameter fracture model for concrete. *ASCE J. Engng. Mech.*, **111(10)**: 1227-1241.
6. Nallathambi, P. and Karihaloo, B.L. (1986). Determination of the specimen size independent fracture toughness of plain concrete. *Mag. Conc. Res.*, **38(135)**: 67-76.
7. Bazant, Z.P. and Kazemi, M.T. (1990). Determination of fracture energy, process zone length, and brittleness number from size effect with application to rock and concrete. *Int. J. of Fract.*, **44(2)**: 111-131.
8. Xu, S. and Reinhardt, H.W. (1999). Determination of double-K criterion for crack propagation in quasi-brittle fracture, Part II: Analytical evaluating and practical measuring methods for three-point bending notched beams. *Int. J. of Fract.*, **98**: 151-177.
9. Brühwiler, E. and Wittmann, F.H. (1990). The wedge splitting test, a method of performing stable fracture tests. *Engng. Fract. Mech.*, **35**: 117-126.
10. Wittmann, F.H., Rokugo, K., Brühwiler, E., Mihashi, H. and Simonin, P. (1988). Fracture energy and strain softening of concrete as determined by means of compact tension specimens. *Mater. and Struct.*, **21**: 21-32.
11. Ince, R. (2010). Determination of concrete fracture parameters based on two-parameter and size effect models using split-tension cubes. *Engng. Fract. Mech.*, **77**: 2233-2250.
12. Ince, R. (2012). Determination of concrete fracture parameters based on peak-load method with diagonal split-tension cubes. *Engng. Fract. Mech.*, **82**: 100-114.
13. Ince, R. (2012). Determination of the fracture parameters of the Double-K model using weight functions of split-tension specimens. *Engng. Fract. Mech.*, **96**: 416-432.
14. Karihaloo, B.L. and Nallathambi, P. (1989). An improved effective crack model for the determination

of fracture toughness of concrete. *Cem. Conc. Res.*, **19**: 603-610.

15. ACI-318. (2002). Building code requirements for structural concrete and commentary. Farmington Hills, Michigan.

16. Tada, H., Paris, P.C. and Irwin, G.R. (2000). The Stress Analysis of Cracks Handbook. ASME Press.

17. Neville, A.M. (1995). Properties of Concrete. Fourth Edition, Longman, London.

18. Bazant, Z.P. and Becq-Giraudon, E. (2002). Statistical prediction of fracture parameters of

concrete and implications for choice of testing standard. *Cem. Conc. Res.*, **32**: 529-556.

19. Ince, R. (2004). Prediction of fracture parameters of concrete by artificial neural networks. *Engng. Fract. Mech.*, **71**: 2143-2159.

20. Ince, R. (2010). Artificial neural network-based analysis of effective crack model in concrete fracture. *Fatigue. Fract. Engng. Mater. Struct.*, **33(9)**: 595-606.