http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 15.01.2018 *Year*: **2018**, *Number*: **21**, *Pages*: **59-67**
Published: 02.03.2018 *Original Article*

On Topology of Fuzzy Strong $b-$Metric Spaces

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Abstaract − In this study, we introduce and investigate the concept of fuzzy strong b-metric space such that is a fuzzy analogy of strong b-metric spaces. By using the open balls, we define a topology on these spaces which is Hausdorff and first countable. Later we show that open balls are open and closed balls are closed. After defining the standard fuzzy strong b-metric space induced by a strong b-metric, we show that these spaces have same topology. We also note that every separable fuzzy strong b-metric space is second countable. Moreover, we give the uniform convergence theorem for these spaces.

Keywords − *Fuzzy strong b-metric space, strong b-metric space, b-metric spaces, uniform convergence.*

1 Introduction and Preliminaries

The concept of b-metric space obtained by modifying the triangle inequality has been introduced by many authors.

Definition 1.1 ([3, 14, 8, 4, 13])**.** An ordered triple (X, D, K) is called b-metric (metric type) space and D is called b-metric on X if X is a nonempty set, $K \geq 1$ is a given real number and $D{:}X \times X \to [0, \infty)$ satisfies the following conditions for all $x, y, z \in X$
1) $D(x, y) = 0$ if and only if $x = y$,
2) $D(x, y) = D(y, x)$,
3) $D(x, z) \leq K[D(x, y) + D(y, z)]$.

For a b-metric space (X, D, K), the b-metric D need not be continuous, an open ball is not necessarily open and a closed ball is not necessarily closed where $B(x, r) = \{y : D(x, y) < r\}$ is an open ball, $B[x, r] = \{y : D(x, y) \leq r\}$ is a closed ball and A is an open set if for any $x \in A$ there exists an open ball $B(x, r)$ such $B(x, r) \subset A$ [15, 16, 11].

This fact suggests a strengthening of the notion of b-metric spaces.

Definition 1.2 ([16])**.** An ordered triple (X, D, K) is called strong b-metric space and D is called strong b-metric on X if X is a nonempty set, $K \geq 1$ is a given real number and $D: X \times X \to [0, \infty)$ satisfies the following conditions for all $x, y, z \in X$
1) $D(x, y) = 0$ if and only if $x = y$,
2) $D(x, y) = D(y, x)$,
3) $D(x, z) \leq D(x, y) + KD(y, z)$.

Remark 1.3 ([16])**.** Let (X, D, K) be a strong b-metric space.
(1) The strong b-metric D is continuous.
(2) Every open ball $B(x, r)$ is open.

After Zadeh [6] introduced the theory of fuzzy sets, many authors have introduced and studied several notions of metric fuzziness [1, 9, 17, 7, 10] from different points of view.

Fuzzy metric type spaces, which is a generalization of fuzzy metric space in sense of George and Veeramani [1] have been introduced and studied in [12] as a fuzzy analogy of b-metric spaces.

Definition 1.4 ([2])**.** A binary operation $* : [0, 1] \times [0, 1] \longrightarrow [0, 1]$ is a continuous t-norm if $*$ satisfies the following conditions;
1) $*$ is associative and commutative,
2) $*$ is continuous,
3) $a * 1 = a$ for all $a \in [0, 1]$,
4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $a, b, c, d \in [0, 1]$.

Definition 1.5 ([12])**.** A 4-tuple $(X, M, *, K)$ is called a fuzzy metric type (fuzzy b-metric) space and M is called fuzzy metric type (fuzzy b-metric) on X if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, and M is a fuzzy set on $X \times X \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,
1) $M(x, y, t) > 0$,
2) $M(x, y, t) = 1$ if and only if $x = y$,
3) $M(x, y, t) = M(y, x, t)$,
4) $M(x, y, t) * M(y, z, s) \leq M(x, z, K(t + s))$ for some constant $K \geq 1$,
5) $M(x, y, .) : (0, \infty) \to [0, 1]$ is continuous.

In a similar manner, in this study, we introduce a new concept, fuzzy strong b-metric space, as a fuzzy analogy of strong b-metric spaces and present some elementary results.

Remark 1.6 ([1])**.** For any $r_1 > r_2$, we can find a r_3 such that $r_1 * r_3 \geq r_2$ and for any r_4 we can find a r_5 such that $r_5 * r_5 \geq r_4$ (r_1,r_2,r_3,r_4,$r_5 \in (0, 1)$).

2 Fuzzy strong b-metric space

Definition 2.1. Let X be a non-empty set, $K > 1$, $*$ is a continuous t-norm and M be a fuzzy set on $X \times X \times (0, \infty)$ such that for all $x, y, z \in X$ and $t, s > 0$,
1) $M(x, y, t) > 0$,
2) $M(x, y, t) = 1$ if and only if $x = y$,

3) $M(x,y,t) = M(y,x,t)$,
4) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+Ks)$,
5) $M(x,y,.) : (0,\infty) \to [0,1]$ is continuous.
Then M is called a fuzzy strong b-metric on X and $(X,M,*,K)$ is called a fuzzy strong b-metric space.

Example 2.2. Let (X,D,K) be a strong b-metric space. Define

$$M_D(x,y,t) = \frac{t}{t+D(x,y)}$$

for $t > 0$ and $x,y \in X$. Then $(X,M_D,\cdot,K)$ is a fuzzy strong b-metric space and is called standard fuzzy strong b-metric space induced by D. Here (1)–(3) and (5) are obvious and we show (4).

$$\begin{aligned} M_D(x,z,t)\cdot M_D(z,y,s) &= \frac{t}{t+D(x,z)}\cdot\frac{s}{s+D(z,y)} \\ &= \frac{1}{1+\frac{D(x,z)}{t}}\cdot\frac{1}{1+\frac{D(z,y)}{s}} \\ &\leq \frac{1}{1+\frac{D(x,z)}{t+Ks}}\cdot\frac{1}{1+\frac{KD(z,y)}{t+Ks}} \\ &\leq \frac{1}{1+\frac{D(x,z)+KD(z,y)}{t+Ks}} \\ &\leq \frac{1}{1+\frac{D(x,z)}{t+Ks}} \\ &= \frac{t+Ks}{t+Ks+D(x,z)} \\ &= M_D(x,y,t+Ks) \end{aligned}$$

Proposition 2.3. Let $(X,M,*,K)$ be a fuzzy strong b-metric space. Then $M(x,y,\_) : (0,\infty) \longrightarrow [0,1]$ is nondecreasing for all $x,y \in X$.

Proof. Assume that $M(x,y,t) > M(x,y,s)$, for $s > t > 0$. We have $M(x,y,t) * M\left(y,y,\frac{s-t}{K}\right) \leq M(x,y,s) < M(x,y,t)$. Since $M(y,y,s-t) = 1$, we have $M(x,y,t) < M(x,y,t)$ that is a contradiction. □

Definition 2.4. Let $(X,M,*,K)$ be a fuzzy strong b-metric space. For $t > 0$, the open ball $B(x,r,t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B(x,r,t) = \{y \in X : M(x,y,t) > 1-r\}.$$

A subset $A \subset X$ is called open if for any $x \in A$, there exist $r \in (0,1)$ and $t > 0$ such that $B(x,r,t) \subset A$.

Proposition 2.5. Let $(X,M,*,K)$ be a fuzzy strong b-metric space and τ_M be the family of all open sets in X. Then τ_M is a topology on X.

Proof. 1. Clearly $\emptyset, X \in \tau_M$.

2. Let $A, B \in \tau_M$ and $x \in A \cap B$. Then $x \in A$ and $x \in B$, so there exist $t_1, t_2 > 0$ and $r_1, r_2 \in (0,1)$ such that $B(x, r_1, t_1) \subset A$ and $B(x, r_2, t_2) \subset B$. Let $t = \min\{t_1, t_2\}$ and $r = \min\{r_1, r_2\}$. Then $B(x, r, t) \subset B(x, r_1, t_1) \cap B(x, r_2, t_2) \subset A \cap B$. Thus $A \cap B \in \tau_M$.

3. Let $A_i \in \tau_M$ for each $i \in I$ and $x \in \bigcup_{i \in I} A_i$. Then there exists $i_0 \in I$ such that $x \in A_{i_0}$. So, there exist $t > 0$ and $r \in (0,1)$ such that $B(x, t, r) \subset A_{i_0}$. Since $A_{i_0} \subset \bigcup_{i \in I} A_i$, $B(x, r, t) \subset \bigcup_{i \in I} A_i$. Thus $\bigcup_{i \in I} A_i \in \tau_M$. Hence, τ_M is a topology on X. □

Proposition 2.6. Let $(X, M, *, K)$ be a fuzzy strong b-metric space. Then an open ball is an open set.

Proof. We will show that an open ball $B(x, r, t)$ is an open set. Let $y \in B(x, r, t)$. Then we have $M(x, y, t) > 1 - r$. Since $M(x, y, \_)$ is nondecreasing and continuous, there exists $t_0 \in (0, t)$ such that $M(x, y, t_0) > 1 - r$. Let $r_0 = M(x, y, t_0)$. Therefore $r_0 > 1 - r$ and we can find a $s, 0 < s < 1$ such that $r_0 > 1 - s > 1 - r$. For r_0 and s such that $r_0 > 1 - s$ we can find $r_1, 0 < r_1 < 1$ such that $r_0 * r_1 \geq 1 - s$. Now we will show that $B(y, 1 - r_1, \frac{t - t_0}{K}) \subset B(x, r, t)$. $z \in B(y, 1 - r_1, \frac{t - t_0}{K})$ implies that $M(y, z, \frac{t - t_0}{K}) > r_1$. Hence we have

$$\begin{aligned} M(x, z, t) &\geq M(x, y, t_0) * M(y, z, \frac{t - t_0}{K}) \\ &\geq r_0 * r_1 \geq 1 - s > 1 - r. \end{aligned}$$

Therefore $z \in B(x, r, t)$ and $B(y, 1 - r_1, \frac{t - t_0}{K}) \subset B(x, r, t)$. □

Proposition 2.7. Let $(X, M, *, K)$ be a fuzzy strong b-metric space. Then (X, τ_M) is Hausdorff.

Proof. Let x, $y \in X$ such that $x \neq y$. From the definition of fuzzy strong b-metric space, $1 > M(x, y, t) > 0$ say $M(x, y, t) = r$. For all r_0 such that $1 > r_0 > r$ we can find $r_1 \in (0, 1)$ such that $r_1 * r_1 > r_0$. Now consider, the sets $B(x, 1 - r_1, \frac{t}{2})$ and $B(y, 1 - r_1, \frac{t}{2K})$. Clearly $B(x, 1 - r_1, \frac{t}{2}) \cap B(y, 1 - r_1, \frac{t}{2K}) = \varnothing$.
Otherwise, if there exists $z \in B(x, 1 - r_1, \frac{t}{2}) \cap B(y, 1 - r_1, \frac{t}{2K})$. Then

$$\begin{aligned} r &= M(x, y, t) \geq M(x, z, \frac{t}{2}) * M(z, y, \frac{t}{2K}) \\ &\geq r_1 * r_1 \geq r_0 > r \end{aligned}$$

which is a contradiction. □

Proposition 2.8. Let $(X, M, *, K)$ be a fuzzy strong b-metric space. Then (X, τ_M) is first countable.

Proof. Let $x \in X$. We need to show that $\mathcal{B}_x = \{B(x, \frac{1}{n}, \frac{1}{n}) : n \in \mathbb{N}\}$ is a local basis for $x \in X$. Let $U \in \tau_M$ such that $x \in U$. Since U is open, then there exists $r \in (0, 1)$ and $t > 0$ such that $B(x, r, t) \subset U$. Choose $n \in \mathbb{N}$ such that $\frac{1}{n} < r$ and $\frac{1}{n} < t$. Now we need to show $B(x, \frac{1}{n}, \frac{1}{n}) \subset B(x, r, t)$. Let $z \in B(x, \frac{1}{n}, \frac{1}{n})$. Then

$M(x,z,\frac{1}{n}) > 1-\frac{1}{n} > 1-r$. Since $\frac{1}{n} < t$, we have $1-r < M(x,z,\frac{1}{n}) \leq M(x,z,t)$. Hence $z \in B(x,r,t)$ which implies $B(x,\frac{1}{n},\frac{1}{n}) \subset B(x,r,t) \subset U$. Consequently, $\mathcal{B}_x$ is countable local basis for x. Hence (X,τ_M) is first countable topological space. □

Definition 2.9. Let $(X,M,*,K)$ be a fuzzy strong b-metric space, $x \in X$ and $\{x_n\}$ be a sequence in X. Then
i) $\{x_n\}$ is said to converge to x if for any $t>0$ and any $r \in (0,1)$ there exists a natural number n_0 such that $M(x_n,x,t) > 1-r$ for all $n \geq n_0$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.
ii) $\{x_n\}$ is said to be a Cauchy sequence if for any $r \in (0,1)$ and any $t>0$ there exists a natural number n_0 such that $M(x_n,x_m,t) > 1-r$ for all $n,m \geq n_0$.
iii) $(X,M,*,K)$ is said to be a complete fuzzy strong b-metric space if every Cauchy sequence is convergent.

Theorem 2.10. Let $(X,M,*,K)$ be a fuzzy strong b-metric space, $x \in X$ and $\{x_n\}$ be a sequence in X. $\{x_n\}$ converges to x if and only if $M(x_n,x,t) \to 1$ as $n \to \infty$, for each $t>0$.

Proof. ($\Rightarrow$:) Suppose that, $x_n \to x$. Then, for each $t>0$ and $r \in (0,1)$, there exists a natural number n_0 such that $M(x_n,x,t) > 1-r$ for all $n \geq n_0$. We have $1-M(x_n,x,t) < r$. Hence $M(x_n,x,t) \to 1$ as $n \to \infty$.

($\Leftarrow$:) Now, suppose that $M(x_n,x,t) \to 1$ as $n \to \infty$. Then, for each $t>0$ and $r \in (0,1)$, there exists a natural number n_0 such that $1-M(x_n,x,t) < r$ for all $n \geq n_0$. In that case, $M(x_n,x,t) > 1-r$. Hence $x_n \to x$ as $n \to \infty$. □

Let X be a first countable space. Then X is Hausdorff if and only if sequential limits in X are unique [5]. Then the following is obvious.

Proposition 2.11. Let $(X,M,*,K)$ be a fuzzy strong b-metric space and $\{x_n\} \subset X$. If $\{x_n\}$ is convergent, then the limit point of $\{x_n\}$ is unique.

Proposition 2.12. Let $(X,M,*,K)$ be a fuzzy strong b-metric space and $\{x_n\} \subset X$. If $\{x_n\}$ is convergent, then $\{x_n\}$ is Cauchy.

Proof. Let r and t be arbitrary real number such that $r \in (0,1)$, $t>0$ and $\lim\limits_{n\to\infty} x_n = x$ for $x \in X$. Since $r \in (0,1)$, there exists $r_0 \in (0,1)$ such that

$$(1-r_0)*(1-r_0) > 1-r.$$

Since $\lim\limits_{n\to\infty} x_n = x$, for $\frac{t}{2K} > 0$ and $r_0 \in (0,1)$ there exists $n_0 \in \mathbb{N}$ such that

$$n \geq n_0 \Longrightarrow M(x_n,x,\frac{t}{2K}) > 1-r_0.$$

Therefore we have

$$\begin{aligned} M(x_n,x_m,t) &\geq M(x_n,x,\frac{t}{2}) * M(x,x_m,\frac{t}{2K}) \\ &\geq M(x_n,x,\frac{t}{2K}) * M(x,x_m,\frac{t}{2K}) \\ &> (1-r_0)*(1-r_0) > 1-r \end{aligned}$$

for $m, n \geq n_0$ which means $\{x_n\}$ is Cauchy. □

Definition 2.13. Let $(X, M, *)$ be a fuzzy strong b-metric space. For $t > 0$, the closed ball $B[x, r, t]$ with center x and radius $r \in (0, 1)$ is defined by $B[x, r, t] = \{y \in X : M(x, y, t) \geq 1 - r\}$.

Proposition 2.14. Let $(X, M, *, K)$ be a fuzzy strong b-metric space. Then a closed ball is a closed set.

Proof. Let $y \in \overline{B[x, r, t]}$. We need to show that $y \in B[x, r, t]$. Since X is first countable space, there exists a sequence $\{y_n\}$ in $B[x, r, t]$ such that $y_n \to y$. Hence $M(y_n, y, t) \to 1$ for all $t > 0$. For a given $\epsilon > 0$

$$M(x, y, t + \epsilon) \geq M(x, y_n, t) * M(y_n, y, \tfrac{\epsilon}{K}).$$

Hence

$$\begin{aligned} M(x, y, t + \epsilon) &\geq \lim_{n \to \infty} M(x, y_n, t) * \lim_{n \to \infty} M(y_n, y, \tfrac{\epsilon}{K}) \\ &\geq (1 - r) * 1 = 1 - r. \end{aligned}$$

(If $M(x, y_n, t)$ is bounded, the sequence $\{y_n\}$ has a subsequence, which we again denote by $\{y_n\}$ for which $\lim_{n \to \infty} M(x, y_n, t)$ exists.) In particular for $n \in \mathbb{N}$, take $\epsilon = \dfrac{t}{n}$. Then we have

$$M(x, y, t + \frac{t}{n}) \geq (1 - r)$$

and

$$M(x, y, t) \geq \lim_{n \to \infty} M(x, y, t + \frac{t}{n}) \geq 1 - r.$$

Therefore $y \in B[x, r, t]$. □

Proposition 2.15. Let (X, D, K) be a strong b-metric space and $(X, M_D, \cdot, K)$ be the standard fuzzy strong b-metric space induced by D. Then the topology τ_D induced by D and the topology τ_{M_D} induced by M_D are the same.

Proof. ($\Rightarrow$) Let $A \in \tau_D$. For every $x \in A$, there exists $\epsilon > 0$ such that $B(x, \epsilon) \subset A$. For a fixed $t > 0$, we have

$$M_D(x, y, t) = \frac{t}{t + D(x, y)} > \frac{t}{t + \epsilon}.$$

If we write $1 - r = \frac{t}{t+\epsilon}$, then we have $M_D(x, y, t) > 1 - r$ which means $B(x, r, t) \subset A$ and $A \in \tau_{M_D}$.
($\Leftarrow$). Let $A \in \tau_{M_D}$. For every $x \in A$, there exists $0 < r < 1$ and $t > 0$ such that $B(x, r, t) \subset A$. We have

$$\begin{aligned} M_D(x, y, t) &= \frac{t}{t + D(x, y)} > 1 - r \\ t &> (1 - r)t + (1 - r)D(x, y) \\ D(x, y) &< \frac{rt}{1 - r} \end{aligned}$$

If we write $\epsilon = \frac{rt}{1-r}$ where $0 < \epsilon < 1$, then we have $D(x, y) < \epsilon$ which means $B(x, \epsilon) \subset A$ and $A \in \tau_D$. Therefore $\tau = \tau_D$. □

Theorem 2.16. Let $(X, M, *, K)$ be a fuzzy strong b-metric space. If (X, τ_M) is separable then (X, τ_M) is second countable.

Proof. Let $A = \{a_n : n \in \mathbb{N}\}$ be a countable dense subset of X. Consider

$$\mathcal{B} = \{B(a_j, \frac{1}{k}, \frac{1}{k}) : j, k \in \mathbb{N}\}.$$

We will show that B is a countable base for τ_M. Clearly B is countable. Let U be an open set in X. For any $x \in U$, there exists $r \in (0,1)$ and $t > 0$ such that $B(x, r, t) \subset U$. For $r \in (0,1)$, we can find an $s \in (0,1)$ such that $(1-s) * (1-s) > (1-r)$. Let $m \in \mathbb{N}$ such that $\frac{1}{m} < s$ and $\frac{1}{m} < \frac{t}{2K}$. Since A is dense in X, there exists $a_j \in A$ such that $a_j \in B(x, \frac{1}{m}, \frac{1}{m})$. If $y \in B(a_j, \frac{1}{m}, \frac{1}{m})$ then,

$$\begin{aligned} M(x, y, t) &\geq M(x, a_j, \frac{t}{2}) * M(y, a_j, \frac{t}{2K}) \\ &\geq M(x, a_j, \frac{1}{m}) * M(y, a_j, \frac{1}{m}) \\ &\geq (1 - \frac{1}{m}) * (1 - \frac{1}{m}) \\ &\geq (1-s) * (1-s) \\ &> (1-r). \end{aligned}$$

Hence $y \in B(x, r, t)$ and $\mathcal{B}$ is a basis. □

Definition 2.17. Let X be a topological space, $(Y, M, *, K)$ be a fuzzy strong b-metric space and $f_n : X \to Y$ be a sequence of functions. Then $\{f_n\}$ is said to converge uniformly to a function f from X to Y if for given $r \in (0,1)$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(f_n(x), f(x), t) > 1 - r$ for all $n \geq n_0$ and for all $x \in X$.

Theorem 2.18. Let X be a topological space, $(Y, M, *, K)$ be a fuzzy strong b-metric space and $f_n : X \to Y$ be a sequence of continuous functions. If $\{f_n\}$ converges uniformly to f then f is continuous.

Proof. Let V be an open set in Y, $x_0 \in f^{-1}(V)$ and let $y_0 = f(x_0)$. Then there exist $r \in (0,1)$ and $t > 0$ such that $B(y_0, r, t) \subset V$. For $r \in (0,1)$, we can find an $s \in (0,1)$ such that $(1-s) * (1-s) * (1-s) > 1 - r$. Since $\{f_n\}$ converges uniformly to f, for given $s \in (0,1)$ and $t > 0$, there exists $n_0 \in N$ such that $M(f_n(x), f(x), \frac{t}{4K^2}) > 1 - s$ for all $n \geq n_0$ which also implies $M(f_n(x), f(x), \frac{t}{2}) > 1 - s$. Since f_n is continuous for all $n \in \mathbb{N}$, we can find a neighborhood U of x_0, for a fixed $n \geq n_0$, such that $f_n(U) \subset B(f_n(x_0), s, \frac{t}{4K})$. Therefore $M(f_n(x), f_n(x_0), \frac{t}{4K}) > 1 - s$ for all x in U an we have

$$\begin{aligned} M(f(x), f(x_0), t) &\geq M(f(x), f_n(x), \tfrac{t}{2}) * M(f_n(x), f_n(x_0), \tfrac{t}{4K}) \\ &\quad * M(f_n(x_0), f(x_0), \tfrac{t}{4K^2}) \\ &\geq (1-s) * (1-s) * (1-s) \\ &\geq 1 - r. \end{aligned}$$

Hence, $f(x) \in B(f(x_0), r, t) \subset V$ for all $x \in U$ which means $f(U) \subset V$ and f is continuous. □

Acknowledgement

This paper has been granted by the Mugla Sitki Kocman University Research Projects Coordination Office. Project Grant Number: 16/083 and title ”On the topology of fuzzy strong b-metric spaces”.

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