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Finite Element Solution of the Contact Problem

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Abstract

In this paper, continuous and discontinuous contact problems for an elastic layer loaded by symmetrical distributed loads whose lengths are 2a on an elastic semi-infinite plane is solved using finite element method. The elastic layer also subjected to uniform vertical body force because of effect of the gravity. Thickness in z direction is taken to be unit. It is assumed that the contact surfaces are frictionless, only normal tractions can be transmitted through the contact areas. The contact along the interface between elastic layer and half plane is continuous if the value of load factor is less than a critical value. In continuous and discontinuous contact cases, the stress distribution on the contact interface are plotted for different dimensionless quantities. The finite element model of the problem is constituted using ANSYS software and analysis of the problem is carried out. Finally, the results obtained from the finite element solution are verified by comprasion with the analytical results.

Keywords: Mechanic, Continuous contact, Discontinuous contact, Elasticity, Half plane, Finite element analysis.

Temas Probleminin Sonlu Elemanlar Çözümü

Özet

Bu çalışmada, simetrik 2a genişliğindeki yayılı yük ile yüklenmiş elastik tabakanın sürekli ve süreksiz temas problem sonlu elemanlar yöntemi kullanılarak çözülmüştür. Elastik tabakaya ayrıca yer çekimi etkisinden doğan kütle kuvvetleri etkimektedir. z yönündeki kalınlık birim olarak alınmıştır ve temas yüzeylerinde sürtünmenin etkisi dikkate alınmamıştır. Yük faktörünün kritik değerden küçük olması durumunda tabaka ve yarım düzlem arasında sürekli temas durumu oluşmaktadır. Sürekli ve süreksiz temas durumları için temas yüzeylerindeki gerilme dağılımları çeşitli boyutsuz büyüklükler için elde edilmiştir. Problemin sonlu eleman modeli ANSYS yazılımı kullanarak oluşturulmuş ve problemin analizi gerçekleştirilmiştir. Son olarak sonlu elemanlar yönteminden elde edilen sonuçlar analitik sonuçlarla karşılaştırılarak doğrulanmıştır.

Anahtar Kelimeler: Mekanik, Sürekli temas, Süreksiz temas, Elastisite, Yarım düzlem, Sonlu elemanlar metodu.

1. Introduction

Since contact problems have possible application to a variety of structures of practical interest such as foundation grillages, pavements in roads and runways, railway ballats and other structures consisting of layered media, there has been an increasing attention on the contact problems. There is large body of literature concerned with contact problems both analytically [1-9] and numerically [10-15].

Except for these papers, El-Borgi et al. investigated a receding contact problem between a functionally graded layer and a homogeneous substrate and they studied the effect of the material nonhomogeneity parameter and the thickness of the graded layer on the contact pressure and on the contact lengths. Kahya et al. studied a receding contact problem for an anisotropic elastic medium consisting of a layer and a half plane and they obtained contact stresses and contact lengths for different fibre orientations. A axisymmetric double receding contact problem between a functionally graded layer and a homogeneous half plane is solved Rhimi et al.. Effects of surface tension on axisymmetric Hertzian contact problem and they obtained helpful results to characterize and measure the mechanical features of soft materials or biomaterials through micro-indentation. El-Borgi et al. considered a frictional receding contact problem between a functionally graded layer and a homogeneous semi-infinite plane and they analyzed the effect of friction coefficient and nonhomogeniety factor on the contact pressure distribution and contact lengths. Gun and Gao presented a quadratic boundary element formulation for continuously nonhomogeneous, isotropic and linear elastic functionally graded material contact problems with friction. Yan and Li analyzed double receding contact problem between functionally graded layer and elastic layer. Li et al. are presented fundamental contact solutions of a magneto-electro-elastic half space indented by a smooth and rigid half infinite punch. Comez studied moving contact problem for a rigid cylindrical punch and a functionally graded layer and he obtained an effect of relative moving velocity for contact problem of functionally graded layer.

In this paper, continuous and discontinuous contact problem for infinite layer on an elastic semi-infinite plane is solved using FEM. The load which will occur by initial separation and the bigger values than this load, the stress distribution on the contact surfaces are plotted for different dimensionless quantaties. Finally, the results obtained from FEM are verified by comprasion the analytical results.

2. Definition of the Problem

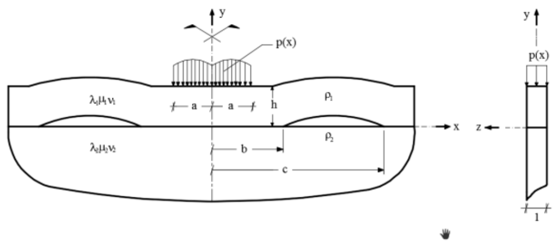

Figure 1. Geometry and the loading of the contact problem

In this paper continuous and discontinuous contact problems for an elastic layer on an elastic semi-infinite plane is solved using finite element method. The layer are loaded by symmetrical distributed loads whose lengths are 2a and thickness in z direction is taken to be unit. It is assumed that the contact surfaces along the interfaces is frictionless, only normal tractions can be transmitted through the contact surfaces, body forces of elastic layer is taken into account and body force of semi-infinite plane is neglected.

As shown in Fig. 1, consider the symmetric plane strain problem consist of an infinitely long homogeneous layer of thickness h in smooth contact with semi-infinite plane. μ_1, ν_1 and ρ_1 are the shear modulus, poisson ratios and density of a layer respectively. Similarly μ_2, ν_2 and ρ_2 are the shear modulus, poisson ratios and density of a half space respectively.

3. The Finite Element Solution

The finite element method is a numerical method for solving problems of engineering and mathematical physics. In this method, problem divides into simpler parts that are called finite elements and the model transform into large system of equations. With recent developments in computer technology and package programs for FEM solution of large system of equations is fulfilled.

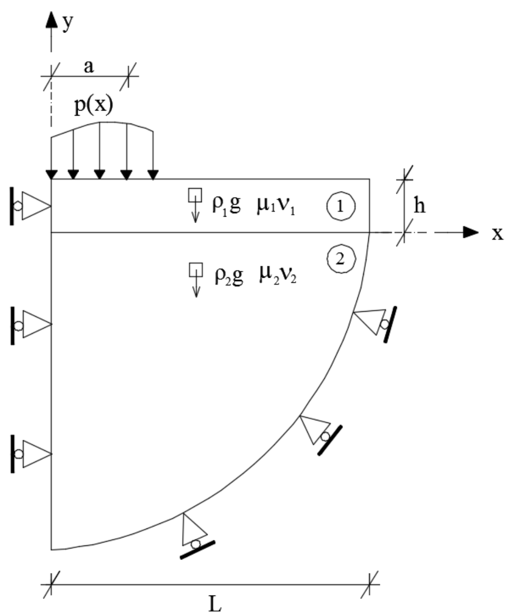

Figure 2. Schematic model of the contact problem (symmetric finite element model)

In this study, the finite element analysis is performed by ANSYS software program. The geometrical model is created with the standart tools in ANSYS software. Because of the problem exhibits symmetry in geometry, material proportions and loading, only half of the problem is modeled. The geometry and the applied load are shown symmetrically in Fig. 2. The layer and semi-infinite plane are considered as lineer elastic and isotropic. In the analysis, geometric properties are taken as L=12 m (length of the layer in x direction), h_1=1m (thickness of the layer in y direction). Other parameters are choosen such that $p_0/(\rho gh)$, μ_1/μ_2, κ_1/κ_2. Are compatible with analytical values.

PLANE183 element is used for the modeling elastic layer and semi-infinite plane. PLANE183 element consist of eigth nodes having two degrees of freedom: translations in the nodal x and y directions. Frictionless surface-to-surface 2D contact elements are used to model the interaction between the contact surfaces: CONTA172 and TARGE169. CONTA172 is used to represent contact. Target surfaces is defined by TARGE169 for the associated contact elements CONTA172. Several numerical solution methods have been proposed to solve the variational equation of elastic contact problem, including penalty method, augmented Lagrangian method, Lagrange multiplier method method and augmented Lagrangian multiplier method. These methods incorporated to general finite element analysis (FEA) technology, are applied to solve the contact problem that involves complex geometry shapes. In the penalty method, the accuracy of the solution depends on the choice the penalty parameter. Too small a penalty parameter may cause unacceptable error in the solution. Also the penalty method suffers from ill-conditioning as the penalty parameter becomes large. The augmented Lagrangian method is an iterative series of penalty methods. The contact tractions (pressure and frictional stresses) are augmented during equilibrium iterations so that the final penetration is smaller than the allowable tolerance. Compared to penalty method, the augmented Lagrangian method usually leads to better conditioning and is less sensitive to the magnitude to contact stiffness.The Langrange multiplier method introduces new unknowns for each constraint. Therefore, it always increases the dimension of the system equations to be solved. For large scale problems where the contact surface consist of a large number of nodes, the number of unknowns introduced by the Lagrange multiplier method is also large. This increases the CPU time to solve the problem. For the augmented Lagrangian multiplier method, both penalty parameters and Lagrangian multipliers are applied, and penetration is admissible but controlled by allowable tolerance [28]. In this study, Augmented Lagrangian method is used as the contact algorithm. And the deformed shape after analysis is shown in Fig. 3.

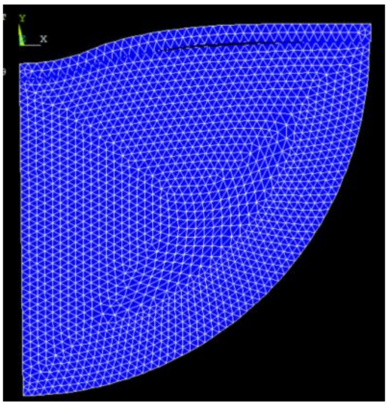

Figure 3. Deformed shape after finite element analysis.

4. Conclusions

In this paper, continuous and discontinuous contact problem for an infinite layer and a semi-infinite plane is solved by using finite element method and results obtained from finite element method are verified by comparing with analytical results in literature. Note that all quantaties are dimentionless.

Fig. 4-5 shows contact stress distributions for the continuous contact cases. It can be seen that the initial separation point x_{cr} seems to increase as a/h increases.

The variation of the normalized contact stress $\sigma_y(x,0)/(\rho_1 gh)$ between the infinite layer and semi-infinite plane for the discontinuous contact case is shown Fig. 6-7. As it can be seen in graphics, there are three regions in the discontinuous contact between the layer and semi-infinite plane. These are the continuous contact region, separation zone, and the discontinuous contact region where the effect of the external load (P) decreases and disappears infinitely. If the a/h increases, initial separation point occurs a longer distance from the origin and separation zone decreases for similar material proportions.

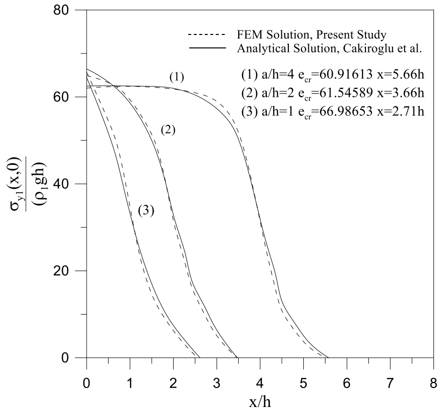

Figure 4. Contact stress distrubutions for the cases of continuous (e=e_{cr}) for the first loading condition.

$(\mu_1/\mu_2 = 1/3,\ e_{cr} = p_0/(\rho g h))$

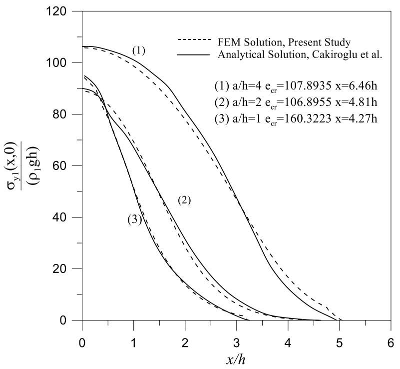

Figure 5. Contact stress distrubutions for the cases of continuous (e=e_{cr}) for the second loading condition.

$(\mu_1/\mu_2 = 3,\ e_{cr} = p_0/(\rho g h))$

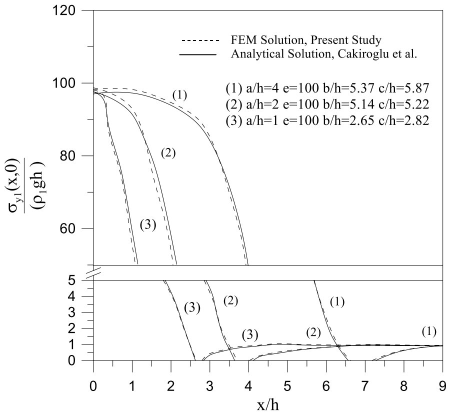

Figure 6. Contact stress distrubutions for the cases of discontinuous (e>e_{cr}) for the first loading condition.

$(\mu_1/\mu_2 = 1/3,\ e_{cr} = p_0/(\rho g h))$

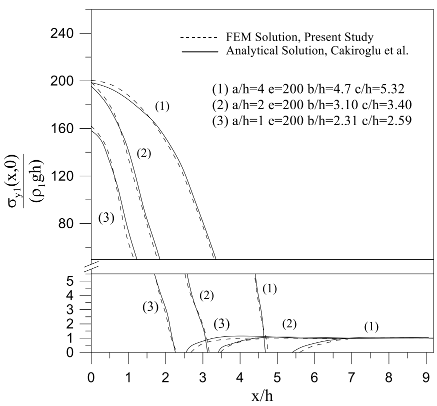

Figure 7. Contact stress distrubutions for the cases of discontinuous (e>e_{cr}) for the second loading condition.

$(\mu_1/\mu_2 = 1/3,\ e_{cr} = p_0/(\rho g h))$

Finally, based of the comprasion of numerical values obtained from finite element solution and analytical solution in literature [11], difference between finite element solution and analytical solution in literature is in a acceptable range.

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