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MALATYA FUNCTIONS: SYMMETRIC FUNCTIONS OBTAINED BY APPLYING FRACTIONAL ORDER DERIVATIVE TO KARCI ENTROPY

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Abstract: Shannon applied derivative to a special probability function and obtained entropy definition. Karcı converted the derivative with fractional order derivative and obtained a new definition for entropy. In this study, the fractional order of derivative were selected as complex number and symmetric function were obtained. Some of them were illustrated in this study, and it is known that there are infinite symmetric functions obtained by this way.

Keywords. Fractional Calculus, Fractional Order Derivative, Entropy, Symmetric Functions.

1. Introduction

The fractional calculus (variational calculus) is a three centuries old concept and the fractional calculus consists of fractional order derivatives and fractional order integrals. The fractional order derivative concept was defined by many scientists such as Euler, Caputo, Riemman-Lioville, etc. [1]. There is an idea such that the fractional calculus may depict the behaviours of nature almost in real behaviours of nature [1]. The recent definition for fractional order derivative was done by Karcı [2,3,4,5,6].

Shannon entropy is based on probability and derivative. The information and its management can be illustrated by entropy, the most of information management techniques are based on entropy. The entropy is the measurement of uncertainty (or information) in physical (digital) systems. Due to this case, there are many methods for definitions of entropy such as Shannon, Tsallis, Renyi, etc [7,8].

The entropy is defined as a derivative of the following equation [8].

$$S = \lim_{t \to -1} \frac{d}{dt} \sum_i p_i^{-t} \tag{1}$$

This definition of entropy is regarded as Shannon entropy. Tsallis entropy can be defined as in similar way

$$S = \lim_{t \to -1} D_q^t \sum_i p_i^{-t} \tag{2}$$

where D_q^t is regarded as Jackson [8] derivative. Karcı [2,3,4,5,6] defined the fractional order derivative concept after that he also defined entropy based on his definition for fractional order derivative [9,10].

The fractional order derivative and entropy based binomial distribution yield a series of symmetric functions. This concept was handled in this paper, and its experimental results were illustrated by graphs in this paper.

The organization of this paper can be emphasized as follow. Section 2 describes fractional order derivatives and Karcı entropy. Section 3 gives details about the symmetric functions obtained in this study, and finally Section 4 concludes this paper and gives some important point about the obtained symmetric functions.

2. Fractional Order Derivatives and Karcı Entropies

The meaning of derivative is the rate of change in the dependent variable versus the changes in the independent variables. At this aim, the derivative of f(x)=x is that

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x} = \lim_{\Delta x \to 0} \frac{x+\Delta x - x}{\Delta x} = 1 \tag{3}$$

The fractional order derivative consists of four terms such as f(x+Δx), f(x), (x+Δx) and x. The idea for fractional order derivative can be given in the following definitions [2,3,4,5,6].

Definition 1: f(x):R→R is a function, α∈R and the FOD can be considered as

$$f^{(\alpha)}(x) = \lim_{\Delta x \to 0} \frac{f^{\alpha}(x+\Delta x) - f^{\alpha}(x)}{(x+\Delta x)^{\alpha} - x^{\alpha}} \tag{4}$$

The limit in the Definition 1 concludes in indefinite limit,

$$f^{(\alpha)}(x) = \lim_{\Delta x \to 0} \frac{f^{\alpha}(x+\Delta x) - f^{\alpha}(x)}{(x+\Delta x)^{\alpha} - x^{\alpha}} = \frac{0}{0} \tag{5}$$

so, L'Hospital method can be applied to this definition. After this application, Definition 2 can be obtained and it is a new definition for fractional order derivative.

Definition 2: Assume that f(x):R→R is a function, α∈R and L(.) be a L'Hospital process. The FOD of f(x) is

$$f^{(\alpha)}(x) = \lim_{\Delta x \to 0} L\left(\frac{f^{\alpha}(x+\Delta x) - f^{\alpha}(x)}{(x+\Delta x)^{\alpha} - x^{\alpha}} \right) = \lim_{\Delta x \to 0} \frac{\frac{d\left(f^{\alpha}(x+\Delta x) - f^{\alpha}(x)\right)}{\Delta x}}{\frac{d((x+\Delta x)^{\alpha} - x^{\alpha})}{\Delta x}} = D_t^{(\alpha)} f(x) \tag{6}$$

The fractional order derivative in Definition 2 can be applied to equation 3 and the definition for Karcı (fractional order entropy) entropy was obtained (equation 7).

Definition 3: Assume that p_x is a probability of x^{th} event. Then the first fractional order entropy (Karcı Entropy - KE) is

$$Entropy = KE = D_t^{(\alpha)} \sum_x \left| p_x^{-t} \right| = \sum_x \left| (-p_x)^{\alpha} p_x \ln p_x \right| \tag{7}$$

In order to compute entropy, any probability distribution can be used. In subsequent section, binomial distribution was used to obtain complex functions.

3 Symmetric Functions

Shannon entropy is defined by using probability, and Karcı entropy is fractional entropy by using Shannon method with fractional order derivative defined by Karcı. The binomial distribution can be used to define Karcı entropy, and the binomial probability mass function is

$$f(x)=\binom{n}{x}p^{x}(1-p)^{n-x} \qquad 0\leq x\leq n \tag{8}$$

and the binomial probability distribution function is

$$p_x=F(x)=\binom{n}{x}p^{x}(1-p)^{n-x} \qquad 0\leq x\leq n \tag{9}$$

Based on this probability distribution function, Karcı entropy can be redefined with complex orders, and these functions were called as Malatya functions (M(..)) or Karcı function (K(..)).

$$M(p)=K(p)=D_t^{(\alpha)}\sum_{x=1}^{n}\left|p_x^{-t}\right|=-\sum_{x=1}^{n}\left|(-p_x)^{\alpha}p_x\ln p_x\right| \tag{10}$$

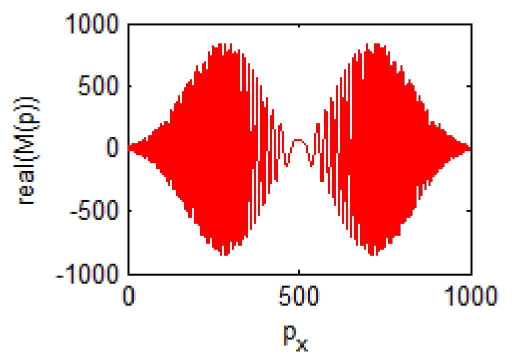
By substituting p_x for binomial distribution into equation (10) and the equation (11) can be yielded.

$$M(p)=K(p)=-\sum_{x=0}^{n}(-1)^{\alpha}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1+\alpha}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right] \tag{11}$$

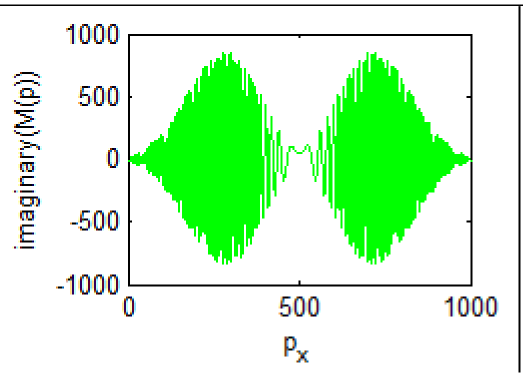
The fractional order of derivative α can be any complex/real number. In this paper, some complex values of α were selected, and then the obtained functions are all symmetrical functions. In order to obtain these symmetric functions, four different values of α were used.

Case 1: α=-i

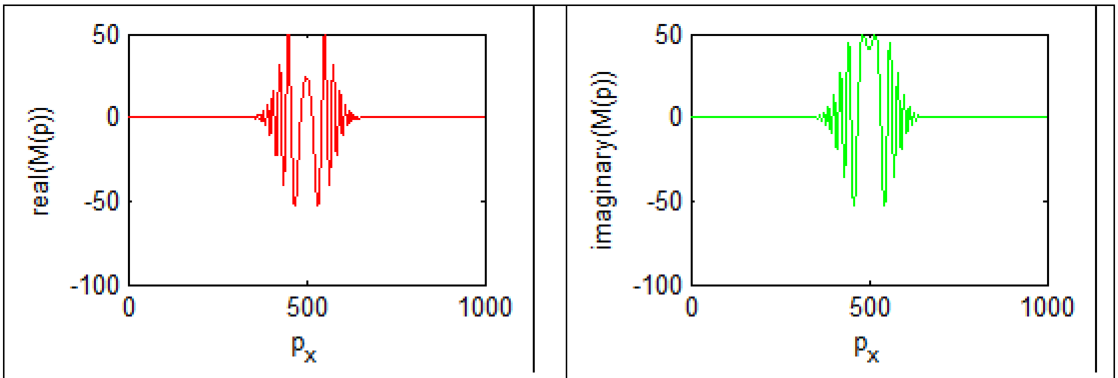
$$M(p)=K(p)=-\sum_{x=0}^{n}(-1)^{\alpha}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1+\alpha}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$
$$=-\sum_{x=0}^{n}(-1)^{0.01-i}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1.01-i}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$



a) α=-i+0.01



b) α=-i+0.01



c) α=-i+0.04 d) α=-i+0.04

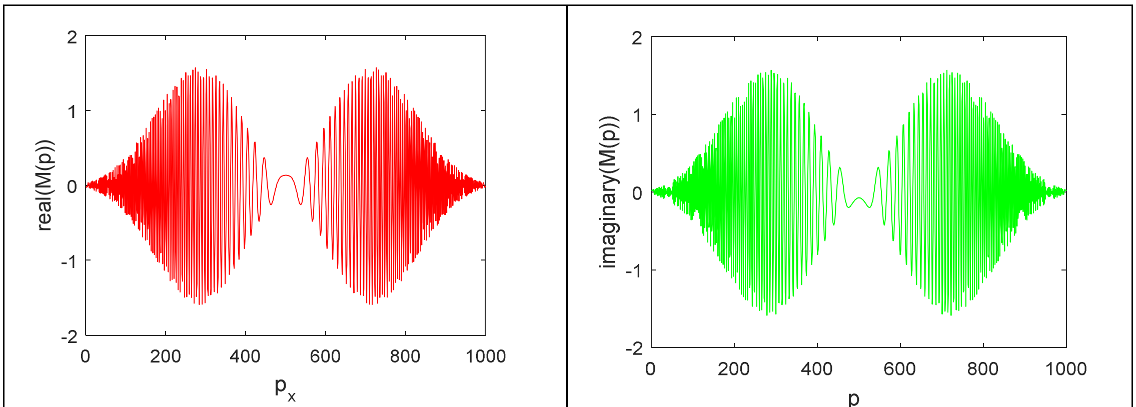
Figure 1. Symmetric functions examples.

As seen in Fig.1 a, Fig.1 b, Fig.1 c and Fig.1 d, the obtained functions (relations) are symmetric functions having complex powers. The complex powers make these functions (relations) be complex functions; their real parts are seen in Fig.1 a and Fig.1 c. The imaginary parts are seen in Fig.1 b and Fig.1 d. The similar cases are seen in Fig.2 a, Fig.2 b, Fig.2 c and Fig.2 d for complex and real parts.

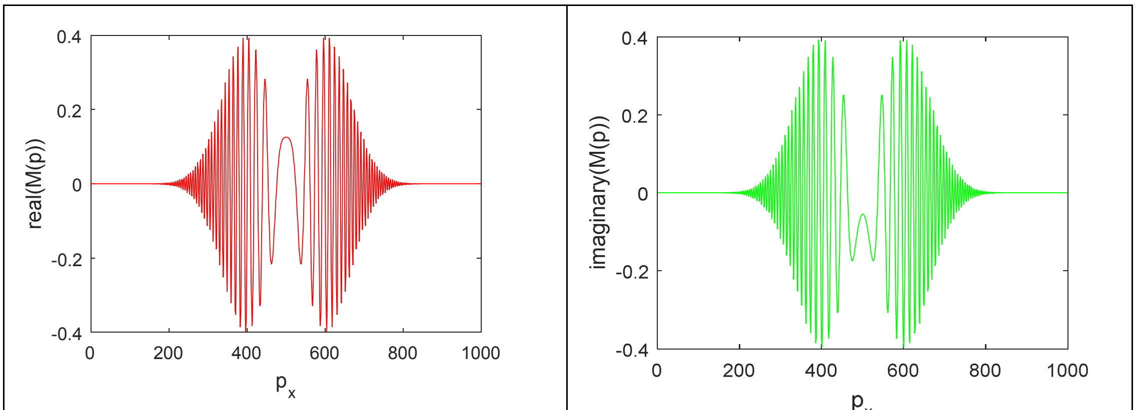
Case 2: α=i

$$M(p)=K(p)=-\sum_{x=0}^{n}(-1)^{\alpha}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1+\alpha}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$

$$=-\sum_{x=0}^{n}(-1)^{0.01+i}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1.01+i}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$



a) α=i+0.01 b) α=i+0.01



c) α=i+0.04 d) α=i+0.04

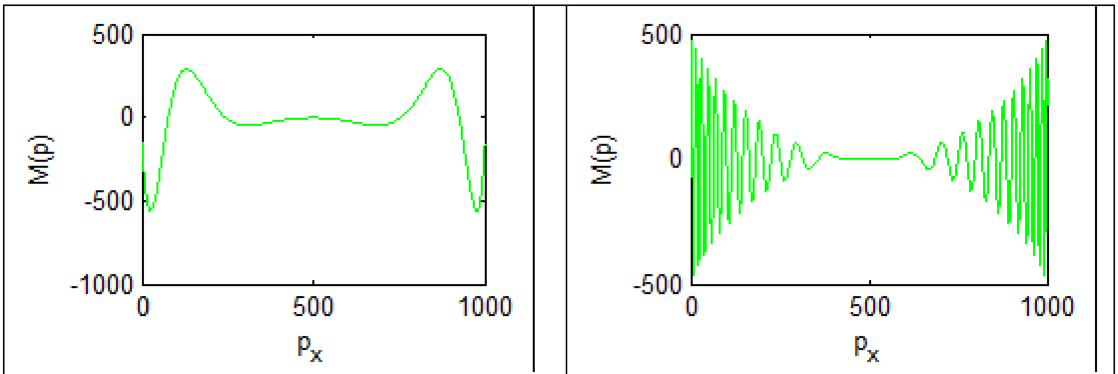
Figure 2. Symmetric functions for fractional order of α=-2i.

Case 3: α=-0.01i

$$M(p)=K(p)=-\sum_{x=0}^{n}(-1)^{\alpha}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1+\alpha}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$

$$=-\sum_{x=0}^{n}(-1)^{-0.011i}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1-0.011i}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$

As seen in Fig.3 a and Fig.3 b, the obtained functions (relations) are symmetric functions having fully complex powers. The complex powers make these functions (relations) be complex functions; their frequency are changing with respect to x-axis values. They do not have constant frequencies.



a) α=-0.011i b) α=-0.03i

Figure 3. Symmetric functions for fractional order completely complex.

Case 4: α=-30i

$$M(p)=K(p)=-\sum_{x=0}^{n}(-1)^{\alpha}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1+\alpha}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$

$$=-\sum_{x=0}^{n}(-1)^{-30i}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1-30i}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$

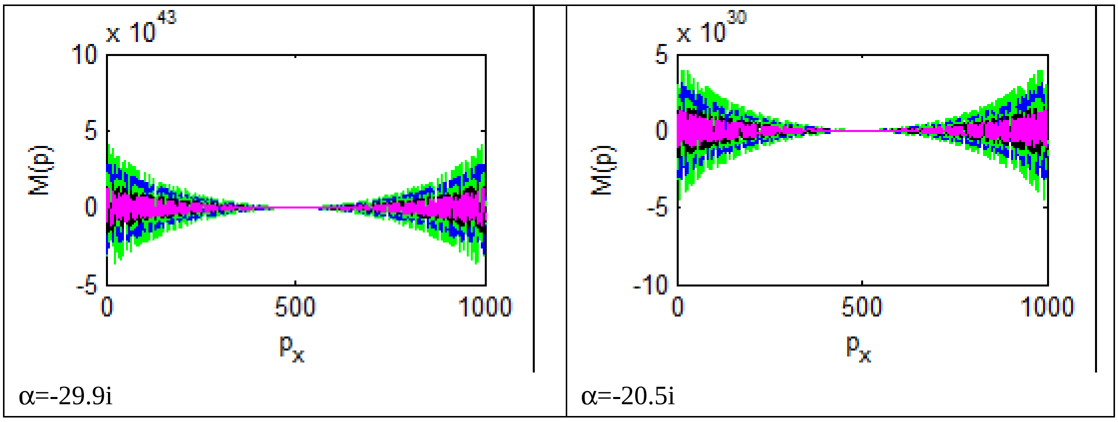


Figure 4. Symmetric functions were drawn on the same plane.

Fig.4 a and Fig.4 b illustrate fives functions drawn on the same plane. Their fractional orders are α=-29.9i, α=-29.8i, α=-29.7i, α=-29.6i, α=-29.5i for Fig.4 a. For Fig.4 b, their fractional orders are α=-20.4i, α=-20.3i, α=-20.2i, α=-20.1i, α=-20i.

Case 5: α=-0.001i

$$M(p)=K(p)=-\sum_{x=0}^{n}(-1)^{\alpha}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1+\alpha}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$

$$=-\sum_{x=0}^{n}(-1)^{-0.001i}\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]^{1-0.001i}\ln\left[\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}\right]$$

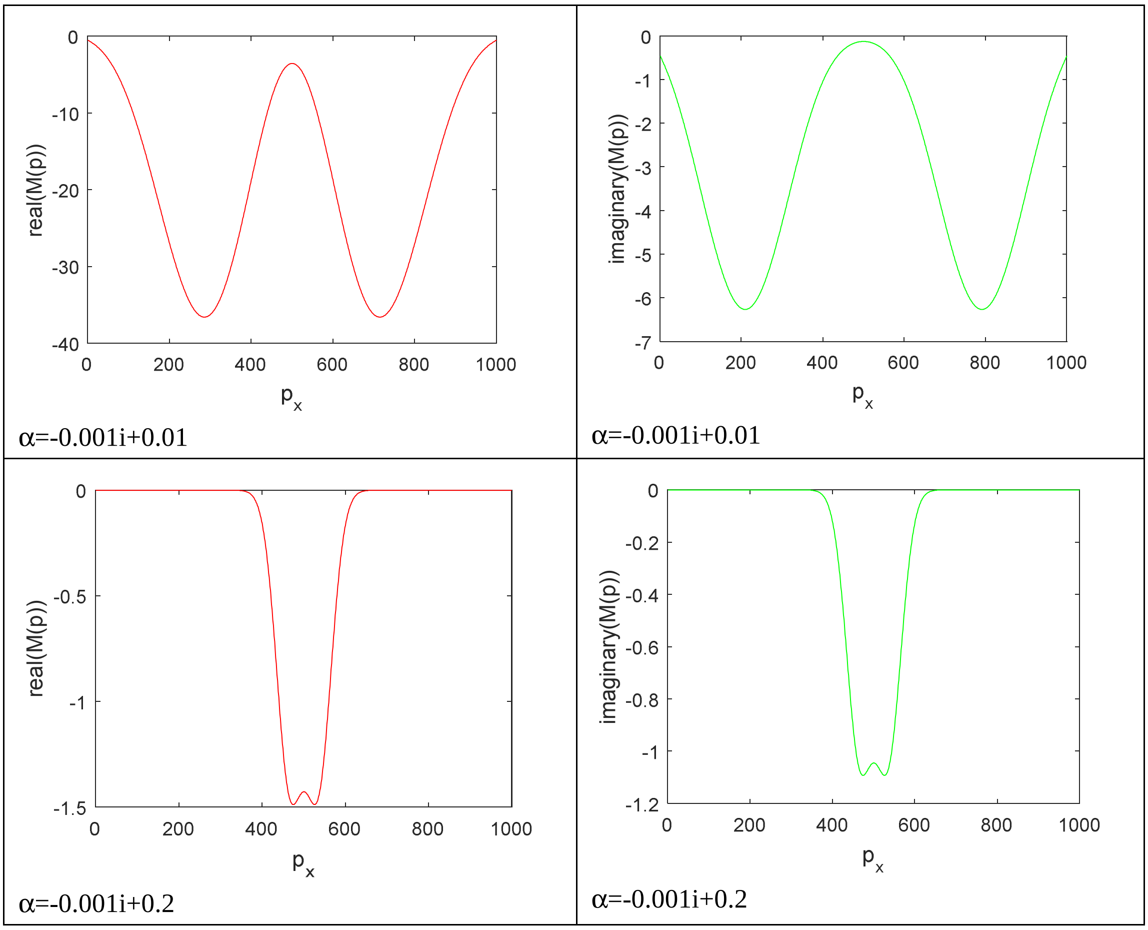
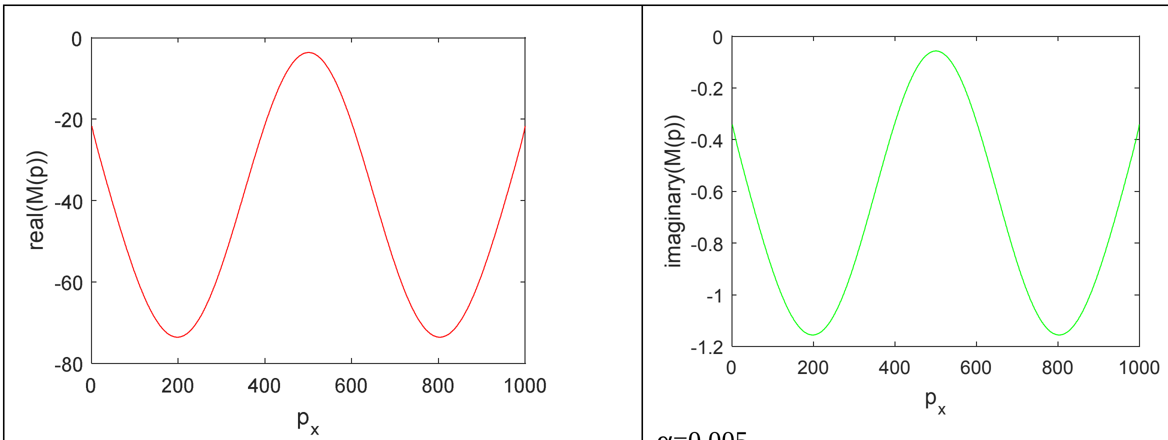


Figure 5. Symmetric function with less frequencies.

When fractional order goes to zero, the frequency of symmetric function decreases as seen in Fig.5. The same case can be observed in Fig.6.

Case 6: α=-0.005

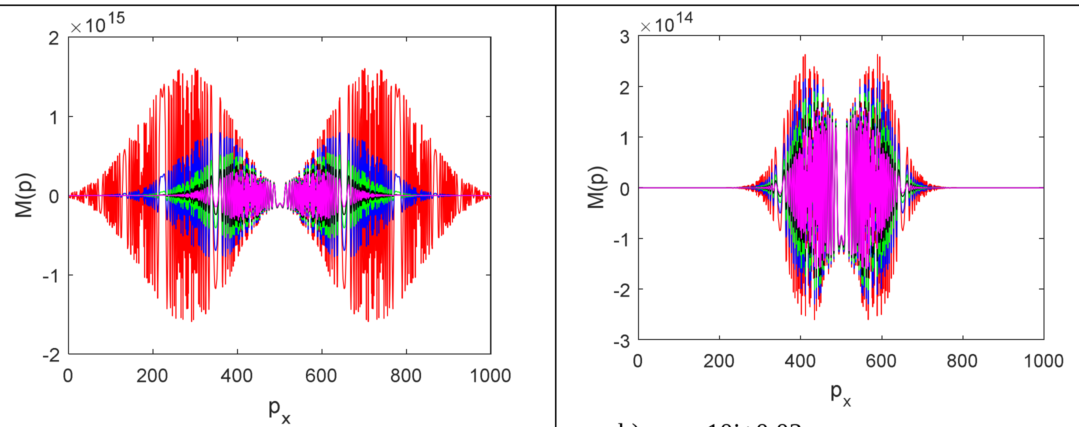


α=0.005 α=0.005

Figure 6. Symmetric functions with less frequencies.

Fig.7.a and Fig.7 b illustrate the functions drawing on the same plane. The symmetric functions obtained by applying fractional order α=-10i+0.01, α=-10i+0.02, α=-10i+0.03, α=-10i+0.04 and α=-10i+0.05 are seen in Fig.7 a. The symmetric functions obtained by applying fractional order α=-10i+0.06, α=-10i+0.07, α=-10i+0.08, α=-10i+0.09 and α=-10i+0.1 are seen in Fig.7 b. The symmetric functions obtained by applying fractional order α=0.0001i+0.06, α=0.0001i+0.07, α=0.0001i+0.08, α=0.0001i+0.09 and α=0.0001i+0.1 are seen in Fig.8 a. The symmetric functions obtained by applying fractional order α=0.0001i+0.01, α=0.0001i+0.02, α=0.0001i+0.03, α=0.0001i+0.04 and α=0.0001i+0.05 are seen in Fig.8 b.

Case 7: α=-10i;



a) α=-10i+0.01 b) α=-10i+0.02

c) d)

Figure 7. Symmetric functions for fractional orders α=-10i+0.01, α=-10i+0.02, α=-10i+0.03, α=-10i+0.04, α=-10i+0.05, α=-10i+0.06, α=-10i+0.07, α=-10i+0.08, α=-10i+0.09 and α=-10i+0.1.

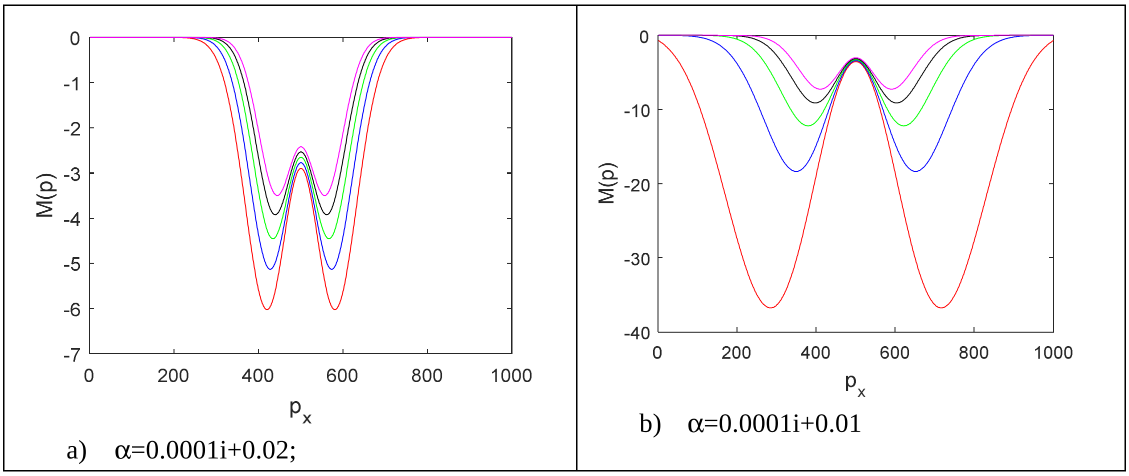


Figure 8. Symmetric functions for fractional orders α=0.0001i+0.06, α=0.0001i+0.07, α=0.0001i+0.08, α=0.0001i+0.09, α=0.0001i+0.1, α=0.0001i+0.01, α=0.0001i+0.02, α=0.0001i+0.03, α=0.0001i+0.04 and α=0.0001i+0.05.

4 Conclusions

The Shannon entropy is obtained by applying derivative to function seen in Eq.1. The derivative in this step was converted to fractional order derivative by Karcı, and new entropy types were obtained. Those entropy definitions can be regarded as Karcı entropy. In this study, binomial distribution was selected as probability distribution and fractional orders were selected as complex numbers, then the symmetric functions were obtained. Some of them were given in this study, there are infinite such functions.

The symmetric functions obtained in this paper demonstrates an important point. The complex powers of any constant number is exist and trigonometric. For example, $e^{i\theta} = \cos\theta + i\sin\theta$. The natural logarithm base "e" is a constant.

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