Hacettepe Journal of Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 54 (2) (2025), 779 – 796
DOI : 10.15672/hujms.1580875

Research Article

Variance estimation using Bernoulli auxiliary variable for time-scaled survey

Prayas Sharma*, Mamta Kumari

Department of Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow, India

Abstract

The diversity in the qualities under investigation is necessary to comprehend any phenomenon, whether in a real-world or practical setting. It is crucial to know the differences between current and previous circumstances. Therefore, to estimate the population variance with dichotomous auxiliary information, the exponentially weighted moving average statistic is used. This manuscript suggests a generalized class of memory-type estimators for the estimation of population variance using the Bernoulli auxiliary variable under time-scaled survey. The properties of the suggested class of memory type estimator and exponentially weighted moving average version of the usual ratio, regression, and exponential estimators are derived up to the first order of approximation. It has been shown through empirical and simulation study that the suggested estimator is more efficient than the usual estimators and the exponentially weighted moving average version of the estimators in the literature.

Mathematics Subject Classification (2020). 62D05

Keywords. Auxiliary attribute, exponentially weighted moving average statistic, mean squared error, memory type estimator, variance estimation.

1. Introduction

In survey sampling, the estimation of population parameters is often improved by incorporating auxiliary information. When used effectively, it can improve the precision of estimators and reduce sampling errors, making the estimation process more efficient. Auxiliary information typically involves continuous variables that are related to the study variable. Using auxiliary information, estimators can be constructed that take advantage of this correlation to provide more accurate estimates of population variance. The auxiliary variable must have a strong linear relationship with the variable of interest in order to be used as auxiliary information. This relationship can be used to generate variance estimators that are more effective than those produced by simple random sampling, for example, through the use of ratio or regression estimating techniques. Ratio estimators can be used to estimate population parameters while accounting for known auxiliary information when there is a linear relationship between the auxiliary variable and the variable

*Corresponding Author.
Email addresses: prayassharma02@gmail.com (P. Sharma), mamta.rs.stats@email.bbau.ac.in (M. Kumari)
Received: 07.12.2024; Accepted: 27.03.2025

of interest. Regression modeling allows for the modeling of the relationship between the auxiliary variable and the variable of interest, leading to more accurate estimates of population variance. Isaki [8] proposed a ratio estimator for the estimation of population variance using auxiliary information. Bhat et al. [5] and Das [7] have made an improvement in the variance estimator for the estimation of population variance, using known values of auxiliary information. Sharma and Singh [18] suggested estimators for population variance using auxiliary information in the quartile, and Adichwa et al. [1] developed a generalized class of estimators for population variance using information on two auxiliary variables.

In certain instances, the supplemental data may not be continuous, but rather binary. It is known as an auxiliary attribute. Incorporating auxiliary attributes in simple random sampling enhances the estimation of population parameters, such as mean, variance, quartile, etc., by leveraging the additional information encoded in categorical or binary variables. This method improves precision and reduces sampling errors, especially when the auxiliary attribute is strongly correlated with the variable of interest. For example, in a health survey where some individuals do not respond, auxiliary variables such as age, gender, or prior health history can be used to adjust for non-response. By incorporating auxiliary attributes as variables correlated with the likelihood of response, we can adjust the same weights or impute missing data, resulting in more accurate estimates of health outcomes. For a second example, if we are conducting a household survey and we know the number of households with Internet access from previous census data, the auxiliary attributes are taken to adjust the weights of the sampled units to match known totals in the population and to calibrate the survey weights to improve the precision of the estimated total.

For the estimation of population mean, Koyuncu [9] suggested an efficient estimator using auxiliary attribute, Sharma and Singh [17] improved the estimators in simple random sampling when the study variable is an attribute, and Sharma and Singh [16] suggested some exponential ratio product type estimators using information on auxiliary attributes under second order approximation. Kadılar [23] proposed a new exponential type estimator for the population mean in simple random sampling and Özel and Kadılar [24] suggested modified exponential type estimators for the population mean in stratified random sampling. For the estimation of population variance, Özel et al. [22] suggested separate ratio estimators for population variance in stratified random sampling. Singh et al. [19] proposed a family of estimators using information on auxiliary attributes, then [10] improved an estimator using two auxiliary attributes. Singh and Malik [20] suggested an improved estimate of population variance using information on an auxiliary attribute in simple random sampling, and Adichwal et al. [2] suggested a generalized class of estimators for population variance using an auxiliary attribute. Tariq et al. [25] have performed variance estimation using memory type estimators based on the exponentially weighted moving average (EWMA) statistic for time-scaled surveys in stratified sampling.

Memory-type estimators use information from previous survey rounds, which helps in reducing variance by exploiting the time-series correlation. Using auxiliary information and past data, these estimators can significantly improve the efficiency of survey estimates, often resulting in lower mean squared errors compared to traditional estimators. These methods can be adapted to various types of survey and can accommodate complex survey designs, including those with unequal probability sampling and missing data. The memory type ratio and product estimators based on EWMA statistics were examined by [13], whereas Noor-ul Amin [12] used hybrid exponentially weighted moving average for estimation of population mean using memory type ratio and product estimators for time-based surveys. Aslam et al. [4] proposed the memory type ratio and product estimators under ranked-based sampling schemes. Bhushan et al. [6] evaluated the performance of memory-type logarithmic estimators under simple random sampling. To estimate population variance, Qureshi et al. [15] developed an EWMA-based memory type ratio and product estimators for population variance under simple random sampling. Noor-ul Amin et al. [14] suggested a variable acceptance sampling plan based on exponentially weighted hybrid moving averages. Zaman and Bulut [21] proposed an efficient family of robust-type estimators for population variance in simple and stratified random sampling. Aslam et al. [3] proposed a new memory-based ratio estimator for survey sampling. Despite the benefits of employing auxiliary attributes to estimate population variance, there is a notable gap in the research when it comes to using auxiliary attributes to estimate population variance using the EWMA statistic. This approach, which uses the EWMA statistic to estimate population variance using auxiliary attributes, has not been investigated in any research to the best of our knowledge.

In this paper, we have proposed a family of memory-type estimators for population variance and also developed the EWMA version of the usual ratio, regression, and exponential estimator for the estimation of population variance when the auxiliary variable is dichotomous in nature. In addition, the paper is divided into eight sections. The first section contains the introduction on variance estimation and then about the estimation using auxiliary attribute and also on the memory type estimator and EWMA statistic. Then, in Section 2 we define the usual ratio, regression, and exponential type estimators for the estimation of population variance in the literature. We have proposed a generalized family of memory-type estimators and also the EWMA version of the usual estimators in the literature for the population variance of the study variable Y in Section 3. Then we compared the efficiency of the estimators in Section 4. In Section 5, we conducted an empirical study and compared the efficiency of the estimators, and in Section 6 we conducted a simulation study to evaluate the performance of the estimators. In Section 7 we discuss the results of tables and figures. Finally, in the last section, that is, in Section 8 we have discussed the conclusion.

2. Review of estimators

Consider a sample of size n, drawn by simple random sampling without replacement (SRSWOR) from a population of size N and let y_i and ϕ_i denote observations on variables y and ϕ, respectively, for the unit i^{th} $(i = 1, 2, ..., N)$. The attributes can be defined as

$$\phi(i) = \begin{cases} 1 & \text{if } i^{th} \text{ unit of the population possesses attribute,} \\ 0 & \text{otherwise.} \end{cases}$$

The EWMA statistic to estimate population variance based on sample variance for $t > 0$ for the study variable and auxiliary attribute is defined as $V_t = \delta s^2_{y_t} + (1-\delta)V_{t-1}$ and $W_t = \delta s^2_{\phi_t} + (1-\delta)W_{t-1}$ where δ is weight given to the data known as the weight parameter or smoothing constant of the current sample observation. The expected value and variance of the EWMA statistic V_t are, respectively, given by

$$E(V_t) = S^2_y, \tag{2.1}$$

$$\begin{aligned} \text{Var}(V_t) &= \text{Var}\left[\delta s^2_{y_t} + (1-\delta)\left(\delta s^2_{y_{t-1}} + (1-\delta)V_{t-2}\right)\right] \\ &= \text{Var}\left[\delta s^2_{y_t} + (1-\delta)\delta s^2_{y_{t-1}} + (1-\delta)^2\delta s^2_{y_{t-2}} + ...\right] \\ &= \text{Var}(s^2_{y_t})\delta^2\left[1 + (1-\delta)^2 + (1-\delta)^4 + ...\right] \\ &= \text{Var}(s^2_{y_t})\delta^2 \sum_{n=0}^{\infty}(1-\delta)^{2n}. \end{aligned} \tag{2.2}$$

The limiting variance of the EWMA statistic V_t is given by

$$\mathrm{Var}(V_t) = \frac{\delta^2 \mathrm{Var}(s_{y_t}^2)}{2\delta - \delta^2} = \psi \mathrm{Var}(s_{y_t}^2). \tag{2.3}$$

similarly, the mean and limiting variance of EWMA statistic W_t can be expressed as

$$\mathrm{E}(W_t) = S_\phi^2 \tag{2.4}$$

$$\mathrm{Var}(W_t) = \frac{\delta^2 \mathrm{Var}(s_\phi^2)}{2\delta - \delta^2} = \psi \mathrm{Var}(s_{\phi_t}^2) \tag{2.5}$$

where $\psi = \frac{\delta}{2-\delta}$.

The variance of the usual unbiased estimator S_y^2 is given by

$$\mathrm{V}(S_y^2) = S_y^4 \left(\frac{\lambda_{40} - 1}{n} \right), \tag{2.6}$$

where

$$\lambda_{rs} = \frac{\mu_{rq}}{\mu_{20}^{r/2} \mu_{02}^{q/2}} \tag{2.7}$$

$$\mu_{rq} = \frac{\sum_{i=1}^{N} (y_i - \bar{Y})^r (\phi_i - P)^q}{N-1} \tag{2.8}$$

To derive the MSE of the proposed memory-type estimators, we consider the following notation:

$$V_t = S_y^2(1 + e_{0t}) \text{ and } W_t = S_\phi^2(1 + e_{1t})$$

such that

$$\begin{aligned} E(e_{0t}) &= E(e_{1t}) = 0, \\ E(e_{0t}^2) &= \psi \left(\frac{\lambda_{40} - 1}{n} \right), \\ E(e_{1t}^2) &= \psi \left(\frac{\lambda_{04} - 1}{n} \right), \\ E(e_{0t} e_{1t}) &= \psi \left(\frac{\lambda_{22} - 1}{n} \right), \end{aligned} \tag{2.9}$$

where $s_\phi^2 = \sum_{i=1}^{n} (\phi_i - \bar{\phi})^2/(n-1)$ and $s_y^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2/(n-1)$ are the sample variance of auxiliary attribute and study variable, respectively. $S_\phi^2 = \sum_{i=1}^{N} (\phi_i - \mu_\phi)^2/(N-1)$ and $S_y^2 = \sum_{i=1}^{N} (Y_i - \mu_y)^2/(N-1)$ are the population variances of the auxiliary attribute and the study variable, respectively. $\mu_\phi = \sum_{i=1}^{N} \phi_i/N$ and $\mu_y = \sum_{i=1}^{N} Y_i/N$ are the population means of the auxiliary attribute and the study variable, respectively. $\bar{\phi} = \sum_{i=1}^{n} \phi_i/n$ and $\bar{y} = \sum_{i=1}^{n} y_i/n$ are the sample means of the auxiliary attribute and the study variable, respectively.

Singh et al. [19] proposed the ratio, regression and exponential type estimators for the estimation of population variance using auxiliary attribute under simple random sampling. The ratio estimator t_1 is given by

$$t_1 = s_y^2 \left(\frac{S_{\phi^2}}{s_\phi^2} \right). \tag{2.10}$$

The MSE of the ratio estimator t_1 is obtained by

$$\mathrm{MSE}(t_1) = S_y^4 \left[\frac{(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)}{n} \right]. \tag{2.11}$$

The regression estimator t_2 is defined as

$$t_2 = s_y^2 + b_\phi \left(S_\phi^2 - s_\phi^2 \right). \tag{2.12}$$

The MSE of the regression estimator t_2 is given by

$$\mathrm{MSE}(t_2) = \frac{S_y^4\left(\lambda_{40}-1\right)+b_\phi^2 S_\phi^4\left(\lambda_{04}-1\right)-2b_\phi S_y^2 S_\phi^2\left(\lambda_{22}-1\right)}{n} \tag{2.13}$$

on differentiating (2.4) with respect to b_ϕ and equating to zero, we get

$$b_\phi = \frac{S_y^2\left(\lambda_{22}-1\right)}{S_\phi^2\left(\lambda_{04}-1\right)}. \tag{2.14}$$

Then, substituting the optimum value of b_ϕ in (2.4), we get the minimum variance of estimator t_2 as

$$\mathrm{MSE}(t_2)_{min} = \frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)-\frac{\left(\lambda_{22}-1\right)^2}{\left(\lambda_{04}-1\right)}\right]. \tag{2.15}$$

The exponential estimator t_3 is defined as

$$t_3 = s_y^2 \exp\left(\frac{S_{\phi^2}-s_\phi^2}{S_{\phi^2}+s_\phi^2}\right). \tag{2.16}$$

The MSE of the exponential estimator t_3 is obtained by

$$\mathrm{MSE}(t_3) = \frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+\frac{\left(\lambda_{04}-1\right)}{4}-\left(\lambda_{22}-1\right)\right]. \tag{2.17}$$

3. Proposed estimator

In this study, motivated by [19], we developed the memory type ratio, regression, and exponential estimators for population variance using auxiliary attributes under simple random sampling. Firstly, the memory type ratio estimator t_{1e} is defined as

$$t_{1e} = V_t\left(\frac{S_{\phi^2}}{W_t}\right). \tag{3.1}$$

The MSE of the memory type ratio estimator t_{1e} is obtained by

$$\mathrm{MSE}(t_{1e}) = S_y^4\psi\left[\frac{\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)-2\left(\lambda_{22}-1\right)}{n}\right]. \tag{3.2}$$

The memory type regression estimator t_{2e} is defined as

$$t_{2e} = V_t + b_\phi\left(S_\phi^2 - W_t\right). \tag{3.3}$$

The MSE of the memory type regression estimator t_{2e} is obtained by

$$\mathrm{MSE}(t_{2e}) = \psi\left[\frac{S_y^4\left(\lambda_{40}-1\right)+b_\phi^2 S_\phi^4\left(\lambda_{04}-1\right)-2b_\phi S_y^2 S_\phi^2\left(\lambda_{22}-1\right)}{n}\right]. \tag{3.4}$$

On differentiating (3.4) with respect to b_ϕ and equating to zero, we get

$$b_\phi = \frac{S_y^2\left(\lambda_{22}-1\right)}{S_\phi^2\left(\lambda_{04}-1\right)}. \tag{3.5}$$

Then, substituting the optimum value of b_ϕ in (3.4), we obtain the minimum variance of the estimator t_{2e} as

$$\mathrm{MSE}(t_{2e})_{min} = \frac{S_y^4\psi}{n}\left[\left(\lambda_{40}-1\right)-\frac{\left(\lambda_{22}-1\right)^2}{\left(\lambda_{04}-1\right)}\right]. \tag{3.6}$$

The memory type exponential estimator t_{3e} is defined as

$$t_{3e} = V_t \exp\left(\frac{S_{\phi^2}-W_t}{S_{\phi^2}+W_t}\right). \tag{3.7}$$

The MSE of the memory type exponential estimator t_{3e} is obtained by

$$\mathrm{MSE}(t_{3e}) = \frac{S_y^4\psi}{n}\left[(\lambda_{40}-1) + \frac{(\lambda_{04}-1)}{4} - (\lambda_{22}-1)\right]. \tag{3.8}$$

In this study, we propose the enhanced generalized family of memory type estimator for the estimation of population variance using auxiliary attribute as

$$t_a = r_1 V_t \left(\frac{W_t}{S_\phi^2}\right)^p \exp\left(\frac{\eta(S_\phi^2 - W_t)}{\eta(S_\phi^2 + W_t) + 2q}\right) + r_2 V_t\left[1 + \log\left(\frac{W_t}{S_\phi^2}\right)\right] \tag{3.9}$$

where r_1 and r_2 are suitable constants to be determined such that the MSE of t_a is minimum, η, p and q are either real numbers or functions of the known parameters of auxiliary variables and $r_1 + r_2 \neq 1$. Expressing (3.9) in terms of e_{0t} and e_{1t}, we get

$$\begin{aligned} t_a &= r_1 S_y^2(1+e_{0t})\left(\frac{S_\phi^2(1+e_{1t})}{S_\phi^2}\right)^p \exp\left(\frac{\eta\left(S_\phi^2 - S_\phi^2(1+e_{1t})\right)}{\eta\left(S_\phi^2 + S_\phi^2(1+e_{1t})\right) + 2q}\right) \\ &\quad + r_2 S_y^2(1+e_{0t})\left[1+\log(1+e_{1t})\right] \\ &= r_1 S_y^2(1+e_{0t})(1+e_{1t})^p \exp\left(\frac{-ve_{1t}}{2+ve_{1t}}\right) + r_2 S_y^2(1+e_{0t})\left[1+\log(1+e_{1t})\right] \end{aligned} \tag{3.10}$$

where $v = \frac{\eta S_\phi^2}{2(\eta S_\phi^2 + q)}$. Expanding (3.10) using Taylor's series expansion up to first order approximation, we have

$$\begin{aligned} t_a - S_y^2 &= (r_1 + r_2 - 1)S_y^2 + r_1 S_y^2\Bigg(e_{0t} + e_{1t}\left(p - \frac{v}{2}\right) + e_{1t}^2\left(\frac{3v^2}{8} - \frac{vp}{2} + \frac{p(p-1)}{2}\right)\Bigg) \\ &\qquad + e_{0t}e_{1t}\left(p - \frac{v}{2}\right) + r_2 S_y^2\left(e_{0t} + e_{1t} + e_{0t}e_{1t} - \frac{e_{1t}^2}{2}\right). \end{aligned} \tag{3.11}$$

Then, we have

$$\begin{aligned} (t_a - S_y^2)^2 &= S_y^4 + r_1^2 S_y^4\left(1 + e_{0t}^2 + e_{1t}^2\left(p^2 + \frac{v^2}{4} + \frac{3v^2}{4} + p(p-1)\right) + 4e_{0t}e_{1t}\left(p - \frac{v}{2}\right)\right) \\ &\quad + r_2^2 S_y^4\left(1 + e_{0t}^2 + 4e_{0t}e_{1t}\right) + 2r_1 r_2 S_y^4\left(1 + e_{0t}^2 + e_{0t}e_{1t}\left(2 + 2p - v\right)\right. \\ &\quad \left. + e_{1t}^2\left(p - \frac{v}{2} + \frac{3v^2}{8} - \frac{vp}{2} + \frac{p(p-1)}{2}\right)\right) \\ &\quad - 2r_1 S_y^4\left(1 + e_{1t}^2\left(\frac{3}{8}v^2 - \frac{vp}{2} + \frac{p(p-1)}{2}\right) + e_{0t}e_{1t}\left(p - \frac{v}{2}\right)\right) \\ &\quad - 2r_2 S_y^4\left(1 + e_{0t}e_{1t} + \frac{e_{1t}^2}{2}\right). \end{aligned} \tag{3.12}$$

Taking expectations both sides in (3.12), we obtain

$$\mathrm{MSE}(t_a) = S_y^4 + r_1^2 A + r_2^2 B + 2r_1 r_2 C - 2r_1 D - 2r_2 E. \tag{3.13}$$

where

$$A = S_y^4 \left(1 + \psi \left[\left(\frac{\lambda_{40} - 1}{n}\right) + 4\left(\frac{\lambda_{22} - 1}{n}\right)\left(p - \frac{v}{2}\right) \right.\right.$$
$$\left.\left. + \left(\frac{\lambda_{04} - 1}{n}\right)\left(p^2 + v^2 + p(p-1)\right)\right]\right), \tag{3.14}$$

$$B = S_y^4 \left(1 + \psi \left[\left(\frac{\lambda_{40} - 1}{n}\right) + 4\left(\frac{\lambda_{22} - 1}{n}\right)\right]\right), \tag{3.15}$$

$$C = S_y^4 \left(1 + \psi \left[\left(\frac{\lambda_{40} - 1}{n}\right) + \left(\frac{\lambda_{22} - 1}{n}\right)(2 + 2p - v) \right.\right.$$
$$\left.\left. + \left(\frac{\lambda_{04} - 1}{n}\right)\left(p - \frac{v}{2} + \frac{3v^2}{8} - \frac{vp}{2} + \frac{p(p-1)}{2}\right)\right]\right), \tag{3.16}$$

$$D = S_y^4 \left(1 + \psi \left[\left(\frac{\lambda_{22} - 1}{n}\right)\left(p - \frac{v}{2}\right) + \left(\frac{\lambda_{04} - 1}{n}\right)\left(\frac{3v^2}{8} - \frac{vp}{2} + \frac{p(p-1)}{2}\right)\right]\right), \tag{3.17}$$

$$E = S_y^4 \left(1 + \psi \left[\left(\frac{\lambda_{22} - 1}{n}\right) + \left(\frac{\lambda_{04} - 1}{2n}\right)\right]\right). \tag{3.18}$$

The expressions A, B, C, D, and E in Equations (3.14)–(3.18) are intermediate components used to derive the minimum MSE of the proposed memory-type estimator t_a. Specifically, A captures the impact of higher-order moments and interactions between p and v. B is a simplified form of A, excluding p and v, and reflects the contributions of the baseline moment. C extends B by incorporating the linear and quadratic effects of p and v, often related to the covariance structure. D adjusts for interaction effects using moment ratios λ_{22} and λ_{04}. Finally, E provides a compact adjustment term involving the effects of the auxiliary attributes and the memory parameter ψ.

Differentiating (3.13) from r_1, r_2 and equating it with zero, we have

$$r_1 = \frac{BD - CE}{AB - C^2}, r_2 = \frac{AE - CD}{AB - C^2}. \tag{3.19}$$

The minimum MSE is given by substituting the values of r_1 and r_2 in (3.13) as

$$\mathrm{MSE}_{\min}(t_a) = S_y^4 - \left(\frac{AE^2 + BD^2 - 2CDE}{AB - C^2}\right) \tag{3.20}$$

A set of estimators generated from t_a using suitable values of r_1, r_2, p, η, and q are listed in Table 1. Table 1 contains conventional parameters such as the quartile deviation Q_2, the coefficient of variation C_x, and the coefficient of skewness $B_1(x)$.

4. Efficiency comparison

In this section, we compare the efficiency of the proposed memory type estimator t_a with the ratio estimator t_1, regression estimator t_2, and exponential estimator t_3 for variance estimation using an auxiliary attribute to show the superiority of the proposed estimators over the others.

We also compare the efficiency of the proposed memory type estimator t_a with the EWMA version of the ratio estimator t_{1e}, the regression estimator t_{2e}, and the exponential estimator t_{3e}. Table 1 represents the set of estimators generated from the class of the proposed memory type estimator (t_a).

Table 1. Set of estimators generated from the class of estimator (t_a).

Subset of proposed estimator	r_1	r_2	p	η	q
$t_{a(1)} = V_t$	1	0	0	0	0
$t_{a(2)} = V_t\left(\frac{W_t}{S_\phi^2}\right) + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	1	1	1	0	1
$t_{a(3)} = V_t\left(\frac{W_t}{S_\phi^2}\right)^p + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	1	1	p	0	1
$t_{a(4)} = V_t\left(\frac{W_t}{S_\phi^2}\right)^{-1} + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	1	1	-1	0	1
$t_{a(5)} = V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	0	1	1	1	1
$t_{a(6)} = V_t \exp\left(\frac{S_\phi^2-W_t}{S_\phi^2+W_t}\right)$	1	0	0	1	0
$t_{a(7)} = r_1V_t\left(\frac{W_t}{S_\phi^2}\right) + r_2V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	r_1	r_2	1	0	1
$t_{a(8)} = r_1V_t + r_2V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	r_1	r_2	0	0	0
$t_{a(9)} = r_1V_t \exp\left(\frac{(S_\phi^2-W_t)}{(S_\phi^2+W_t)+2}\right)$	r_1	0	0	1	1
$t_{a(10)} = r_1V_t\left(\frac{W_t}{S_\phi^2}\right)\exp\left(\frac{(S_\phi^2-W_t)}{(S_\phi^2+W_t)+2}\right) + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	r_1	1	1	1	1
$t_{a(11)} = V_t\left(\frac{W_t}{S_\phi^2}\right)\exp\left(\frac{(S_\phi^2-W_t)}{(S_\phi^2+W_t)+2}\right) + r_2V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	1	r_2	1	1	1
$t_{a(12)} = r_2V_t\left(1+\log\left(\frac{W_t}{S_\phi^2}\right)\right)$	0	r_2	0	0	0
$t_{a(13)} = r_1V_t \exp\left(\frac{S_\phi^2-W_t}{S_\phi^2+W_t}\right)$	r_1	0	0	1	0
$t_{a(14)} = r_1V_t \exp\left(\frac{S_\phi^2-W_t}{(S_\phi^2+W_t)+2}\right) + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	r_1	1	0	1	1
$t_{a(15)} = V_t\left(\frac{W_t}{S_\phi^2}\right)$	1	0	1	0	0
$t_{a(16)} = V_t\left(\frac{W_t}{S_\phi^2}\right)^{-1}$	1	0	-1	0	0
$t_{a(17)} = V_t\left(\frac{W_t}{S_\phi^2}\right)^{-p}$	1	0	$-p$	0	0
$t_{a(18)} = r_1V_t\left(\frac{W_t}{S_\phi^2}\right)^p$	r_1	0	p	0	0
$t_{a(19)} = r_1V_t\left(\frac{W_t}{S_\phi^2}\right)^p + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	r_1	1	p	0	1
$t_{a(20)} = Q_2V_t + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	Q_2	1	0	0	0
$t_{a(21)} = Q_2V_t\left(\frac{W_t}{S_\phi^2}\right)\exp\left(\frac{S_\phi^2-W_t}{S_\phi^2+W_t}\right) + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	Q_2	1	1	1	0
$t_{a(22)} = Q_2V_t\left(\frac{W_t}{S_\phi^2}\right)\exp\left(\frac{S_\phi^2-W_t}{S_\phi^2+W_t}\right)$	Q_2	0	1	1	0
$t_{a(23)} = Q_2V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	0	Q_2	0	1	1
$t_{a(24)} = V_t\left(\frac{W_t}{S_\phi^2}\right) + Q_2V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	1	Q_2	1	0	1
$t_{a(25)} = r_1V_t \exp\left(\frac{S_\phi^2-W_t}{(S_\phi^2+W_t)+2}\right) + Q_2V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	r_1	Q_2	0	1	1
$t_{a(26)} = C_xV_t\left(\frac{W_t}{S_\phi^2}\right)$	C_x	0	1	0	0
$t_{a(27)} = C_xV_t\left(\frac{W_t}{S_\phi^2}\right)^p \exp\left(\frac{S_\phi^2-W_t}{S_\phi^2+W_t}\right) + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	C_x	1	p	1	0
$t_{a(28)} = C_xV_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	0	C_x	0	0	1
$t_{a(29)} = B_1(x)V_t\left(\frac{W_t}{S_\phi^2}\right)\exp\left(\frac{S_\phi^2-W_t}{(S_\phi^2+W_t)+2}\right) + V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	$B_1(x)$	1	1	1	1
$t_{a(30)} = r_1V_t\left(\frac{W_t}{S_\phi^2}\right)^p \exp\left(\frac{S_\phi^2-W_t}{S_\phi^2+W_t}\right) + B_1(x)V_t\left[1+\log\left(\frac{W_t}{S_\phi^2}\right)\right]$	r_1	$B_1(x)$	p	1	0

$$\begin{aligned}
\mathrm{MSE}(t_1) - \mathrm{MSE}(t_a) &= \frac{S_y^4}{n}\left[(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)\right] \\
&\quad - \left(S_y^4 + r_1^2 A + r_2^2 B + 2r_1 r_2 C - 2r_1 D - 2r_2 E\right) \geq 0 \qquad (4.1)\\
\mathrm{MSE}(t_2)_{min} - \mathrm{MSE}(t_a) &= \frac{S_y^4}{n}\left[(\lambda_{40}-1)-\frac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}\right] \\
&\quad - \left(S_y^4 + r_1^2 A + r_2^2 B + 2r_1 r_2 C - 2r_1 D - 2r_2 E\right) \geq 0 \qquad (4.2)\\
\mathrm{MSE}(t_3) - \mathrm{MSE}(t_a) &= \frac{S_y^4}{n}\left[(\lambda_{40}-1)+\frac{(\lambda_{04}-1)}{4}-(\lambda_{22}-1)\right] \\
&\quad - \left(S_y^4 + r_1^2 A + r_2^2 B + 2r_1 r_2 C - 2r_1 D - 2r_2 E\right) \geq 0 \qquad (4.3)\\
\mathrm{MSE}(t_{1e}) - \mathrm{MSE}(t_a) &= \frac{S_y^4\psi}{n}\left[(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)\right] \\
&\quad - \left(S_y^4 + r_1^2 A + r_2^2 B + 2r_1 r_2 C - 2r_1 D - 2r_2 E\right) \geq 0 \qquad (4.4)\\
\mathrm{MSE}(t_{2e})_{min} - \mathrm{MSE}(t_a) &= \frac{S_y^4\psi}{n}\left[(\lambda_{40}-1)-\frac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}\right] \\
&\quad - \left(S_y^4 + r_1^2 A + r_2^2 B + 2r_1 r_2 C - 2r_1 D - 2r_2 E\right) \geq 0 \qquad (4.5)\\
\mathrm{MSE}(t_{3e}) - \mathrm{MSE}(t_a) &= \frac{S_y^4\psi}{n}\left[(\lambda_{40}-1)+\frac{(\lambda_{04}-1)}{4}-(\lambda_{22}-1)\right] \\
&\quad - \left(S_y^4 + r_1^2 A + r_2^2 B + 2r_1 r_2 C - 2r_1 D - 2r_2 E\right) \geq 0 \qquad (4.6)
\end{aligned}$$

Equations (4.1), (4.2), (4.3), (4.4), (4.5), and (4.6) will always be greater than zero.

5. Empirical study

In this section, two data sets are performed to illustrate the efficiency of the proposed estimator. The first data set is based on the household of the village [27]. The size of the household in each village household is taken as the study variable y and the size of the household that consists of more than five is taken as the auxiliary attribute ϕ. The second data set is based on the household of the village [26]. The number of villages in the circle is taken as the study variable y and a circle consisting of more than five villages is taken as an auxiliary attribute ϕ. We have selected different sample sizes $n = 5, 10, 15, 20$ and different values of δ such as $\delta = 0.2, 0.5, 0.7, 0.9$. The population characteristics are given below in Table 2. To show the stepwise procedure of empirical investigation of the proposed estimators over the others, a flow chart is presented in Figure 1.

Table 2. Population characteristics.

	N	S_y^2	S_ϕ^2	C_y	C_p	λ_{22}	λ_{40}	λ_{04}
Population 1	35	4.232	0.252	0.346	0.897	0.952	4.977	1.052
Population 2	89	4.074	0.11	0.601	2.678	3.996	3.811	6.162

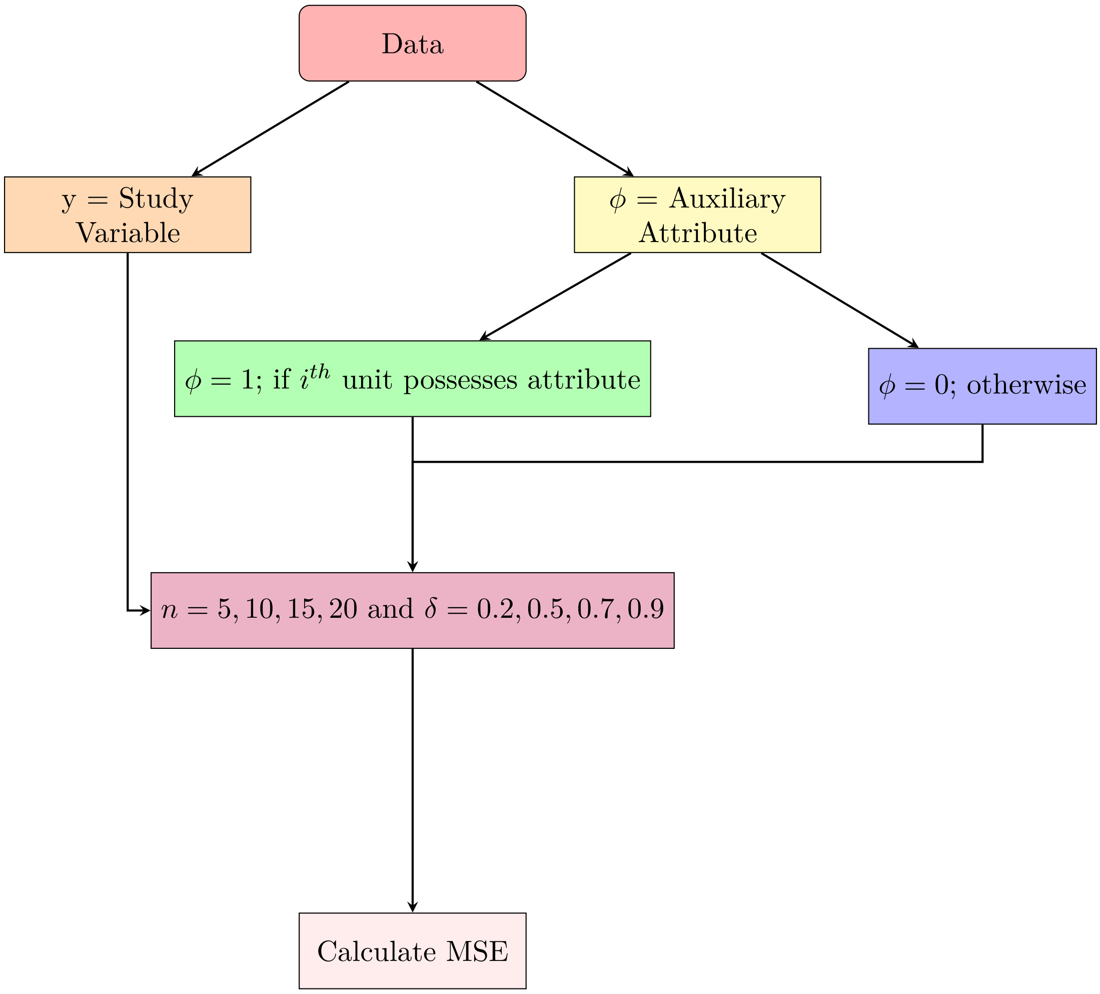

Figure 1. Flow chart of the empirical study

For a better and easier understanding of the properties of the proposed estimators along with other estimators considered in this paper, we present the exploratory data analysis of Tables 3 and 4. Tables 3 and 4 exhibit the MSEs of the estimators in the literature and their EWMA version and also the MSE of proposed generalized class of memory type estimators for Population 1 and 2 respectively. Here, we have taken different values of $n = 5, 10, 15, 20$ and $\delta = 0.2, 0.5, 0.7, 0.9$ and then obtained the MSE of the estimators. Tables 3 and 4 show that as the value of n increases, the values of MSE decrease for the estimators. So, there is a decreasing trend in the MSE values for the increase in the sample size from 5 to 20. For the fixed value of n, as the values of δ increase from 0.2 to 0.9, the MSE values also increase. The values of δ are used to assign weight to the current and past observations, which can lead to the efficiency of the estimators, as shown in Tables 3 and 4. At different values of n and δ, we see that the MSEs of the EWMA version of estimators in the literature are more efficient than the usual estimators, as the MSEs of the EWMA version of estimators are less than the usual estimators in the literature. In addition, the proposed generalized class of memory-type estimators is more efficient than all the EWMA versions of estimators and the usual estimators in the literature using Bernoulli auxiliary information.

Table 3. The MSEs of estimators at different values of n and δ for Population 1

Estimator		MSE				Estimator	MSE
	n	$\boldsymbol{\delta = 0.2}$	$\boldsymbol{\delta = 0.5}$	$\boldsymbol{\delta = 0.7}$	$\boldsymbol{\delta = 0.9}$		
t_{1e}	5	1.6417	4.9252	7.9561	12.0891	t_1	14.7756
	10	0.8209	2.4626	3.9780	6.0446		7.3878
	15	0.5472	1.6417	2.6520	4.0297		4.9252
	20	0.4104	1.2313	1.9890	3.0223		3.6939
t_{2e}	5	1.4241	4.5898	7.5119	11.4967	t_2	14.0868
	10	0.7121	2.2949	3.7560	5.7483		7.0434
	15	0.4947	1.5299	2.5040	3.8322		4.6956
	20	0.3560	1.1474	1.8780	2.8742		3.5217
t_{3e}	5	1.6071	4.8213	7.7883	11.8342	t_3	14.4640
	10	0.8036	2.4107	3.8941	5.9171		7.2320
	15	0.5357	1.6071	2.5961	3.9447		4.8213
	20	0.4018	1.2053	1.9471	2.9585		3.6160
t_a	5	1.3871	3.4635	4.9800	6.6346	t_a	7.5194
	10	0.7187	1.8944	2.8444	3.9851		4.6465
	15	0.4850	1.3035	1.9900	2.8460		3.3591
	20	0.3660	0.9935	1.5302	2.2130		2.6298

Table 4. The MSEs of estimators at different values of n and δ for Population 2.

Estimator		MSE				Estimator	MSE
	n	$\boldsymbol{\delta = 0.2}$	$\boldsymbol{\delta = 0.5}$	$\boldsymbol{\delta = 0.7}$	$\boldsymbol{\delta = 0.9}$		
t_{1e}	5	0.7307	2.1920	3.5409	5.3803	t_1	6.5759
	10	0.3653	1.0960	1.7704	2.6901		3.2879
	15	0.2436	0.7307	1.1803	1.7934		2.1919
	20	0.1827	0.5480	0.8852	1.3451		1.6439
t_{2e}	5	0.3954	1.1863	1.9164	2.9119	t_2	3.5589
	10	0.1977	0.5932	0.9582	1.4559		1.7794
	15	0.1318	0.3954	0.6388	0.9706		1.1863
	20	0.0989	0.2966	0.4791	0.7280		0.8897
t_{3e}	5	0.4077	1.2232	1.9760	3.0025	t_3	3.6697
	10	0.2039	0.6116	0.9880	1.5012		1.8348
	15	0.1359	0.4077	0.6587	1.0008		1.2232
	20	0.1019	0.3058	0.4940	0.7506		0.9174
t_a	5	0.2919	1.0651	1.2585	2.6453	t_a	3.2004
	10	0.0537	0.3480	0.8149	1.3244		1.6213
	15	0.0474	0.1919	0.4478	0.8176		1.0987
	20	0.0138	0.0711	0.1100	0.3431		0.8415

6. Simulation Study

A simulation study is conducted to evaluate the performance of the proposed memory-type estimators over the other estimators in the literature. The computation steps for the MSEs of the estimators are listed below:

(1) A population of size 1000 are generated using the bivariate normal distribution with parameters as $(Y, X) \sim N_2(5, 6, 2, 5, \rho)$

(2) The different values of smoothing constant and correlation coefficient are taken as $\delta = 0.2, 0.5, 0.7, 0.9$ and $\rho = 0.01, 0.05, 0.10, 0.50$, respectively.
(3) Select 10000 samples of different sizes as $n = 100, 200, 500, 800$.
(4) The MSEs for each sample size are obtained using the formula given as

$$MSE(\theta) = \frac{1}{10000} \sum_{i=1}^{10000} (\theta_i - S_y^2)^2 \tag{6.1}$$

Figures 2 and 3 depict the MSEs of the memory-type estimators across n and δ, respectively, for Population 1. Figure 2 shows the MSEs of the memory-type estimators for Population 1 across different sample sizes (n), under four correlation levels: 0.2, 0.5, 0.7, and 0.9. In all settings, the estimator t_a consistently produces the lowest MSE, indicating a better efficiency compared to the others. In contrast, t_{1e} shows the highest MSE values in each scenario. The estimators t_{2e} and t_{3e} perform similarly, closely following each other in all subplots. As the correlation increases, the overall MSE values decrease, suggesting that a higher correlation leads to improved estimator performance. Additionally, the distinction between estimators becomes more pronounced at higher correlation levels.

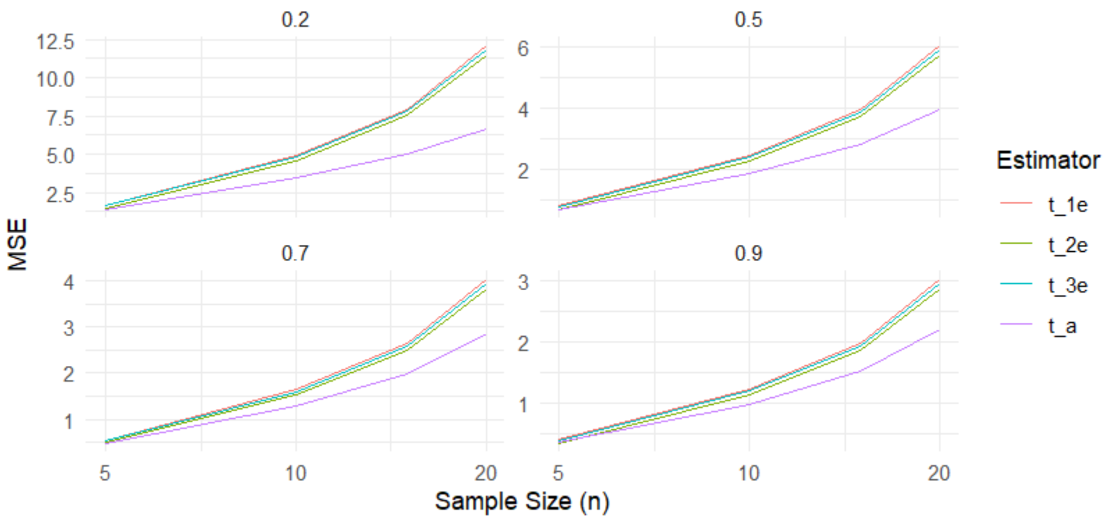

Figure 2. MSEs of memory type estimators across n for Population 1

Figure 3 illustrates the MSEs of memory-type estimators for Population 1 across different values of δ, under four sample sizes: 5, 10, 15, and 20. For all sample sizes, MSE values decrease as δ increases. This indicates that higher values of δ improve the accuracy of the estimation for all estimators. Among the estimators, t_a consistently yields the lowest MSE, confirming its superior efficiency. In contrast, t_{1e} exhibits the highest MSE in all settings. The performance of t_{2e} and t_{3e} is quite similar and generally falls between t_a and t_{1e}. As the sample size increases, the differences in the MSEs between the estimators become more pronounced, especially highlighting the advantage of t_a.

Figures 4 and 5 depict the MSEs of the memory-type estimators across n and δ, respectively, for Population 2 across different sample sizes (n), under four correlation levels: 0.2, 0.5, 0.7, and 0.9.

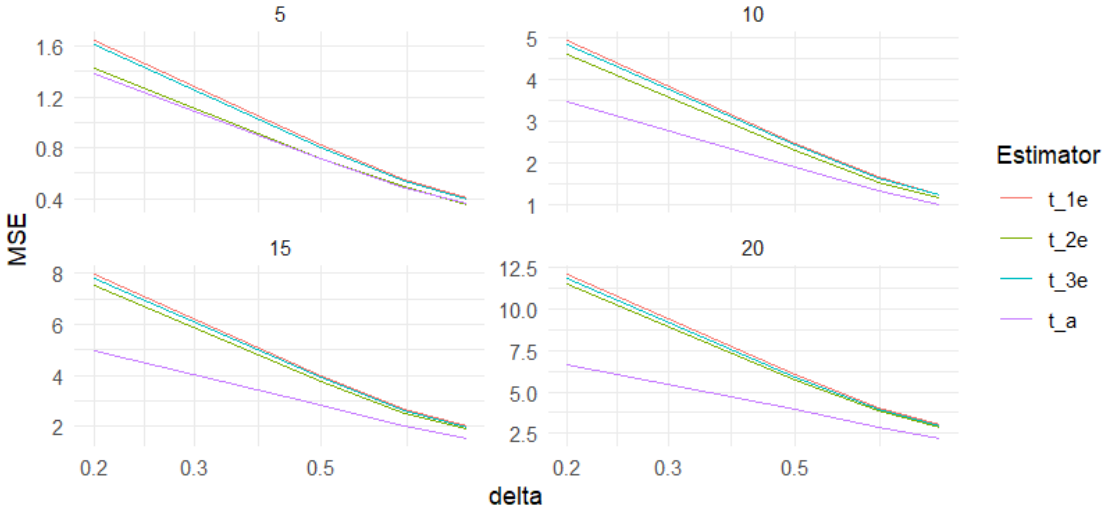

Figure 3. MSEs of memory type estimators across δ for Population 1

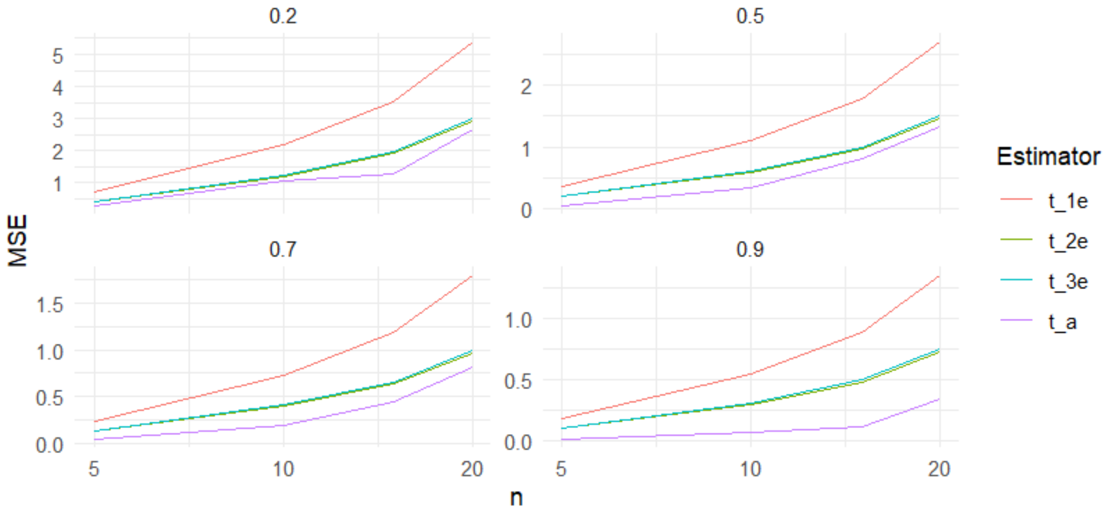

Figure 4. MSEs of memory type estimators across n for Population 2

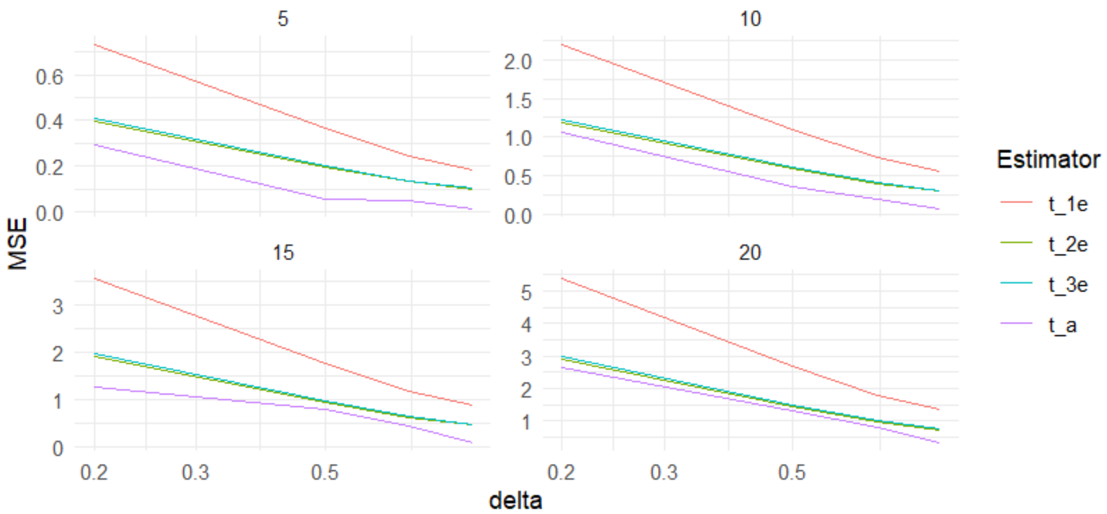

Figure 5. MSEs of memory type estimators across δ for Population 2

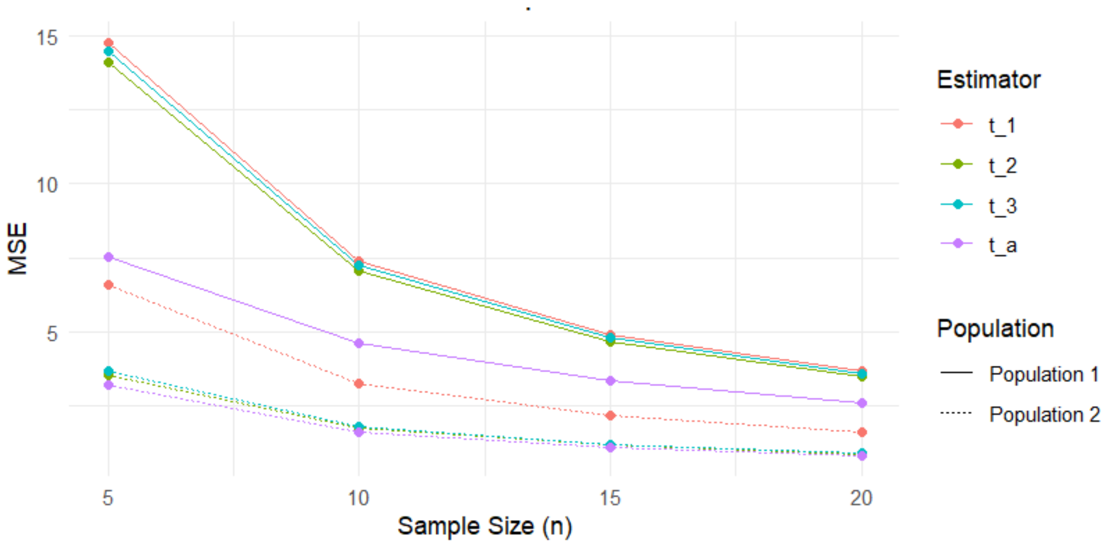

Figure 6. MSEs of estimators at different values of n for Populations 1 and 2

In Figure 4, the MSE values increase with the sample size at all correlation levels (0.2, 0.5, 0.7, 0.9). Among the estimators, t_a consistently yields the lowest MSE, followed by t_{3e} and t_{2e}, while t_{1e} performs the worst. The benefit of correlation becomes more apparent at higher levels, with all estimators showing relatively lower MSEs. Figure 5 shows that MSEs decrease as δ increases, indicating that larger values of δ lead to more accurate estimations. Again, t_a remains the most efficient estimator across all sample sizes, and t_{1e} shows the poorest performance. The difference in performance between estimators becomes more distinct at larger sample sizes. Figure 6 shows the MSEs of the usual estimators and the proposed memory type estimator at different values of n for Populations 1 and 2. Table 5 presents the MSEs of the estimators at several values of n and ρ. From Table 5, it can be seen that as the value of n increases from 100 to 800 , the values of MSE decrease.

Table 5. MSEs of estimators at different values n and ρ.

ρ	Sample size (n)	t_1	t_2	t_3
0.01	100	0.074474030	0.073616359	0.073852798
	200	0.037489942	0.037350319	0.037386925
	500	0.007583014	0.007570541	0.007574743
	800	0.001843933	0.001844344	0.001843954
0.05	100	0.080231597	0.079511934	0.079663978
	200	0.034853948	0.034682245	0.034732202
	500	0.008144596	0.008134262	0.008137447
	800	0.002324964	0.002324080	0.002324392
0.10	100	0.068778504	0.068144944	0.068298237
	200	0.029927543	0.029800984	0.029830596
	500	0.008311328	0.008302342	0.008304616
	800	0.001942500	0.001942375	0.001942286
0.50	100	0.061881766	0.061412506	0.061461588
	200	0.024129249	0.023984164	0.024028232
	500	0.007053652	0.007048166	0.007049023
	800	0.001986096	0.001986221	0.001985995

Table 6. MSEs of memory type estimators at different values of n, ρ and δ.

ρ	n	δ	t_{1e}	t_{2e}	t_{3e}	t_a
0.01	100	0.2	0.008070695	0.008157621	0.007996600	0.007968853
		0.5	0.025139498	0.025248435	0.024892906	0.024785149
		0.7	0.039547104	0.040135219	0.039194730	0.039049582
		0.9	0.060098173	0.061353568	0.059646846	0.059471866
	200	0.2	0.003121118	0.003142532	0.003107786	0.003102919
		0.5	0.009663651	0.009789600	0.009638463	0.009632072
		0.7	0.016032223	0.016235195	0.015987198	0.015973613
		0.9	0.024423161	0.024770919	0.024367786	0.024353948
	500	0.2	0.000955665	0.000958673	0.000955167	0.000955085
		0.5	0.002867827	0.002867864	0.002864374	0.002862950
		0.7	0.004583108	0.004586237	0.004578351	0.004576452
		0.9	0.006920719	0.006931447	0.006914867	0.006912785
	800	0.2	0.000232497	0.000231652	0.000232277	0.000232156
		0.5	0.000682781	0.000682677	0.000682569	0.000682479
		0.7	0.001108105	0.001109038	0.001107932	0.001107895
		0.9	0.001650170	0.001647717	0.001649263	0.001648800
0.05	100	0.2	0.007294725	0.007232105	0.007191152	0.007144751
		0.5	0.020849600	0.021497283	0.020696645	0.020652966
		0.7	0.034621683	0.035413472	0.034309477	0.034192589
		0.9	0.051451909	0.052560065	0.051019224	0.050851817
	200	0.2	0.003505380	0.003520028	0.003492192	0.003487248
		0.5	0.010067906	0.010223116	0.010048743	0.010046038
		0.7	0.016438497	0.016546146	0.016375082	0.016351367
		0.9	0.025296262	0.025502353	0.025206709	0.025173276
	500	0.2	0.000934758	0.000935868	0.000933865	0.000933538
		0.5	0.002702813	0.002710472	0.002700990	0.002700546
		0.7	0.004190165	0.004197515	0.004185838	0.004184287
		0.9	0.006617622	0.006643919	0.006614614	0.006614435
	800	0.2	0.000220116	0.000220804	0.000220169	0.000220216
		0.5	0.000684491	0.000683023	0.000684029	0.000683786
		0.7	0.001110035	0.001111724	0.001110018	0.001110073
		0.9	0.001664942	0.001667011	0.001664833	0.001664866
0.10	100	0.2	0.008445069	0.008420667	0.008354160	0.008313983
		0.5	0.025044947	0.025688222	0.024874052	0.024822400
		0.7	0.040806295	0.041833041	0.040572986	0.040496798
		0.9	0.062273696	0.063657284	0.061820251	0.061638118
	200	0.2	0.003596798	0.003649247	0.003588857	0.003587546
		0.5	0.010351823	0.010477647	0.010323714	0.010316494
		0.7	0.017545582	0.017648135	0.017482370	0.017457346
		0.9	0.025481799	0.025802523	0.025401345	0.025378481
	500	0.2	0.001174675	0.001172851	0.001173203	0.001172546
		0.5	0.003747531	0.003753736	0.003745183	0.003744497
		0.7	0.005919403	0.005932769	0.005916102	0.005915332
		0.9	0.009030532	0.009043229	0.009023034	0.009020422
	800	0.2	0.000243781	0.000243512	0.000243670	0.000243615
		0.5	0.000737599	0.000738841	0.000737608	0.000737655
		0.7	0.001184690	0.001185721	0.001184569	0.001184555
		0.9	0.001733259	0.001733878	0.001732884	0.001732749
0.50	100	0.2	0.008809943	0.008870927	0.008734070	0.008704825
		0.5	0.026666422	0.027428546	0.026522655	0.026491662
		0.7	0.043890202	0.045109507	0.043672955	0.043619884
		0.9	0.064863281	0.067049324	0.064603703	0.064566737
	200	0.2	0.003676004	0.003716788	0.003664672	0.003661272
		0.5	0.011163133	0.011294014	0.011132897	0.011124303
		0.7	0.018253305	0.018510915	0.018215299	0.018207382
		0.9	0.027449633	0.027680124	0.027361425	0.027329136
	500	0.2	0.000808593	0.000812158	0.000808141	0.000808080
		0.5	0.002272645	0.002282460	0.002270869	0.002270494
		0.7	0.003626710	0.003653547	0.003626432	0.003627294
		0.9	0.005787633	0.005819542	0.005785099	0.005785158
	800	0.2	0.000246984	0.000246721	0.000246879	0.000246826
		0.5	0.000699952	0.000700261	0.000699792	0.000699735
		0.7	0.001132896	0.001134379	0.001132823	0.001132843
		0.9	0.001760733	0.001762220	0.001760497	0.001760450

Table 6 presents the MSE values of various estimators. It is clearly observed that, for a fixed value of δ, the MSE values decrease steadily as the sample size n increases from 100 to 800. This behavior is consistent across all estimators considered in the study and aligns with the general statistical expectation that larger sample sizes yield more precise estimates due to reduced variability. Conversely, when the sample size n is held constant, an increase in the parameter δ from 0.2 to 0.9 leads to a gradual increase in the MSE values for all estimators. This suggests that higher values of δ, which may be associated with increased variability or dependence in the data structure, negatively affect the estimation accuracy.

A comparative evaluation of Tables 5 and 6 reveals a significant finding: the proposed generalized class of memory-type estimators consistently achieves lower MSE values compared to both the traditional estimators available in the literature and their EWMA versions. This clearly demonstrates the superior performance of the proposed estimator.

7. Conclusion

In this study, we have suggested a generalized class of memory-type estimators for the estimation of population variance using Bernoulli auxiliary information. In addition, some known estimators of population variance, such as the usual ratio and exponential ratio type estimators, are found to be members of a generalized class of memory-type estimators. The set of estimators are also generated from the proposed generalized class of memory type estimators. Additionally, we calculated the MSE of the suggested estimator up to the first order of approximation. The results of the empirical study showed that, in different sample sizes, the suggested memory type estimators outperformed the other memory-type estimators discussed above and the estimators in the literature. According to simulation research results, the suggested memory type estimator outperformed the literature estimators at various values of n, δ and ρ. The suggested estimator had the lowest MSE in both empirical and simulated studies, indicating that the suggested memory-type estimator is more effective and beneficial for the estimation of population variance with Bernoulli auxiliary information specifically for time-scaled surveys. Conventional estimators often make the assumption that the auxiliary data is constant or independent over time.

The proposed method utilizes an exponentially weighted moving average (EWMA), which gives more weight to recent data while preserving some of the impact of previous observations. This increases accuracy in dynamic contexts where population characteristics change over time. Therefore, the suggested approach addresses the limitation of existing studies. Future studies could concentrate on applying our suggested estimator to different sampling schemes, as well as to situations involving measurement error and non response, where the data from auxiliary attributes can be used to more precisely estimate population variance.

Acknowledgements

We would like to extend our sincere gratitude to the Editor and the anonymous reviewers for their insightful comments and suggestions, which have significantly contributed to improving the quality of our manuscript.

Author contributions. All the co-authors have contributed equally in all aspects of the preparation of this submission.

Conflict of interest statement. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding. The authors declare that no funds or other grants were received for the preparation of this manuscript.

Data availability. All data applied is included in the manuscript.

References

[1] N.K. Adichwal, P. Sharma, and R. Singh, *Generalized class of estimators for population variance using information on two auxiliary variables*, Int. Jour. of Appl. and Comp.Math. **3**, 651661, 2017.

[2] N.K. Adichwal, P. Sharma, H.K. Verma, and R. Singh, *Generalized class of estimators for population variance using auxiliary attribute*, Int. Jour. of Appl. and Comp.Math. **2**, 499508, 2016.

[3] I. Aslam, M.N. Amin, A. Mahmood, and P. Sharma, *New memory-based ratio estimator in survey sampling*, Natu.and Appl.Sci.Int. Jour. (NASIJ) **5** (1), 168181, 2024.

[4] I. Aslam, M. Noor-ul Amin, M. Hanif, and P. Sharma, *Memory type ratio and product estimators under ranked-based sampling schemes*, Comm.in Stat.-Theo.and Meth. **52** (4), 11551177, 2023.

[5] M.A. Bhat, S. Maqbool, and M. Subzar, *An improvement in variance estimator for the estimation of population variance, using known values of auxiliary information*, Int. Jour.of Pure & App. Biosci.**6** (5), 135138, 2018.

[6] S. Bhushan, A. Kumar, A. Alrumayh, H.A. Khogeer, and R. Onyango, *Evaluating the performance of memory type logarithmic estimators using simple random sampling*, Plos One **17** (12), e0278264, 2022.

[7] A.K. Das, *Use of auxiliary information in estimating the finite population variance*, Sankhya, C **40**, 139148, 1978.

[8] C.T. Isaki, *Variance estimation using auxiliary information*, JASA **78** (381), 117123, 1983.

[9] N. Koyuncu, *Efficient estimators of population mean using auxiliary attributes*, App. Math. and Comp. **218** (22), 1090010905, 2012.

[10] S. Malik and R. Singh, *An improved estimator using two auxiliary attributes*, App.Math.and Comp.**219** (23), 1098310986, 2013.

[11] P. Mukhopadhyay, *Theory and methods of survey sampling*, PHI Learning Pvt. Ltd., 2008.

[12] M. Noor-ul Amin, *Memory type ratio and product estimators for population mean for time-based surveys*, Jour. of Stat. Comp. and Simu.**90** (17), 30803092, 2020.

[13] M. Noor-ul Amin, *Memory type estimators of population mean using exponentially weighted moving averages for time scaled surveys*, Comm.in Stat.-Theo.and Meth. **50** (12), 27472758, 2021.

[14] M. Noor-ul Amin, A. Safeer, and P. Sharma, *Variable acceptance sampling plan based on hybrid exponentially weighted moving averages*, Comm.in Stat.-Simu. and Comp. **51** (12), 75447553, 2022.

[15] M.N. Qureshi, M.U. Tariq, and M. Hanif, *Memory-type ratio and product estimators for population variance using exponentially weighted moving averages for time-scaled surveys*, Comm.in Stat.-Simu. and Comp. **53** (3), 14841493, 2024.

[16] P. Sharma and R. Singh, *Efficient estimator of population mean in stratified random sampling using auxiliary attribute*, Wor.App.Sci Jour. **27** (12), 17861791, 2013.

[17] P. Sharma and R. Singh, *Improved estimators in simple random sampling when study variable is an attribute*, Jour.of Stat. App. and Pro. Lett. **2** (1), 5158, 2015.

[18] P. Sharma, H.K. Verma, S. Singh, and C.N. Bouza, *Estimators for population variance using auxiliary information on quartile*, Inves.Oper.**39** (4), 2018.

[19] R. Singh, M. Kumar, A.K. Singh, and F. Smarandache, *A family of estimators of population variance using information on auxiliary attribute*, IN SAM ME SER, 63, 2011.

[20] R. Singh and S. Malik, *Improved estimation of population variance using information on auxiliary attribute in simple random sampling*, Appl. Math. and Comp. **235**, 4349, 2014.

[21] T. Zaman and H. Bulut, *An efficient family of robust-type estimators for the population variance in simple and stratified random sampling*, Comm.in Stat.-Theo.and Meth. **52** (8), 26102624, 2023.

[22] G. Özel, H. Çıngı, and M. Oğuz, *Separate ratio estimators for the population variance in stratified random sampling*, Comm.in Stat.-Theo.and Meth. **43** (22), 47664779, 2014.

[23] G. Ö. Kadılar, *A new exponential type estimator for the population mean in simple random sampling*, JMASM **15**, 207214, 2016.

[24] G. Özel and C. Kadılar, *Modified Exponential Type Estimators for Population Mean in Stratified Random Sampling: An Application on the Geometric Distributed Aftershocks*, Co-chair, 63.

[25] M. U. Tariq, M. N. Qureshi, O. A. Alamri, S. Iftikhar, B. S. Alsaedi, and M. Hanif, *Variance estimation using memory type estimators based on EWMA statistic for time scaled surveys in stratified sampling*, Sci.Rep. **14** (1), 26700, 2024.

[26] P. V. Sukhatme and B. V. Sukhatme, *Sampling theory of surveys with applications*, 1970.

[27] P. Mukhopadhyay, *Theory and methods of survey sampling*, PHI Learning Pvt. Ltd., 2008.