



Comparison of Genetic Crossover Operators for Traveling Salesman Problem

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Highlights

- This paper focuses on crossover operators, one of the basic operators of genetic algorithms.
- A Critical difference diagram was used to statistically compare the operators' performances.
- With the improved greedy crossover, a more efficient accuracy rate was achieved.

Article Info

Received: 10 Nov 2024

Accepted: 10 Feb 2025

Keywords

Travelling salesman problem,
Genetic algorithms,
Crossover operators,
Critical difference diagram

Abstract

The traveling salesman problem (TSP) is an NP-hard problem that has been the subject of intensive study by researchers and academics in the field of optimization for many years. Genetic algorithms (GA) are one of the most effective methods for solving various NP-hard problems, including TSP. Recently, many crossover operators have been proposed to solve the TSP problem using GA. However, it remains unclear which crossover operator performs better for the particular problem. In this study, ten crossover operators, namely; Partially-Mapped Crossover (PMX), Cycle Crossover (CX), Order Crossover (OX1), Order Based Crossover (OX2), Position Based Crossover (POS), Edge Recombination Crossover (ERX), Maximal Preservative Crossover (MPX), Extended Partially-Mapped Crossover (EPMX), Improved Greedy Crossover (IGX), and Sequential Constructive Crossover (SCX) have been empirically evaluated. 30 TSP data sets have been used to comprehensively evaluate the selected crossover operators, and the experiments have been repeated 30 times to make our results statistically sound. Likewise, how successful the operators are, has been found through critical diagrams and statistical tests. Among tested operators, the IGX and SCX methods were the best operators in terms of convergence rate. On the other hand, PMX outperformed other operators in terms of computational cost.

1. INTRODUCTION

Traveling Salesman Problem (TSP) has been the most studied combinatorial optimization problem since it was proposed by Leonhard Euler in 1759 and defined mathematically by Karl Menger in the early 1930s [1]. TSP is an easy problem to define and difficult to solve and belongs to the class of non-polynomial (NP)-hard optimization problems.

TSP has important applications in real-world problems such as route planning, punching of printed circuit boards, scheduling, computer wiring, X-ray crystallography, placing goods in warehouses, and overhauling gas turbine engines [2]. For instance, Kavlak et al. [3] conducted studies on determining routes and optimizing time usage for drones. To solve network problems, they formulated a mixed integer mathematical model called Multi-Drone Capacitated Vehicle Routing Problem (mDroneCVRP), which is a vehicle routing problem transformed from TSP. Liu et al. [4] conducted research on package delivery in the urban traffic network and stated that situations such as bad weather conditions and traffic accidents may cause the delivery time to extend. They formulated the packet distribution problem as Steiner TSP (STSP), which takes traffic uncertainties into account and minimized the cost with the Retrace algorithm they proposed. Lai et al. [5] stated that tests on prototype air conditioners are costly for companies. They showed that they reduced electricity costs by up to 41% by reducing the task planning and scheduling problem to a TSP. Groba et al. [6] demonstrated how constantly moving targets in fish harvesting devices on tuna vessels can improve route optimization. They modeled the problem as Dynamic TSP (DTSP) and showed that ships

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reduce fuel and CO₂ emissions by shortening their routes. Liu et al. [7] studied the hot rolling planning used in steel production. They reduced the problem to the Multi-Objective TSP model and optimized parameters such as minimum rolling unit layout and minimum power consumption per ton of steel. Özgür et al. (2021) [8] examined articles on hot rolling planning and scheduling methods in steel mills between 1989 and 2020. They showed that among the 90 studies they examined, the most frequently used method for the formulation of scheduling problems was TSP and its derivatives with 22 studies.

Exact, heuristic and metaheuristic algorithms are used to solve TSP. Simulated annealing, ant colony, neural networks, tabu search, particle swarm optimization (PSO), and genetic algorithms (GA) are examples of meta-heuristics algorithms [9, 10]. GA have become a preferred method in many sectors, especially because they offer an effective solution method for complex and multivariable problems. Öztürk et al. [11] used GA and electromagnetic analysis software to calculate core loss coefficients of M19 steel material used in electrical machine design. In the proposed study, they showed that they minimized the deviation between the original loss value and the calculated loss value. In 2014, GA was applied to adjust the fuzzy logic controller (FLC) parameters of permanent magnet synchronous motor (PMSM) drive in control system design. It is aimed to improve the speed response profile of PMSM in response to speed reference and load torque changes [12]. In a study conducted by Çelik et al. [13] in 2017, they proposed a new reference current generation technique based on a fuzzy logic estimator (FLE) that allows controlling the current in Brushless DC motors (BLDCM) to prevent a significant drop in motor power when the current regulators reach a certain saturation. They used GA to identify rule-based parameters with a suitable simulation model while other FLE parameters were fixed.

Recent works have focused on the effect of crossover operators on the performance of GA. Nevertheless, the number of crossover operators that have been compared using the TSPLIB data sets [14] is relatively low. In instances where the number of crossover operators being compared is high, the number of data sets on which they are compared is not. For further details, please refer to section 2 and Table 1. Furthermore, only a limited number of studies have employed statistical analysis to evaluate the performance of operators. The objective of the proposed study is to conduct a more comprehensive analysis of this topic, with the following contributions to the field of study.

1. In our study, we compare ten different crossover operators, 11 of which are symmetric and 19 of which are asymmetric, on 30 different datasets. This comprehensive analysis provides a rich variety of data to determine the performance of the operators.
2. The performance of the operators has been analyzed in detail within the framework of convergence rate and computational cost.
3. Contrary to the statistical methods used in the literature (ANOVA and t-test), Friedman test has been used to reject the null hypothesis and rank the algorithms and then post-hoc test, Wilcoxon signed rank test with Holm's alpha (5%) correction, has been performed to calculate the critical differences. Differences in statistical significance has been then visualized and critical difference diagrams have been created.
4. The study results provide researchers with new perspectives by providing the opportunity to examine the behavior of crossover operators in certain data sets.

This study is organized as follows: A review of recent literature related to crossover operators is described in section 2 and the underlying principles and mechanisms of how these operators work is described in section 3. Statistical analyses performed on the experimental data are given in Section 4. Our experimental results with findings from the recent studies are discussed in section 5. Finally, in Section 6, the in-depth experimental results are analyzed.

2. RELATED WORKS

Many crossover methods have been proposed for the solution of TSP in the literature. But here we mostly focus on the literature that has compared different crossover operators on TSP problems.

Table 1. *Related works on Crossover Operators*

References	Dataset	Crossover Operator	References	Dataset	Crossover Operator
Ref [15], 2011	1	6	Ref [26], 2017	23	2
Ref [16], 2017	12	3	Ref [27], 1998	12	2
Ref [17], 2018	10	5	Ref [28], 2002	3	2
Ref [18], 2019	18	6	Ref [29], 2007	3	1
Ref [19], 2019	15	3	Ref [30], 2010	26	3
Ref [20], 2019	15	6	Ref [31], 2024	18	7
Ref [21], 2010	10	4	Ref [32], 2009	8	3
Ref [9], 2020	18	6	Ref [33], 2015	3	3
Ref [22], 2020	12	8	Ref [34], 2022	13	5
Ref [23], 2020	28	6	Ref [35], 2015	12	4
Ref [24], 2021	2	3	Ref [36], 2017	11	6
Ref [25], 2023	6	10	Ref [37], 2020	36	4

A summary of the literature from 1998 to 2024 can be seen in Table 1, including the number of datasets and operators used. The most commonly used crossover operators are Uniform Crossover (UX) [15], Single Point Crossover (1p) [24], Two Point Crossover (2p) [24], Partially-Mapped Crossover (PMX) [9, 15, 16, 17, 18, 20, 22, 24, 31, 38, 39], Uniform Partially-Mapped Crossover (UPMX) [15], Extended Partially-Mapped Crossover (EPMX) [31, 38], Multi-Offspring PMX (MO-PMX) [18], Multi-Offspring Genetic Algorithm (MO-GA) [18], Order Crossover (OX1) [9, 15, 16, 17, 18, 20, 22, 24, 31], Order Based Crossover (OX2) [31], Non-Wrapping Ordered Crossover (NWOX) [9, 15], Cycle Crossover (CX) [9, 15, 16, 17, 20, 22, 31], Modified Cycle Crossover (CX2) [9, 16, 17, 18, 19], Improved Cycle Crossover (ICX) [17], Position Based Crossover (POS) [31], Edge Recombination Crossover (ERX) [20, 31], A Variant of the ERX (ERX6) [21], Maximal Preservative Crossover (MPX) [31], A Variant of the MPX (MPX3) [21], Greedy Sub-Tour Crossover (GSX2) [21, 38], Greedy Crossover (GX) [22, 38], Improved Greedy Crossover (IGX) [39], Very Greedy Crossover (VGX) [23], Sequential Constructive Crossover (SCX) [19, 20, 23, 25, 26], Adaptive SCX (ASCX) [22, 23], Bidirectional Circular SCX (BCSCX) [22], Comprehensive SCX (CSCX) [23], Modified Heuristic Crossover (MHX) [23], Greedy SCX(GSCX) [23], Reverse Greedy SCX (RGSCX) [23], Zoning Crossover (ZX) [25], Enhanced SCX (ESCX) [26], Triple Crossover Operator (TCX) [19], Alternate Position crossover (AEX) [20, 22], Generalized N-point crossover (GNX) [30], Shuffle Crossover (SX) [40], Uniform Order-Based Crossover (UOX) [40], Sub-tour Exchange Crossover (SEX) [40], Unnamed Heuristic Crossover (UHX) [38], Distance Preserving Operator (DPX) [38], Circular Shift Reversal Crossover (CSRX) [41], Best Order Crossover (BOX) [41].

Abdoun et al. [15] compared six different crossover operators namely UX, CX, PMX, UPMX, NWOX, OX1, over berlin52 data set for solving TSP problem. According to the experimental results, it was observed that the OX1 operator performed better than other methods.

Hussain et al. [16] proposed the CX2 inspired by CX in 2017. They tested the proposed operator by comparing it with operators PMX and OX1 using twelve symmetric and asymmetric data sets. Experimental results showed that the CX2 method had higher accuracy even for large dimensional problems.

In 2018, Hussain et al. [17] proposed the ICX inspired by CX2 and CX. Five different operators PMX, OX1, CX, CX2, and ICX were compared over the ten symmetric and asymmetric data sets. As a result of the t-test performed, statistical results showed that the ICX method offered the best solution quality and had a %95 confidence level.

Hussain et al. [18] performed a comparative study in 2019, proposing the MO-PMX method. They compared the proposed operator with operators OX1, MO-GA, CX, CX2, and PMX to analyze efficiency on both symmetric and asymmetric data sets. Statistical analysis showed that the MO-PMX method was the best in terms of t-test, standard deviation, and convergence rates.

In a study by Akter et al. [19] in 2019, they demonstrated the problem that the CX2 method could not find variation in child chromosomes in some cases. To overcome the limitations, they proposed a new crossover technique in which the process of creating the next generation depended on the minimum cost between cities. They showed that the proposed operator was the least time-consuming when compared to the TCX and SCX.

In 2019, a comparative study was conducted by Al Ommer et al. [20] using six different crossover operators: OX1, PMX, ERX, CX, AEX, and SCX. According to the experimental results, while the ERX operator had the lowest performance (90.86%) in terms of error percentage, the SCX operator showed the best performance (19.52%) among the six operators. In addition, it was observed that the PMX operator was the best method in terms of computation time.

Kuroda et al. [21] proposed the SRX and compared the proposed method with MPX3, GSX2, and ERX6. In their experiments on 10 different TSP problems, the proposed method has shown that it gives faster and better results.

In 2020, the PMX2 operator was proposed by Hussain et al. [9] inspired by the working principle of the PMX method. In PMX2, cities match outside the breakpoints, while in PMX, cities match in the inner region of the breakpoints. The proposed operator was compared with six operators: OX1, NWOX, CX, CX2, PMX and PMX2. They showed that as the number of cities in the data sets increased, the relative error increased and the method they proposed resulted in lower error than other methods in both symmetric and asymmetric datasets.

The ASCX, an improved version of SCX, was proposed by Ahmed [22] in 2020. The proposed method was compared with PMX, OX1, AEX, CX, GX, SCX, and BCSCX. The results of the testing revealed that, among the eight operators that were subjected to analysis, ASCX was identified as the most effective method, BCSCX and SCX were classified as the second-best methods, and CX was identified as the least effective method.

In 2020, distance-based crossover methods were compared and the CSCX method was proposed by Ahmed [23]. The proposed method showed the best performance among MHX, VGX, ASCX, GSCX, and RGSCX. Although the CSCX method obtained the best solution quality, it showed that it remained at the local minimum in a certain iteration.

In 2021, the effect of GA on the performance of GA was investigated using different combinations of crossover and mutation operators by Bye et al. [24]. While PMX, OX1, 1p, 2p, and UX were used as a crossover, twors mutation (TM), centre inverse mutation (CIM), reverse sequence mutation (RSM), and partial shuffle mutation (PSM) were used as a mutation. In the two data sets used, it was observed that the combination of OX1 with CIM performed better than the other combinations in terms of solution quality.

In 2023, Dou et al. [25] compared ten crossover operators in six TSP samples to conduct a systematic study of crossover operators. The experimental results indicated that the SCX and ZX operators yielded the most optimal outcomes.

As given in Table 1, while the maximum "36 data sets with 4 operators" [37] were used in terms of the number of data sets, the maximum "10 operators with 6 data sets" [25] were used in terms of operator numbers. In this study, 10 operators and 30 data sets were used to deepen this comparison. Thus, researchers are provided with a wide range of possibilities in terms of operator selection according to the size of the data set.

The aim of this study is to evaluate the effectiveness of GA crossover operators in solving TSP problems. A multitude of crossover methodologies have been proposed in the existing literature. The present study employs a comprehensive investigation utilizing critical difference diagrams with a larger number of operators and datasets, thus providing a more robust analysis.

3. MATERIALS AND METHODS

One of the most used metaheuristic algorithms, the GA is inspired by Darwin's theory of evolution. GA, proposed by John Holland in the early 1970s, is based on the principle that while good generations survive, bad generations perish [42–44]. To represent the TSP using a GA, various representations can be employed such as binary, path, adjacency, ordinal, and matrix. The path representation is often used, where each chromosome represents a sequence of cities to visit in a specific order. Let's assume that we have a chromosome structure with 8 cities as given in Figure 1:

(1 2 7 3 4 6 8 5)

Figure 1. Chromosome structure with 8 cities

In the path notation for TSP, each city must be visited exactly once and there must be no gene duplication in the chromosomes. To address this issue, selection, crossover, and mutation operators have been developed specifically for TSP using path notation [45, 46]. Several important factors can affect the performance of the GA as shown in Table 2.

Table 2. Parameters that determine the optimization performance of the GA

Parameters	Parameters
Mutation Operator	Mutation Rate
Crossover Operator	Crossover Rate
Population Pool Size	Selection Method

These factors include the population size, mutation rate, crossover rate, selection method, and crossover method. The population size is the number of candidate solutions (chromosomes) present in each generation. Larger populations may increase diversity but require more computational resources. The mutation rate determines the probability that a gene (part of a chromosome) will be subject to mutation. Mutation introduces random changes to the population, helping exploration. The crossover rate is the probability of applying crossover (recombination) on a pair of parent chromosomes to produce offspring. Crossover is the primary genetic operator for exploring promising areas of the search space. The selection method determines which chromosomes will be chosen as parents for the next generation based on their fitness. Good selection methods help maintain diversity and favor better-performing individuals.

Algorithm 1: Genetic Algorithm

Data: Population with random candidate solutions

Result: Best chromosomes

Begin;

t=0;

Generate the initial population randomly;

Calculate the objective function values of all chromosomes in the population;

while the stop condition is reached **do**

 Select individuals from the current population based on objective function values for reproduction;

 Apply the reproduction, crossover, and mutation operators;

 Find the objective function values of each new chromosome created;

 Chromosomes with poor objective function values are removed from the population;

 t=t+1;

end

The choice of crossover operator is crucial as it affects the exploration-exploitation balance in the GA. Crossover transmits genetic information from parents to offspring, influencing the population's convergence to better solutions. In standard crossover, genetic material from two parent solutions is exchanged at one or two points to create offspring solutions. However, in the context of the TSP, this approach leads to offspring that violate the requirement of visiting each city exactly once. The standard

crossover also destroys the order and connectivity of cities, which are crucial elements in TSP solutions. Researchers continuously work on developing and refining crossover operators to improve the performance of GAs for specific problem domains. The pseudo-code of GA and its working principle are given in Algorithm 1. In this section, the crossover operators developed to solve the TSP will be focused and it will be observed how they affect the performance of the GA.

In our previous research, we have implemented our operators PMX, OX1, OX2, CX, EPMX, POS and MPX on 5 symmetric, 10 asymmetric and steel production planning datasets. For a detailed description of our operators, please refer to the relevant literature [31]. While the crossover methods used in the previous study were methods that adopted a more direct approach (i.e., non-heuristic methods) without using a specific strategy or predetermined rules in the problem-solving process, methods using distance and edge matrices (ERX, SCX, IGX) were added in this study.

3.1. Genetic Edge Recombination Crossover-ERX

The Edge Recombination crossover method proposed by Whitley et al. in 1989 [47] is implemented by considering the number of edges for each city. An edge map is used to perform crossover operations and is given in Table 3 [44, 48]. The working principle of the crossover method is given in Appendix 1.

Table 3. Edge Map

City	Connected Cities
1	2, 6, 3, 5
2	1, 3, 4, 6
3	2, 4, 1
4	3, 5, 2
5	4, 6, 1
6	1, 5, 2

3.2. Sequential Constructive Crossover- SCX

Sequential Constructive Crossover proposed by Ahmad in 2010 is based on the generation of offspring chromosomes by selecting nodes that have minimal costs from the parents [30, 49]. This approach aims to preserve the good sequential structure of the parent chromosomes. The cost matrix between cities is shown in Table 4 and the steps of the SCX operator are given in Appendix 2.

Table 4. Cost Matrix Between Cities

	1	2	3	4	5	6	7	8
1	0	50	40	70	80	20	10	70
2	50	0	30	20	60	50	20	40
3	40	30	0	40	30	60	30	10
4	70	20	40	0	60	30	10	50
5	80	60	30	60	0	20	60	20
6	20	50	60	30	20	0	40	40
7	10	20	30	10	60	40	0	30
8	70	40	10	50	20	40	30	0

3.3. Improved Greedy Crossover- IGX

The IGX method, introduced by Ismkhan et al. [39] in 2012, works by considering the distance matrix between cities. The first city is selected from the parent and the distances of this selected city to the neighboring cities are compared. The city that has the closest distance is selected. Using the double-linked list on the parents, when a selected city is copied to the child, it is removed from the candidate list on linked parent lists. By using Table 4, the working principle of the IGX crossover is given in Figure 2 and Appendix 3.

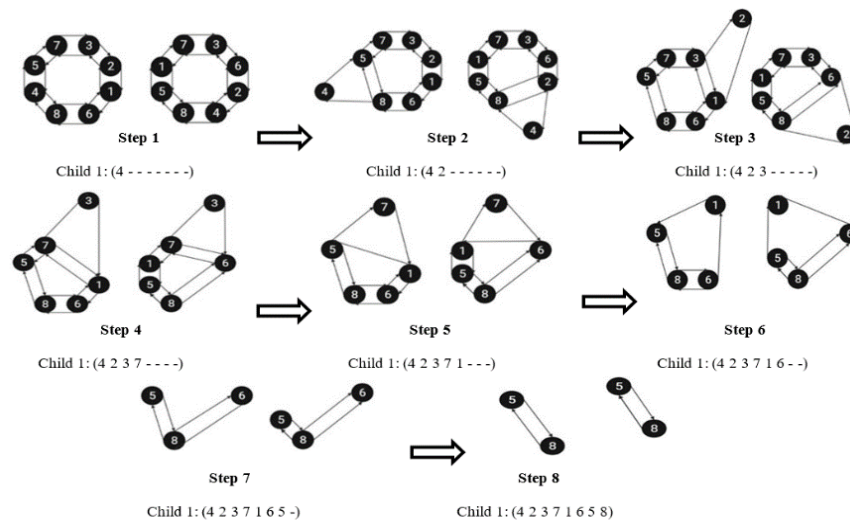


Figure 2. Child chromosomes after IGX crossover

4. RESULTS

4.1. Datasets Used in The Study

The GA has been run using 10 different crossover operators. To compare the performances of the operators, a total of 30 TSPLIB samples have been tested, 11 of which are symmetrical and 19 are asymmetrical. In a symmetrical data set, the distance from city A to city B is equal to the distance from city B to city A. In an asymmetrical data set the distance from city A to city B might not be the same as the distance from city B to city A. Asymmetric TSPs are generally more challenging than symmetric TSPs because the solution space is larger due to the directionality of the edges in the graph. The characteristics of the data sets used are given in Appendix 4.

4.2. Experiment Parameters and Computer

The commonly used and chosen parameters of the GA are shown in Table 5.

Table 5. Selected Parameters for GA

Parameters	Value
Population size	80
Crossover rate	1
Mutation rate	0.02
Mutation type	EM
Selection type	FPS
Maximum Generation	1000

The number of individuals in each generation is 80, the crossover rate is 1, the mutation rate is 0.02, and the termination condition is set to a maximum of 1000 generations. Moreover, Exchange mutation (EM) [48] has been employed as a mutation operator, while Fitness Proportional Selection (FPS) [45] has been selected as the selection operator throughout the simulation study. Each instance has been tested 30 times to evaluate the performance of the GA across different runs and obtain statistically significant results. The experiments are conducted on a computer with an Intel (R) Core i5-7200u CPU running at 2.71 GHz.

4.3. Comparison of Crossover Operator Performances

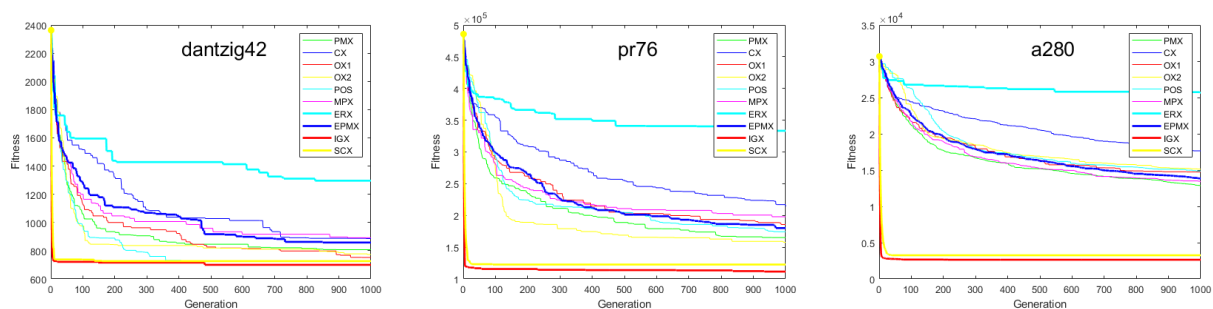
The percentage error indicated in Equation (1) can be ascertained by measuring the relative discrepancy between the calculated solution and the optimal solution. The solution value obtained by the GA or any

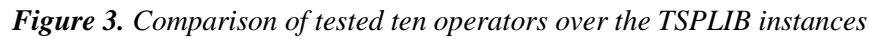
other optimization method represents the solution value for the problem being solved. The optimal solution value, on the other hand, denotes the best known or possible value for the problem, as determined by analytical or comprehensive methods [40]

$$error = \frac{FoundSolution - OptimalSolution}{OptimalSolution} * 100 \quad (1)$$

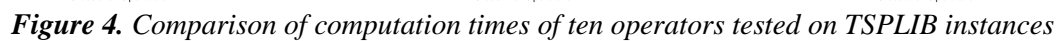
$$accuracy = 100 - error. \quad (2)$$

The percentage errors (%) obtained for each operator in the symmetric and asymmetric datasets, together with the associated best, average convergence rates, computational costs, t-test and Mann-Whitney U test of tested operators are presented in the supplementary Appendix 5 and 6, respectively. According to Appendix 5, the IGX operator resulted in the lowest percentage error rate in 8 of the 11 symmetric data sets; dantzig42 (%0.14), eil51 (%1.77), berlin52 (%0.09), pr76 (%3.15), lin105(%1.33), pr226 (%0.83), a280 (%2.38), att532 (%237.1). In Appendix 6, while the IGX operator had the lowest percentage error rate in 12 out of 19 data sets, ftv38 (%2.74), p43 (%0.12), ftv44 (%3.84), ftv47 (%9.62), ry48p (%1.69), ft53 (%8.71), ftv55 (%5.59), ftv64 (%5.87), ft70 (%4.38), ftv70 (%8.87), kro14p (%8.91), ftv170 (%16.58), SCX method had the lowest percentage error rate in 5 out of 19 data sets ftv35(%3.46), rbg323 (%21.79), rbg358 (%35.68), rbg403 (%34.96), rbg443 (%41.94). Furthermore, it was observed that the IGX operator exhibited the lowest standard deviation, and the PMX operator demonstrated the shortest computation time, in both symmetric and asymmetric data sets. From a statistical perspective, if p-values are less than critical value (0.05), it can be deduced that there is a difference between the methods. Bold t-test ($t \leq -2.00$) values indicate significantly improved performance of the IGX method, while non-bold values ($t \geq 2.00$) indicate significant degradation by IGX. Negative but non-bold t-test values represent slightly improved performance of the IGX operator relative to the average. In Appendix 5, in the gr21 instance, the OX1 method performed better than the IGX method as it had a positive t-test value (0.48). However, in all other instances, the IGX method performed best as there was no method with a positive t-test value. In Appendix 6, given the positive t-test values in the rbg323 (7.06), rbg358 (10.01) and rbg443 (0.50) data sets, it can be concluded that the SCX method outperforms the IGX method in these specific instances. Figure 3 shows the fitness function-generation graphs for small, medium and large datasets. The rationale behind the IGX and SCX methods displaying a swift decline in function values in the initial stages is that these methods possess a problem-specific configuration and seek a more expeditious solution by leveraging the intercity distance matrix. However, this rapid progress may increase the risk of getting stuck in local minima. To observe this risk, all methods were initiated with 30 different random seeds and different starting points were used. The findings of these tests demonstrated that in the majority of trials, both methods converged to the global optimum with greater rapidity and stability.





The requisite computation times for each operator are provided in Figure 4. While the PMX operator is the most efficient method in terms of computation time, the IGX operator demonstrates that it is the most time-consuming method. As children are selected using the inter-city distance matrix in the new generation production process, the computational complexity of heuristic crossover methods (IGX and SCX) is greater than that of non-heuristic methods (PMX, CX, OX1, OX2, POS, MPX, ERX, and EPMX).



Symmetric TSP data set

Dataset	Average computation time (s)
g21	~100
fri26	~100
bmg29	~100
danzig42	~100
ell51	~100
berlin52	~100
pr76	~100
lin105	~3000
pr226	~6000
a280	~6000
att532	~20000

Asymmetric TSP data set

Dataset	Average computation time (s)
br17	~100
fnc33	~100
fnc35	~100
fnc38	~100
p42	~100
fnc44	~100
fnc47	~100
ry48p	~100
fnc53	~100
fnc55	~100
fnc64	~100
fnc70	~100
kro124p	~1000
fnc170	~8000
fnc323	~8000
fnc358	~14000
fnc403	~15000
fnc443	~21000

Figure 5. Average computation times of ten operators tested on TSPLIB instances

4.5. Critical Difference Diagram

To evaluate the performances of crossover methods across various datasets, we first perform the Friedman test [50] to reject the null hypothesis, taking into account the suggestion in Demsar (2006) [51]. The accuracy of the solution is evaluated using the previously presented Equation (2), which allows for the measurement of the proximity of the found solution to the known optimum. Following the Friedman test, we performed the pairwise post-hoc analysis proposed by Benavoli et al. [52], in which the mean rank comparison was replaced by the Wilcoxon signed rank test with Holm's alpha (5%) correction [53]. Finally, we used the critical difference diagram to visualize these statistical results, starting with the rightmost model, projected onto the mean ranking axis indicated next to the models. A lower ranking indicates superior performance, on average, for a given model in comparison to the others. The horizontal lines that connect the groups of models indicate that the connected models are not significantly different from each other.

Table 6. Rankings, statistics and corresponding p-values obtained by the Friedman test are also shown. IGX achieves the best ranking on symmetric and asymmetric datasets

Methods	Symmetric	Asymmetric
PMX	4.04	4.71
CX	7.77	8.18
OX1	5.68	5.76
OX2	5.36	5.28
MPX	7.09	7.18
POS	4.31	4.94
EPMX	5.77	5.63
ERX	10.00	9.76
IGX	2.00	1.55
SCX	2.95	1.97
Friedman Statistics	65.61	125.47
Friedman p-value	1.09e-10	1.0 e-22

Table 6 shows the ranks of the methods, Friedman statistic and Friedman p-value and highlights IGX as the best performing algorithm in comparison with a rank of 2.00, 1.55 on symmetric and asymmetric datasets respectively. The p-values obtained from the Friedman test (1.09e-10, 1.0e-22) strongly suggest the existence of significant differences between the studied algorithms. According to the results obtained in Appendix 5-6, Figures 6 and 7 illustrate the statistical outcomes for both symmetric and asymmetric datasets. The small numbers next to the methods in the diagram represent the average rankings obtained from the Friedman test.

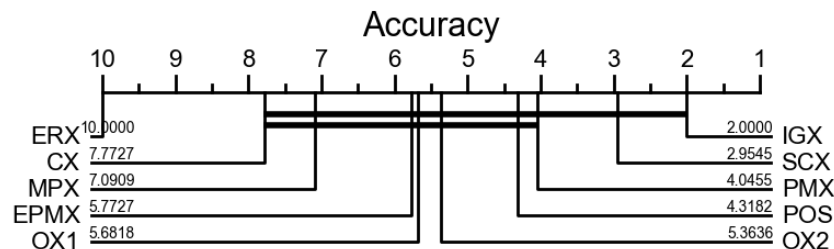


Figure 6. Critical Difference Diagrams for Symmetric Data Sets

In symmetric structured datasets, no statistically significant difference is observed between CX-EPMX-OX1-OX2-POS-PMX-SCX-IGX because they are interconnected. Similarly, CX-MPX-EPMX-OX1-OX2-POS-PMX are interconnected and exhibited statistically similar performance. Therefore, all methods except ERX do not show statistically significant differences. As a consequence of having the lowest ranking degrees, IGX and SCX operators demonstrate the most optimal performance, whereas ERX exhibits the least performance.

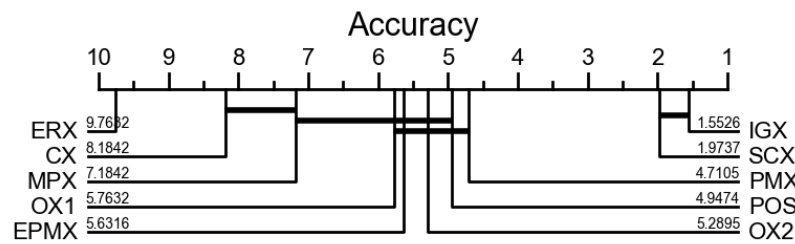


Figure 7. Critical Difference Diagrams for Asymmetric Data Sets

Conversely, given the interconnectivity of the lines in the asymmetric datasets, IGX and SCX exhibit statistically indistinguishable performance. Given that IGX has a slightly better degree, it can be concluded that it is the superior option. Furthermore, as the CX-MPX, MPX-OX1-EPMX-OX2-POS, and PMX-OX1-EPMX-OX2-POS groups are interconnected, they are not statistically distinct from one another. It is notable that ERX is not connected to any other method and exhibits the lowest performance

5. DISCUSSION

The results we obtained on both symmetric and asymmetric TSP data sets have been compared with the results of other studies and summarized in Table 7. The percentage error (%) values in the table measure how close the algorithm's solution is to the optimal solution, and parameter values used in other studies are also given. As a result of comparing our experiments with the literature, IGX and SCX crossover methods showed the best performance using the FPS selection and EM mutation method in terms of percentage error.

Table 7. Comparison of the results obtained in the literature with our experimental results

Literature	Data sets	Parameters	Parameters Value	Operators	Error (%)	Our experiments	Error (%)
Ahmed et al. (2010) [30]	kro124p	Crossover rate	1	ERX	30.96	IGX	8.91
		Mutation rate	0.01	GNX	25.72		
		Population size	200	SCX	4.24		
		Generation	10.000				
Ahmed et al. (2010) [30]	lin105	Crossover rate	1	ERX	30.05	IGX	1.33
		Mutation rate	0.01	GNX	48.13		
		Population size	200	SCX	2.52		
		Generation	10.000				
Hussain et al. (2017) [16]	fri26	Crossover rate	0.8	PMX	12.70	IGX	0.00
		Mutation rate	0.1	OX1	17.29		
		Population size	30	CX2	12.16		
		Generation	10				
Hussain et al. (2017) [16]	ftv170	Crossover rate	0.8	PMX	384.42	IGX	16.58
		Mutation rate	0.1	OX1	451.72		
		Population size	30	CX2	133.06		
		Generation	10				
Khan (2015) [40]	ftv170	Crossover rate	0.8	TPX	77.38	IGX	16.58
		Mutation rate	0.01	PMX	127.38		
		Population size	100	CX	128.46		
		Generation	50000	SX	206.92		
				ERX	249.17		
				UOX	102.06		
				SEX	137.29		
				SCX	29.72		
	eil51	Crossover rate	-	PMX	1.88	IGX	1.77

Ismkhan et al. (2013) [38]		Mutation rate	-	EPMX	1.64			
		Population size	50	GSX-2	0.47			
		Generation	500	GX	0.0			
				UHX	0.0			
				DPX	0.7			
Uray et al. (2023) [41]	eil51	Crossover rate	-	CSRX	3.75	IGX	1.77	
		Mutation rate	0.05	BOX	7.98			
		Population size	100					
		Generation	1000					

6. CONCLUSION

In this study, the GA algorithm, one of the meta-heuristic methods, was employed in the TSP problem, and ten crossover operators affecting its performance were compared. Considering previous studies, 10 different crossover operators in the literature have been analyzed on 11 symmetric and 19 asymmetric TSPLIB data sets. The FPS selection method and the EM mutation method were selected for our experiments. Crossover operators were evaluated in terms of percentage error, computational time and statistical differences. For the first time in this study, a critical difference diagram was used to compare operators. The critical difference diagram provided a visual understanding of the relationships between different groups and showed which groups interacted. While the IGX method performed better in small-medium-large size symmetric data sets, the SCX operator performed better in large asymmetric data sets. From a statistical standpoint, the IGX method was determined to be the most effective overall. In terms of computation time, PMX has been the least time-consuming method. In future studies, the effect of genetic operators on the balance between exploration and exploitation can be examined.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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APPENDIX

Appendix 1. Example of ERX method

Steps of crossover operation	Parent 1: (1 2 3 4 5 6), Parent 2: (2 4 3 1 5 6)
Step 1: The first cities are selected from parent 1. The first cities are 1 and 2 and both have 4 connected edges according to Table 3. Suppose that City 2 is chosen randomly.	Child 1: (2 - - - -)
Step 2: City 2 determines the next candidate's cities which are 1, 3, 4, and 6. According to Table 3, City 3, City 4, and City 6 have two connected edges, while for City 1 there are three connected edges. So, let's assume that city 3 is chosen randomly among cities 3, 4, and 6.	Child 1: (2 3 - - -)
Step 3: Next candidate cities are 1 and 4. City 4 has one connected edge (5), while City 1 has two connected edges (5, 6). Therefore, City 4 is selected	Child 1: (2 3 4 - -)
Step 4: For City 4, there is only one candidate city in the connected edge list (5). City 5 is selected.	Child 1: (2 3 4 5 -)
Step 5: City 5 includes City 1 and City 6. Since City 1 (6) and City 6 (1), each have a connected city, city 1 is randomly selected.	Child 1: (2 3 4 5 1 -)
Step 6: Only city 6 remains in the connected edge list for city 1. Finally, 6 is selected.	Child 1: (2 3 4 5 1 6)

Appendix 2. Example of SCX method

Steps of crossover operation	Parent 1: (4 5 7 3 2 1 6 8), Parent 2: (5 1 7 3 6 2 4 8)
Step 1: The first node from the first parent is selected and copied to the child.	Child 1: (4 - - - - -)
Step 2: For node 4, the legitimate node is 5 in P1 and 8 in P2 ($C_{48}=50$ and $C_{45}=60$). Due to the cost being $C_{45}>C_{48}$, node 8 is selected.	Child 1: (4 8 - - - -)
Step 3: The legitimate node, after 8, is none in both P1 and P2. For P1 and P2, the first legitimate node is considered as 2 from (2, 3, 4, 5, 6, 7, 8) since nodes 4, and 8 are available for the child. Node 2 is accepted.	Child 1: (4 8 2 - - - -)
Step 4: The legitimate node, after 2, is 1 in P1, and 4 in P2. Although the cost is $C_{21}=50 > C_{24}=30$, node 1 is accepted since node 4 is available for the child.	Child 1: (4 8 2 1 - - -)
Step 5: The legitimate node, after 1, is 6 in P1, and 7 in P2. Due to the cost being $C_{16}=20 > C_{17}=10$, node 7 is accepted.	Child 1: (4 8 2 1 7 - -)
Step 6: The legitimate node, after 7, is node 3 both P1 and P2. Node 3 is copied to the child.	Child 1: (4 8 2 1 7 3 -)
Step 7: The legitimate node, after 3, is 2 in P1, and 6 in P2. Although the cost is $C_{32}=30 > C_{36}=60$, node 6 is accepted since node 2 is available for the child.	Child 1: (4 8 2 1 7 3 6 -)
Step 7: The legitimate node, after 6, is 8 in P1, and 2 in P2.	Child 1: (4 8 2 1 7 3 6 5)

Although the cost is $C_{68}=40 > C_{62}=50$, none of them is accepted since nodes 2 and 8 are available for the child. For P1 and P2, the legitimate city is considered as city 5 from (2, 3, 4, 5, 6, 7, 8) since nodes 2, 3, 4, 6, 7, and 8 are available for the child.	
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Appendix 3. Example of IGX method

Steps of crossover operation	Parent 1: (4 5 7 3 2 1 6 8), Parent 2: (5 1 7 3 6 2 4 8)
Step 1: By choosing City 4 from first Parent 1 arbitrarily, it is copied to the child.	Child 1: (4 - - - - -)
Step 2: For City 4 in Parent 1, the neighboring Cities are 5 and 8, in Parent 2, neighboring Cities are 8 and 2. According to Table 7, 20 between City 4 and City 2 is the shortest distance. Therefore, City 2 is chosen.	Child 1: (4 2 - - - - -)
Step 3: For City 2 in Parent 1, the neighboring Cities are 3 and 1, in Parent 2, neighboring Cities are 8 and 6. 30 between City 2 and City 3 is the shortest distance. Therefore, City 3 is chosen.	Child 1: (4 2 3 - - - -)
Step 4: For City 3 in Parent 1, the neighboring Cities are 7 and 1, in Parent 2, neighboring Cities are 7 and 6. 30 between City 3 and City 7 is the shortest distance. Therefore, City 7 is chosen.	Child 1: (4 2 3 7 - - -)
Step 5: For City 7 in Parent 1, the neighboring Cities are 5 and 1, in Parent 2, neighboring Cities are 7 and 1. 10 between City 7 and City 1 is the shortest distance. Therefore, City 1 is chosen.	Child 1: (4 2 3 7 1 - -)
Step 6: For City 1 in Parent 1, the neighboring Cities are 5 and 6, in Parent 2, neighboring Cities are 5 and 6. 20 between City 1 and City 6 is the shortest distance. Therefore, City 6 is chosen.	Child 1: (4 2 3 7 1 6 -)
Step 7: For City 6 in Parent 1, the neighboring Cities are 5 and 8, in Parent 2, neighboring Cities are 5 and 8. 20 between City 6 and City 5 is the shortest distance. Therefore, City 5 is chosen.	Child 1: (4 2 3 7 1 6 5)
Step 8: For City 5 in Parents 1 and 2, the only neighboring City is 8. Therefore, City 8 is chosen.	Child 1: (4 2 3 7 1 6 5 8)

Appendix 4. TSPLIB data sets

Name	Cities	Type	Optimal Tour	Problem Type	Problem Size
gr21	21	Matrix	2707	Symmetric	Small
fri26	26	Matrix	937	Symmetric	Small
bayg29	29	Geo	1610	Symmetric	Small
dantzig42	42	Matrix	699	Symmetric	Small
eil51	51	Euclidean distance	426	Symmetric	Small
berlin52	52	Euclidean distance	7542	Symmetric	Small
pr76	76	Euclidean distance	108159	Symmetric	Medium
lin105	105	Euclidean distance	14379	Symmetric	Medium
pr226	226	Euclidean distance	80369	Symmetric	Large
a280	280	Euclidean distance	2579	Symmetric	Large
att532	532	Att	27686	Symmetric	Large
br17	17	Matrix	39	Asymmetric	Small
ftv33	34	Matrix	1286	Asymmetric	Small
ftv35	36	Matrix	1473	Asymmetric	Small

ftv38	39	Matrix	1530	Asymmetric	Small
p43	43	Matrix	5620	Asymmetric	Small
ftv44	45	Matrix	1613	Asymmetric	Small
ftv47	48	Matrix	1776	Asymmetric	Small
ry48p	48	Matrix	14422	Asymmetric	Small
ft53	53	Matrix	6905	Asymmetric	Small
ftv55	56	Matrix	1608	Asymmetric	Small
ftv64	65	Matrix	1839	Asymmetric	Small
ft70	70	Matrix	38673	Asymmetric	Medium
ftv70	71	Matrix	1950	Asymmetric	Medium
kro124p	100	Matrix	36230	Asymmetric	Medium
ftv170	171	Matrix	2755	Asymmetric	Medium
rbg323	323	Matrix	1326	Asymmetric	Large
rbg358	358	Matrix	1163	Asymmetric	Large
rbg403	403	Matrix	2465	Asymmetric	Large
rbg443	443	Matrix	2720	Asymmetric	Large

Appendix 5. Best, average convergence rates, computational costs, t-test and Mann-Whitney test of operators over the symmetric TSPLIB

Instance	Results	PMX	CX	OX1	OX2	POS	MPX	ERX	EPMX	IGX	SCX
gr21 (2707)	Best	2707.00 (%0.00)	2707.00 (%0.00)	2707.00 (%0.00)	2707.00 (%0.00)	2707.00 (%0.00)	2767.00 (%2.21)	2853.00 (%5.39)	2707.00 (%0.00)	2707.00 (%0.00)	2707.00 (%0.00)
	Average	2961.63 (%9.40)	3023.77 (%11.70)	2716.70 (%0.35)	2806.77 (%3.68)	2890.00 (%6.76)	2941.10 (%8.64)	3187.43 (%17.74)	2968.00 (%9.64)	2721.90 (%0.55)	2837.30 (%4.81)
	SD	219.98	225.81	53.12	145.68	191.91	105.35	172.95	188.25	25.79	90.66
	t-test statistic (p value)	-5.92 (1.78e-07)	-7.27 (1.01-09)	0.48 (0.63)	-3.14 (0.002)	-4.75 (1.35e-05)	-11.06 (6.29e-16)	-14.5 (4.67e-21)	-7.09 (2.03e-09)	-	-6.70 (9.15e-09)
	Mann-Whitney statistic (p value)	118.5 (2.77 e-07)	50.0 (1.16 e-09)	565.5 (0.008)	351.0 (0.086)	234.0 (0.0004)	3.5 (1.52e-11)	0.0 (1.06e-11)	137.0 (1.06e-06)	-	91.5 (4.30e-08)
	Time	11.531	14.541	13.272	26.575	35.636	23.389	31.030	14.136	135.098	100.890
fri26 (937)	Best	937.00 (%0.00)	937.00 (%0.00)	937.00 (%0.00)	937.00 (%0.00)	937.00 (%0.00)	1019.00 (%8.75)	1168.00 (%24.65)	974.00 (%3.94)	937.00 (%0.00)	937.00 (%0.00)
	Average	1030.03 (%9.92)	1071.13 (%14.31)	966.70 (%3.16)	1005.33 (%7.29)	1023.73 (%9.25)	1121.57 (%19.69)	1263.17 (%34.80)	1074.03 (%14.62)	937.00 (%0.00)	956.83 (%2.11)
	SD	52.26	61.98	22.30	43.30	61.00	56.47	58.19	67.31	0.00	15.26
	t-test statistic (p value)	-9.74 (7.89e-14)	-11.85 (3.94 e-17)	-7.29 (9.45e-10)	-8.64 (5.18e-12)	-7.78 (1.39e-10)	-17.89 (2.70e-25)	-30.70 (1.45e-37)	-11.15 (4.69e-16)	-	-7.11 (1.85e-09)
	Mann-Whitney statistic (p value)	30.0 (1.65e-11)	15.0 (4.56 e-12)	45.0 (5.27e-11)	30.0 (1.64e-11)	15.0 (4.56e-12)	0.0 (1.21e-12)	0.0 (1.21e-12)	0.0 (1.21e-12)	-	90.0 (1.92e-09)
	Time	14.200	18.920	15.573	36.505	49.247	28.434	37.510	15.823	207.546	126.102
bayg29 (1610)	Best	1610 (%0.00)	1639 (%1.80)	1610 (%0.00)	1620 (%0.62)	1610 (%0.00)	1872 (%16.27)	1978 (%22.85)	1618 (%0.49)	1620 (%0.62)	1654 (%2.73)
	Average	1815.26 (%12.74)	1886.40 (%17.16)	1656.70 (%2.90)	1760.80 (%9.36)	1791.63 (%11.28)	2021.96 (%25.58)	2298.00 (%42.73)	1791.16 (%11.25)	1642.43 (%2.01)	1765.50 (%9.65)
	SD	133.58	125.79	45.49	95.64	90.40	87.97	121.49	92.05	12.79	63.59
	t-test statistic (p value)	-7.05 (2.37 e-09)	-10.56 (3.84 e-15)	-1.65 (0.103)	-6.71 (8.69e-09)	-8.94 (1.6e-12)	-23.38 (3.26e-31)	-29.39 (1.57e-36)	-8.76 (3.24e-12)	-	-10.39 (7.32 e-15)
	Mann-Whitney statistic (p value)	79.5 (4.41e-08)	16.0 (1.43 e-10)	400.0 (0.463)	126.5 (1.75e-06)	57.0 (6.37e-09)	0.0 (2.94e-11)	0.0 (2.94e-11)	30.0 (5.45e-10)	-	11.0 (8.76e-11)
	Time	56.331	70.798	67.648	155.822	207.190	117.403	157.280	66.933	865.269	590.873
dantzig42 (699)	Best	807 (%15.45)	886 (%26.75)	752 (%7.58)	773 (%10.58)	728 (%4.14)	892 (%27.61)	1296 (%85.40)	858 (%22.74)	700 (%0.14)	726 (%3.86)

	Average	973.53 (%39.27)	1064.73 (%52.32)	897.93 (%28.45)	886.80 (%26.86)	915.03 (%30.90)	1094.47 (%56.57)	1506.47 (%115.51)	998.47 (%42.84)	730.30 (%4.47)	793.70 (%13.54)
	SD	88.91	83.07	62.98	70.38	83.58	81.28	88.70	80.46	20.22	47.58
	t-test statistic (p value)	-14.61 (4.26 e-21)	-21.42 (3.10 e-29)	-13.87 (4.37 e-20)	-11.70 (6.60 e-17)	-11.76 (5.34 e-17)	-23.8 (1.24 e-31)	-46.72 (9.93 e-48)	-17.70 (4.66 e-25)		-6.71 (8.77 e-09)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	7.0 (6.03e-11)	0.0 (3.01e- 11)	15.0 (1.32e- 10)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	0.0 (3.00e- 11)		90.0 (1.06e-07)
	Time	58.340	93.302	75.129	237.208	328.956	151.372	205.517	80.104	1990.976	1208.474
eil51 (426)	Best	504.38 (%18.40)	590.81 (%38.68)	564.09 (%32.41)	489.73 (%14.96)	491.09 (%15.27)	628.83 (%47.61)	856.77 (%101.12)	556.27 (%30.58)	433.54 (%1.77)	457.33 (%7.35)
	Average	582.70 (%36.78)	665.47 (%56.21)	615.93 (%44.58)	577.94 (%35.66)	580.27 (%36.21)	699.55 (%64.21)	920.96 (%116.18)	603.43 (%41.65)	443.15 (%4.02)	484.24 (%13.64)
	SD	35.07	36.05	33.50	49.29	58.45	49.25	38.07	28.52	6.10	16.11
	t-test statistic (p value)	-21.46 (2.78 e-29)	-33.29 (1.67 e-39)	-27.78 (3.29 e-35)	-14.86 (1.93 e-21)	-12.77 (1.65 e-18)	-28.29 (1.23 e-35)	-67.86 (5.83 e-57)	-30.09 (4.36 e-37)		-13.05 (6.50 e-19)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)		1.0 (3.33e- 11)
berlin52 (7542)	Time	15.713	25.938	19.235	76.208	115.601	42.714	57.035	25.149	705.844	405.421
	Best	9413.55 (%24.81)	10771.57 (%42.82)	9777.88 (%29.64)	9465.73 (%25.50)	8608.91 (%14.14)	10399.26 (%37.88)	14283.66 (%89.38)	9347.68 (%23.94)	7548.99 (%0.09)	8029.10 (%6.45)
	Average	10471.53 (%38.84)	11775.55 (%56.13)	10641.30 (%41.07)	10594.10 (%40.48)	10320.57 (%36.84)	12024.50 (%59.43)	15577.62 (%106.54)	10778.50 (%42.91)	7548.99 (%0.09)	8458.94 (%12.15)
	SD	536.06	713.59	476.69	775.65	873.27	759.40	662.95	693.26	4.62E-12	291.29
	t-test statistic (p value)	-29.86 (6.62 e-37)	-32.44 (7.07 e-39)	-35.53 (4.63 e-41)	-21.50 (2.52 e-29)	-17.38 (1.13 e-24)	-32.27 (9.31 e-39)	-66.33 (2.16 e-56)	-25.51 (3.21 e-33)		-17.10 (2.45 e-24)
pr76 (108159)	Mann-Whitney statistic (p value)	0.0 (1.21e-12)	0.0 (1.21e-12)	0.0 (1.21e-12)	0.0 (1.21e-12)	0.0 (1.21e-12)	0.0 (1.21e-12)	0.0 (1.21e-12)	0.0 (1.21e- 12)		0.0 (1.21e-12)
	Time	16.012	28.612	21.021	80.114	123.002	45.789	64.917	32.367	746.187	419.372
	Best	165188.2 (%52.72)	214336.16 (%98.16)	185704.96 (%71.69)	157741.91 (%45.84)	174341.38 (%61.18)	197908.77 (%82.97)	333288.26 (%208.14)	180066.39 (%66.48)	111566.07 (%3.15)	122646.6 (%13.39)
	Average	196375.5 (%81.56)	240958.04 (%122.78)	217855.94 (%101.42)	209293.28 (%93.50)	210191.14 (%94.33)	221261.95 (%104.57)	353820.21 (%227.12)	207115.54 (%91.49)	115581.30 (%6.86)	132545.9 (%22.54)
	SD	10680.86	13872.41	14403.20	23885.01	16667.10	13312.46	11558.39	11496.64	1639.85	4992.13
	t-test statistic (p value)	-40.95 (1.67 e-44)	-49.16 (5.63 e-49)	-38.64 (4.32 e-43)	-21.43 (2.99 e-29)	-30.94 (9.49 e-38)	-43.15 (8.83 e-46)	-111.77 (1.97 e-69)	-43.17 (8.64 e-46)		-17.68 (4.91 e-25)

	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)		0.0 (3.01e-11)
	Time	19.644	36.619	23.956	142.736	231.373	70.657	93.808	39.840	1659.563	898.876
lin105 (14379)	Best	32172.41 (%123.7)	42656.67 (%196.65)	37137.34 (%158.27)	33117.13 (%130.31)	31153.95 (%116.66)	35198.41 (%144.79)	72242.21 (%402.41)	36862.97 (%156.36)	14570.89 (%1.33)	16568.88 (%15.22)
	Average	37707.64 (%162.2)	48781.53 (%239.25)	42849.72 (%198.00)	41681.78 (%189.87)	39053.03 (%171.59)	41739.12 (%190.27)	77608.14 (%439.73)	40132.67 (%179.10)	14701.95 (%2.24)	18279.64 (%27.12)
	SD	3059.82	3015.85	2346.14	4209.87	4576.94	3092.39	2637.50	1954.05	62.24	662.28
	t-test statistic (p value)	-41.17 (1.24 e-44)	-61.88 (1.15 e-54)	-65.68 (3.78 e-56)	-35.09 (9.15 e-41)	-29.13 (2.51 e-36)	-47.87 (2.51 e-48)	-130.59 (2.46 e-73)	-71.24 (3.60 e-58)		-29.45 (1.38 e-36)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)		0.0 (3.01e-11)
	Time	97.205	230.008	136.884	1022.902	1724.052	337.152	444.031	257.318	17780.69	7097.201
pr226 (80369)	Best	538313.6 (%569.8)	766256.38 (%853.42)	625261.28 (%677.98)	603901.17 (%651.41)	559486.81 (%596.14)	554853.93 (%590.38)	1218622.4 (%1416.2)	590992.04 (%635.34)	81040.53 (%0.83)	95110.62 (%18.34)
	Average	597236.8 (%643.1)	833778.30 (%937.43)	663799.40 (%725.93)	757261.29 (%842.23)	755923.06 (%840.56)	602929.84 (%650.20)	1251152.2 (%1456.7)	645038.91 (%702.59)	82239.78 (%2.32)	102535.4 (%27.58)
	SD	33433.76	30710.66	22259.53	60249.541	72784.75	32043.63	16947.88	27895.69	1037.08	4423.61
	t-test statistic (p value)	-84.32 (2.24e-62)	-133.96 (5.66 e-74)	-142.94 (1.32e-75)	-61.35 (1.87e-54)	-50.69 (9.89e-50)	-88.95 (1.03e-63)	-377.06 (5.25e-100)	-110.42 (3.98e-69)		-24.46 (2.98e-32)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)		0.0 (3.01e-11)
	Time	164.891	575.15	250.616	4250.25	6771.410	644.188	845.768	875.461	47469.14	19633.17
a280 (2579)	Best	12853.00 (%398.3)	17648.45 (%584.31)	14749.21 (%471.89)	15131.71 (%486.72)	14859.38 (%476.16)	13402.69 (%419.68)	25651.84 (%894.64)	13847.99 (%436.95)	2640.41 (%2.38)	3256.67 (%26.27)
	Average	13765.74 (%433.7)	18772.39 (%627.89)	15363.21 (%495.70)	16818.76 (%552.14)	17248.26 (%568.79)	14207.77 (%450.90)	26241.04 (%917.48)	14662.50 (%468.53)	2695.62 (%4.52)	3467.66 (%34.45)
	SD	507.99	595.18	379.50	1037.53	1089.11	440.12	382.19	482.33	28.13	116.98
	t-test statistic (p value)	-119.17 (4.87e-71)	-147.7 (1.93e-76)	-182.32 (1.01e-81)	-74.53 (2.71e-59)	-73.16 (7.86e-59)	-142.97 (1.31-75)	-336.51 (3.84e-97)	-135.66 (2.73e-74)		-35.14 (8.50e-41)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)		0.0 (3.01e-11)
	Time	61.452	305.613	88.050	963.903	1800.935	119.179	120.700	179.284	37800.04	21996.99
att532 (27686)	Best	698926.7 (%2424)	971839.48 (%3410.2)	705370.56 (%2447.7)	855759.33 (%2990.9)	832197.39 (%2905.8)	681591.62 (%2361.8)	1292158.7 (%4567.1)	775546.43 (%2701.2)	93340.05 (%237.1)	115664.7 (%317.7)

Average	737043.4 (%2562)	1025301.2 (%3603.32)	738398.55 (%2567.04)	938418.32 (%3289.50)	938146.67 (%3288.52)	718559.92 (%2495.39)	1319978.22 (%4667.67)	805220.76 (%2808.40)	95495.14 (%244.9)	123104.69 (%344.64)
SD	18339.61	23244.98	19154.31	30420.69	45977.59	25994.32	14710.03	20519.42	846.56	3263.10
t-test statistic (p value)	-191.39 (6.10e-83)	-218.94 (2.53e-86)	-183.66 (6.66e-82)	-151.70 (4.24e-77)	-100.36 (9.87e-67)	-131.21 (1.87e-73)	-455.17 (9.54e-105)	-189.28 (1.16e-82)	-	-44.85 (9.97e-47)
Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	0.0 (3.01e- 11)	-	0.0 (3.01e-11)
Time	121.706	557.048	187.613	7432.941	11243.477	358.552	512.807	1011.013	96019.058	63765.646

Appendix 6. Best, average convergence rates, computational costs, t-test and Mann-Whitney test of tested ten operators over the asymmetric TSPLIB

Instance	Results	PMX	CX	OX1	OX2	POS	MPX	ERX	EPMX	IGX	SCX
br17 (39)	Best	39 (%0.00)	39 (%0.00)	39 (%0.00)	39 (%0.00)	39 (%0.00)	39 (%0.00)	39 (%0.00)	39 (%0.00)	39 (%0.00)	39 (%0.00)
	Average	39.60 (%1.53)	44.00 (%12.82)	39.00 (%0.00)	39.53 (%1.36)	39.63 (%1.62)	39.00 (%0.00)	39.60 (%1.53)	41.50 (%6.41)	39.00 (%0.00)	39.03 (%0.08)
	SD	1.45	6.66	0.00	1.33	1.42	0.00	0.77	4.76	0.00	0.18
	t-test statistic (p value)	-2.26 (0.02)	-4.10 (0.0001)	nan (nan)	-2.19 (0.032)	-2.43 (0.018)	nan (nan)	-4.26 (7.40e- 05)	-2.87 (0.005)	-	-0.99 (0.32)
	Mann-Whitney statistic (p value)	345.0 (0.005)	165.0 (3.41e-07)	450.0 (1.0)	360.0 (0.011)	330.0 (0.002)	450.0 (1.0)	255.0 (6.30e- 05)	270.0 (0.0001)	-	435.0 (0.33)
	Time	10.840	13.284	13.193	23.134	28.793	21.441	26.762	11.903	148.937	67.543
ftv33 (1286)	Best	1415 (%10.0)	1431 (%11.27)	1286 (%0.00)	1415 (%10.03)	1459 (%13.45)	1532 (%19.12)	1982 (%54.12)	1494 (%16.17)	1316 (%2.33)	1406 (%9.33)
	Average	1653.93 (%28.6)	1749.03 (%36.00)	1519.37 (%18.14)	1614.97 (%25.58)	1644.50 (%27.87)	1837.43 (%42.87)	2194.67 (%70.65)	1700.27 (%32.21)	1353.27 (%5.23)	1470.23 (%14.32)
	SD	106.05	128.01	124.06	117.67	115.95	94.72	103.12	103.00	16.24	42.01
	t-test statistic (p value)	-15.34 (4.36e-22)	-16.79 (5.99e-24)	-7.27 (1.02e-09)	-12.06 (1.87e-17)	-13.62 (1.00e-19)	-27.59 (4.80e-35)	-44.14 (2.462e-46)	-18.22 (1.10e- 25)	-	-14.22 (1.45e-20)
	Mann-Whitney statistic (p value)	0.0 (1.76e-11)	0.0 (1.76e- 11)	95.0 (1.13e- 07)	0.0 (1.76e-11)	0.0 (1.75e-11)	0.0 (1.76e-11)	0.0 (1.76e- 11)	0.0 (1.76e- 11)	-	0.0 (1.75e-11)
	Time	14.844	21.532	17.971	46.527	63.921	32.502	42.151	18.617	314.465	190.318
ftv35 (1473)	Best	1616 (%9.70)	1768 (%20.02)	1534 (%4.14)	1647 (%11.81)	1542 (%4.68)	1796 (%21.92)	2315 (%57.16)	1563 (%6.10)	1532 (%4.00)	1524 (%3.46)
	Average	1845.10 (%25.2)	1973.87 (%34.00)	1741.37 (%18.21)	1847.30 (%25.41)	1857.13 (%26.07)	2108.03 (%43.11)	2542.03 (%72.57)	1877.70 (%27.47)	1550.03 (%5.22)	1632.20 (%10.80)
	SD	113.78	141.94	121.73	121.92	152.44	136.29	125.94	139.69	7.47	69.78

	t-test statistic (p value)	-14.17 (1.70e-20)	-16.33 (2.31e-23)	-8.59 (6.26e-12)	-13.32 (2.62e-19)	-11.02 (7.48e-16)	-22.39 (3.14e-30)	-43.06 (9.93e-46)	-12.82 (1.39e-18)		-6.41 (2.82e-08)
	Mann-Whitney statistic (p value)	0.0 (2.57e-11)	0.0 (2.57e-11)	41.0 (1.34e-09)	0.0 (2.57e-11)	27.0 (3.64e-10)	0.0 (2.57e-11)	0.0 (2.57e-11)	0.0 (2.57e-11)		103.0 (2.72e-07)
	Time	56.070	85.506	71.843	199.885	275.274	133.614	180.315	75.271	1376.630	960.381
ftv38 (1530)	Best	1753 (%14.5)	1896 (%23.92)	1696 (%10.84)	1802 (%17.77)	1648 (%7.71)	2014 (%31.63)	2581 (%68.69)	1740 (%13.72)	1572 (%2.74)	1621 (%5.94)
	Average	1965.13 (%28.4)	2126.50 (%38.98)	1942.80 (%26.98)	1992.80 (%30.24)	1958.93 (%28.03)	2267.40 (%48.19)	2777.17 (%81.51)	2008.33 (%31.26)	1606.53 (%5.00)	1689.07 (%10.39)
	SD	136.98	144.89	134.05	144.67	154.33	124.67	111.21	117.69	7.01	39.20
	t-test statistic (p value)	-14.31 (1.06e-20)	-19.63 (2.68e-27)	-13.72 (7.31e-20)	-14.60 (4.32e-21)	-12.49 (4.33e-18)	-28.98 (3.32e-36)	-57.53 (7.34e-53)	-18.66 (3.38e-26)		-11.35 (2.29e-16)
	Mann-Whitney statistic (p value)	0.0 (9.36e-12)	0.0 (9.36e-12)	0.0 (9.36e-12)	0.0 (9.35e-12)	0.0 (9.35e-12)	0.0 (9.35e-12)	0.0 (9.34e-12)	0.0 (9.34e-12)		0.0 (9.34e-12)
	Time	13.307	21.680	19.723	54.259	78.213	34.477	46.550	20.591	392.011	253.967
p43 (5620)	Best	5658 (%0.67)	5702 (%1.45)	5663 (%0.76)	5657 (%0.65)	5658 (%0.67)	5679 (%1.04)	5859 (%4.25)	5657 (%0.65)	5627 (%0.12)	5646 (%0.46)
	Average	5875.00 (%4.53)	8062.07 (%43.45)	5714.20 (%1.67)	6065.27 (%7.92)	5704.40 (%1.50)	5745.70 (%2.23)	5927.73 (%5.47)	7482.77 (%33.14)	5636.63 (%0.29)	5656.80 (%0.65)
	SD	970.64	2695.58	20.85	1357.70	25.44	42.56	34.97	2556.96	3.66	5.22
	t-test statistic (p value)	-1.34 (0.183)	-4.92 (7.27e-06)	-20.06 (9.01e-28)	-1.72 (0.089)	-14.44 (7.29e-21)	-13.98 (3.12e-20)	-45.33 (5.50e-47)	-3.95 (0.0002)	-	-17.31 (1.36e-24)
	Mann-Whitney statistic (p value)	0.0 (2.82e-11)	0.0 (2.82e-11)	0.0 (2.82e-11)	0.0 (2.82e-11)	0.0 (2.82e-11)	0.0 (2.81e-11)	0.0 (2.81e-11)	0.0 (2.82e-11)		0.0 (2.66e-11)
	Time	64.001	102.361	83.124	261.871	380.430	162.367	216.043	92.180	1513.001	1213.568
ftv44 (1613)	Best	2026 (%25.6)	2219 (%37.56)	1945 (%20.58)	1847 (%14.50)	1955 (%21.20)	2354 (%45.93)	3069 (%90.26)	1964 (%21.76)	1675 (%3.84)	1784 (%10.60)
	Average	2279.80 (%57.2)	2516.90 (%72.78)	2299.07 (%66.64)	2273.20 (%61.87)	2287.53 (%71.97)	2626.77 (%76.93)	3449.27 (%127.9)	2332.17 (%61.99)	1710.60 (%8.92)	1855.77 (%20.58)
	SD	120.50	125.95	163.88	213.75	211.12	119.37	174.58	162.46	30.96	47.85
	t-test statistic (p value)	-25.05 (8.41e-33)	-34.04 (4.91e-40)	-19.32 (5.94e-27)	-14.26 (1.26e-20)	-14.80 (2.29e-21)	-40.69 (2.40e-44)	-53.71 (3.69e-51)	-20.58 (2.42e-28)		-13.94 (3.48e-20)
	Mann-Whitney statistic (p value)	0.0 (2.58e-11)	0.0 (2.58e-11)	0.0 (2.58e-11)	0.0 (2.58e-11)	0.0 (2.58e-11)	0.0 (2.58e-11)	0.0 (2.58e-11)	0.0 (2.58e-11)		0.0 (2.58e-11)
	Time	61.602	99.325	78.721	268.410	377.632	155.370	210.030	89.160	2138.027	1417.122

ftv47 (1776)	Best	2230 (%25.5)	2514 (%41.55)	2309 (%30.01)	2137 (%20.32)	2028 (%14.18)	2612 (%47.07)	3722 (%109.5)	2359 (%32.82)	1947 (%9.62)	1958 (%10.24)
	Average	2516.27 (%41.6)	2844.20 (%60.14)	2644.33 (%48.89)	2530.67 (%42.49)	2505.60 (%41.08)	2868.63 (%61.52)	3970.40 (%123.5)	2620.73 (%47.56)	2011.17 (%13.24)	2145.53 (%20.80)
	SD	148.02	182.18	151.92	205.25	269.32	159.64	130.96	113.58	16.33	78.12
	t-test statistic (p value)	-18.57 (4.30e-26)	-24.94 (1.07e-32)	-22.69 (1.55e-30)	-13.81 (5.30e-20)	-10.03 (2.71e-14)	-29.26 (1.98e-36)	-81.30 (1.82e-61)	-29.09 (2.73e-36)	-	-9.22 (5.74e-13)
	Mann-Whitney statistic (p value)	0.0 (2.82e-11)	0.0 (2.82e-11)	0.0 (2.82e-11)	0.0 (2.82e-11)	2.0 (3.45e-11)	0.0 (2.82e-11)	0.0 (2.82e-11)	0.0 (2.82e-11)	-	57.0 (6.18e-09)
	Time	64.906	108.293	86.458	322.350	481.811	190.737	255.892	109.738	2419.355	1733.169
ry48p (14422)	Best	17152 (%18.9)	19368 (%34.39)	16967 (%17.64)	16054 (%11.31)	15647 (%8.49)	19266 (%33.58)	25215 (%74.83)	17104 (%18.59)	14666 (%1.69)	15414 (%6.87)
	Average	19423.0 (%34.6)	21383.43 (%48.26)	18516.13 (%28.38)	18337.17 (%27.14)	19063.63 (%32.18)	21951.97 (%52.21)	27985.97 (%94.05)	19017.07 (%31.86)	14772.13 (%2.42)	16445.33 (%14.02)
	SD	1142.08	1047.51	904.16	1260.49	1576.08	1345.30	1370.17	1345.84	53.38	411.22
	t-test statistic (p value)	-22.28 (4.06e-30)	-34.52 (2.27e-40)	-22.64 (1.76e-30)	-15.47 (2.95e-22)	-14.90 (1.70e-21)	-29.20 (2.20e-36)	-52.78 (9.96e-51)	-17.26 (1.59e-24)	-	-22.10 (6.20e-30)
	Mann-Whitney statistic (p value)	0.0 (2.88e-11)	0.0 (2.88e-11)	0.0 (2.88e-11)	0.0 (2.88e-11)	0.0 (2.88e-11)	0.0 (2.88e-11)	0.0 (2.88e-11)	0.0 (2.88e-11)	-	0.0 (2.88e-11)
	Time	70.201	117.562	92.724	331.558	474.997	185.067	244.187	104.345	2908.600	1760.452
ft53 (6905)	Best	8907 (%28.9)	10234 (%48.21)	9744 (%41.11)	9410 (%36.27)	9646 (%39.69)	9860 (%42.79)	13720 (%98.69)	9478 (%37.26)	7507 (%8.71)	7815 (%13.17)
	Average	10303.4 (%49.2)	11837.67 (%71.43)	10522.87 (%52.39)	10922.90 (%58.18)	10815.07 (%56.62)	11106.60 (%60.84)	14559.90 (%110.8)	10859.27 (%57.26)	7860.33 (%13.83)	8219.27 (%19.03)
	SD	790.65	718.34	365.82	847.16	809.98	655.14	394.14	633.94	171.86	221.17
	t-test statistic (p value)	-16.53 (1.26e-23)	-29.49 (1.29e-36)	-36.08 (1.97e-41)	-19.40 (4.83e-27)	-19.54 (3.36e-27)	-26.25 (7.01e-34)	-85.34 (1.12e-62)	-25.00 (9.34e-33)	-	-7.01 (2.72e-09)
	Mann-Whitney statistic (p value)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	-	93.0 (1.35e-07)
	Time	67.878	94.287	86.648	347.366	495.952	178.345	231.684	105.291	2765.092	1824.633
ftv55 (1608)	Best	2236 (%39.0)	2487 (%54.66)	2395 (%48.94)	2220 (%38.05)	2156 (%34.07)	2773 (%72.45)	4012 (%149.5)	2337 (%45.33)	1698 (%5.59)	1790 (%11.31)
	Average	2627.40 (%63.3)	3059.47 (%90.26)	2840.87 (%76.67)	2722.03 (%69.28)	2705.13 (%68.22)	3039.80 (%89.04)	4318.23 (%168.5)	2707.00 (%68.34)	1725.87 (%7.33)	1919.20 (%19.35)
	SD	190.03	227.09	168.71	262.74	332.39	173.03	138.64	193.92	16.84	79.11

	t-test statistic (p value)	-25.88 (1.49e-33)	-32.07 (1.31e-38)	-36.01 (2.17e-41)	-20.72 (1.71e-28)	-16.11 (4.37e-23)	-41.39 (9.15e-45)	-101.66 (4.69e-67)	-27.60 (4.67e-35)		-13.09 (5.77e-19)
	Mann-Whitney statistic (p value)	0.0 (1.58e-11)	0.0 (1.58e-11)	0.0 (1.58e-11)	0.0 (1.58e-11)	0.0 (1.58e-11)	0.0 (1.58e-11)	0.0 (1.58e-11)	0.0 (1.58e-11)		0.0 (1.58e-11)
	Time	72.085	130.861	96.459	426.853	626.283	209.519	272.997	122.716	3761.014	2217.655
ftv64 (1839)	Best	2894 (%57.3)	3323 (%80.69)	3216 (%74.87)	2666 (%44.97)	2737 (%48.83)	3354 (%82.38)	5052 (%174.7)	2987 (%62.42)	1947 (%5.87)	2065 (%12.28)
	Average	3231.13 (%75.7)	3776.77 (%105.37)	3628.57 (%97.31)	3537.87 (%92.37)	3437.60 (%86.92)	3743.10 (%103.5)	5469.10 (%197.3)	3364.40 (%82.94)	1969.63 (%7.10)	2202.70 (%19.77)
	SD	188.79	193.60	212.63	355.16	338.14	208.65	194.07	175.07	13.59	75.83
	t-test statistic (p value)	-36.50 (1.03e-41)	-50.99 (7.02e-50)	-42.64 (1.72e-45)	-24.16 (5.73e-32)	-23.75 (1.40e-31)	-46.45 (1.38e-47)	-98.52 (2.87e-66)	-43.50 (5.61e-46)		-16.56 (1.15e-23)
	Mann-Whitney statistic (p value)	0.0 (2.40e-11)	0.0 (2.40e-11)	0.0 (2.40e-11)	0.0 (2.40e-11)	0.0 (2.40e-11)	0.0 (2.40e-11)	0.0 (2.40e-11)	0.0 (2.40e-11)		0.0 (2.40e-11)
	Time	70.670	145.642	105.063	533.851	781.402	229.781	310.241	142.478	4946.616	3028.843
ft70 (38673)	Best	44632 (%15.4)	47335 (%22.39)	46119 (%19.25)	44399 (%14.80)	44690 (%15.55)	46624 (%20.55)	54132 (%39.97)	44853 (%15.98)	40368 (%4.38)	40897 (%5.75)
	Average	46397.8 (%19.9)	49680.40 (%28.46)	47721.13 (%23.39)	48093.00 (%24.35)	48736.73 (%26.02)	47962.17 (%24.01)	55952.37 (%44.68)	47642.10 (%23.19)	40669.17 (%5.16)	41813.40 (%8.12)
	SD	1074.08	1079.99	909.01	1584.59	1607.56	958.47	901.84	1010.26	138.83	556.43
	t-test statistic (p value)	-28.97 (3.43e-36)	-45.32 (5.54e-47)	-42.00 (4.03e-45)	-25.56 (2.90e-33)	-27.38 (7.22e-35)	-41.24 (1.12e-44)	-91.73 (1.75e-64)	-37.45 (2.47e-42)		-10.92 (1.04e-15)
	Mann-Whitney statistic (p value)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)		1.0 (3.32e-11)
	Time	73.973	148.315	101.423	550.267	798.727	221.930	289.246	136.036	4615.406	2586.271
ftv70 (1950)	Best	3207 (%64.4)	3906 (%100.30)	3479 (%78.41)	3047 (%56.25)	2971 (%52.35)	3538 (%81.43)	5296 (%171.5)	3491 (%79.02)	2123 (%8.87)	2195 (%12.56)
	Average	3641.50 (%86.74)	4332.87 (%122.198)	4052.80 (%107.8)	3881.60 (%99.05)	3903.57 (%100.1)	4190.83 (%114.9)	6161.20 (%215.9)	3771.03 (%93.38)	2170.03 (%11.28)	2325.33 (%19.24)
	SD	202.40	260.50	259.51	337.42	357.16	281.95	239.93	198.15	19.70	89.13
	t-test statistic (p value)	-39.63 (1.05e-43)	-45.34 (5.43e-47)	-39.62 (1.06e-43)	-27.73 (3.64e-35)	-26.54 (3.87e-34)	-39.16 (2.05e-43)	-90.80 (3.16e-64)	-44.03 (2.83e-46)		-9.31 (3.97e-13)
	Mann-Whitney statistic (p value)	0.0 (2.93e-11)	0.0 (2.93e-11)	0.0 (2.93e-11)	0.0 (2.93e-11)	0.0 (2.94e-11)	0.0 (2.94e-11)	0.0 (2.93e-11)	0.0 (2.93e-11)		10.0 (7.94e-11)
	Time	67.511	140.905	94.856	540.342	805.015	225.886	297.539	144.966	5549.571	3443.251

kro124p (36230)	Best	65102 (%79.6)	75632 (%108.75)	73093 (%101.7)	65054 (%79.55)	61456 (%69.62)	69739 (%92.48)	118497 (%227.0)	66569 (%83.74)	39459 (%8.91)	40625 (%12.13)
	Average	70382.70 (%94.26)	86827.70 (%139.65)	80115.73 (%121.1)	74183.17 (%104.7)	74172.10 (%104.7)	79825.53 (%120.3)	125536.6 (%246.4)	74544.57 (%105.7)	40730.33 (%12.42)	43519.27 (%20.11)
	SD	3808.91	3894.20	3376.59	6118.41	7502.13	4482.85	3239.60	3526.62	535.57	1190.76
	t-test statistic (p value)	-42.22 (3.00 e-45)	-64.23 (1.36e-55)	-63.09 (3.77e-55)	-29.83 (6.97e-37)	-24.35 (3.81e-32)	-47.42 (4.28e-48)	-141.46 (2.42e-75)	-51.92 (2.53e-50)		-11.69 (6.73e-17)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)		19.0 (1.95e-10)
	Time	85.759	199.707	120.140	936.352	1430.037	295.453	432.325	223.050	13338.874	6654.634
ftv170 (2755)	Best	9538 (%246.2)	12597 (%357.24)	11684 (%324.1)	11268 (%309.0)	9921 (%260.1)	10861 (%294.2)	18705 (%578.9)	10025 (%263.8)	3212 (%16.58)	3465 (%25.77)
	Average	10467.07 (%279.9)	13746.50 (%398.96)	12483.80 (%353.1)	12839.03 (%366.0)	12707.17 (%361.2)	11726.83 (%325.6)	19445.80 (%605.8)	11278.90 (%309.3)	3334.17 (%21.02)	3854.97 (%39.92)
	SD	400.06	486.99	393.45	810.38	857.46	438.28	408.81	493.66	63.97	156.69
	t-test statistic (p value)	-96.42 (9.92e-66)	-116.11 (2.19e-70)	-125.71 (2.22e-72)	-64.04 (1.6e-55)	-59.70 (8.89e-54)	-103.78 (1.43e-67)	-213.26 (1.16e-85)	-87.41 (2.82e-63)		-16.85 (5.10e-24)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)		0.0 (3.01e-11)
	Time	133.689	415.633	204.601	2660.052	4083.593	494.605	676.936	540.908	48947.34	20062.825
rbg323 (1326)	Best	2943 (%121.9)	3899 (%194.04)	3299 (%148.7)	3793 (%186.0)	3848 (%190.1)	3114 (%134.8)	4968 (%274.6)	3153 (%137.7)	1729 (%30.39)	1615 (%21.79)
	Average	3074.10 (%131.8)	4013.00 (%202.64)	3406.37 (%156.8)	4002.57 (%201.8)	4006.83 (%202.1)	3230.60 (%143.6)	5120.97 (%286.1)	3249.30 (%145.0)	1754.43 (%32.31)	1695.33 (%27.85)
	SD	67.29	66.43	64.96	128.69	97.50	71.67	62.43	52.27	13.26	43.82
	t-test statistic (p value)	-105.37 (5.94e-68)	-182.60 (9.28e-82)	-136.46 (1.94e-74)	-95.17 (2.10e-65)	-125.37 (2.60e-72)	-110.91 (3.08e-69)	-288.89 (2.66e-93)	-151.82 (4.05e-77)		7.06 (2.23e-09)
	Mann-Whitney statistic (p value)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.01e-11)		792.0 (4.42e-07)
	Time	61.211	241.006	92.955	1896.984	3242.363	220.781	300.312	389364	46742.72	23066.243
rbg358 (1163)	Best	3177 (%173.1)	4270 (%267.15)	3583 (%208.0)	4310 (%270.5)	4190 (%260.2)	3264 (%180.6)	5602 (%381.6)	3373 (%190.0)	1667 (%43.33)	1578 (%35.68)
	Average	3328.03 (%186.1)	4451.90 (%282.79)	3701.90 (%218.30)	4458.07 (%283.32)	4452.23 (%282.8)	3483.87 (%199.55)	5724.63 (%392.23)	3552.63 (%205.47)	1737.67 (%49.41)	1655.23 (%42.32)
	SD	85.95	112.51	71.79	83.67	127.04	80.90	50.82	81.26	25.20	37.38

rbg403 (2465)	t-test statistic (p value)	-97.24 (6.10e-66)	-128.93 (5.16e-73)	-141.38 (2.50e-75)	-170.50 (4.93e-80)	-114.78 (4.25e-70)	-112.86 (1.13e-69)	-384.91 (1.59e-100)	-116.83 (1.53e-70)	-	10.01 (2.94e-14)
	Mann-Whitney statistic (p value)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (2.99e-11)	0.0 (3.00e-11)	0.0 (2.99e-11)	0.0 (2.99e-11)	0.0 (3.00e-11)	-	871.5 (4.80e-10)
	Time	67.512	281.052	104.122	2318.058	3853.989	238.597	329.122	463.936	92362.618	34153.716
	Best	4223 (%71.31)	5205 (%111.15)	4527 (%83.65)	5200 (%110.95)	5088 (%106.40)	4199 (%70.34)	6390 (%159.22)	4435 (%79.91)	3391 (%37.56)	3327 (%34.96)
	Average	4344.70 (%76.25)	5353.50 (%117.18)	4652.70 (%88.75)	5298.23 (%114.93)	5315.50 (%115.6)	4471.07 (%81.38)	6496.67 (%163.5)	4552.40 (%84.68)	3461.10 (%40.40)	3464.0 (%40.5)
	SD	73.37	79.70	57.31	62.13	97.96	90.74	46.43	57.11	39.83	64.81
	t-test statistic (p value)	-57.96 (4.81e-53)	-116.32 (1.97e-70)	-93.50 (5.86e-65)	-136.33 (2.05e-74)	-96.04 (1.24e-65)	-55.81 (4.13e-52)	-271.75 (9.25e-92)	-85.83 (8.10e-63)	-	-0.20 (0.83)
rbg443 (2720)	Mann-Whitney statistic (p value)	0.0 (3.00e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.00e-11)	0.0 (3.01e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	0.0 (3.00e-11)	-	422.0 (0.68)
	Time	80.329	347.886	126.736	2910.703	4728.761	262.226	379.246	621.005	92359.13	46628.668
	Best	4693 (%72.53)	5610 (%106.25)	4981 (%83.12)	5717 (%110.1)	5611 (%106.2)	4800 (%76.47)	6937 (%155.0)	4919 (%80.84)	3872 (%42.35)	3861 (%41.94)
	Average	4814.90 (%77.0)	5863.50 (%115.56)	5128.53 (%88.54)	5845.25 (%114.8)	5819.03 (%113.9)	4941.93 (%81.68)	7070.00 (%159.9)	5049.90 (%85.65)	3952.70 (%45.31)	3946.50 (%45.09)
	SD	64.47	90.98	84.12	98.32	112.90	89.02	51.51	70.86	39.22	54.95
	t-test statistic (p value)	-62.57 (6.09e-55)	-105.63 (5.16e-68)	-69.38 (1.64e-57)	-105.19 (6.57e-68)	-85.52 (9.98e-63)	-55.69 (4.67e-52)	-263.71 (5.26e-91)	-74.19 (3.51e-59)	-	0.50 (0.61)
	Mann-Whitney statistic (p value)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	0.0 (3.01e-11)	-	490.5 (0.55)
	Time	194.331	925.876	320.229	7934.883	13342.60	754.944	977.282	1714.381	73779.26	91363.523