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ON ALMOST IDEAL CONVERGENCE WITH RESPECT TO AN ORLICZ FUNCTION

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Abstract. In this article, we define new classes of ideal convergent and ideal bounded sequence spaces combining an infinite matrix, an Orlicz function and invariant mean. We investigate some linear topological structures and algebraic properties of the resulting spaces. Also we find out some relations related to these spaces.

1. Introduction

By ω and ℓ_∞, we denote the space of all *complex valued sequences* and *bounded sequences*, respectively. $\mathbb{N}$ and $\mathbb{C}$ stand for the set of *natural numbers* and *complex numbers* and $e = (1, 1, 1, ...)$.

The notion of *ideal convergence* which is a generalization of statistical convergence (see [1, 2]) was introduced by Kostyrko et al. [3].

A family $\mathcal{I}$ of subsets of a non-empty set X is called an *ideal* on X if for each $A, B \in \mathcal{I}$, we have $A \cup B \in \mathcal{I}$ and for each $B \in \mathcal{I}$ and $B \subseteq A$, we have $B \in \mathcal{I}$. If $X \notin \mathcal{I}$, it is called a non-trivial ideal. A non-trivial ideal is said to be admissible if it contains all the finite subsets of X.

A sequence $x = (x_k)$ in $\mathbb{R}$ is called ideal convergent to a real number l if for every $\varepsilon > 0$ the set $\{k \in \mathbb{N} : |x_k - l| \geq \varepsilon\}$ belongs to the ideal [3].

A sequence $x = (x_k)$ of real numbers is said to be ideal bounded if there is a $K > 0$ such that $\{k \in \mathbb{N} : |x_k| > K\} \in \mathcal{I}$ [4].

Later, many authors studied on ideal convergence. See for example [5, 6, 7]. Also, ideal convergence is studied on normed spaces and topological spaces in [8, 9, 10, 11, 12].

Let σ be an injective mapping from the set of the positive integers to itself such that $\sigma^p(n) \neq n$ for all positive integers n and p, where $\sigma^p(n) = \sigma(\sigma^{p-1}(n))$. An invariant mean or a σ-mean is a continuous linear functional defined on the space ℓ_∞ such that for all $x = (x_n) \in \ell_\infty$:

(1) If $x_n \geq 0$ for all n, then $\varphi(x) \geq 0$,

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(2) $\varphi(e) = 1$,
(3) $\varphi(Sx) = \varphi(x)$, where $Sx = (x_{\sigma(n)})$.

V_σ denotes the set of bounded sequences all of whose invariant means are equal which is also called as the space of σ-convergent sequences. In [13], it is defined by

$$V_\sigma = \{x \in \ell_\infty : \lim_k t_{kn}(x) = l, \text{ uniformly in } n, l = \sigma - \lim x\},$$

where $t_{kn}(x) = \frac{x_n + x_{\sigma^1(n)} + ... + x_{\sigma^k(n)}}{k+1}$.

σ-mean is called a Banach limit if σ is the translation mapping $n \to n+1$. In this case, V_σ becomes the set of almost convergent sequences which is denoted by $\hat{c}$ and defined in[14] as

$$\hat{c} = \{x \in \ell_\infty : \lim_k d_{kn}(x) \text{ exists uniformly in } n\},$$

where $d_{kn}(x) = \frac{x_n + x_{n+1} + ... + x_{n+k}}{k+1}$.

The space of strongly almost convergent sequences was introduced by Maddox [15] as follow:

$$[\hat{c}] = \{x \in \ell_\infty : \lim_k d_{kn}(|x - le|) \text{ exists uniformly in } n \text{ for some } l\}.$$

A function $M : [0, \infty) \to [0, \infty)$ is called an *Orlicz function* if M is continuous, nondecreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. By convexity of M and $M(0) = 0$, we have $M(\lambda x) \leq \lambda M(x)$ for all $\lambda \in (0, 1)$.

It is said that M satisfies Δ_2-condition for all $x \in [0, \infty)$ if there exists a constant $K > 0$ such that $M(Lx) \leq KLM(x)$, where $L > 1$ (see [16]).

By using the idea of Orlicz function, Lindenstrauss and Tzafriri [17] defined *Orlicz sequence space*

$$\ell_M = \left\{x \in \omega : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0\right\}$$

which is a Banach space with the norm

$$\|x\| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1\right\}.$$

Several authors used the concept of an Orlicz function to define a new sequence space. For some of the related papers, one can see [19, 20, 21, 22].

Let $p = (p_k)$ be a sequence of positive real numbers such that $0 < h = \inf p_k \leq p_k \leq H = \sup p_k < \infty$. For each $k \in \mathbb{N}$ the inequalities

$$|\alpha_k + \beta_k|^{p_k} \leq D\left\{|\alpha_k|^{p_k} + |\beta_k|^{p_k}\right\} \tag{1.1}$$

and

$$|\alpha|^{p_k} \leq \max\{1, |\alpha|^H\}$$

hold, where $\alpha, \alpha_k, \beta_k \in \mathbb{C}$ and $D = \max\left\{1, 2^{H-1}\right\}$.

Let $A = (a_{ij})$ be an infinite matrix of complex numbers a_{ij}, where $i, j \in \mathbb{N}$. We write $Ax = (A_i(x))$ if $A_i(x) = \sum_{j=1}^{\infty} a_{ij}x_j$ converges for each $i \in \mathbb{N}$. Throughout the text, by $t_{kn}(Ax)$, we mean

$$t_{kn}(Ax) = \frac{A_n(x) + A_{\sigma^1(n)}(x) + ... + A_{\sigma^k(n)}(x)}{k+1}$$

for all $k, n \in \mathbb{N}$.

A sequence space X is called as *solid (or normal)* if $(\gamma_k x_k) \in X$ whenever $(x_k) \in X$ and (γ_k) is a sequence of scalars such that $|\gamma_k| \leq 1$ for all $k \in \mathbb{N}$.

Let X be a sequence space and $K = \{k_1 < k_2 < ...\} \subseteq \mathbb{N}$. The sequence space $Z_K^X = \{(x_{k_n}) \in \omega : (x_n) \in X\}$ is called *K-step space* of X.

A *canonical preimage* of a sequence $(x_{k_n}) \in Z_K^X$ is a sequence $(y_n) \in \omega$ defined by

$$y_n = \begin{cases} x_n, & \text{if } n \in \mathbb{N}, \\ 0, & \text{otherwise.} \end{cases}$$

A sequence space X is *monotone* if it contains the canonical preimages of all its step spaces.

Lemma 1.1. ([18],p.53) *If a sequence space X is solid, then X is monotone.*

Recently, strongly almost ideal convergent sequence spaces in 2-normed spaces defined via an Orlicz function was introduced by Esi [23]. Quite recently, Hazarika [24] defined a new class of strongly almost ideal convergent sequence spaces using an infinite matrix, Orlicz functions and a new generalized difference matrix in locally convex spaces and proved some results about this notion. Further in [25, 26, 27], the authors defined new spaces by combining ideal convergence, Orlicz functions and infinite matrices.

The purpose of this paper is to introduce and study some new ideal convergent sequence spaces with respect to an Orlicz function and an infinite matrix.

2. Main results

In this section, by combining ideal convergence, an infinite matrix, an Orlicz function and invariant means, we define some new sequence spaces.

From now on, by $\mathcal{I}$, we denote an admissible ideal of $\mathbb{N}$.

Let M be an Orlicz function, A be an infinite matrix and $p = (p_k)$ be a bounded sequence of positive real numbers.

For every $\varepsilon > 0$ and some $\rho > 0$, we introduce the spaces as follows:

$$\mathcal{I}{-}c_0^\sigma(M, A, p) = \left\{ u \in \omega : \left\{ k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\} \in \mathcal{I} \text{ for all } n \in \mathbb{N} \right\},$$

$$\mathcal{I}{-}c^\sigma(M, A, p) = \left\{ u \in \omega : \left\{ k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au - le)|}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\} \in \mathcal{I} \text{ for all } n \in \mathbb{N} \text{ and some } l \in \mathbb{C} \right\},$$

$$\mathcal{I}{-}\ell_\infty^\sigma(M, A, p) = \left\{ u \in \omega : \exists K > 0 \text{ such that } \left\{ k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho} \right) \right]^{p_k} > K \right\} \in \mathcal{I} \text{ for all } n \in \mathbb{N} \right\}.$$

If we take $p_k = 1$ for all $k \in \mathbb{N}$, then the above spaces are denoted by $\mathcal{I}{-}c_0^\sigma(M, A)$, $\mathcal{I} - c^\sigma(M, A)$, $\mathcal{I} - \ell_\infty^\sigma(M, A)$, respectively.

Theorem 2.1. *The spaces $\mathcal{I} - c_0^\sigma(M, A, p)$, $\mathcal{I} - c^\sigma(M, A, p)$ and $\mathcal{I} - \ell_\infty^\sigma(M, A, p)$ are linear spaces.*

Proof. The result will be proved only for $\mathcal{I}{-}c_0^\sigma(M, A, p)$. The others follow similarly. Take any $u, v \in \mathcal{I} - c_0^\sigma(M, A, p)$. Then for given $\varepsilon > 0$ the sets

$$S_1 = \left\{ k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho_1} \right) \right]^{p_k} \geq \frac{\varepsilon}{2D} \right\}$$

and

$$S_2=\left\{k\in\mathbb{N}:\left[M\left(\frac{|t_{kn}(Av)|}{\rho_2}\right)\right]^{p_k}\geq\frac{\varepsilon}{2D}\right\}$$

are contained in $\mathcal{I}$ for some $\rho_1,\rho_2>0$.

By using the inequality (1.1) and the fact that M is nondecreasing and convex, one can see the following inequality:

$$\begin{aligned}\left[M\left(\frac{|t_{kn}(A(\lambda u+\mu v)|}{\rho}\right)\right]^{p_k}&\leq\left[M\left(\frac{|t_{kn}(A(u)|}{\rho_1}\right)+M\left(\frac{|t_{kn}(A(v)|}{\rho_2}\right)\right]^{p_k}\\&\leq D\left\{\left[M\left(\frac{|t_{kn}(A(u)|}{\rho_1}\right)\right]^{p_k}+\left[M\left(\frac{|t_{kn}(A(v)|}{\rho_2}\right)\right]^{p_k}\right\},\end{aligned}$$

where $\rho=\max\{2|\lambda|\rho_1,2|\mu|\rho_2\}$ and $\lambda,\mu\in\mathbb{C}$.

If we choose a positive integer k' from $\mathbb{N}\backslash S_1\cup S_2$, we obtain

$$\left[M\left(\frac{|t_{kn}(A(\lambda u+\mu v)|}{\rho}\right)\right]^{p_k}<\varepsilon.$$

Hence the set

$$\left\{k\in\mathbb{N}:\left[M\left(\frac{|t_{kn}(A(\lambda u+\mu v))|}{\rho}\right)\right]^{p_k}\geq\varepsilon\right\}$$

belongs to the ideal which implies $\lambda u+\mu v\in\mathcal{I}-c_0^\sigma(M,A,p)$. This completes the proof.

□

Theorem 2.2. *The inclusions*

$$\mathcal{I}-c_0^\sigma(M_1,A,p)\cap\mathcal{I}-c_0^\sigma(M_2,A,p)\subseteq\mathcal{I}-c_0^\sigma(M_1+M_2,A,p),$$

$$\mathcal{I}-c^\sigma(M_1,A,p)\cap\mathcal{I}-c^\sigma(M_2,A,p)\subseteq\mathcal{I}-c^\sigma(M_1+M_2,A,p),$$

$$\mathcal{I}-\ell_\infty^\sigma(M_1,A,p)\cap\mathcal{I}-\ell_\infty^\sigma(M_2,A,p)\subseteq\mathcal{I}-\ell_\infty^\sigma(M_1+M_2,A,p)$$

hold for any Orlicz functions M_1 and M_2.

Proof. Let u belong to the intersection of $\mathcal{I}-c_0^\sigma(M_1,A,p)$ and $\mathcal{I}-c_0^\sigma(M_2,A,p)$. Since the inequality

$$\begin{aligned}\left[(M_1+M_2)\left(\frac{|t_{kn}(A(u)|}{\rho}\right)\right]^{p_k}&=\left[M_1\left(\frac{|t_{kn}(A(u)|}{\rho}\right)+M_2\left(\frac{|t_{kn}(A(u)|}{\rho}\right)\right]^{p_k}\\&\leq D\left\{\left[M_1\left(\frac{|t_{kn}(A(u)|}{\rho}\right)\right]^{p_k}+\left[M_2\left(\frac{|t_{kn}(A(u)|}{\rho}\right)\right]^{p_k}\right\}\end{aligned}$$

holds, the result is obvious.

The other inclusions can be shown similarly. □

Theorem 2.3. *Let M_2 satisfy Δ_2 condition. Then the inclusions*

$$\mathcal{I}-c_0^\sigma(M_1,A,p)\subseteq\mathcal{I}-c_0^\sigma(M_1\circ M_2,A,p),$$

$$\mathcal{I}-c^\sigma(M_1,A,p)\subseteq\mathcal{I}-c^\sigma(M_1\circ M_2,A,p),$$

$$\mathcal{I}-\ell_\infty^\sigma(M_1,A,p)\subseteq\mathcal{I}-\ell_\infty^\sigma(M_1\circ M_2,A,p)$$

hold for any Orlicz functions M_1 and M_2.

Proof. We prove the theorem in two parts. Firstly, let $M_1\left(\frac{|t_{kn}(A(u)|}{\rho}\right) > \delta$. By using the properties of an Orlicz function and the fact that M_2 satisfies Δ_2 condition, we have

$$\left[M_2\left(M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right)\right]^{p_k} \leq (K\delta^{-1}M_2(2))^{p_k}\left[M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k}$$
$$\leq \max\left\{1,(K\delta^{-1}M_2(2))^H\right\}\left[M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k},$$

where $K \geq 1$ and $\delta < 1$. From the last inequality, the inclusion

$$\left\{k\in\mathbb{N}:\left[M_2\left(M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right)\right]^{p_k}\geq\varepsilon\right\}\subseteq\left\{k\in\mathbb{N}:\left[M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k}\geq\frac{\varepsilon}{\max\{1,(K\delta^{-1}M_2(2))^H\}}\right\}$$

is obtained. If $u \in \mathcal{I}-c_0^\sigma(M_1,A,p)$, then the set in the right side of the above inclusion belongs to the ideal and so $\left\{k\in\mathbb{N}:\left[M_2\left(M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right)\right]^{p_k}\geq\varepsilon\right\}\in\mathcal{I}$.

Secondly, Suppose that $M_1\left(\frac{|t_{kn}(A(u)|}{\rho}\right) \leq \delta$. Since M_2 is continuous, we have $M_2\left(M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right) < \varepsilon$ for all $\varepsilon > 0$ which implies $\mathcal{I}-\lim_k\left[M_2\left(M_1\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right)\right]^{p_k} = 0$ as $\varepsilon \to 0$. This completes the proof.

The other inclusions can be shown similarly. $\square$

Theorem 2.4. *If* $\sup_k[M(t)]^{p_k} < \infty$ *for all* $t > 0$*, then we have*

$$\mathcal{I}-c^\sigma(M,A,p)\subseteq\mathcal{I}-\ell_\infty^\sigma(M,A,p).$$

Proof. Let $x \in \mathcal{I}-c^\sigma(M,A,p)$. The inequality

$$\left[M\left(\frac{|t_{kn}(A(u)|}{\rho}\right)\right]^{p_k}\leq D\left\{\left[M\left(\frac{|t_{kn}(Au-le)|}{\rho_1}\right)\right]^{p_k}+\left[M\left(\frac{|t_{kn}(le)|}{\rho_1}\right)\right]^{p_k}\right\}$$

holds by (1.1), where $\rho = 2\rho_1$. Hence we have

$$\left\{k\in\mathbb{N}:\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k}\geq K\right\}\subseteq\left\{k\in\mathbb{N}:\left[M\left(\frac{|t_{kn}(Au-le)|}{\rho_1}\right)\right]^{p_k}\geq\varepsilon\right\}$$

for all n and some $K > 0$. Since the set in the right side of the above inclusion belogs to the ideal, all of its subsets are in the ideal. So

$$\left\{k\in\mathbb{N}:\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k}\geq K\right\}\in\mathcal{I}$$

which completes the proof. $\square$

Theorem 2.5. *Let* $0 < p_k \leq q_k < \infty$ *for each* $k \in \mathbb{N}$ *and* $(\frac{q_k}{p_k})$ *be bounded. Then we have*

$$\mathcal{I}-W(M,A,q)\subseteq\mathcal{I}-W(M,A,p),$$

where $W = c_0^\sigma, c^\sigma$.

Proof. Suppose that $u \in \mathcal{I}-c_0^\sigma(M,A,q)$. Write $\alpha_k = \frac{p_k}{q_k}$. By hypothesis, we have $0 < \alpha \leq \alpha_k \leq 1$. If $\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{q_k} \geq 1$, the inequality $\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k} \leq \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{q_k}$ holds. This implies the inclusion

$$\left\{k\in\mathbb{N}:\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k}\geq\varepsilon\right\}\subseteq\left\{k\in\mathbb{N}:\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{q_k}\geq\varepsilon\right\}$$

and so the result is obvious. Conversely, if $\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{q_k} < 1$, we obtain the following inclusion

$$\left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \subseteq \left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{q_k} \geq \varepsilon^{1/\alpha}\right\}$$

since then the inequality $\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k} \leq \left(\left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{q_k}\right)^{\alpha}$ holds. Hence we conclude that $u \in \mathcal{I} - c_0^{\sigma}(M, A, p)$. □

Theorem 2.6.

(1) *If* $0 < \inf p_k \leq p_k \leq 1$ *for each* $k \in \mathbb{N}$, *then* $\mathcal{I} - W(M, A, p) \subseteq \mathcal{I} - W(M, A)$, *where* $W = c_0^{\sigma}, c^{\sigma}$.

(2) *If* $1 \leq p_k \leq \sup p_k < \infty$ *for each* $k \in \mathbb{N}$, *then* $\mathcal{I} - W(M, A) \subseteq \mathcal{I} - W(M, A, p)$, *where* $W = c_0^{\sigma}, c^{\sigma}$.

Proof.

(1) Let $u \in \mathcal{I} - c_0^{\sigma}(M, A, p)$. Suppose that $k \notin \left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\}$ for $0 < \varepsilon < 1$. By hypothesis, the inequality $M\left(\frac{|t_{kn}(Au)|}{\rho}\right) \leq \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k}$ holds. Then we have $k \notin \left\{k \in \mathbb{N} : M\left(\frac{|t_{kn}(Au)|}{\rho}\right) \geq \varepsilon\right\}$ which implies

$$\left\{k \in \mathbb{N} : M\left(\frac{|t_{kn}(Au)|}{\rho}\right) \geq \varepsilon\right\} \subseteq \left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\}.$$

Hence $u \in \mathcal{I} - c_0^{\sigma}(M, A)$ since the set $\left\{k \in \mathbb{N} : M\left(\frac{|t_{kn}(Au)|}{\rho}\right) \geq \varepsilon\right\}$ in $\mathcal{I}$.

(2) The proof is similar to the first part.

□

Theorem 2.7. *The spaces* $\mathcal{I} - c_0^{\sigma}(M, A, p)$ *and* $\mathcal{I} - \ell_{\infty}^{\sigma}(M, A, p)$ *are solid.*

Proof. Let $u \in \mathcal{I} - c_0^{\sigma}(M, A, p)$. Then we have $\left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \in \mathcal{I}$ for all n. If $\gamma = (\gamma_k)$ is a sequence of scalars such that $|\gamma_k| \leq 1$ for all $k \in \mathbb{N}$, then the following holds:

$$\left[M\left(\frac{|t_{kn}(A\gamma u)|}{\rho}\right)\right]^{p_k} \leq \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k}.$$

Hence we obtain $\left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(A\gamma u)|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \subseteq \left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au)|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\}$ and so $\left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(A\gamma u)|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \in \mathcal{I}$ which means $\gamma u \in \mathcal{I} - c_0^{\sigma}(M, A, p)$. We conclude that the space $\mathcal{I} - c_0^{\sigma}(M, A, p)$ is solid.

□

Corollary 2.1. *The spaces* $\mathcal{I} - c_0^{\sigma}(M, A, p)$ *and* $\mathcal{I} - \ell_{\infty}^{\sigma}(M, A, p)$ *are monotone.*

Proof. The proof follows from Lemma 1.1. □

Theorem 2.8. *If* $\lim_k p_k > 0$ *and* $u \to u_0(\mathcal{I} - c^{\sigma}(M, A, p))$, *then* u_0 *is unique.*

Proof. Let $\lim_k p_k = p_0 > 0$. We assume that $u \to u_0(\mathcal{I} - c^\sigma(M, A, p))$ and $u \to v_0(\mathcal{I} - c^\sigma(M, A, p))$. Then there exist $\rho_1, \rho_2 > 0$ such that

$$\left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au - u_0e)|}{\rho_1}\right)\right]^{p_k} \geq \frac{\varepsilon}{2D}\right\} \in \mathcal{I}$$

and

$$\left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au - v_0e)|}{\rho_2}\right)\right]^{p_k} \geq \frac{\varepsilon}{2D}\right\} \in \mathcal{I}$$

for all $n \in \mathbb{N}$. Put $\rho = \max\{2\rho_1, 2\rho_2\}$. Then the inequality

$$\left[M\left(\frac{|u_0 - v_0|}{\rho}\right)\right]^{p_k} \leq D\left\{\left[M\left(\frac{|t_{kn}(Au - u_0e)|}{\rho_1}\right)\right]^{p_k} + \left[M\left(\frac{|t_{kn}(Au - v_0e)|}{\rho_2}\right)\right]^{p_k}\right\}$$

holds. Hence we have for all $n \in \mathbb{N}$

$$\left\{k \in \mathbb{N} : \left[M\left(\frac{|u_0 - v_0|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \subseteq \left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au - u_0e)|}{\rho_1}\right)\right]^{p_k} \geq \frac{\varepsilon}{2D}\right\}$$
$$\cup \left\{k \in \mathbb{N} : \left[M\left(\frac{|t_{kn}(Au - v_0e)|}{\rho_2}\right)\right]^{p_k} \geq \frac{\varepsilon}{2D}\right\}.$$

By this inclusion, we obtain $\left\{k \in \mathbb{N} : \left[M\left(\frac{|u_0 - v_0|}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \in \mathcal{I}$ which means $\mathcal{I} - \lim \left[M\left(\frac{|u_0 - v_0|}{\rho}\right)\right]^{p_k} = 0$. Also we have

$$\left[M\left(\frac{|u_0 - v_0|}{\rho}\right)\right]^{p_k} \to \left[M\left(\frac{|u_0 - v_0|}{\rho}\right)\right]^{p_0}$$

as $k \to \infty$ since the limit of the sequence (p_k) is p_0 and so $\left[M\left(\frac{|u_0 - v_0|}{\rho}\right)\right]^{p_0} = 0$. This implies that $u_0 = v_0$.

□

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