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Research Article

Incorporating past data in enhancing coefficient of variation estimation: A practical approach

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Abstract

Measurement of variability is a crucial issue in survey sampling. This problem takes on a practical perspective when considering relative variability, as real-life scenarios often involve comparing two different characteristics or the same characteristic on different scales. For researchers and professionals working in a variety of fields, including agriculture, the stock market, economics, public health, finance, and more, its ability to improve decision-making processes makes it an essential tool. In this paper, we employ the exponentially weighted moving average statistic to propose memory-type estimators for the coefficient of variation that take advantage of auxiliary information in time-scaled surveys. These estimators are based on the concept of memory-type statistics, which integrate both past and present data to enhance the estimation of population parameters. Designed to dynamically incorporate historical data, they improve the accuracy and efficiency of estimation in longitudinal surveys. To validate their effectiveness, we establish the necessary mathematical conditions and explicitly derive expressions for bias and mean square error. Observations from a simulation study indicate that incorporating historical sample data alongside current data significantly enhances estimator performance. In addition, two real-life case studies are presented to demonstrate the efficacy and superiority of the proposed estimators.

Mathematics Subject Classification (2020). MSC 2020

Keywords. Bias, coefficient of variation, exponentially weighted moving average statistic, mean square error, memory type estimators, simulation.

1. Introduction

The coefficient of variation (CV) is a statistical measure of the highest relevance in sectors such as agriculture, economics, and finance because it measures relative variability. Requiring an improved estimation of CV is essential in order to increase the accuracy and reliability of statistical analysis, which is vital in applications that take place in the real world. In order to obtain accurate population estimates, which in turn assist researchers in achieving efficient outcomes, they require a CV that has been accurately predicted when we discuss survey sampling. There are several different industries in which CV plays an

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important role, including medical, manufacturing processes, agriculture, and finance. The CV is utilized in the field of finance for the purpose of risk assessment, which enables investors to evaluate the volatility of assets in relation to the returns they anticipate receiving. An accurate conclusion is reached as a consequence of a well-structured CV estimation, which in turn leads to the formulation of appropriate judgments for economic forecasting, strategies, and policy choices. Estimation of CV is an extremely important part of the field of medicine, since clinical trials rely on CV to quantify the variability in drug response. This helps ensure that new therapies are both successful and consistent. To the same extent, CV is utilized in manufacturing processes in order to identify instances of uneven quality, which can result in increased defect rates and financial losses. Even in the field of agriculture, to better prepare for the future, farmers can use CV to quantify the impact of weather and soil conditions on crop yields.

The use of supplementary information improves the estimating process by delivering estimates that are more effective, thus increasing the accuracy of the outcomes that are acquired. Several researchers have contributed to the development of various estimators in different sampling frameworks. Kadılar and Cıngı [5] suggested ratio estimators under simple random sampling, while Perri [20] and Aslam et al. [9] proposed improved ratio-cum-product type estimators. In a finite population context, Ozturk [16] focused on estimating the mean and total population. Özel Kadılar [7] proposed a new exponential-type estimator for the population mean in simple random sampling.

Yaqub and Shabbir [15] introduced an improved class of estimators for finite population variance. In stratified random sampling, Özel et al. [6] proposed distinct ratio estimators for population variance. Chatterjee et al. [1] provided a general procedure to estimate the variance of the population in successive samplings, and Sharma and Singh [22] proposed a generalized class of estimators for the median population. Furthermore, Ozturk [17] explored the population mean and total using the ranks of the sampling units, while Bhushan and Kumar [29] developed novel log-type estimators under the sampling of the ranked set, which utilize auxiliary information and include derived expressions for bias and MSE. These authors have also conducted comparative analyzes with existing estimators under a variety of sampling strategies. Various authors have highlighted the importance of CV in various areas of research, such as [23] in the business field, [18] as a means of measuring changes in inequality status and [14] in evolutionary studies. Das and Tripathi [2] were the pioneers in proposing an estimator for the estimation of population coefficient of variation under simple random sampling without replacement. Furthermore, the works of Rajyaguru and Gupta [3,4], Patel, and Rina [19] contribute to the field of CV estimation using auxiliary information.

The usual estimator for estimating the coefficient of variation is given by

$$t_p^0 = \hat{c}_y. \tag{1.1}$$

Archana [32] introduced the ratio estimator to estimate the CV using auxiliary information such as

$$t_p^r = \frac{\hat{c}_y}{\hat{c}_x} C_x. \tag{1.2}$$

Similarly, the product estimator to estimate CV when study and auxiliary variables are negatively correlated is defined as

$$t_p^p = \frac{\hat{c}_y}{C_x} \hat{c}_x. \tag{1.3}$$

The expression of mean square error (MSE) for the estimators given in Equations (1.1), (1.2) and (1.3) is given as

$$\mathrm{MSE}(t_p^0) = C_y^2\theta\left[C_y^2 + \frac{u_{40}-1}{4} - C_y u_{30}\right], \tag{1.4}$$

$$\text{MSE}(t_p^r) = C_y^2\,\theta\left[C_y^2 + \frac{u_{40}-1}{4} + \frac{u_{04}-1}{4} + C_x^2 - 2\rho C_y C_x \right.$$
$$\left. - C_y u_{30} + C_y u_{12} - \frac{u_{22}-1}{2} + C_x u_{21} - C_x u_{03}\right], \tag{1.5}$$

$$\text{MSE}(t_p^p) = C_y^2\,\theta\left[C_y^2 + \frac{u_{40}-1}{4} + \frac{u_{04}-1}{4} + C_x^2 + 2\rho C_y C_x \right.$$
$$\left. - C_y u_{30} - C_y u_{12} + \frac{u_{22}-1}{2} - C_x u_{21} - C_x u_{03}\right]. \tag{1.6}$$

Here,

$$\theta = \frac{N-n}{Nn}$$

and C_y and C_x are the population correlation coefficients of Y and X, respectively. Additionally, we have

$$u_{rs} = \frac{v_{rs}}{v_{20}^{r/2} {v_{02}}^{s/2}}$$

where

$$v_{rs} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^r(x_i-\bar{x})^s$$

ρ is the correlation coefficient between the study and the auxiliary variable.

Shabbir and Gupta [10] worked on the estimation of the CV of the population in simple and stratified sampling under two-phase sampling, Singh et al. [24] proposed improved estimators of the CV of the population using an auxiliary variable, Singh and Mishra [25] and Kadav et al. [30] also worked on the estimation of the CV utilizing auxiliary information in various sampling schemes.

Generalized ratio estimator given by Singh et al. [24] for CV estimation is

$$t_p^{\gamma'} = \hat{c}_y\left(\frac{C_x}{\hat{c}_x}\right)^{\gamma'}. \tag{1.7}$$

The MSE expression for the estimator given in Equation (1.7) is given by

$$MinMSE(t_p^{\gamma'}) = \theta C_y^2\left[C_y^2 + \frac{u_{40}-1}{4} - C_y u_{30} - \frac{\left(\rho C_y C_x - \frac{C_x u_{21}}{2} - \frac{C_y \mu_{12}}{2} + \frac{u_{22}-1}{4}\right)^2}{\left(C_x^2 + \frac{u_{04}-1}{4} - C_x u_{03}\right)}\right]. \tag{1.8}$$

Singh et al. [24] also gave a difference-ype estimator to estimate CV as

$$t_p^{reg} = \hat{c}_y + \beta\left(C_x - \hat{c}_x\right). \tag{1.9}$$

The MSE expression for the estimator given in Equation (1.9) is

$$MinMSE(t_p^{reg}) = \theta C_y^2\left[C_y^2 + \frac{u_{40}-1}{4} - C_y u_{30} - \frac{\left(\rho C_y C_x - \frac{C_x u_{21}}{2} - \frac{C_y \mu_{12}}{2} + \frac{u_{22}-1}{4}\right)^2}{\left(C_x^2 + \frac{u_{04}-1}{4} - C_x u_{03}\right)}\right] \tag{1.10}$$

These studies provide valuable information for cross-sectional surveys. Roberts [31] introduced the exponentially weighted moving average (EWMA) statistic to monitor changes in the process means. Noor-ul-Amin [11] applied EWMA statistics to estimate population mean in time-scale surveys, highlighting the importance of utilizing both existing and current sample data. In a subsequent contribution, Noor-ul-Amin [12] proposed estimators that incorporate the EWMA statistic within the stratified sampling framework, with the objective of enhancing the efficiency of the estimator by considering variations between different strata. Building on his work, several authors, including Singh et al. [26], proposed ratio estimators using the EWMA statistics. Bhushan et al. [27] focused on mean estimation for time-scaled surveys through log-type estimators, while Kumar et al. [28] introduced generalized memory-type estimators. Aslam et al. [8] and Sharma et al. [21] contributed to improving the estimation of population means in time-scaled surveys across various sampling frameworks. Furthermore, [13] proposed estimators for population variance utilizing the EWMA statistic. In fact, CV estimation using EWMA statistics is still relatively unexplored territory.

There are a number of limitations associated with traditional techniques of determining CV, which can often result in findings that are erroneous or unstable. These estimators assume continuous variability and do not take into account any trends or patterns that may emerge over time, which is a significant difficulty. When it comes to sectors such as finance, economics, and manufacturing, where data frequently move as a result of market swings, economic shifts, or seasonal influences, this can be an issue. A further problem is that they make the assumption that every data point is independent, neglecting the fact that previous values frequently impact future trends, such as the prices of stocks, the rates of inflation, or the yields of crops. By providing greater weight to recent data while still taking historical values into account, the EWMA statistic enhances the estimation, which is how this study attempts to address these issues. In contrast to conventional techniques, EWMA helps identify underlying patterns and smooths out short-term variations, increasing the estimate's stability and dependability. This is particularly helpful in time-sensitive industries such as banking, where current stock values are more significant than historical ones, or in economic forecasting, where making decisions may be aided by identifying early indications of changes in inflation.

The CV is a widely used measure because it provides a standardized measurement and enables comparisons to be made across data sets that contain different units. For example, in biological studies, CV is utilized to compare the variability in body sizes between different species, despite the fact that their average sizes are different. CV is an essential statistic in the financial sector for evaluating the relative risk of investments. In economics, the coefficient of variation serves as a tool to assess the variability of income levels across various countries or regions, regardless of differing average income levels. The importance of CV estimation in various practical applications highlights the overlooked potential of EWMA to improve this estimation process. Taking into account all these factors, we have created memory-type estimators for CV estimation. These estimators are designed to assess relative variability with greater precision, which is beneficial in multiple domains, including agriculture, economics, health, and finance.

The structure of the article is outlined as follows: Section 1 offers an in-depth review of the literature, establishing the essential context for the study. Section 2 presents the proposed EWMA statistic, utilized to create innovative estimators for CV. In Section 3, the proposed estimators are meticulously derived, and their corresponding MSE are calculated to evaluate their theoretical properties. Section 4 conducts a mathematical comparison to improve the understanding of the performance of these estimators, followed by an empirical application that uses agricultural data, demonstrating the practical importance of the estimators. Finally, Sections 5 and 6 provide a comprehensive simulation study to assess the robustness of the proposed methods, culminating in a thorough discussion and summary of the findings in the conclusion.

2. Proposed EWMA statistic for CV estimation

Consider a finite population of size N represented by $\phi = \{\phi_1, \phi_2, \phi_3, \ldots, \phi_N\}$. Let Y represent the study variable and X denote the auxiliary variable, both defined in N identifiable and distinct units. Then n pairs of observations are obtained using simple random sampling without replacing two variables, X and Y.

Using the Roberts [31] EWMA statistic to monitor changes in the mean of the process, we have introduced the EWMA statistic for the study variables y and x, which is used for CV estimation as outlined below:

$$W_t = \alpha\, \hat{c}_y + (1-\alpha)\, W_{t-1}, \quad 0 < \alpha < 1 \tag{2.1}$$

$$V_t = \alpha\, \hat{c}_x + (1-\alpha)\, V_{t-1}, \quad 0 < \alpha < 1 \tag{2.2}$$

In this context, $\hat{c}_y$ and $\hat{c}_x$ represent the coefficient of variation for the study variable y and the auxiliary variable x, respectively. The parameter α denotes the smoothing constant, indicating the weight attributed to the observations. A higher value of α indicates that more weight is assigned to the current values, while less weight is given to the past observations, and the opposite is true in the other scenario. When α=1, the current observations are given full weight, resulting in the EWMA statistic being equivalent to the conventional sample mean. In this context, t denotes the number of samples, while W_{t-1} and V_{t-1} signify the previous observations. The initial values W_0 and V_0 are established on the basis of the anticipated coefficient of variation or the coefficient of variation from the previous sample.

$E(W_t)$, $E(V_t)$, $\mathrm{Var}(W_t)$, and $\mathrm{Var}(V_t)$ of the EWMA statistic are given by

$$\begin{aligned} E(W_t) &= C_y \quad \text{and} \quad E(V_t) = C_x, \\ \mathrm{Var}(W_t) &= \mathrm{var}(\hat{c}_y)\left[\frac{\alpha}{2-\alpha}\left(1-(1-\alpha)^{2t}\right)\right], \\ \mathrm{Var}(V_t) &= \mathrm{var}(\hat{c}_x)\left[\frac{\alpha}{2-\alpha}\left(1-(1-\alpha)^{2t}\right)\right]. \end{aligned}$$

The limiting forms of $\mathrm{Var}(W_t)$ and $\mathrm{Var}(V_t)$ are given by

$$\begin{aligned} \mathrm{Var}(W_t) &= \theta\left(\frac{\alpha}{2-\alpha}\right) C_y^2\left[C_y^2 + \frac{u_{40}-1}{4} - C_y u_{30}\right], \\ \mathrm{Var}(V_t) &= \theta\left(\frac{\alpha}{2-\alpha}\right) C_x^2\left[C_x^2 + \frac{u_{04}-1}{4} - C_x u_{03}\right]. \end{aligned}$$

3. Proposed memory type estimators

3.1. Memory type class of estimator for CV estimation

The proposed class of memory-type estimators is defined using the EWMA statistic as outlined in Equations (2.1) and (2.2).

$$t_{pm}^{\gamma} = W_t\left(\frac{C_x}{V_t}\right)^{\gamma} \tag{3.1}$$

where $\gamma = 0, 1\, or - 1$ and the class of estimators t_{pm}^{γ} turns out to be the estimator t_{pm}^{0}, t_{pm}^{r} and t_{pm}^{p} for different values of $\gamma = 0, 1\, and - 1$, respectively.
For $\gamma = 0$, we obtain the usual estimator as

$$t_{pm}^{0} = W_t\, (M - \text{Usual estimator})$$

For $\gamma = 1$, we get the ratio estimator as

$$t_{pm}^{r} = W_t \frac{C_x}{V_t}\, (M - \text{Ratio estimator})$$

For $\gamma = -1$, we have the product estimator

$$t_{pm}^{p} = W_t \left(\frac{C_x}{V_t}\right)^{-1} (M - \text{Product estimator})$$

In order to derive the MSE expressions for the proposed estimators, we define

$$e_{0m} = \frac{\overline{y}}{\overline{Y}} - 1, \quad e_{1m} = \frac{\overline{x}}{\overline{X}} - 1, \quad e_{2m} = \frac{s_y^2}{S_Y^2} - 1, \quad e_{3m} = \frac{s_x^2}{S_x^2} - 1$$

such that

$$E(e_{0m}) = E(e_{1m}) = E(e_{2m}) = E(e_{3m}) = 0$$

$$E(e_{0m}^2) = \theta\left(\frac{\alpha}{2-\alpha}\right) C_y^2, \quad E(e_{1m}^2) = \theta\left(\frac{\alpha}{2-\alpha}\right) C_x^2,$$

$$E(e_{2m}^2) = \theta\left(\frac{\alpha}{2-\alpha}\right) (u_{40}^*), \quad E(e_{0m}e_{2m}) = \theta\left(\frac{\alpha}{2-\alpha}\right) C_y u_{30},$$

$$E(e_{3m}^2) = \theta\left(\frac{\alpha}{2-\alpha}\right) (u_{04}^*), \quad E(e_{0m}e_{1m}) = \theta\left(\frac{\alpha}{2-\alpha}\right) \rho C_y C_x,$$

$$E(e_{0m}e_{3m}) = \theta\left(\frac{\alpha}{2-\alpha}\right) C_y u_{12}, \quad E(e_{1m}e_{2m}) = \theta\left(\frac{\alpha}{2-\alpha}\right) C_x u_{21},$$

$$E(e_{2m}e_{3m}) = \theta\left(\frac{\alpha}{2-\alpha}\right) (u_{22}^*), \quad E(e_{1m}e_{3m}) = \theta\left(\frac{\alpha}{2-\alpha}\right) C_x u_{03},$$

where $u_{40}^* = u_{40} - 1$, $u_{04}^* = u_{04} - 1$ and $u_{22}^* = u_{22} - 1$. Therefore, we obtain

$$W_t = C_y(1+e_2)^{\frac{1}{2}}(1+e_0)^{-1}, \quad V_t = C_x(1+e_3)^{\frac{1}{2}}(1+e_1)^{-1}.$$

Expressing t_{pm}^{γ} given in Equation (3.1) in terms of e's, we get

$$t_{pm}^{\gamma} = C_y \left(1 - e_{om} + e_{om}^2 + \frac{e_{2m}}{2} - \frac{e_{om}e_{2m}}{2} - \frac{e_{2m}^2}{8}\right)(1+e_{1m})^{\gamma}(1+e_{3m})^{-\frac{\gamma}{2}}. \tag{3.2}$$

Simplifying Equation (3.2), we have

$$\begin{aligned} t_{pm}^{\gamma} = C_y \Bigg[& 1 - e_{om} + e_{om}^2 + \frac{e_{2m}}{2} - \frac{e_{om}e_{2m}}{2} - \frac{e_{2m}^2}{8} \\ & + \gamma\left(-\frac{e_{3m}}{2} - \frac{e_{2m}e_{3m}}{4} + \frac{e_{0m}e_{3m}}{2} + \frac{e_{3m}^2}{8}\right. \\ & \left. + e_{1m} + \frac{e_{1m}e_{2m}}{2} - e_{om}e_{1m} + \frac{e_{1m}e_{3m}}{2} - e_{1m}^2\right)\Bigg] \end{aligned} \tag{3.3}$$

To get MSE, subtractC_y, square, and then taking the expectation. Then, we have

$$\begin{aligned} E(t_{pm}^{\gamma} - C_y)^2 = C_y^2 E \Bigg[& e_{om}^2 - e_{om}e_{2m} + \frac{e_{2m}^2}{4} + \gamma^2\left(e_{1m}^2 + \frac{e_{3m}^2}{4} - e_{1m}e_{3m}\right) \\ & + \gamma\left(-\frac{e_{2m}e_{3m}}{2} + e_{1m}e_{2m} + e_{0m}e_{3m} - 2e_{om}e_{1m}\right)\Bigg]. \end{aligned} \tag{3.4}$$

Simplifying Equation (3.4) by taking expectations, it becomes

$$\begin{aligned} \text{MSE}(t_{pm}^{\gamma}) = C_y^2\theta\left(\frac{\alpha}{2-\alpha}\right) \Bigg[& C_y^2 + \frac{u_{40}^*}{4} + \frac{\gamma^2}{4}(u_{04}^*) + \gamma^2 C_x^2 - 2\gamma\rho C_y C_x \\ & - C_y u_{30} + \gamma C_y u_{12} - \frac{\gamma}{2}(u_{22}^*) + \gamma C_x u_{21} - \gamma^2 C_x u_{03}\Bigg]. \end{aligned} \tag{3.5}$$

Equation (3.5) can also be written as

$$\mathrm{MSE}(t_{pm}^{\gamma}) = C_y^2\theta\left(\frac{\alpha}{2-\alpha}\right)\left[C_y^2 + \frac{u_{40}^*}{4} - C_y u_{30} + \gamma^2\left(C_x^2 + \frac{(u_{04}^*)}{4} - C_x u_{03}\right)\right.$$
$$\left. -\gamma\left(2\rho C_y C_x - C_y u_{12} + \frac{(u_{22}^*)}{2} - C_x u_{21}\right)\right]. \tag{3.6}$$

Substituting values of $\gamma = 0, 1,$ and -1 in Equation (3.6), we get MSE(t_{pm}^0), MSE(t_{pm}^r), and MSE(t_{pm}^p) , respectively, as

$$\mathrm{MSE}(t_{pm}^0) = C_y^2\theta\left(\frac{\alpha}{2-\alpha}\right)\left[C_y^2 + \frac{u_{40}^*}{4} - C_y u_{30}\right] \tag{3.7}$$

$$\mathrm{MSE}(t_{pm}^r) = C_y^2\,\theta\left(\frac{\alpha}{2-\alpha}\right)\left[C_y^2 + \frac{u_{40}^*}{4} + \frac{u_{04}^*}{4} + C_x^2 - 2\rho C_y C_x\right.$$
$$\left. -C_y u_{30} + C_y u_{12} - \frac{u_{22}^*}{2} + C_x u_{21} - C_x u_{03}\right] \tag{3.8}$$

$$\mathrm{MSE}(t_{pm}^p) = C_y^2\,\theta\left(\frac{\alpha}{2-\alpha}\right)\left[C_y^2 + \frac{u_{40}^*}{4} + \frac{u_{04}^*}{4} + C_x^2 + 2\rho C_y C_x\right.$$
$$\left. -C_y u_{30} - C_y u_{12} + \frac{u_{22}^*}{2} - C_x u_{21} - C_x u_{03}\right] \tag{3.9}$$

To obtain a mininum $MSE(t_{pm}^{\gamma})$, differentiate equation (3.6) with respect to γ and equate it to zero,

$$\mathrm{Min.MSE}(t_{pm}^{\gamma}) = C_y^2\theta\left(\frac{\alpha}{2-\alpha}\right)\left[C_y^2 + \frac{u_{40}^*}{4} - C_y u_{30} + \gamma^{*2}\left(\frac{(u_{04}^*)}{4} + C_x^2 - C_x u_{03}\right)\right.$$
$$\left. -\gamma^*\left(2\rho C_y C_x - C_y u_{12} + \frac{(u_{22}^*)}{2} - C_x u_{21}\right)\right] \tag{3.10}$$

where

$$\gamma = \frac{1}{2}\,\frac{2\rho C_y C_x - C_y u_{12} + \frac{(u_{22}^*)}{2} - C_x u_{21}}{C_x^2 + \frac{(u_{04}^*)}{4} - C_x u_{03}} = \gamma^*. \tag{3.11}$$

Substituting value of γ given in Equation (3.11) in Equation (3.10), we get Min MSE expression as

$$MinMSE(t_{pm}^{\gamma}) = \theta\left(\frac{\alpha}{2-\alpha}\right)C_y^2\left[C_y^2 + \frac{u_{40}^*}{4} - C_y u_{30} - \frac{\left(\rho C_y C_x - \frac{C_x u_{21}}{2} - \frac{C_y \mu_{12}}{2} + \frac{u_{22}^*}{4}\right)^2}{\left(C_x^2 + \frac{u_{04}^*}{4} - C_x u_{03}\right)}\right]. \tag{3.12}$$

3.2. Memory type regression estimator for CV estimation

In this subsection, we propose the current regression estimator through the use of EWMA statistic as a memory-type regression estimator for CV estimation as follows:

$$t_{pm}^{reg} = W_t + \beta^*\left(C_x - V_t\right). \tag{3.13}$$

Expressing the Equation (3.13) in terms of $e's$, we get

$$t_{pm}^{reg} = C_y\left[\left(1 - e_{om} + \frac{e_{2m}}{2} + e_{om}^2 - \frac{e_{om}e_{2m}}{2} - \frac{e_{2m}^2}{8}\right)\right.$$
$$\left. +\beta^*\left(C_x - C_x\left(1 + \frac{e_{3m}}{2} - e_{1m} - \frac{e_{3m}^2}{8} - \frac{e_{1m}e_{3m}}{2} + e_{1m}^2\right)\right)\right]. \tag{3.14}$$

Further simplifying Equation (3.14) and subtracting C_y, we obtain

$$t_{pm}^{reg} - C_y = C_y \left[\left(1 - e_{om} + \frac{e_{2m}}{2} + e_{om}^2 - \frac{e_{om}e_{2m}}{2} - \frac{e_{2m}^2}{8} \right) \right.$$
$$\left. + \beta^* C_x \left(\frac{e_{3m}}{2} - e_{1m} - \frac{e_{3m}^2}{8} - \frac{e_{1m}e_{3m}}{2} + e_{1m}^2 \right) \right]. \tag{3.15}$$

By squaring Equation (3.15) and considering the expectation, we are able to obtain the MSE formulation as follows:

$$E(t_{pm}^{reg} - C_y)^2 = E \left[C_y^2 \left(e_{0m}^2 + \frac{e_{2m}^2}{4} - e_{om}e_{2m} \right) + \beta^{*2} C_x^2 \left(e_{1m}^2 + \frac{e_{3m}^2}{4} - e_{1m}e_{3m} \right) \right.$$
$$\left. + 2\beta^* C_y C_x \left(-e_{0m}e_{1m} + \frac{e_{1m}e_{2m}}{2} + \frac{e_{om}e_{3m}}{2} - \frac{e_{2m}e_{3m}}{2} \right) \right] \tag{3.16}$$

Through further simplification of Equation (3.16), we obtain

$$MSE(t_{pm}^{reg}) = \theta \left(\frac{\alpha}{2-\alpha} \right) \left[C_y^2 \left(C_y^2 + \frac{u_{40} - 1}{4} - C_y u_{30} \right) + \beta^{*2} C_x^2 \left(C_x^2 + \frac{u_{04} - 1}{4} - C_x u_{03} \right) \right.$$
$$\left. + 2\beta^* C_y C_x \left(-\rho C_y C_x + \frac{C_x u_{21}}{2} + \frac{C_y \mu_{12}}{2} - \frac{u_{22} - 1}{4} \right) \right] \tag{3.17}$$

where

$$\beta^* = \frac{C_y \left(\rho C_y C_x - \frac{C_x u_{21}}{2} - \frac{C_y \mu_{12}}{2} + \frac{u_{22}^*}{4} \right)}{C_x \left(C_x^2 + \frac{u_{04}^*}{4} - C_x u_{03} \right)} = \beta^{*\prime}. \tag{3.18}$$

The value of β^* is substituted in Equation (3.17), resulting in the calculation of $MinMSE(t_{pm}^{reg})$ as follows:

$$MinMSE(t_{pm}^{reg}) = \theta \left(\frac{\alpha}{2-\alpha} \right) C_y^2 \left[C_y^2 + \frac{u_{40}^*}{4} - C_y u_{30} - \frac{\left(\rho C_y C_x - \frac{C_x u_{21}}{2} - \frac{C_y \mu_{12}}{2} + \frac{u_{22}^*}{4} \right)^2}{\left(C_x^2 + \frac{u_{04}^*}{4} - C_x u_{03} \right)} \right] \tag{3.19}$$

4. Mathematical comparison

A mathematical comparison is made between the estimators suggested for time-scaled surveys and the estimators that are already in use with regard to CV estimation. In order to illustrate the effectiveness of the suggested typical memory type estimator in comparison to the existing one, we have investigated the possibility of locating the condition where

$$\text{MSE}(t_{pm}^0) < \text{MSE}(t_p^0)$$
$$\Rightarrow C_y^2 \theta \left(\frac{\alpha}{2-\alpha} \right) \left[C_y^2 + \frac{u_{40}^*}{4} - C_y u_{30} \right] < C_y^2 \theta \left[C_y^2 + \frac{u_{40}^*}{4} - C_y u_{30} \right]$$
$$\Rightarrow \frac{\alpha}{2-\alpha} < 0$$
$$\Rightarrow \alpha < 1$$

This demonstrates that the memory type usual mean estimator for estimating CV will always be more efficient than the simple usual mean estimator. This is because the value of α is between 0 and 1, and when $\alpha = 1$, t_p^0 and t_{pm}^0 are identical. However, when $\alpha < 1$, t_{pm}^0 will always be more effective than t_p^0.

In order to illustrate the efficacy of the memory type ratio estimator that was recommended compared to the ratio type estimator that is currently in use for CV estimation, we have investigated the possibility of finding the condition where

$$\mathrm{MSE}(t^r_{pm}) < \mathrm{MSE}(t^r_p)$$

$$\Rightarrow$$

$$C_y^2\theta\left(\frac{\alpha}{2-\alpha}\right)\left[C_y^2+\frac{u^*_{40}}{4}+\frac{u^*_{04}}{4}+C_x^2-2\rho C_yC_x-C_yu_{30}+C_yu_{12}-\frac{u^*_{22}}{2}+C_xu_{21}-C_xu_{03}\right]$$

$$< C_y^2\theta\left[C_y^2+\frac{u^*_{40}}{4}+\frac{u^*_{04}}{4}+C_x^2-2\rho C_yC_x-C_yu_{30}+C_yu_{12}-\frac{u^*_{22}}{2}+C_xu_{21}-C_xu_{03}\right]$$

$$\Rightarrow \frac{\alpha}{2-\alpha} < 0$$

$$\Rightarrow \alpha < 1$$

This illustrates that the memory type ratio estimator will always be more efficient than the ratio estimator when it comes to calculating CV. This occurs because the value of α falls within the range of 0 to 1, and when α is equal to 1, the values of t^r_p and t^r_{pm} will be identical. On the other hand, when α is less than one, t^r_{pm} will invariably be revealed to be more effective than t^r_p.

In order to illustrate the effectiveness of the suggested memory type product estimator for estimating CV in comparison to the existing one, we have investigated the possibility of locating the condition where,

$$\mathrm{MSE}(t^p_{pm}) < \mathrm{MSE}(t^p_p)$$

$$\Rightarrow$$

$$C_y^2\theta\left(\frac{\alpha}{2-\alpha}\right)\left[C_y^2+\frac{u^*_{40}}{4}+\frac{u^*_{04}}{4}+C_x^2+2\rho C_yC_x-C_yu_{30}-C_yu_{12}+\frac{u^*_{22}}{2}-C_xu_{21}-C_xu_{03}\right]$$

$$< C_y^2\theta\left[C_y^2+\frac{u^*_{40}}{4}+\frac{u^*_{04}}{4}+C_x^2+2\rho C_yC_x-C_yu_{30}-C_yu_{12}+\frac{u^*_{22}}{2}-C_xu_{21}-C_xu_{03}\right]$$

$$\Rightarrow \frac{\alpha}{2-\alpha} < 0$$

$$\Rightarrow \alpha < 1$$

The fact that this is the case illustrates that the memory-type product estimator for calculating CV will always be more efficient than the product estimator that is now operational. This occurs because the value of α falls within the range of 0 to 1, and when α is equal to 1, the values of t^p_p and t^p_{pm} will be identical. Consequently, t^p_{pm} is guaranteed to consistently demonstrate more efficacy than t^p_p. Similarly as above,

$$\mathrm{MSE}(t^\gamma_{pm}) < \mathrm{MSE}(t^{\gamma'}_p)$$

$$\Rightarrow$$

$$\theta\left(\frac{\alpha}{\alpha-1}\right)C_y^2\left[\left(C_y^2+\frac{u^*_{40}}{4}-C_yu_{30}\right)-\frac{\left(\rho C_yC_x-\frac{C_xu_{21}}{2}-\frac{C_y\mu_{12}}{2}+\frac{u^*_{22}}{4}\right)^2}{\left(C_x^2+\frac{u^*_{04}}{4}-C_xu_{03}\right)}\right]$$

$$< \theta C_y^2\left[\left(C_y^2+\frac{u^*_{40}}{4}-C_yu_{30}\right)-\frac{\left(\rho C_yC_x-\frac{C_xu_{21}}{2}-\frac{C_y\mu_{12}}{2}+\frac{u^*_{22}}{4}\right)^2}{\left(C_x^2+\frac{u^*_{04}}{4}-C_xu_{03}\right)}\right]$$

$$\Rightarrow \frac{\alpha}{2-\alpha} < 0$$

$$\Rightarrow \alpha < 1$$

always for $\alpha < 1$. Similarly for

$$\mathrm{MSE}(t^{reg}_{pm}) < \mathrm{MSE}(t^{reg}_p)$$

$\Rightarrow$

$$\theta\left(\frac{\alpha}{\alpha-1}\right)C_y^2\left[\left(C_y^2+\frac{u_{40}^*}{4}-C_yu_{30}\right)-\frac{\left(\rho C_yC_x-\frac{C_xu_{21}}{2}-\frac{C_y\mu_{12}}{2}+\frac{u_{22}^*}{4}\right)^2}{\left(C_x^2+\frac{u_{04}^*}{4}-C_xu_{03}\right)}\right]$$
$$<\theta C_y^2\left[\left(C_y^2+\frac{u_{40}^*}{4}-C_yu_{30}\right)-\frac{\left(\rho C_yC_x-\frac{C_xu_{21}}{2}-\frac{C_y\mu_{12}}{2}+\frac{u_{22}^*}{4}\right)^2}{\left(C_x^2+\frac{u_{04}^*}{4}-C_xu_{03}\right)}\right]$$
$$\Rightarrow\frac{\alpha}{2-\alpha}<0$$
$$\Rightarrow\alpha<1$$

Given that $\alpha<1$ is always true, it follows that MSE(t_{pm}^{reg}) will always be less than MSE(t_p^{reg}). Similarly for

$$\mathrm{MSE}(t_{pm}^{\gamma})<\mathrm{MSE}(t_p^{reg})$$

$\Rightarrow$

$$\theta\left(\frac{\alpha}{\alpha-1}\right)C_y^2\left[\left(C_y^2+\frac{u_{40}^*}{4}-C_yu_{30}\right)-\frac{\left(\rho C_yC_x-\frac{C_xu_{21}}{2}-\frac{C_y\mu_{12}}{2}+\frac{u_{22}^*}{4}\right)^2}{\left(C_x^2+\frac{u_{04}^*}{4}-C_xu_{03}\right)}\right]$$
$$<\theta C_y^2\left[\left(C_y^2+\frac{u_{40}^*}{4}-C_yu_{30}\right)-\frac{\left(\rho C_yC_x-\frac{C_xu_{21}}{2}-\frac{C_y\mu_{12}}{2}+\frac{u_{22}^*}{4}\right)^2}{\left(C_x^2+\frac{u_{04}^*}{4}-C_xu_{03}\right)}\right]$$
$$\Rightarrow\frac{\alpha}{2-\alpha}<0$$
$$\Rightarrow\alpha<1$$

Given that $\alpha<1$ is always true, it follows that MSE(t_{pm}^{γ}) will always be less than MSE(t_p^{reg}).

5. Application

5.1. Application I

This empirical study uses an extensive data set that encompasses agricultural metrics related to rice, specifically focusing on cultivated area, yield, and production in various Indian states over a 6-year period, from 2016 to 2021. The data set is carefully sourced from the respected annual publication Agricultural Statistics at a Glance, released by the Directorate of Economics & Statistics, Department of Agriculture Cooperation & Farmers Welfare (DAC&FW), Government of India.

The main variables of focus are defined as follows: The study variable (Y) is specified as rice yield, precisely measured in kilograms per hectare (Kg/Ha). The Auxiliary variable (X) is defined as the area under rice cultivation, quantified in millions of hectares, which functions as an additional metric to clarify its association with yield variability. To quantify the inherent variability within our primary variables, the coefficient of variation was calculated for both Y and X over the years sampled. The population CV for each variable was obtained by averaging the coefficients calculated on an annual basis, effectively summarizing the overall variability throughout the specified timeframe. As a result, the CV values derived for Y and X, which indicate the statistical dispersion of the yield and area, are given in Table 1, respectively, each highlighting the degree of variability surrounding their respective means.

An EWMA statistic, using a smoothing parameter $\alpha=0.5$, was applied to monitor and manage the trends of yield variability over time. The selected value of α reflects the sensitivity of the EWMA statistic, balancing the weighting of recent observations with the retention of influence from previous data points. This method, recognized for its strength in identifying changes in process behavior, enables a detailed understanding of fluctuations in rice yield over time.

Table 1 presents the calculated CV estimates for each unique sample in data set I, providing a detailed examination of the variability of the yield and the dynamics of the cultivated area over the years. The 1 presents the values of the proposed estimators in conjunction with the CV values, acting as a comparative reference. The effectiveness and precision of these estimators, designed to improve the measurement of variability in rice yield, are thoroughly assessed to determine their dependability to reflect the nuances of annual variations.

Using data set I, the MSE of our proposed estimator is $MSE(t_{pm}^{reg}) = MSE(t_{pm}^{\gamma}) = 0.0001792$, $MSE(t_{pm}^{r}) = 0.000342$ and the MSE of the existing ratio estimator is $MSE(t_p^r) = 0.000972$, $MSE(t_p^{reg}) = MSE(t_p^{\gamma}) = 0.0005723$. The population coefficient of variation for the study variable is $C_y = 0.37254$. We have not computed MSE for t_{pm}^{p}, as the variables we have considered here for the study are positively correlated. Figure 1, clearly demonstrated that t_{pm}^{r} is more efficient than existing estimators.

Table 1. The results of the sample-wise CV estimations for the data set-I

Year	$\hat{c}_y = t_p^0$	$\hat{c}_x$	$W_t = t_{pm}^0$	V_t	t_p^r	t_{pm}^r	t_p^{γ}	t_{pm}^{γ}
2016	0.37188	0.43188	0.37221	0.41144	0.33668	0.35372	0.34556	0.36782
2017	0.42055	0.37693	0.39622	0.40441	0.43624	0.38308	0.38223	0.37884
2018	0.38673	0.39599	0.40364	0.38646	0.38185	0.40838	0.40012	0.38884
2019	0.3623	0.38959	0.37452	0.39279	0.3636	0.3728	0.36452	0.37117
2020	0.34775	0.38226	0.35502	0.38593	0.35569	0.35968	0.35282	0.36122
2021	0.34601	0.36931	0.34688	0.37578	0.36633	**0.36092**	0.36633	0.3732

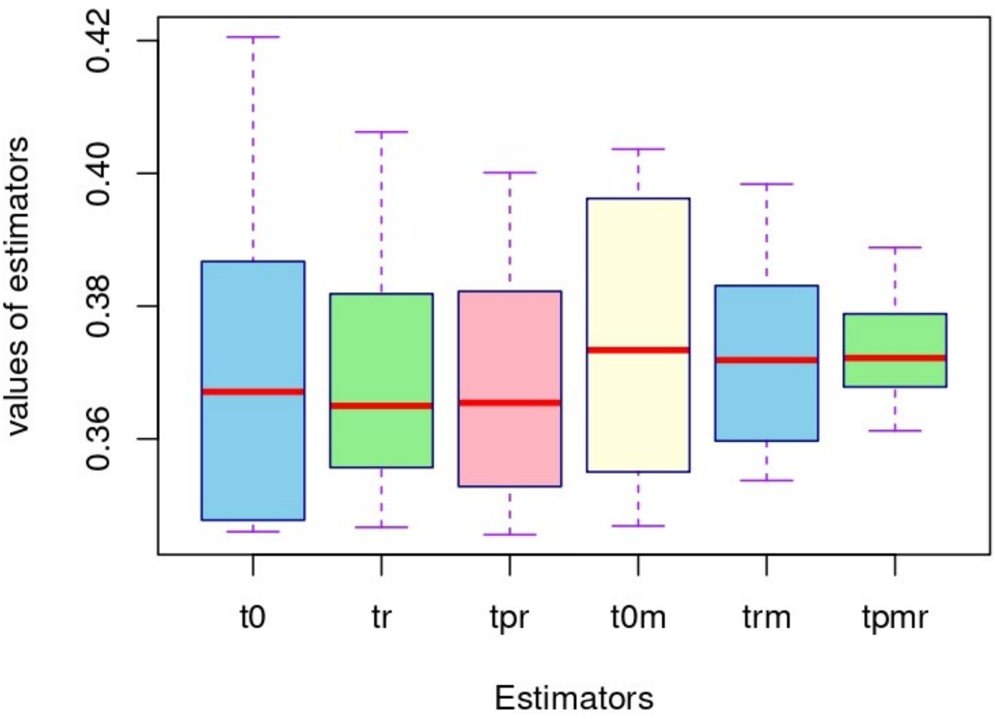

Figure 1. Box-plot for the analysis of data Set-I

5.2. Application II

This empirical analysis uses a secondary dataset from Yan and Su [33], which offers strong real-world data that are well suited for the evaluation of statistical estimators. This data set includes 25 samples, each sample consisting of 5 observations, collected at regular intervals through a Simple Random Sampling Without Replacement (SRSWOR) method. A detailed summary of the variables, population parameters, and important statistical conclusions drawn from these data may be found below.

The study variable (Y) is the density and the auxiliary variable (X) is the stiffness. The population parameters are defined as follows: The study concentrates on several parameters found in this dataset, which form the basis for additional statistical calculations and estimator evaluations. Population refers to $\bar{Y} = 15.47$ and $\bar{X} = 34666.83$. The variances are $S_y^2 = 34.023$, $S_x^2 = 643,900,000$ and $\rho_{YX} = 0.89$.

The strong linear relationship indicated by the high correlation coefficient $\rho_{YX} = 0.89$ between density (Y) and stiffness (X) suggests that X could be a valuable auxiliary variable for enhancing the precision of estimators for Y.

The MSE and the percentage relative efficiency (PRE) of the proposed estimators were calculated to determine their reliability and precision. The metrics offer valuable insight into how the estimators perform compared to the standard approach, facilitating a thorough evaluation. The results of these computations are detailed in Table 2, highlighting the values of the estimators. Using data set II, $MSE(t_{pm}^{reg}) = MSE(t_{pm}^{\gamma}) = 0.00379$, $MSE(t_p^{reg}) = MSE(t_p^{\gamma}) = 0.01214$, $MSE(t_{pm}^{r}) = 0.015857$ and $MSE(t_p^{r}) = 0.02186$ Population coefficient of variation for the study variable is $C_y = 0.37747$ We have not computed the MSE for t_{pm}^{p}, as the variables we have considered here for the study are positively correlated. Figure 2, clearly demonstrated that t_{pm}^{r} is more efficient than existing estimators.

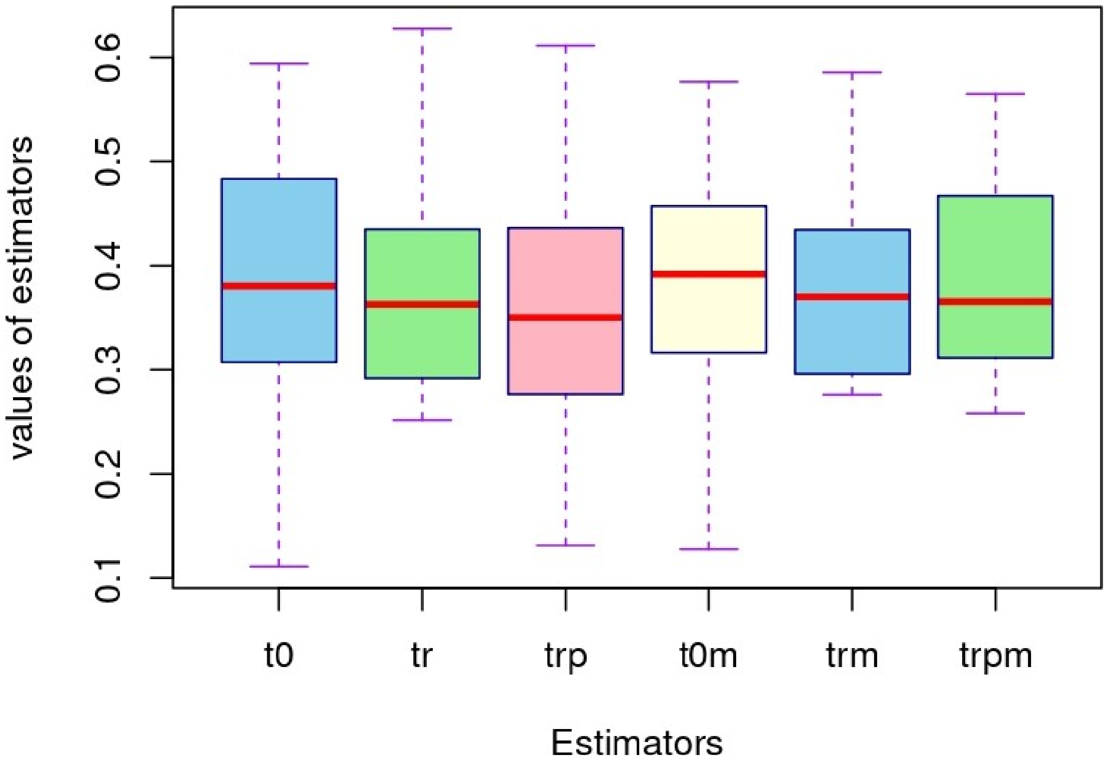

Figure 2. Box-plot for the analysis of data set-II

Table 2. The results of the sample-wise CV estimations for the data set-II.

Sample No.	$\hat{c_y} = t_p^0$	$W_t = t_{pm}^0$	t_p^r	t_{pm}^r	t_p^γ	t_{pm}^γ
1.	0.51984	0.39171	0.33791	0.3717	0.33125	0.35285
2.	0.44747	0.5126	0.42777	0.34423	0.33609	0.39170
3.	0.54371	0.45709	0.30204	0.40759	0.32323	0.34240
4.	0.26305	0.51564	0.38087	0.30527	0.27323	0.49296
5.	0.26979	0.26372	0.28623	0.36841	0.26053	0.21789
6.	0.36831	0.27964	0.31586	0.28981	0.33071	0.28759
7.	0.4256	0.37404	0.62774	0.33479	0.44960	0.36530
8.	0.38071	0.42111	0.34968	0.58564	0.36096	0.48364
9.	0.38599	0.38124	0.37582	0.35216	0.37966	0.36194
10.	0.54594	0.40199	0.40889	0.37999	0.43941	0.38359
11.	0.4833	0.53967	0.88881	0.42967	0.52635	0.51552
12.	0.17419	0.45239	0.30083	0.8266	0.35018	0.54165
13.	0.20594	0.17736	0.26522	0.29621	0.27893	0.30838
14.	0.38049	0.22339	0.55467	0.29109	0.46443	0.28556
15.	0.11111	0.35355	0.54226	0.55427	0.21792	0.40220
16.	0.27526	0.12752	0.27706	0.44941	0.27644	0.25792
17.	0.492	0.29693	0.41872	0.29351	0.39273	0.29033
18.	0.37192	0.47999	0.25161	0.39823	0.25610	0.43026
19.	0.59393	0.39412	0.60706	0.27596	0.41119	0.35251
20.	0.42219	0.57675	0.29766	0.56414	0.38644	0.57484
21.	0.31685	0.41165	0.27624	0.29589	0.29882	0.36367
22.	0.31239	0.31641	0.48172	0.28839	0.31858	0.31470
23.	0.50297	0.33145	0.44194	0.47523	0.44086	0.46752
24.	0.3366	0.48634	0.4034	0.43903	0.36646	0.46691
25.	0.30724	0.33367	0.28298	0.38819	0.28787	0.36540

6. Simulation study

To confirm the results obtained for CV estimation, we carried out a simulation study following these steps.

(1) Initially, a bivariate normal population was created with a size of $N = 10000$, with a mean of $\mu_{XY} = [2, 3]$, $S_y^2 = 4$, and $S_x^2 = 9$.
(2) Subsequently, samples of varying sizes ($n = 50, 100, 200, 500$) were chosen using the SRSWOR population scheme established in Step 1.
(3) Choose the various values for the weight constant $\alpha = 0.1, 0.25, 0.5, 0.75$.
(4) Execute Step 2 a total of 50,000 times and calculate the estimators t_{pm}^γ.
(5) The MSE and PRE values are calculated using the formula provided below:

$$\text{MSE}(t_{pm}^\gamma) = \frac{1}{50000} \sum_{i=1}^{50000} (t_i - C_y)^2$$

Conducting 50,000 replications improves statistical reliability by reducing Monte Carlo standard error (MCSE) and ensuring stable estimates. This approach also facilitates the convergence of essential metrics, such as bias, mean squared error (MSE), and coverage probability, according to the Law of Large Numbers (LLN), while maintaining a balance between computational efficiency and accuracy. To corroborate our theoretical conclusions, we conducted simulation research, with results shown in Tables 3 and 4. The principal outcomes are discussed in the discussion part.

Table 3. The MSE values for the proposed and existing ratio estimators for ($\alpha = 0.1, 0.25, 0.50, 0.75, 0.95$ and 1) for $\rho = (0.25, 0.50, 0.75$ and $0.95)$

				$\alpha = 0.1$		$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$		$\alpha = 0.95$	
ρ	n	t_p^r	t_p^γ	t_{pm}^r	t_{pm}^γ	t_{pm}^r	t_{pm}^γ	t_{pm}^r	t_{pm}^γ	t_{pm}^r	t_{pm}^γ	t_{pm}^r	t_{pm}^γ
0.25	50	0.0917	0.0384	0.0057	0.0023	0.0136	0.0057	0.0305	0.0129	0.0546	0.0230	0.0828	0.0347
	100	0.0403	0.0172	0.0023	0.0009	0.0058	0.0025	0.0134	0.0057	0.0241	0.0103	0.0364	0.0156
	200	0.0190	0.0081	0.0011	0.0004	0.0027	0.0012	0.0063	0.0027	0.0114	0.0049	0.0172	0.0073
	500	0.0073	0.0032	0.0004	0.0002	0.0011	0.0005	0.0025	0.0011	0.0044	0.0019	0.0066	0.0029
0.5	50	0.1098	0.0312	0.0064	0.0018	0.0156	0.0045	0.0355	0.0104	0.0646	0.0186	0.0990	0.0282
	100	0.0472	0.0142	0.0026	0.0008	0.0067	0.0020	0.0155	0.0047	0.0281	0.0085	0.0426	0.0128
	200	0.0222	0.0067	0.0012	0.0004	0.0031	0.0010	0.0073	0.0022	0.0132	0.0040	0.0200	0.0061
	500	0.0085	0.0027	0.0005	0.0002	0.0012	0.0004	0.0028	0.0009	0.0051	0.0016	0.0077	0.0024
0.75	50	0.1341	0.0186	0.0072	0.0011	0.0182	0.0027	0.0422	0.0062	0.0778	0.0111	0.1205	0.0168
	100	0.0562	0.0087	0.0030	0.0005	0.0079	0.0013	0.0183	0.0029	0.0332	0.0052	0.0507	0.0079
	200	0.0262	0.0042	0.0014	0.0002	0.0037	0.0006	0.0086	0.0014	0.0156	0.0025	0.0236	0.0038
	500	0.0100	0.0017	0.0006	0.0001	0.0015	0.0003	0.0033	0.0006	0.0060	0.0010	0.0090	0.0015
0.95	50	0.1584	0.0042	0.0081	0.0003	0.0208	0.0006	0.0488	0.0014	0.0910	0.0025	0.1421	0.0038
	100	0.0651	0.0020	0.0034	0.0001	0.0090	0.0003	0.0210	0.0007	0.0383	0.0012	0.0587	0.0018
	200	0.0301	0.0010	0.0016	0.0001	0.0042	0.0002	0.0098	0.0003	0.0178	0.0006	0.0271	0.0009
	500	0.0114	0.0004	0.0006	0.0000	0.0017	0.0001	0.0038	0.0002	0.0068	0.0003	0.0103	0.0004

Table 4. MSE table for the proposed and existing product estimators for ($\alpha = 0.1, 0.25, 0.50, 0.75, 0.95$ and 1) for $\rho = (-0.25, -0.50, -0.75$ and $-0.95)$.

		$\alpha = 0.1$		$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$		$\alpha = 0.95$		$\alpha = 1$	
ρ	n	t_p^r	t_{pm}^r	t_p^r	t_{pm}^r	t_p^r	t_{pm}^r	t_p^r	t_{pm}^r	t_p^r	t_{pm}^r	t_p^r	t_{pm}^r
-0.25	50	0.0670	0.0048	0.0670	0.0108	0.0670	0.0233	0.0670	0.0408	0.0670	0.0607	0.0670	0.0670
	100	0.0307	0.0019	0.0307	0.0046	0.0307	0.0104	0.0307	0.0185	0.0307	0.0278	0.0307	0.0307
	200	0.0145	0.0008	0.0145	0.0021	0.0145	0.0049	0.0145	0.0087	0.0145	0.0131	0.0145	0.0145
	500	0.0057	0.0003	0.0057	0.0008	0.0057	0.0019	0.0057	0.0034	0.0057	0.0052	0.0057	0.0057
-0.5	50	0.0574	0.0043	0.0574	0.0095	0.0574	0.0204	0.0574	0.0353	0.0574	0.0521	0.0574	0.0574
	100	0.0268	0.0017	0.0268	0.0041	0.0268	0.0092	0.0268	0.0163	0.0268	0.0243	0.0268	0.0268
	200	0.0127	0.0008	0.0127	0.0019	0.0127	0.0043	0.0127	0.0077	0.0127	0.0115	0.0127	0.0127
	500	0.0051	0.0003	0.0051	0.0008	0.0051	0.0017	0.0051	0.0030	0.0051	0.0046	0.0051	0.0051
-0.75	50	0.0510	0.0040	0.0510	0.0087	0.0510	0.0184	0.0510	0.0316	0.0510	0.0464	0.0510	0.0510
	100	0.0242	0.0016	0.0242	0.0038	0.0242	0.0084	0.0242	0.0147	0.0242	0.0219	0.0242	0.0242
	200	0.0115	0.0007	0.0115	0.0017	0.0115	0.0039	0.0115	0.0070	0.0115	0.0104	0.0115	0.0115
	500	0.0046	0.0003	0.0046	0.0007	0.0046	0.0016	0.0046	0.0028	0.0046	0.0042	0.0046	0.0046
-0.95	50	0.0481	0.0038	0.0481	0.0083	0.0481	0.0176	0.0481	0.0300	0.0481	0.0438	0.0481	0.0481
	100	0.0229	0.0015	0.0229	0.0036	0.0229	0.0080	0.0229	0.0140	0.0229	0.0208	0.0229	0.0229
	200	0.0110	0.0007	0.0110	0.0017	0.0110	0.0037	0.0110	0.0066	0.0110	0.0099	0.0110	0.0110
	500	0.0044	0.0003	0.0044	0.0007	0.0044	0.0015	0.0044	0.0026	0.0044	0.0040	0.0044	0.0044

7. Discussion

The findings of the empirical investigation that we conducted in Section 5 using two real data sets are presented in Tables 1 and 2, respectively.

It is evident from the data shown in table 1 that the estimated values of our suggested estimators are quite close to the population CV for each test sample. We used the EWMA statistic for the calculation and the corresponding findings are presented in the box plots shown in Figures 1 and 2. The distribution of the suggested and current estimator values over several samples is depicted in the boxplots; a smaller interquartile range suggests that the EWMA-based estimators provide better stability and efficiency than conventional techniques. For each sample, the calculation of estimated value involves information from the previous sample and the EWMA statistic is employed. Compared to the other values, the box plot that is created for the suggested estimator demonstrates the least amount of variation in the proposed estimates.

In both data sets discussed, the MSE values that were computed for the provided estimators were found to be the lowest possible values. This demonstrates the practical implications of the recommended CV estimation technique in agricultural sectors as well as other fields.

Table 3 presents the simulated results of the mean squared error (MSE) for the proposed estimators (t_p^r and t_{pm}^r)and current estimators across many values of ρ = (0.25,0.50,0.75 and 0.95) and α = (0.25,0.50,0.75,0.95 and 1). Our examination discloses the subsequent findings as follows: As ρ increases from 0.25 to 0.95, the MSE of the suggested estimators consistently decreases across all sample sizes ($n = 50, 100, 200$ and 500), indicating improved estimator precision with elevated values of ρ. This pattern underscores the resilience of the estimator and its appropriateness for data sets exhibiting greater correlations.

We notice a constant drop in MSE as the sample size increases from n = 50 to n = 100,000 and 500, indicating a considerable improvement in estimator efficiency across all correlation coefficients. This trend demonstrates that the precision and reliability of estimators are improved by higher sample sizes, regardless of the amount of correlation between variables. In the process of increasing the value of α from 0.1 to 0.25,0.50,0.75,0.95 and 1; we find a continuous increase in the MSE, which indicates that the data from the past provide valuable stability. When α is equal to one, the MSE of the memory type estimator is identical to that of the existing estimator for all values of ρ and n. This indicates that the estimate depends only on the present data, without any contribution from the information from the past.

We have just included the MSE of t_{pm}^{γ} in Table 3 since the MSE of the suggested estimators (t_{pm}^{γ} and t_{pm}^{reg}) turns out to be the same and is always less than t_p^{reg}. Table 3 makes it clear that the suggested estimators have the lowest MSE. This will make using them to estimate CV in time-scaled surveys the optimal course of action.

Figures 3, 4, 5 and 6 are multiple bar charts that show the MSE of the proposed and current estimators with respect to α. In these figures, c$_r atioisestimator$t$_p^r$, c$_r egisestimator$t$_p^{\gamma}$ or t_p^{reg}, m$_r atioisestimator$t$_{pm}^r$, m$_p ropisestimator$t$_{pm}^{\gamma}$, and m$_r egisestimator$t$_{pm}^{reg}$. These figures show the mean squared error (MSE) for various sample sizes and ρ values; it is easy to see that the MSEs of the suggested estimators decrease as the values of α. The rationale for this is that a higher weight is given to current values and less weight to previous observations when α is greater, and vice versa in the other case. The EWMA statistic is the same as the traditional sample mean when α=1, because all current observations are given full weight. Thus, when $\alpha = 1$, the proposed and current states are identical. For $\rho = 0.25, 0.50, 0.75$, and 0.95, the figures demonstrate that the MSE of the estimators decreases as the correlation coefficients grow, indicating that these estimators are superior options for highly correlated data.

In Table 4 presents the numerical results of the MSE for both the proposed and current product estimators with many values of ρ =(-0.25,-0.50,-0.75 and -0.95) and α =(0.1,0.25,0.50,0.75,0.95 and 1), we observed that the findings are identical to those found in Table 3.

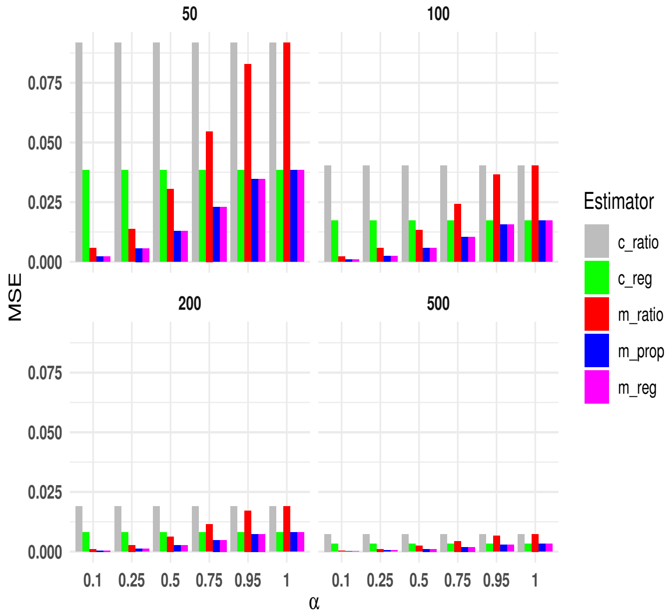

Figure 3. The comparisons for MSE of estimators for $\rho = 0.25$

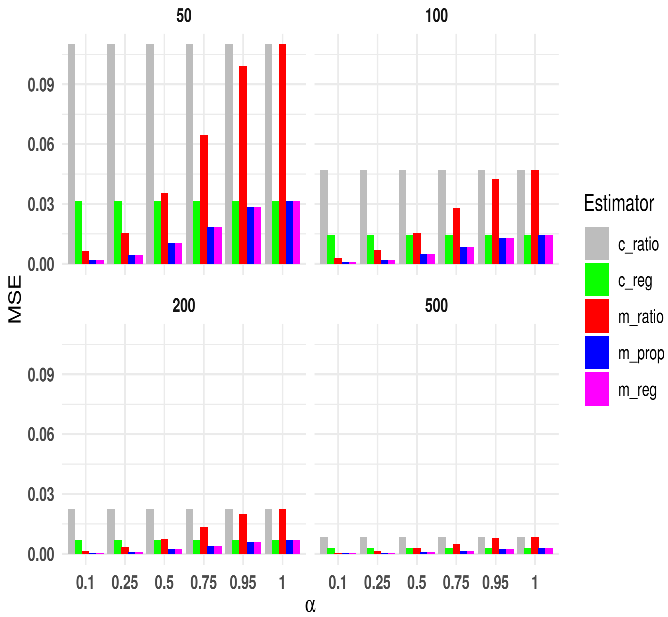

Figure 4. The comparisons MSE of estimators for $\rho = 0.50$

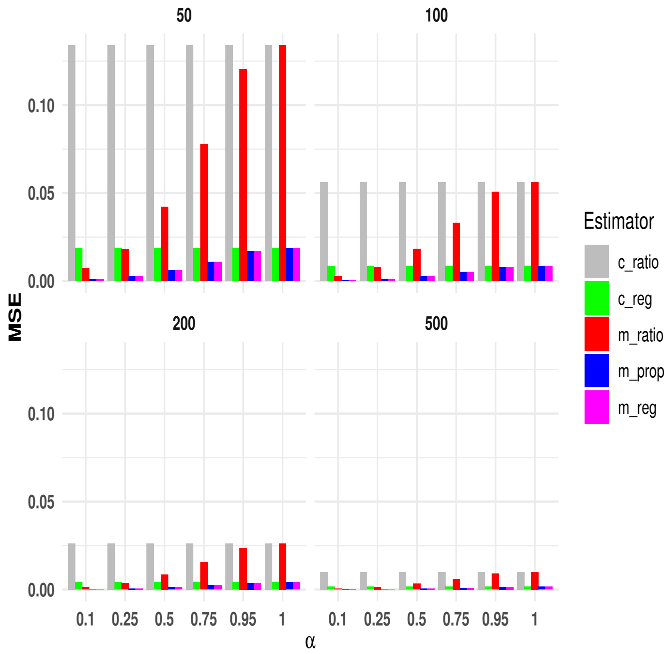

Figure 5. The comparisons for MSE of estimators for $\rho = 0.75$

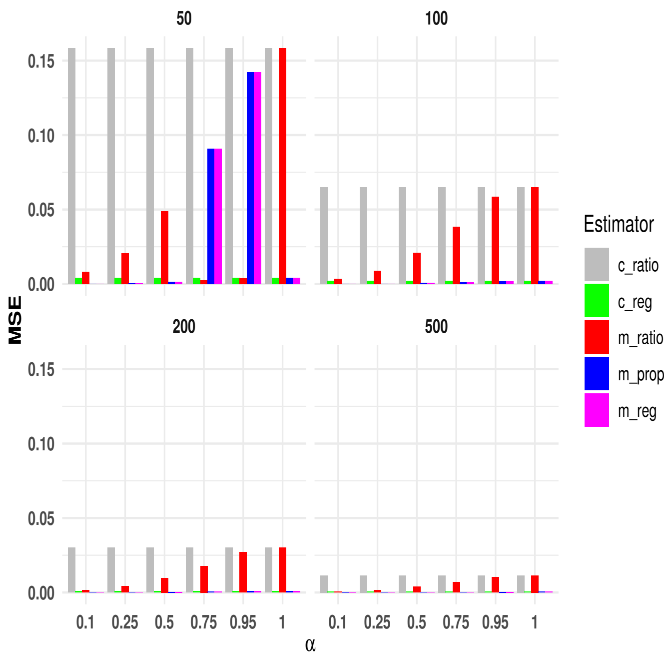

Figure 6. The comparisons for MSE of estimators for $\rho = 0.95$

8. Conclusion

The assessment of CV is critically significant in several domains, especially in agriculture. Although much research has been concentrated on longitudinal surveys, our work presents specialized estimators designed to calculate the coefficient of variation in time-scaled surveys. We propose these novel estimators to improve accuracy in dynamic survey environments, where conventional longitudinal approaches may not adequately address the intricacies of time-dependent variability.

The efficiency of our proposed memory-type CV estimators for time-scaled surveys, which incorporate the EWMA statistic to retain information from past observations, is assessed by computing their MSE. We conducted an empirical analysis using two real datasets to demonstrate the practical applicability of the proposed methods, which proved to be effective, as evidenced by the results presented in the corresponding tables and figures. In addition, to validate the findings in different scenarios, we performed simulation studies that successfully illustrated the core objectives of this investigation. The results indicate that the proposed approach consistently outperforms others in various contexts, making it a reliable choice for time-scaled surveys. Therefore, we recommend the adoption of our memory-type estimators to improve the accuracy of CV estimation.

This research provides a basis for future studies by supporting the creation of more effective estimators to calculate the CV. By examining various sampling frameworks, including ranked set sampling, and incorporating multiple auxiliary variables in time-scaled surveys, the study helps improve the precision and reliability of estimation methods in different analytical contexts.

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