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Medical Waste Management Based on an Interval-Valued Fermatean Fuzzy Decision-Making Method

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Abstract

Due to its infectious and hazardous nature, medical waste poses risks to people and the environment. For patients to receive medical attention and recover in a safe environment, waste must be disposed of correctly. Improper medical waste disposal poses a severe risk to society, which can accelerate the development of various pandemics and epidemics. In this case, medical waste disposal should be handled appropriately. This study presents an integrated multi-criteria decision-making method consisting of entropy, the Pivot Pairwise Relative Criteria Importance Assessment, and Measurement of Alternatives and Ranking according to Compromise Solution methods based on an interval-valued Fermatean fuzzy set. This method can guarantee high safety and security for health practitioners and society through effective modeling and ranking of risks associated with medical waste disposal. Five alternatives and eight criteria were determined. According to the results, incineration is the most suitable disposal process for medical waste. The performance was then assessed and validated using a sensitivity analysis. A sensitivity analysis has been conducted across the range of values for the α parameter. It was examined whether the rankings of the alternatives changed when the α values in the integrated weight determination model for sensitivity analysis were altered. When the different α values were reviewed with the selected α value in the application example, it was seen that incineration was the first alternative. In addition, the study's findings and their consequences for lawmakers, businesspeople, technologists, and practitioners are examined. In the future, these stakeholders can concentrate on these deficiencies and provide long-term remedies.

1. Introduction

Concerns over the health of people and animals are legitimate, given the world's population growth and the rise in pandemic diseases. One of these issues is the appropriate disposal of the enormous volumes of medical waste (MW) produced by hospitals, labs, and other healthcare facilities. Managing the MW generated by their operations is among the most significant issues facing healthcare facilities worldwide [1]. Inappropriately disposing of hazardous goods, such as used needles and personal protective equipment, is risky because MW is communicable. Several pandemic infections can spread more quickly when MW is not managed correctly, which is a severe worry for the general public. The problem of how to dispose of MW in this situation needs careful consideration [2].

MW impacts the environment, the general public, employees, and patients. Selecting a technique for disposing of MW is among the most crucial decisions healthcare institutions must make. This situation is a complicated problem with multiple competing requirements and options. Nevertheless, decision specialists could feel uncertain about themselves when assessing these possibilities. Further, one of the biggest challenges that healthcare organizations confront globally is managing the MW that their activities generate. The improper disposal of hazardous waste poses a significant risk to society and can hasten the spread of several pandemic illnesses. The problem of deciding how to get rid of MW needs to be given serious thought right away. Because a MW method selection problem has several criteria and options, it can be classified as a multi-criteria decision-making(MCDM) problem. However, specialists assessing these issues are limited to linguistically assessing the qualitative criteria. Fuzzy logic addresses the ambiguity in these verbal formulations, while MCDM approaches

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allow for converting these evaluations into a numerical format. This research offers novel fuzzy MCDM approaches for evaluating MW disposal options. This method includes the interval-valued Fermatean Fuzzy Set(IVFFS)-defined entropy, PIPCERIA(the PIvot Pairwise RElative Criteria Importance Assessment), and MARCOS(the Measurement of Alternatives and Ranking based on COmpromise Solution) methods. In the MCDM process, attribute weights play a vital role. In this study, IVFFS-entropy and IVFFS-PIPCERIA methods will be used to calculate objective and subjective weights, and IVFFS-MARCOS methods will be used to rank the alternatives.

1.1. Motivation

Pythagorean Fuzzy sets(PFS) introduced by Yager [3] are more efficient in modeling problems with incomplete information than intuitionistic fuzzy sets(IFS), which is due to the flexibility given in the case of PFSs that the sum of squares of membership and non-membership degrees of any element in the PFSs must be less than or equal to 1. The efficiency obtained using PFSs was further improved by utilizing FFSs introduced by Senapati and Yager [4]. In a practical scenario, many real-life problems involve incomplete and vague information. These problems can be modeled better using FFSs than PFSs. However, it may only be possible to give precise Fermatean fuzzy values for some problems with incomplete information in real-life problems.

The IVFF environment-based DM model can reveal obscure information more flexibly and in detail due to its broad scope of application. Therefore, the DM model within the IVFF context demands more attention. The main advantage of IVFFS is that it can be used to model the problem with incomplete and vague information much better than IVPFSs. For example, let us assume that a decision expert (DE) defines the membership value (support) of an alternative as $[0.58, 0.66]$ and DE defines the non-membership value of an alternative as $[0.69, 0.78]$. Here, the sum of squares of an upper bound of the membership value (0.66^3) and an upper bound of the non-membership value (0.78^3) is greater than 1, the given value is neither IVIFS nor IVPFS. However, they can be considered IVFFS since $0.66^3 + 0.78^3 \leq 1$. Hence, the IVFFSs are more capable of modeling problems with incomplete and imprecise information than the IVIFSs and IVPFSs. The main advantage of IVFFS is that it can be used to model the problem with incomplete and vague information much better than IVPFSs. Studies in the literature show that the IVFFS structure is a more flexible and superior way of modeling the vagueness and imprecision of complex MCDM problems. Experts can use the proposed methodology to assign a two-point interval from a predetermined linguistic scale. The interval values are then converted into IVFFNs, which describe the confirmed and undetermined sections of the assessment in lower and upper approximations. The defined structure of IVFFS offers vast opportunities for uncertainty assessment in MW management. It is very easy to determine with the help of IVFFNs what the main criteria are to choose the most suitable MW method with uncertain information. Again, finding the essential criteria in MW disposal evaluation, selecting the most appropriate method for a sustainable world, and ranking the methods can be quickly done with IVFFNs.

1.2. Literature

In human cognitive and DM activities, quantifying the degrees of membership ($\mathscr{M}$) and non-membership degree($\mathscr{N}$) in a single numeric value is only partially justifiable or technically sound. The usage of interval numbers might be involved when there is a need to provide information as intervals instead of single-valued numbers. Instead, it is convenient for the decision-expert(DE) to employ intervals to express his/her preference for $\mathscr{M}$ and $\mathscr{N}$. In some real DM problems, it may be difficult for DEs to precisely quantify their opinions with a crisp number due to insufficient information. Still, they can be represented by an interval number within $[0, 1]$. Therefore, it is essential to present the idea of IVFFSs, which permit the $\mathscr{M}$ and $\mathscr{N}$ to a given set to have an interval value.

The $\mathscr{M}$'s ambiguity and vagueness were illustrated using [5] 's concept of an FS. Atanassov's intuitionistic FS (IFS) [6] links an element's $\mathscr{N}$ to an item, providing a more comprehensive explanation of assessment data. Kirisci [7] has defined the concept of Fibonacci statistical convergence on intuitionistic fuzzy normed space. Yager [3], [8] developed the Pythagorean FS(PFS) idea to broaden the range of $\mathscr{M}$ and $\mathscr{N}$ so that $\mathscr{M}^2 + \mathscr{N}^2 \leq 1$ in response to the IFS vulnerability previously described. Because of this, PFS offers professionals more evaluation opportunities to express their opinions on various objectives. The complexity of the DM framework increases the difficulties specialists have in producing reliable evaluation data. IFS and PFS have been developed to overcome the ambiguity and vagueness brought on by the intricate subjectivity of human cognition. By adding the cubic sum of $\mathscr{M}$ and $\mathscr{N}$, the FFS was the first to broaden the reach of information assertions. As a result, FFS handles ambiguous choice problems more effectively and practically than IFS and PFS. Senapati and Yager [9] initiated the FFS. Senapati and Yager [4], [10] were the first to give the basic features of FFS.

Ejegwa and Onyeke [11] proposed a three-way approach by adding the degree of hesitation to the correlation coefficients related to IFS. Ejegwa et al. [12] modified the distance operators between IFSs belonging to Szmidt and Kacprzyk. Theorems related to the modified distance operators are proven. To overcome the shortcomings of the distance and similarity measurements given in PFSs, Ejegwa et al. [13] gave new measurements with more reliable performance. In the article by Ejegwa et al. [14], a three-way approach to calculating the correlation coefficients between PFSs is proposed using the concepts of variance and covariance. Ejegwa et al. [15] established an FF-composite relation based on a max-average rule to enhance the viability of FFSs in machine learning via a soft computing approach. An innovative Spearman's type FF correlation coefficient method is built to enhance trustworthy insecurity assessment by Ejegwa et al. [16]. Garg et al. [17] have established general aggregation operators, based on Yager's t-norm and t-conorm, to cumulate the FF data in decision-making environments. In [18], a hybrid MCDM based on IVFF was proposed for risk analysis related to autonomous vehicle driving systems. Kirisci [19] defined new correlation coefficients based on the Fermatean hesitant fuzzy elements and interval-valued Fermatean hesitant fuzzy elements. The least common multiple expansion was used in the newly defined correlation coefficients. In [20], a three-way method for computing the correlation coefficients between FFSs has been given using the notions of variance and covariance. New distance and cosine similarity measures amongst FFSs have been defined [21]. A method was established to construct similarity measures between FFSs based on the cosine similarity and Euclidean distance measures. In [22], a new correlation coefficient and weighted correlation coefficient formularization to evaluate the affair between two FFSs have been proposed. In [23], an extended version of the ELECTRE-I model called the FF ELECTRE-I method for multi-criteria group decision-making with FF human assessments has been presented. Kirisci [24] defined the Fermatean hesitant fuzzy set and gave aggregation operations based on the Fermatean hesitant fuzzy set. The interval-valued

Fermatean fuzzy linguistic Kernel Principal Component Analysis model has been given in [25]. The definition of FF soft sets and some properties were introduced [26]. Furthermore, the Fermatean fuzzy soft entropy and the formulas for standard distance measures, such as Hamming and Euclidean distance, were defined [26]. Riaz et al [27] Fermatean developed fuzzy prioritized weighted average and geometric operators. A new model for group decision-making methods in which experts' preferences can be expressed as incomplete FF-preference relations has been presented [28]. A multi-criteria decision-making strategy to evaluate the risk probabilities of autonomous vehicle driving systems by combining the AHP technique with interval-valued FFSs has been proposed in [29]. First, the interval-valued IFS was described in [30]. It represented the $\mathscr{M}$ and $\mathscr{N}$ by the closed subinterval of the interval $[0,1]$. The interval-valued PFS (IVPFS), whose $\mathscr{M}$ and $\mathscr{N}$ are represented by an interval number, was further proposed by [31]. Several operations and relations of IVPFS are also examined. Jeevaraj defined the IVFFS [32].

PIPRECIA is a subjective weighting model, and MARCOS is a ranking model. Objective and subjective criterion weighting models are the two models found in the literature. The contrast intensity of each criterion and conflicts between criteria are used in this approach to describe the objective relevance of the criteria. A relatively new weight-determining tool that avoids the drawbacks of the SWARA tool while retaining its advantages is the PIPRECIA ([33]). The PIPRECIA model's main advantage is that it allows criteria to be evaluated without being sorted by significance rating. Stevic et al. [34] provided an extension of PIPRECIA under FSs and used it to identify the SWOT matrix's components. Demir et al. [35] used the fuzzy SWARA for prioritizing and ranking the criteria in the wind farm installation and used fuzzy MARCOS to determine the most suitable location for the wind farm. Stevic et al. [36] invented the MARCOS, a tool for DM. Combining the ideal and anti-ideal solutions is the foundation for its development. Additionally, the alternatives' utility is measured, and their rank is then determined by computing various utility functions depending on the value of the alternative utilities. Demir et al. [37] reviewed the studies conducted on MARCOS between 2020 and 2024. Mishra et al. [38] gave an integrated MCDM with PF-fairly operator-based entropy, PIPRECIA, and MARCOS methods to solve the sustainable circular supplier selection problem. Farit et al. [39] prepared a hybrid q-step orthopair fuzzy-based methodology including CRITIC and EDAS and proposed this method as a sustainable approach for smart waste management of road freight vehicles. In [40], new cosine and distance measures were defined with cubic m-polar fuzzy sets, and an application was carried out for a sustainable solid waste treatment and recycling approach.

1.3. Necessity

MW produced by healthcare facilities has the potential to endanger patients, staff, the environment, and the general public. One of the most essential choices that healthcare organizations have to make is how to dispose of MW, and there are several conflicting factors and options to consider. On the other hand, when assessing these options, decision experts could be somewhat unsure. Municipal authorities in developing countries need help in selecting appropriate MW disposal strategies. This process can be framed as an MCDM problem involving tangible and intangible criteria.

The following research topics are intended to be addressed by the hybrid framework established by this study:

i. Which factors must be considered when choosing waste disposal techniques in the healthcare sector?

ii. How can the existing methodologies derive the significance weights of the established evaluation criteria?

iii. How effective is it to rank healthcare waste disposal options using the IVFF-MARCOS method while objectively determining the criteria weights using the IVFF-entropy and IVFF-PIPCERIA approaches?

iv. Which approach is best for eliminating medical waste in general?

v. How do the suggested approaches fare in scenarios involving the distribution of decision expert weights and other criteria?

vi. Compared to other well-known MCDM techniques, how consistent are the results?

1.4. Originality

MW impacts the environment, the general public, employees, and patients. Selecting a technique for disposing of MW is among the most crucial decisions healthcare institutions must make. This is a complicated problem with multiple competing requirements and options. Nevertheless, decision specialists could feel uncertain about themselves when assessing these possibilities. This research offers novel fuzzy MCDM approaches for evaluating MW disposal options. This method includes the IVFFS-defined entropy, PIPCERIA, and MARCOS methods. One of the biggest challenges that healthcare organizations confront globally is managing the MW that their activities generate. The improper disposal of hazardous waste poses a significant risk to society and can hasten the spread of several pandemic illnesses. The problem of deciding how to get rid of MW needs to be given serious thought right away. Because a MW method selection problem has several criteria and options, it can be classified as an MCDM problem. However, specialists assessing these issues are limited to linguistically assessing the qualitative criteria. Fuzzy logic addresses the ambiguity in these verbal formulations, while MCDM approaches allow for converting these evaluations into a numerical format.

1.5. Research gap

This study uses MCDM techniques based on IVFFLs to rank MW disposal techniques, simulate the associated uncertainty, and optimize their advantages. One urgent issue that the recommended methodology addresses is the planning of MW disposal.

The methodology developed for this work goals to fill in the following research gaps that we identify:

(i) Which criteria will be used to assess the effectiveness of the methods to be used in the MW disposal planning?

(ii) Is utilizing IVFFLs the most effective way to eliminate MW?

(iii) In comparison to other well-known MCDM approaches, how consistent are the results?

(iv) What are the primary and secondary requirements required to evaluate the risk of disposing of MW?

(v) Considering the variables considered, which criterion depends on which?

(vi) Which criteria can be critical in the risk assessment by proving their criticality inside the constructed system?

(vii) What are the best practices for eliminating MW?

(viii) What findings can be made based on the relevant analysis and application that have been completed?

1.6. Contribution

The methodological component of the work states that a strategy based on IVFFLs has been developed to rank the possibilities for disposing of MW. An MCDM issue has been examined in a risk assessment of MW disposal. Furthermore, comparative analyses are performed to confirm the accuracy of the recommended decisions and procedures. The application part also makes it clear that the danger issue surrounding the disposal of MW was solely given technical considerations based on assessments of earlier research publications. It is more challenging for safety specialists to take preventive measures later since they must allow the social setting. Therefore, potential issues could arise.

The main contributions:

1. Using the MCDM framework model, which integrates IVFF-entropy, -PIPCERIA, and -MARCOS methodologies, ensures high safety and security for healthcare practitioners and society by effectively modeling and ranking the hazards associated with MW disposal. Using the proposed methodology, the DEs can identify a range of two scale points from the preset language scale. Once the interval data have been translated into IVFFNs, the confirmed and indeterminate components of the evaluation are further described in lower and higher approximation, respectively.

2. Each alternative's criterion separately assesses the risks associated with MW disposal plans.

3. A comparative study is offered to determine how well the suggested model ranks MW disposal methods.

4. In-depth ramifications are provided based on the findings.

2. Preliminaries

Definition 2.1 ([9]). *Let $\ddot{E}$ be the universal set. The FFS is defined as the set $A=\{(x,\mu_A(x),\upsilon_A(x)) : x\in\ddot{E}\}$, where with $0\le\mu_A^3+\upsilon_A^3\le 1$ and $\mu_A,\upsilon_A\in[0,1]$. The hesitation degree has been shown with $\theta_A=(1-\mu_A^3+\upsilon_A^3)^{1/3}$.*

Definition 2.2 ([32]). *Let $\mathscr{I}[0,1]$ show the set of all closed subintervals of the unit interval. The IVFFS is defined as $A=\{(x,\mu_A(x),\upsilon_A(x)) : x\in\ddot{E}\}$, where $\mu_A(x),\upsilon_A(x)\in\mathscr{I}[0,1]$ with $0<\sup_x(\mu_A(x))^3+\sup_x(\upsilon_A(x))^3\le 1$.*

The set $A=\left\{(x,[\mu_A^-(x),\mu_A^+(x)],[\upsilon_A^-(x),\upsilon_A^+(x)]) : x\in\ddot{E}\right\}$, is also defined as IVFFS. Here, $0\le(\mu_A^+(x))^3+(\upsilon_A^+(x))^3\le 1$ and $\theta_A=[\theta_A^-,\theta_A^+]=[(1-(\mu_A^-)^3+(\upsilon_A^-)^3)^{1/3},(1-(\mu_A^+)^3+(\upsilon_A^+)^3)^{1/3}]$.

Definition 2.3 ([32]). *For IVFFSs $A=([\mu_A^-(x),\mu_A^+(x)],[\upsilon_A^-(x),\upsilon_A^+(x)])$, $A_1=([\mu_{A1}^-(x),\mu_{A1}^+(x)],[\upsilon_{A1}^-(x),\upsilon_{A1}^+(x)])$, $A_2=([\mu_{A2}^-(x),\mu_{A2}^+(x)],[\upsilon_{A2}^-(x),\upsilon_{A2}^+(x)])$,*

- $A_1\cup A_2=\left([\max(\mu_{A1}^-,\mu_{A2}^-),\max(\mu_{A1}^+,\mu_{A2}^+)],[\min(\upsilon_{A1}^-,\upsilon_{A2}^-),\min(\upsilon_{A1}^+,\upsilon_{A2}^+)]\right)$
- $A_1\cap A_2=\left([\min(\mu_{A1}^-,\mu_{A2}^-),\min(\mu_{A1}^+,\mu_{A2}^+)],[\max(\upsilon_{A1}^-,\upsilon_{A2}^-),\max(\upsilon_{A1}^+,\upsilon_{A2}^+)]\right)$
- $A^c=([\upsilon_A^-,\upsilon_A^+],[\mu_A^-,\mu_A^+])$
- $A_1\oplus A_2=\Bigg(\Big[\sqrt[3]{(\mu_{A1}^-(x))^3+(\mu_{A2}^-(x))^3-(\mu_{A1}^-(x))^3.(\mu_{A2}^-(x))^3},$ $\sqrt[3]{(\mu_{A1}^+(x))^3+(\mu_{A2}^+(x))^3-(\mu_{A1}^+(x))^3.(\mu_{A2}^+(x))^3}\Big],[\upsilon_{A1}^-\upsilon_{A2}^-,\upsilon_{A1}^+\upsilon_{A2}^+]\Bigg)$
- $A_1\otimes A_2=\Bigg([\mu_{A1}^-\mu_{A2}^-,\mu_{A1}^+\mu_{A2}^+],\Big[\sqrt[3]{(\upsilon_{A1}^-(x))^3+(\upsilon_{A2}^-(x))^3-(\upsilon_{A1}^-(x))^3.((\upsilon_{A2}^-)(x))^3},$ $\sqrt[3]{(\upsilon_{A1}^+(x))^3+(\upsilon_{A2}^+(x))^3-(\upsilon_{A1}^+(x))^3.(\upsilon_{A2}^+(x))^3}\Big]\Bigg)$

- $\lambda A = \left(\left[\sqrt[3]{1-\left(1-(\mu_A^-)^3\right)^{\lambda}}, \sqrt[3]{1-\left(1-(\mu_A^+)^3\right)^{\lambda}} \right], \left[(\upsilon_A^-)^{\lambda}, (\upsilon_A^+)^{\lambda} \right] \right)$
- $A^{\lambda} = \left(\left[(\mu_A^-)^{\lambda}, (\mu_A^+)^{\lambda} \right], \left[\sqrt[3]{1-\left(1-(\upsilon_A^-)^3\right)^{\lambda}}, \sqrt[3]{1-\left(1-(\upsilon_A^+)^3\right)^{\lambda}} \right] \right)$

Definition 2.4 ([41]). *For the IVFFS $A = ([\mu_A^-(x), \mu_A^+(x)], [\upsilon_A^-(x), \upsilon_A^+(x)])$,*

$$\begin{aligned} SC(A) &= \frac{1}{2}\left([(\mu_A^-(x))^3 + (\mu_A^+(x))^3] - [(\upsilon_A^-(x))^3 + (\upsilon_A^+(x))^3] \right) \\ AC(A) &= \frac{1}{2}\left([(\mu_A^-(x))^3 + (\mu_A^+(x))^3] + [(\upsilon_A^-(x))^3 + (\upsilon_A^+(x))^3] \right) \\ \overline{SC}(A) &= \frac{1}{2}\left(SC(A) + 1 \right) \end{aligned}$$

are called score, accuracy, and normalized score functions, respectively, where $SC(A) \in [-1,1]$, $AC(A) \in [0,1]$, and $\overline{SC}(A) \in [0,1]$.

3. Fairly Aggregation Operators for IVFFSs

The fair aggregation operators for FFNs have been defined by [42]. The fair aggregation operators on interval-valued PFSs are presented by [43]. Based on these two studies, fair aggregation operators based on IVFFSs will be defined, and their basic properties will be examined.

For IVFFSs $A_1 = ([(\mu_{A1}^-), (\mu_{A1}^+)], [(\upsilon_{A1}^-), (\upsilon_{A1}^+)])$, $A_2 = ([(\mu_{A2}^-), (\mu_{A2}^+)], [(\upsilon_{A2}^-), (\upsilon_{A2}^+)])$, the fairly operations are defined on FFNs [42], which as

$$\begin{aligned} A_1 \times A_2 = \Bigg\{ \Bigg[& \sqrt[3]{\left(\frac{(\mu_{A1}^-)^3 (\mu_{A2}^-)^3}{(\mu_{A1}^-)^3 (\mu_{A2}^-)^3 + (\upsilon_{A1}^-)^3 (\upsilon_{A2}^-)^3} \right) \times \left(1 - \left(1 - (\mu_{A1}^-)^3 - (\upsilon_{A1}^-)^3\right) \left(1 - (\mu_{A2}^-)^3 - (\upsilon_{A2}^-)^3\right) \right)}, \\ & \sqrt[3]{\left(\frac{(\mu_{A1}^+)^3 (\mu_{A2}^+)^3}{(\mu_{A1}^+)^3 (\mu_{A2}^+)^3 + (\upsilon_{A1}^+)^3 (\upsilon_{A2}^+)^3} \right) \times \left(1 - \left(1 - (\mu_{A1}^+)^3 - (\upsilon_{A1}^+)^3\right) \left(1 - (\mu_{A2}^+)^3 - (\upsilon_{A2}^+)^3\right) \right)} \Bigg], \\ \Bigg[& \sqrt[3]{\left(\frac{(\upsilon_{A1}^-)^3 (\upsilon_{A2}^-)^3}{(\mu_{A1}^-)^3 (\mu_{A2}^-)^3 + (\upsilon_{A1}^-)^3 (\upsilon_{A2}^-)^3} \right) \times \left(1 - \left(1 - (\mu_{A1}^-)^3 - (\upsilon_{A1}^-)^3\right) \left(1 - (\mu_{A2}^-)^3 - (\upsilon_{A2}^-)^3\right) \right)}, \\ & \sqrt[3]{\left(\frac{(\upsilon_{A1}^+)^3 (\upsilon_{A2}^+)^3}{(\mu_{A1}^+)^3 (\mu_{A2}^+)^3 + (\upsilon_{A1}^+)^3 (\upsilon_{A2}^+)^3} \right) \times \left(1 - \left(1 - (\mu_{A1}^+)^3 - (\upsilon_{A1}^+)^3\right) \left(1 - (\mu_{A2}^+)^3 - (\upsilon_{A2}^+)^3\right) \right)} \Bigg] \Bigg\} \end{aligned}$$

$$\begin{aligned} \lambda * A_i = \Bigg\{ \Bigg[& \sqrt[3]{\left(\frac{(\mu_{Ai}^-)^{3\lambda}}{(\mu_{Ai}^-)^{3\lambda} + (\upsilon_{Ai}^-)^{3\lambda}} \right) \times \left(1 - \left(1 - (\mu_{Ai}^-)^3 - (\upsilon_{Ai}^-)^3\right)^{\lambda} \right)}, \\ & \sqrt[3]{\left(\frac{(\mu_{Ai}^+)^{3\lambda}}{(\mu_{Ai}^+)^{3\lambda} + (\upsilon_{Ai}^+)^{3\lambda}} \right) \times \left(1 - \left(1 - (\mu_{Ai}^+)^3 - (\upsilon_{Ai}^+)^3\right)^{\lambda} \right)} \Bigg], \\ \Bigg[& \sqrt[3]{\left(\frac{(\upsilon_{Ai}^-)^{3\lambda}}{(\upsilon_{Ai}^-)^{3\lambda} + (\upsilon_{Ai}^-)^{3\lambda}} \right) \times \left(1 - \left(1 - (\mu_{Ai}^-)^3 - (\upsilon_{Ai}^-)^3\right)^{\lambda} \right)}, \\ & \sqrt[3]{\left(\frac{(\upsilon_{Ai}^+)^{3\lambda}}{(\upsilon_{Ai}^+)^{3\lambda} + (\upsilon_{Ai}^+)^{3\lambda}} \right) \times \left(1 - \left(1 - (\mu_{Ai}^+)^3 - (\upsilon_{Ai}^+)^3\right)^{\lambda} \right)} \Bigg] \Bigg\}, \quad \lambda > 0 \end{aligned}$$

Proposition 3.1. *Take two IVFFSs $A_1 = ([(\mu_{A1}^-), (\mu_{A1}^+)], [(\upsilon_{A1}^-), (\upsilon_{A1}^+)])$, $A_2 = ([(\mu_{A2}^-), (\mu_{A2}^+)], [(\upsilon_{A2}^-), (\upsilon_{A2}^+)])$. For $\lambda > 0$, if $\mu_{A_1} = \upsilon_{A_1}$ and $\mu_{A_2} = \upsilon_{A_2}$,then*

i. $\mu_{A_1 \otimes A_2} = \upsilon_{A_1 \otimes A_2}$,
ii. $\mu_{\lambda * A_1} = \upsilon_{\lambda * A_1}$.

Proof. (i.) If $\mu_{A_1} = \upsilon_{A_1}$ and $\mu_{A_2} = \upsilon_{A_2}$,then $\frac{\mu_{A_1 \otimes A_2}}{\upsilon_{A_1 \otimes A_2}} = 1$ and $\mu_{A_1 \otimes A_2} = \upsilon_{A_1 \otimes A_2}$.
(ii.) Using the (i.), $\mu_{\lambda * A_1} = \upsilon_{\lambda * A_1}$ is obtained. □

Proposition 3.2. *For any two IVFFSs $A_1 = ([(\mu_{A1}^-), (\mu_{A1}^+)], [(\upsilon_{A1}^-), (\upsilon_{A1}^+)])$, $A_2 = ([(\mu_{A2}^-), (\mu_{A2}^+)], [(\upsilon_{A2}^-), (\upsilon_{A2}^+)])$ and $\lambda, \lambda_1, \lambda_2 > 0$,*

i. $A_1 \otimes A_2 = A_2 \otimes A_1$,
ii. $\lambda (A_1 \otimes A_2) = (\lambda * A_1) \otimes (\lambda * A_1)$,
iii. $(\lambda_1 + \lambda_2) * A_i = (\lambda_1 * A_i) \otimes (\lambda_2 * A_i)$.

Definition 3.3. *Consider a set of IVFFNs* $A_i = ([(\mu_{Ai}^-),(\mu_{Ai}^+)],[(\upsilon_{Ai}^-),(\upsilon_{Ai}^+)])$ *and let* ω_i *be a weight of* A_i*. Then, the IVFF fairly weighted aggregation operator is given by*

$$IVFFWF(A_1,A_2,\cdots,A_n) = (\omega_1 * A_1)\otimes(\omega_2 * A_2)\otimes\cdots(\omega_n * A_n).$$

Theorem 3.4. *The aggregation with the IVFFWF operator is an IVFFN and presented by*

$$\begin{aligned} IVFFWF(A_1,A_2,\cdots,A_n) \quad = \quad & \Bigg\{\Bigg[\sqrt[3]{\frac{\prod_{i=1}^n(\mu_i^-)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^-)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^-)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^-)^3-(\mu_i^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^n(\mu_i^+)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^+)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^+)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^+)^3-(\mu_i^+)^3\right)^{\omega_i}\right)}\Bigg], \\ & \Bigg[\sqrt[3]{\frac{\prod_{i=1}^n(\upsilon_i^-)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^-)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^-)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^-)^3-(\mu_i^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^n(\upsilon_i^+)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^+)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^+)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^+)^3-(\mu_i^+)^3\right)^{\omega_i}\right)}\Bigg]\Bigg\}. \end{aligned} \tag{3.1}$$

Proof. To prove this theorem, the Mathematical Induction Principle will be used.

For $n=2$, Equation 3.1 becomes $IVFFWF(A_1,A_2) = (\omega_1 * A_1)\otimes(\omega_2 * A_2)$. Then,

$$\begin{aligned} IVFFWF(A_1,A_2) \quad = \quad & \Bigg\{\Bigg[\sqrt[3]{\frac{\prod_{i=1}^2(\mu_i^-)^{3\omega_i}}{\prod_{i=1}^2(\mu_i^-)^{3\omega_i}+\prod_{i=1}^2(\upsilon_i^-)^{3\omega_i}}\times\left(1-\prod_{i=1}^2\left(1-(\mu_i^-)^3-(\mu_i^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^2(\mu_i^+)^{3\omega_i}}{\prod_{i=1}^2(\mu_i^+)^{3\omega_i}+\prod_{i=1}^2(\upsilon_i^+)^{3\omega_i}}\times\left(1-\prod_{i=1}^2\left(1-(\mu_i^+)^3-(\mu_i^+)^3\right)^{\omega_i}\right)}\Bigg], \\ & \Bigg[\sqrt[3]{\frac{\prod_{i=1}^2(\upsilon_i^-)^{3\omega_i}}{\prod_{i=1}^2(\mu_i^-)^{3\omega_i}+\prod_{i=1}^2(\upsilon_i^-)^{3\omega_i}}\times\left(1-\prod_{i=1}^2\left(1-(\mu_i^-)^3-(\mu_i^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^2(\upsilon_i^+)^{3\omega_i}}{\prod_{i=1}^2(\mu_i^+)^{3\omega_i}+\prod_{i=1}^2(\upsilon_i^+)^{3\omega_i}}\times\left(1-\prod_{i=1}^2\left(1-(\mu_i^+)^3-(\mu_i^+)^3\right)^{\omega_i}\right)}\Bigg]\Bigg\}. \end{aligned}$$

That is Equation 3.1 holds for $n=2$. Now, let Equation 3.1 hold for $n=k$. Then, it will be shown that Equation 3.1 is valid for $n=k+1$:

$$\begin{aligned} IVFFWF(A_1,A_2,\cdots,A_n,A_{n+1}) \quad = \quad & \Bigg\{\Bigg[\sqrt[3]{\frac{\prod_{i=1}^n(\mu_i^-)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^-)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^-)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^-)^3-(\mu_i^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^n(\mu_i^+)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^+)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^+)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^+)^3-(\mu_i^+)^3\right)^{\omega_i}\right)}\Bigg], \\ & \Bigg[\sqrt[3]{\frac{\prod_{i=1}^n(\upsilon_i^-)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^-)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^-)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^-)^3-(\mu_i^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^n(\upsilon_i^+)^{3\omega_i}}{\prod_{i=1}^n(\mu_i^+)^{3\omega_i}+\prod_{i=1}^n(\upsilon_i^+)^{3\omega_i}}\times\left(1-\prod_{i=1}^n\left(1-(\mu_i^+)^3-(\mu_i^+)^3\right)^{\omega_i}\right)}\Bigg]\Bigg\} \\ & \otimes\sqrt[3]{\frac{(\mu_{k+1}^-)^{3\omega_{k+1}}}{(\mu_{k+1}^-)^{3\omega_{k+1}}+(\upsilon_{k+1}^-)^{3\omega_{k+1}}}\times\left(1-\left(1-(\mu_{k+1}^-)^3-(\mu_{k+1}^-)^3\right)^{\omega_{k+1}}\right)}, \\ & \sqrt[3]{\frac{(\mu_{k+1}^+)^{3\omega_{k+1}}}{(\mu_{k+1}^+)^{3\omega_{k+1}}+(\upsilon_{k+1}^+)^{3\omega_{k+1}}}\times\left(1-\left(1-(\mu_{k+1}^+)^3-(\mu_{k+1}^+)^3\right)^{\omega_{k+1}}\right)}\Bigg], \\ & \Bigg[\sqrt[3]{\frac{(\upsilon_{k+1}^-)^{3\omega_{k+1}}}{(\mu_{k+1}^-)^{3\omega_{k+1}}+(\upsilon_{k+1}^-)^{3\omega_{k+1}}}\times\left(1-\left(1-(\mu_{k+1}^-)^3-(\mu_{k+1}^-)^3\right)^{\omega_{k+1}}\right)}, \\ & \sqrt[3]{\frac{(\upsilon_{k+1}^+)^{3\omega_{k+1}}}{(\mu_{k+1}^+)^{3\omega_{k+1}}+(\upsilon_{k+1}^+)^{3\omega_{k+1}}}\times\left(1-\left(1-(\mu_{k+1}^+)^3-(\mu_{k+1}^+)^3\right)^{\omega_{k+1}}\right)}\Bigg]\Bigg\}. \end{aligned}$$

Using Definition 3.3, it is seen that Equation 3.1 is valid for $n=k+1$. That is, Equation 3.1 is true for all n. □

Definition 3.5. *Consider a set of IVFFNs $A_i = ([(\mu_{Ai}^-),(\mu_{Ai}^+)],[(\upsilon_{Ai}^-),(\upsilon_{Ai}^+)])$ and let ω_i be a weight of A_i. Let $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ be signify permutation of $(1,2,\cdots,n)$ with $A_{\sigma(i-1)} \geq A_{\sigma(i)}$. Then, the IVFF fairly ordered weighted aggregation operator is given by*

$$IVFFOWF(A_1,A_2,\cdots,A_n) = (\omega_1 * F_{\sigma(1)}) \otimes (\omega_2 * F_{\sigma(2)}) \otimes \cdots (\omega_n * A_{\sigma(n)}).$$

Theorem 3.6. *The aggregation with the IVFFOWFF operator is an IVFFN and presented by*

$$\begin{aligned} IVFFOWF(A_1,A_2,\cdots,A_n) \quad = \quad & \Bigg\{\Bigg[\sqrt[3]{\frac{\prod_{i=1}^n(\mu_{\sigma(i)}^-)^{3\omega_i}}{\prod_{i=1}^n(\mu_{\sigma(i)}^-)^{3\omega_i}+\prod_{i=1}^n(\upsilon_{\sigma(i)}^-)^{3\omega_i}} \times \left(1-\prod_{i=1}^n\left(1-(\mu_{\sigma(i)}^-)^3-(\mu_{\sigma(i)}^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^n(\mu_{\sigma(i)}^+)^{3\omega_i}}{\prod_{i=1}^n(\mu_{\sigma(i)}^+)^{3\omega_i}+\prod_{i=1}^n(\upsilon_{\sigma(i)}^+)^{3\omega_i}} \times \left(1-\prod_{i=1}^n\left(1-(\mu_{\sigma(i)}^+)^3-(\mu_{\sigma(i)}^+)^3\right)^{\omega_i}\right)}\Bigg], \\ & \Bigg[\sqrt[3]{\frac{\prod_{i=1}^n(\upsilon_{\sigma(i)}^-)^{3\omega_i}}{\prod_{i=1}^n(\mu_{\sigma(i)}^-)^{3\omega_i}+\prod_{i=1}^n(\upsilon_{\sigma(i)}^-)^{3\omega_i}} \times \left(1-\prod_{i=1}^n\left(1-(\mu_{\sigma(i)}^-)^3-(\mu_{\sigma(i)}^-)^3\right)^{\omega_i}\right)}, \\ & \sqrt[3]{\frac{\prod_{i=1}^n(\upsilon_{\sigma(i)}^+)^{3\omega_i}}{\prod_{i=1}^n(\mu_{\sigma(i)}^+)^{3\omega_i}+\prod_{i=1}^n(\upsilon_{\sigma(i)}^+)^{3\omega_i}} \times \left(1-\prod_{i=1}^n\left(1-(\mu_{\sigma(i)}^+)^3-(\mu_{\sigma(i)}^+)^3\right)^{\omega_i}\right)}\Bigg]\Bigg\}. \end{aligned}$$

Proof. It can be proven in a similar way to Theorem 3.4. □

Using Theorems 3.4 and 3.6, the following properties can be given:

Idempotency: If all IVFFNs $A_i = ([(\mu_{Ai}^-),(\mu_{Ai}^+)],[(\upsilon_{Ai}^-),(\upsilon_{Ai}^+)])$ are identical, i.e., $A_i = A$, then $IVFFWF(A_1,A_2,\cdots,A_n) = A$ and $IVFFWF(A_1,A_2,\cdots,A_n) = A$.

Boundedness: For a set of IVFFNs $A_i = ([(\mu_{Ai}^-),(\mu_{Ai}^+)],[(\upsilon_{Ai}^-),(\upsilon_{Ai}^+)])$, let $A^- = ([\min_i(\mu_{Ai}^-),\min_i(\mu_{Ai}^+)],[\max_i(\upsilon_{Ai}^-),\max_i(\upsilon_{Ai}^+)])$ and $A^+ = ([\max_i(\mu_{Ai}^-),\max_i(\mu_{Ai}^+)],[\min_i(\upsilon_{Ai}^-),\min_i(\upsilon_{Ai}^+)])$. Then

$$A^- \leq IVFFWF(A_1,A_2,\cdots,A_n) \leq A^+,$$
$$A^- \leq IVFFOWF(A_1,A_2,\cdots,A_n) \leq A^+.$$

Monotonicity: Consider $A_i = ([(\mu_{Ai}^-),(\mu_{Ai}^+)],[(\upsilon_{Ai}^-),(\upsilon_{Ai}^+)])$ and $A_i* = ([(\mu_{Ai}^-)*,(\mu_{Ai}^+)*],[(\upsilon_{Ai}^-)*,(\upsilon_{Ai}^+)]*)$ be collections of IVFFNs. If $(\mu_{Ai}^-)* \leq (\mu_{Ai}^-)$, $(\mu_{Ai}^+)* \leq (\mu_{Ai}^+)$, $(\upsilon_{Ai}^-)* \geq (\upsilon_{Ai}^-)$ and $(\upsilon_{Ai}^+)* \geq (\upsilon_{Ai}^+)$, then

$$IVFFWF(A_1*,A_2*,\cdots,A_n*) \leq IVFFWF(A_1,A_2,\cdots,A_n),$$
$$IVFFOWF(A_1*,A_2*,\cdots,A_n*) \leq IVFFOWF(A_1,A_2,\cdots,A_n).$$

4. IVFF-entropy Measure

Peng and Li [44] have given IVPFS-similarity measures, -distance measures, and -entropy. Kirisci [26] has presented a definition of the FF soft entropy and also acquired the formulae for standard distance measures such as Hamming and Euclidean distance. Based on Kirisci [26] entropy measure for FFSS we develop new entropy measure for IVFFS.

Definition 4.1. *Let A and B be two IVFFSs. A real-valued function $E : IVFFS(\ddot{E}) \to [0,1]$ is called an entropy for IVFFSs with the following properties:*

Ent1. $0 \leq E(A) \leq 1$,
Ent2. $E(A) = 0$ *if and only if A is a crisp set,*
Ent3. $E(A) = 1$ *if and only if* $(\mu_{Ai}^-)(x_i) = (\upsilon_{Ai}^-)(x_i)$, $(\mu_{Ai}^+)(x_i) = (\upsilon_{Ai}^+)(x_i)$ *for each* $x_i \in \ddot{E}$,
Ent4. $E(A) = E(A^c)$,
Ent5. $E(A) \leq E(B)$ *if and only if*

- *If* $(\mu_{Ai}^-)(x_i) \leq (\upsilon_{Ai}^-)(x_i)$, $(\mu_{Ai}^+)(x_i) \leq (\upsilon_{Ai}^+)(x_i)$, *then* $A \subseteq B$,
- *If* $(\mu_{Ai}^-)(x_i) \geq (\upsilon_{Ai}^-)(x_i)$, $(\mu_{Ai}^+)(x_i) \geq (\upsilon_{Ai}^+)(x_i)$, *then* $A \supseteq B$.

Theorem 4.2. *The entropy measure is given as*

$$E_t(A) \quad = \quad 1-\sqrt[t]{\frac{1}{2n}\sum_{i=1}^n\left(\left|((\mu_{Ai}^-)(x_i))^3-((\upsilon_{Ai}^-)(x_i))^3\right|^t+\left|((\mu_{Ai}^+)(x_i))^3-((\upsilon_{Ai}^+)(x_i))^3\right|^t\right)}. \tag{4.1}$$

Proof. It must be shown that $E_t(A)$ satisfies the conditions in Definition 4.1. It is straightforward to show axioms $[Ent1.]-[Ent4.]$. To save space, let's prove only axiom $[Ent5.]$.
1. If $(\mu_{Ai}^-)(x_i) \leq (\upsilon_{Ai}^-)(x_i)$, $(\mu_{Ai}^+)(x_i) \leq (\upsilon_{Ai}^+)(x_i)$, then $A \subseteq B$: We have,

$$(\mu_{Ai}^-)(x_i) \leq \mu_{Bi}^-(x_i) \leq \upsilon_{Bi}^-(x_i) \leq (\upsilon_{Ai}^-)(x_i),$$
$$(\mu_{Ai}^+)(x_i) \leq \mu_{Bi}^+(x_i) \leq \upsilon_{Bi}^+(x_i) \leq (\upsilon_{Ai}^+)(x_i).$$

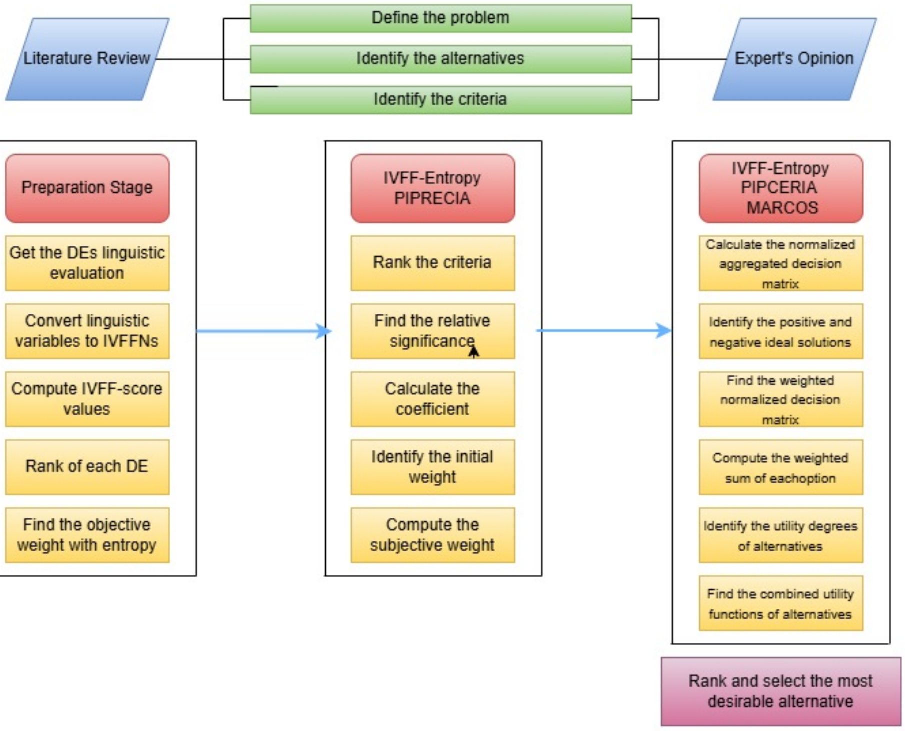

Figure 5.1: Flowchart of method

Therefore,

$$\left|((\mu_{Ai}^-)(x_i))^3 - ((\upsilon_{Ai}^-)(x_i))^3\right| \geq \left|(\mu_{Bi}^-(x_i))^3 - (\upsilon_{Bi}^-(x_i))^3\right|,$$
$$\left|((\mu_{Ai}^+)(x_i))^3 - ((\upsilon_{Ai}^+)(x_i))^3\right| \geq \left|(\mu_{Bi}^+(x_i))^3 - (\upsilon_{Bi}^+(x_i))^3\right|.$$

Thus,

$$1-\sqrt[t]{\frac{1}{2n}\sum_{i=1}^{n}\left(\left|((\mu_{Ai}^-)(x_i))^3 - ((\upsilon_{Ai}^-)(x_i))^3\right|^t + \left|((\mu_{Ai}^+)(x_i))^3 - ((\upsilon_{Ai}^+)(x_i))^3\right|^t\right)}$$
$$\leq 1-\sqrt[t]{\frac{1}{2n}\sum_{i=1}^{n}\left(\left|(\mu_{Bi}^-(x_i))^3 - (\upsilon_{Bi}^-(x_i))^3\right|^t + \left|(\mu_{Bi}^+(x_i))^3 - (\upsilon_{Bi}^+(x_i))^3\right|^t\right)}.$$

So, $E_t(A) \leq E_t(B)$.

2. If $(\mu_{Ai}^-)(x_i) \geq (\upsilon_{Ai}^-)(x_i)$, $(\mu_{Ai}^+)(x_i) \geq (\upsilon_{Ai}^+)(x_i)$, then $A \supseteq B$: Similarly, It can be shown that $E_t(A) \leq E_t(B)$ is the same for this condition. □

5. Proposed Method

When there are several alternative outcomes of a particular event, but their likelihood is unknown, this is known as uncertainty. As a result, the DM must comprehend uncertainty. It takes time and effort to comprehend the likelihood that events will occur in reality. Consequently, there is uncertainty at every stage of the DM process. A solid basis for logical reasoning with vague and imperfect data is provided by fuzzy logic theory. Thanks to fuzzy logic theory, computers can understand human language and apply human knowledge. At this point, it starts employing symbols instead of numerical expressions. Fuzzy sets (FS) are symbolic expressions of this type. FSs are known to include choice variables, such as probability states. FSs are generated when each alternative is assigned an objective membership degree ($\mathscr{M}$) instead of the corresponding probability values.

This study proposes an integrated approach to treating the MCDM problems under IVFFSs by integrating the IVFF-entropy, -PIPRECIA, and -MARCOS tools. The core principles of IVFFSs and the corresponding MCDM techniques have been described independently utilizing pseudo representations (Figure 5.1).

Using the newly created IVFF-entropy and suggested IVPF fairly WFA operator, we provide a hybrid integrated IVFF-entropy-PIPRECIA-MARCOS model. Here, the criteria weights are estimated using the entropy-PIPRECIA model, which is discussed in the context of IVFFS. At the same time, the integrated IVFF-MARCOS model is used to determine the options' rank.

Consider the DEs set $U = \{U_1, U_2, \cdots, U_t\}$ to choice the appropriate option among a set of options $S = \{S_1, S_2, \cdots, S_m\}$ over th criterion set $D = \{D_1, D_2, \cdots, D_n\}$. Suppose that the $M = (\mu_{ij})_{m\times n}$ be a linguistic decision matrix (DEMA) for DEs. Therefore, convert it into an FF-DEMA using linguistic scales.

In order to find the weight of DEs, the significance ratings of DEs are primarily defined by linguistic values and then expressed in the form of FFNs. Let $F_s = ([\mu_{F_{sL}}, \mu_{F_{sU}}], [\upsilon_{F_{sL}}, \upsilon_{F_{sU}}])$ be a IVFFN of sth DE. Hence, the expression for finding the weight is given by

$$\vartheta_s = \frac{1}{2}\left(\frac{\frac{1}{2}\left((\mu_{Fs}^-)^3 + (\mu_{Fs}^+)^3 - (\upsilon_{Fs}^-)^3 - (\upsilon_{Fs}^+)^3\right) + 1}{\sum_{s=1}^t \left[\frac{1}{2}\left((\mu_{Fs}^-)^3 + (\mu_{Fs}^+)^3 - (\upsilon_{Fs}^-)^3 - (\upsilon_{Fs}^+)^3\right) + 1\right]} + \frac{s - \overline{SC}(s) + 1}{\sum_{s=1}^t \left(s - \overline{SC}(s) + 1\right)}\right) \tag{5.1}$$

where $\vartheta_s \geq 0$ and $\sum_{s=1}^t \vartheta_s = 1$.

The steps of our proposed method shown in Figure 5.1 are given below:

Entropy assesses the aspirational information by the substance of confirmed information. The entropy could estimate the vague information. The information entropy can adjust the course of DM because existent contrasts among plentiful details can measure it and clarify the internal information for DEs. A novel entropy approach for calculating the objective weights is presented in Theorem 4.2.

Determine objective weight of each criterion with IVFF-entropy:

For an information entropy $E_j = \dfrac{1}{m}\sum_{i=1}^m E_{ij}$ $j = 1, 2, \cdots, n$ of criteria, then the objective weights

$$\omega_j^O = \frac{1 - E_j}{\sum_{j=1}^n (1 - E_j)}. \tag{5.2}$$

where E_{ij} signifies the entropy and given in Equation 4.1.

We use the IVFF-PIPRECIA model for subjective weights. In this approach, first, the appropriate assessment criteria are considered, and their expected significance using the FF-score function rating is found.

Step 1: Starting with the second criterion, DMs evaluate the criteria in order to obtain the relative importance of the criteria, given as

$$s_j = \begin{cases} 1 + [\overline{SC}(U_j) - \overline{SC}(U_{j-1})] & , \quad \textit{if} \quad U_j > U_{j-1}, \\ 1 & , \quad \textit{if} \quad U_j = U_{j-1}, \\ 1 - [\overline{SC}(U_{j-1}) - \overline{SC}(U_j)] & , \quad \textit{if} \quad U_j < U_{j-1}, \end{cases} \tag{5.3}$$

where U_j and U_{j-1} symbolize the significance rating of the criterion jth and $(j-1)$th criterion, respectively.

Step 2: Based on relative significance, compute the coefficient by

$$K_j = \begin{cases} 1 & , \quad \textit{if} \quad j = 1, \\ 2 - s_j & , \quad \textit{if} \quad j > 1. \end{cases} \tag{5.4}$$

Step 3: Determine the initial weight by

$$Q_j = \begin{cases} 1 & , \quad \textit{if} \quad j = 1, \\ \frac{Q_{j-1}}{K_j} & , \quad \textit{if} \quad j > 1. \end{cases}$$

Step 4: Obtain the subjective weight of jth criterion by

$$\omega_j^s = \frac{Q_j}{\sum_{j=1}^n Q_j}, \quad \forall j.$$

Step 5: An integrated weight-determining model is presented as

$$\omega_j = \alpha\omega_j^O + (1 - \alpha)\omega_j^s \tag{5.5}$$

to get the benefits of objective and subjective weighting models, where $j = 1, 2, \cdots, n$ and $\alpha \in [0, 1]$ represents the strategic coefficient to assess the changes of criterion weights.

The FF-MARCOS method is given, which describes the association between options and the ideal and negative-ideal alternatives on IVFF-information.

Step 6: Normalized the aggregated FF-DEMA by

$$N_{ij} = \begin{cases} ([\mu_{ijF}^-, \mu_{ijF}^+], [\upsilon_{ijF}^-, \upsilon_{ijF}^+]) & , \quad \textit{for benefit-type criteria}, \\ ([\upsilon_{ijF}^-, \upsilon_{ijF}^+], [\mu_{ijF}^-, \mu_{ijF}^+]) & , \quad \textit{for cost-type criteria}. \end{cases}$$

Step 7: Computing positive ideal solutions and negative ideal solutions by

$$N_j^+ = \max_i N_{ij} \quad and \quad N_j^- = \min_i N_{ij}.$$

Step 8: Find the weighted normalized IVFF-DEMA using the Equation 3.1.

Step 9: Obtain the weighted sum of each option using score function by

$$S_i = \sum_{j=1}^{n} \bar{(}SC)(N_{ij}), \tag{5.6}$$

where $\bar{(}SC)(N_{ij})$ signifies the score values of each element of the weighted normalized IVFF-DEMA.

Step 10: Computing the utility degree with the following equations:

$$U_i^- = \frac{S_i}{S_{PIS}} \quad and \quad U_i^+ = \frac{S_i}{S_{NIS}},$$

where S_{PIS} and S_{NIS} signify the sum of score degrees of weighted values N_j^+ and N_j^-.

Step 11: For $f(U_i^+) = \frac{U_i^-}{U_i^- + U_i^+}$ and $f(U_i^-) = \frac{U_i^+}{U_i^- + U_i^+}$, the final values of utility functions by

$$f(U_i) = \frac{U_i^+ + U_i^-}{1 + \frac{1 - f(U_i^+)}{f(U_i^+)} + \frac{1 - f(U_i^-)}{f(U_i^-)}}. \tag{5.7}$$

Step 12: Rank the options according to the Equation 5.7. The appropriate choice is the maximum values obtained from Equation 5.7.

6. Results

6.1. Problem design

In the healthcare industry, waste management is essential. Waste handlers, the public, and medical professionals risk getting sick, suffering negative consequences, getting hurt, or contaminating the environment when MW is not properly managed. To reduce the adverse health effects of destructive behaviors, like exposure to infectious germs and toxic substances, managing MW requires greater attention and commitment. For progress to be universal, long-lasting, and sustainable, government assistance and effort are needed [45]. Because MW infects other animals with infectious diseases, it affects the ecology. Hospital employees risk infection from these MWs, which could harm their health. Along with other essential components and workable solutions, planning for MW management is an important issue that needs to be addressed. Figure 5.1 depicts the MW disposal selection problem in an MCDM architecture.

Incineration is a common disposal technique in underdeveloped countries due to its ease of use, safety, and practicality [46]. More important than 800 degrees Celsius is the temperature at the incinerator's exit. Most organic materials can be burned at this high temperature, eliminating pathogens and converting them to inorganic dust. After incineration, solid waste can be cut by $85-90\%$ [47]. Burning MWs other than radioactive and explosive wastes is an option. Several facilities offer waste processing, incineration, and flue gas purification as additional services for hospital trash combustion. Pyrolysis vaporization, plasma, and rotary kiln incinerators are examples of standard incineration technologies [48].

Chemical disinfection has several uses, and it has been used for a long time. Hospital trash is frequently crushed using mechanical and chemical methods. Chemical disinfectants are frequently mixed with crushed hospital trash and left to sit correctly. Pathogenic bacteria are either killed or put to sleep by disinfection. Organic substances decompose. Because of their low effective concentration, rapid action, homogeneity, and broad sterilizing spectrum, chemical disinfectants effectively eliminate bacteria and germs [49]. Because they are colorless, tasteless, safe, odorless, and readily soluble in water, chemicals like calcium hypochlorite, sodium hypochlorite, and chlorine dioxide are frequently used. Additionally, they have little toxicity, are resistant to both physical and chemical agents, and, once disinfected, pose no concern. When hospital waste volumes are minimal, chemical disinfection methods can be considered.

Encapsulation renders trash immobile by encasing trash in a solid matrix [50]. The nuclear industry's preferred approach to handling low- and intermediate-level radioactive waste has long been encapsulation in cement or its composites. Before disposing of waste in landfills or geological locations, medical sciences enclose it in polyethylene or iron barrels partially filled with inert fillers such as plastic foam, bituminous sand, lime, cement mortar, or clay. This stops sharp things (including scalpels, hypodermic needles, and breakable culture dishes), chemicals, pharmaceutical residues, and incinerator waste from contacting people or the environment.

When burned waste is disposed of, it releases phthalates and other heavy metals like lead, cadmium, and tin into the environment in addition to the dioxin produced during incineration. MW is separated from ordinary municipal solid waste before burning. In addition to being environmentally harmful, burning MW is more costly than disposing of it in a landfill. An alternative to incineration is landfill disposal. However, it is illegal to dispose of biohazardous waste. Particularly in developing nations, MW is routinely dumped in landfills. In a landfill cell, this mechanism isolates MW. To prevent ingress or escape, lime should be placed in MW, and the surrounding area walled off. There are

Figure 6.1: Alternatives

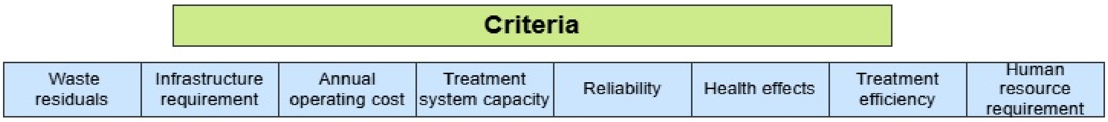

Figure 6.2: Criteria

several things to remember when getting rid of sharp waste. Building a dedicated landfill for MW is advised to properly dispose of hazardous MW and separate it from other forms of waste due to growing public and environmental awareness. During the COVID-19 pandemic, there were severe problems with how to get rid of dangerous medical supplies that affected people left behind.

Microwave technology is a low-temperature, steam-based thermal technique that disinfects with steam and wet heat. Neither water nor steam is used in dry heat treatments. Some use infrared heaters, forced convection, or heated air circulation to heat their waste. Therefore, reverse polymerization occurs in microwave technology at temperatures between 177 and 540 degrees Celsius. High-energy microwaves break down organic molecules. Internal energy is increased when molecules' bonds vibrate or rub against one another due to electromagnetic waves, which have a wavelength of one millimeter to one meter and a frequency of 300–3000 MHz. In a N2 atmosphere, oxygen cannot burn, unlike high-temperature disinfection. Disinfection lowers energy and temperature, preventing heat loss and environmental contamination because it leaves behind a harmless residue.

Alternatives to the application are Figure 6.1: S_1 Incineration, S_2 Encapsulation, S_3 Landfill, S_4 Electromagnetic wave sterilization, S_5 Disinfection with chemicals.

Criteria and their explanations are Figure 6.2: D_1 Waste residuals, D_2 Infrastructure requirement, D_3 Annual operating cost, D_4 Treatment system capacity, D_5 Reliability, D_6 Health effects, D_7 Treatment efficiency, D_8 Human resource requirement. Of these criteria, D_2 and D_3 are cost, and the other criteria are benefits.

6.2. Computation

Table 6.1 shows linguistic terms and their corresponding IVFFNs.

Linguistic Terms	μ_L	μ_U	υ_L	υ_U
Certainly High Importance(CH)	0.95	1	0	0
Very High Importance(VH)	0.8	0.9	0.1	0.2
High Importance(H)	0.7	0.8	0.2	0.3
Slightly More Importance(SM)	0.6	0.65	0.35	0.4
Equally Importance(E)	0.5	0.5	0.5	0.5
Slightly Less Importance(SL)	0.35	0.4	0.6	0.65
Low Importance(L)	0.2	0.3	0.7	0.8
Very Low Importance(VL)	0.1	0.2	0.8	0.9
Certainly Low Importance(CL)	0	0	0.95	1

Table 6.1: Scale Values according to IVFF

Step 1: Consider the alternative set S_i $(1,2,3,4,5)$, the criteria set D_j $(j=1,2,\cdots,8)$, and the DEs set $U=\{U_1,U_2,U_3\}$.

Step 2: The linguistic variables of the DEs' weights are given as $U_1=CH$, $U_2=VH$, $U_3=H$. The IVFFNs related to the linguistic variables are represented in Table 6.1. This information calculates the DEs' weights using Equation 5.1, and $\omega=\{0.38,0.32,0.30\}$ is obtained.

Step 3: Create the aggregated IVFF-DEMA with Definition 3.3 (Tables 6.2, 6.3).

The IVFF-entropy and IVFF-PIPRECIA model will be used to find the criteria weights. The criteria' objective weights are computed using Equation 5.2. Then, we have $\omega_1^O=0.093$, $\omega_2^O=0.123$, $\omega_3^O=0.147$, $\omega_4^O=0.105$, $\omega_5^O=0.1$, $\omega_6^O=0.187$, $\omega_7^O=0.105$, $\omega_8^O=0.14$.

	S_1	§2	S_3	S_4	S_5
D_1	(H, H, E)	(SM, E, H)	(E, SL, VL)	(H, SM, E)	(SL, E, SM)
D_2	(H, SL, SM)	(E, VH, H)	(SL, SM, H)	(H, E, SM)	(L, L, E)
D_3	(SL, E, SM)	(SL, E, H)	(H, VH, H)	(VH, H, VH)	(H, SL, SM)
D_4	(SL, E, H)	(SL, SM, VH)	(VH, E, H)	(SL, SL, VH)	(SL, E, H)
D_5	(SM, E, E)	(SM, SL, E)	(SL, SM, H)	(SL, H, H)	(E, SL, VL)
D_6	(CH, H, SM)	(SL, SM, L)	(VL, SM, L)	(VH, H, E)	(SM, E, H)
D_7	(H, SM, E)	(SL, SM, L)	(L, VL, L)	(E, SM, E)	(H, SL, SM)
D_8	(SL, E, H)	(H, VH, VH)	(E, H, VH)	(SL, E, H)	(E, SM, E)

Table 6.2: IVFFNs of linguistic values given by DEs

	S_1	S_2	S_3	S_4	S_5
D_1	[(0.67, 0.73), (0.3, 0.4)]	[(0.63, 0.68), (0.35, 0.42)]	[(0.3, 0.4), (0.7, 0.76)]	[(0.63, 0.68), (0.35, 0.42)]	[(0.5, 0.57), (0.59, 0.67)]
D_2	[(0.61, 0.67), (0.4, 0.5)]	[(0.72, 0.8), (0.23, 0.35)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.63, 0.68), (0.35, 0.42)]	[(0.29, 0.38), (0.67, 0.74)]
D_3	[(0.5, 0.54), (0.5, 0.54)]	[(0.58, 0.62), (0.45, 0.5)]	[(0.74, 0.84), (0.16, 0.27)]	[(0.77, 0.88), (0.1, 0.23)]	[(0.61, 0.67), (0.4, 0.5)]
D_4	[(0.58, 0.62), (0.45, 0.5)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.72, 0.68), (0.24, 0.35)]	[(0.63, 0.68), (0.35, 0.42)]	[(0.58, 0.62), (0.45, 0.5)]
D_5	[(0.54, 0.56), (0.45, 0.48)]	[(0.5, 0.547), (0.5, 0.54)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.66, 0.73), (0.34, 0.45)]	[(0.3, 0.37), (0.64, 0.71)]
D_6	[(0.84, 1.0), (0.0, 0.0)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.61, 0.28), (0.4, 0.71)]	[(0.71, 0.67), (0.27, 0.38)]	[(0.63, 0.68), (0.35, 0.42)]
D_7	[(0.65, 0.71), (0.32, 0.43)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.16, 0.26), (0.74, 0.84)]	[(0.54, 0.56), (0.45, 0.47)]	[(0.61, 0.67), (0.4, 0.5)]
D_8	[(0.61, 0.67), (0.4, 0.5)]	[(0.77, 0.88), (0.1, 0.23)]	[(0.72, 0.8), (0.23, 0.35)]	[(0.58, 0.62), (0.45, 0.5)]	[(0.54, 0.56), (0.45, 0.47)]

Table 6.3: Aggregated DEMA

The subjective weight of the criteria will be obtained using the IVFF-PIPRECIA model. Equations (5.3)–(5.5) were used for the subjective weights, and the results were shown in Tables 6.4, 6.5.

	U_1	U_2	U_3	aggregated values	crisp values
D_1	E	H	E	[(0.61, 0.64), (0.39, 0.46)]	0.583
D_2	SM	SM	H	[(0.64, 0.71), (0.3, 0.37)]	0.636
D_3	SL	E	SL	[(0.4, 0.44), (0.58, 0.61)]	0.432
D_4	SM	SM	E	[(0.58, 0.61), (0.4, 0.44)]	0.568
D_5	L	SL	E	[(0.35, 0.42), (0.63, 0.68)]	0.388
D_6	H	VH	VH	[(0.77, 0.88), (0.13, 0.23)]	0.752
D_7	E	SM	VH	[(0.7, 0.76), (0.29 0.39)]	0.674
D_8	H	SM	SL	[(0.6, 0.67), (0.4, 0.48)]	0.586

Table 6.4: Significance ratings of criteria

	Crisp degrees	s_j	κ_j	Q_j	ω_j^S
D_1	0.583	-	1.000	1.000	0.12
D_2	0.636	1.053	0.947	1.056	0.12
D_3	0.432	0.796	1.204	0.877	0.1
D_4	0.568	1.136	0.864	1.015	0.116
D_5	0.388	0.820	1.180	0.860	0.1
D_6	0.752	1.364	0.636	1.352	0.16
D_7	0.674	0.922	1.078	1.254	0.15
D_8	0.586	0.912	1.088	1.153	0.134

Table 6.5: The weight of different criteria using IVFF-PIPRECIA

Further, the combined weight of each criterion based on IVFF-entropy and IVFF-PIPRECIA model is evaluated for $\alpha = 0.5$, then $\omega_j = \{0.093, 0.1215, 0.1235, 0.1105, 0.1, 0.1735, 0.1275, 0.137\}$. Since the criteria D_2 and D_3 difficulties are non-beneficial, and the rest are of beneficial criteria, the aggregated IVFF-DEMA is transformed into a normalized aggregated IVFF-DEMA Table 6.6. Positive and negative ideal solutions are as follows:

$$N_j^+ = \Big([(0.67,0.73),(0.3,0.4)],[(0.72,0.8),(0.23,0.35)],[(0.77,0.88),(0.1,0.23)],[(0.72,0.68),(0.24,0.35)],$$
$$[(0.66,0.73),(0.34,0.45)],[(0.84,1.0),(0.0,0.0)],[(0.65,0.71),(0.32,0.43)],[(0.77,0.88),(0.1,0.23)]\Big)$$

$$N_j^- = \Big([(0.3,0.4),(0.7,0.76)],[(0.61,0.67),(0.4,0.5)],[(0.5,0.54),(0.5,0.54)],[(0.58,0.62),(0.45,0.5)],$$
$$[(0.5,0.547),(0.5,0.54)],[(0.61,0.28),(0.4,0.71)],[(0.16,0.26),(0.74,0.84)],[(0.58,0.62),(0.45,0.5)]\Big)$$

The weighted normalized aggregated DEMA is created(Table 6.7) with Definition 3.3, Theorem 3.4 and Table 6.6. Using N_j^+,N_j^- and Table 6.7, the IVFF-score degree of each option, N_j^+, and N_j^- are determined and shown in Table 6.8. Using the Equation 5.6 and utility function values(Table 6.9), the prioritization of options is $S_1 > S_4 > S_5 > S_3 > S_2$, and S_1 is the best choice.

	S_1	§2	S_3	S_4	S_5
D_1	[(0.67, 0.73), (0.3, 0.4)]	[(0.63, 0.68), (0.35, 0.42)]	[(0.3, 0.4), (0.7, 0.76)]	[(0.63, 0.68), (0.35, 0.42)]	[(0.5, 0.57), (0.59, 0.67)]
D_2	[(0.4, 0.5), (0.61, 0.67)]	[(0.23, 0.35), (0.72, 0.8)]	[(0.4, 0.5), (0.61, 0.67)]	[(0.35, 0.42), (0.63, 0.68)]	[(0.67, 0.74), (0.29, 0.38)]
D_3	[(0.5, 0.54), (0.5, 0.54)]	[(0.45, 0.5), (0.58, 0.62)]	[(0.16, 0.27), (0.74, 0.84)]	[(0.1, 0.23), (0.77, 0.88)]	[(0.4, 0.5), (0.61, 0.67)]
D_4	[(0.58, 0.62), (0.45, 0.5)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.72, 0.68), (0.24, 0.35)]	[(0.63, 0.68), (0.35, 0.42)]	[(0.58, 0.62), (0.45, 0.5)]
D_5	[(0.54, 0.56), (0.45, 0.48)]	[(0.5, 0.547), (0.5, 0.54)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.66, 0.73), (0.34, 0.45)]	[(0.3, 0.37), (0.64, 0.71)]
D_6	[(0.84, 1.0), (0.0, 0.0)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.61, 0.28), (0.4, 0.71)]	[(0.71, 0.67), (0.27, 0.38)]	[(0.63, 0.68), (0.35, 0.42)]
D_7	[(0.65, 0.71), (0.32, 0.43)]	[(0.61, 0.67), (0.4, 0.5)]	[(0.16, 0.26), (0.74, 0.84)]	[(0.54, 0.56), (0.45, 0.47)]	[(0.61, 0.67), (0.4, 0.5)]
D_8	[(0.61, 0.67), (0.4, 0.5)]	[(0.77, 0.88), (0.1, 0.23)]	[(0.72, 0.8), (0.23, 0.35)]	[(0.58, 0.62), (0.45, 0.5)]	[(0.54, 0.56), (0.45, 0.47)]

Table 6.6: Normalized DEMA

	S_1	§2	S_3	S_4	S_5
D_1	[(0.16, 0.23), (0.77, 0.81)]	[(0.19, 0.25), (0.75, 0.80)]	[(0.66, 0.72), (0.25, 0.27)]	[(0.19, 0.25), (0.75, 0.80)]	[(0.31, 0.40), (0.88, 0.96)]
D_2	[(0.20, 0.25), (0.80, 0.87)]	[(0.23, 0.29), (0.65, 0.74)]	[(0.20, 0.25), (0.80, 0.87)]	[(0.19, 0.25), (0.75, 0.80)]	[(0.16, 0.23), (0.77, 0.81)]
D_3	[(0.31, 0.39), (0.73, 0.84)]	[(0.91, 0.93), (0.81, 0.87)]	[(0.24, 0.36), (0.66, 0.70)]	[(0.29, 0.42), (0.57, 0.65)]	[(0.20, 0.25), (0.80, 0.87)]
D_4	[(0.79, 0.86), (0.77, 0.73)]	[(0.20, 0.25), (0.80, 0.87)]	[(0.23, 0.29), (0.65, 0.74)]	[(0.19, 0.23), (0.75, 0.80)]	[(0.79, 0.86), (0.77, 0.73)]
D_5	[(0.89, 0.92), (0.77, 0.72)]	[(0.31, 0.88), (0.73, 0.84)]	[(0.20, 0.25), (0.80, 0.87)]	[(0.18, 0.23), (0.76, 0.80)]	[(0.66, 0.72), (0.25, 0.27)]
D_6	[(0.36, 0.43), (0.0, 0.0)]	[(0.20, 0.35), (0.80, 0.87)]	[(0.20, 0.69), (0.80, 0.95)]	[(0.22, 0.25), (0.84, 0.81)]	[(0.19, 0.25), (0.75, 0.80)]
D_7	[(0.18, 0.22), (0.77, 0.80)]	[(0.20, 0.25), (0.80, 0.87)]	[(0.98, 0.91), (0.27, 0.32)]	[(0.39, 0.40), (0.81, 0.87)]	[(0.20, 0.25), (0.80, 0.87)]
D_8	[(0.20, 0.25), (0.80, 0.87)]	[(0.29, 0.42), (0.57, 0.65)]	[(0.23, 0.29), (0.65, 0.74)]	[(0.79, 0.86), (0.77, 0.73)]	[(0.89, 0.92), (0.77, 0.72)]

Table 6.7: Weighted Normalized Aggregated DEMA

	S_1	S_2	S_3	S_4	S_5	N_j^+	N_j^-
D_1	0.26	0.27	0.66	0.27	0.13	0.65	0.33
D_2	0.21	0.34	0.21	0.27	0.26	0.71	0.58
D_3	0.28	0.6	0.36	0.22	0.22	0.78	0.50
D_4	0.57	0.24	0.34	0.27	0.43	0.66	0.55
D_5	0.66	0.43	0.24	0.26	0.66	0.64	0.5
D_6	0.53	0.22	0.24	0.27	0.27	0.9	0.46
D_7	0.26	0.21	0.91	0.17	0.22	0.63	0.26
D_8	0.21	0.22	0.34	0.57	0.34	0.78	0.55

Table 6.8: Score Values

	U_i^-	U_i^+	$f(U_i)$	Ranking
S_1	0.738	1.104	0.581	1
S_2	0.460	0.687	0.217	5
S_3	0.487	704	0.388	4
S_4	0.523	0.782	0.412	2
S_5	0.513	0.767	0.404	3

Table 6.9: Utility Degrees and Utility Functions

Hence, the priority order of MW technique alternatives is found as $S_1 > S_4 > S_5 > S_3 > S_2$ and S_1 is the most desirable alternative. As a result, the final ranking of the options is obtained. Based on the ranking, Incineration is the best MW approach. It is followed by electromagnetic wave sterilization, chemical disinfection, and landfilling. Encapsulation gets the last position.

7. Discussion

The issue of health care has gained much attention in the modern world. It must be taken out and disposed of appropriately. There are numerous options for getting rid of waste. Here, a scenario-based multi-objective mathematical model was presented to design a viable MW chain. There are two benefits to this study. First, by employing a quantitative approach that considers ambiguities and uncertainties, this study can ascertain the causal linkages between these components. Second, we determine which seven are the most important by measuring the causal linkages between each element. These important components most significantly impact the entire factor system. Enhancing these important determinants significantly raises the MW management system's sustainability.

Consequently, it was decided that incineration was the most crucial factor. Handling MW properly safeguards patients, healthcare providers, and staff. It is essential for public health, safety, and the environment. For effective waste management, MW must be handled, stored, transported, processed, and disposed of properly. One crucial component of environmental sustainability is the handling of MW disposal. Healthcare waste is becoming increasingly abundant daily; hence, proper disposal is required. MW can be disposed of in various ways, with

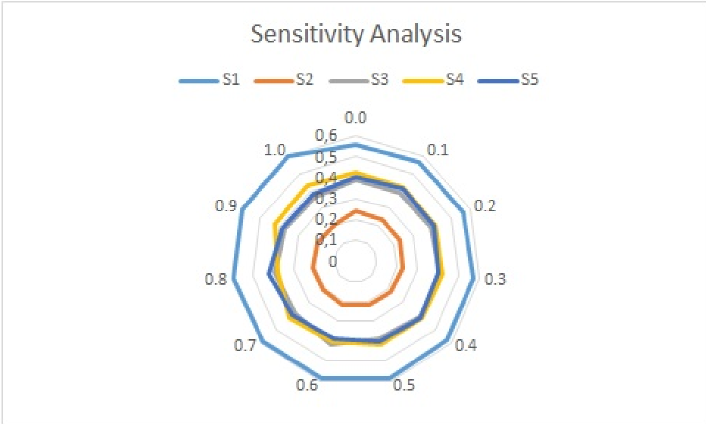

Figure 7.1: Sensitivity Analysis

incineration being one of the methods.

The results show that incinerator technology ranks higher than Electromagnetic Wave Sterilization, which indicates the current scenario. Incineration is one of the best options for disposing of medical waste since it requires less land, has a higher treatment capacity, costs less to transport garbage, and improves waste-to-energy operations [2]. This rating also aligns with the findings of [51] and [2], which indicate that electromagnetic wave sterilization disposal techniques—ranked second and third, respectively—are very successful because they produce non-hazardous residues and emit fewer pollutants than other approaches.

The presented hybrid IVFF model is proficient in producing stable and, simultaneously, flexible prioritization in variation in parameters. The results obtained showed that the developed approach could effectively address the concerns of healthcare professionals regarding the choice of the MW method in IVFFSs.

7.1. Sensitivity:

A sensitivity analysis has been conducted across the range of values for the α parameter. We systematically investigate how the parameters affect health practitioners' and society's high safety and security through effective modeling and ranking of risks associated with medical waste disposal. A range of α values were considered in the sensitivity study. This evaluation is discussed to convey how well the recently developed framework functions. DEs can assess how sensitive the introduced model is to changes by adjusting the α parameter. The best alternative, S_1, is the same for every parameter value, as indicated by the sensitivity analysis results in Table 7.1 and Figure 7.1. Therefore, MW treatment problem analysis depends on and is sensitive to α values. Thus, the proposed model has sufficient stability over various parameter values. Figure 7.1 shows that an alternate S_1 holds the first rank, and an alternate S_2 holds the last for every α. For $\alpha = 0.3, 0.6, 0.8$; S_3, S_4 and S_5 have different rankings. However, it is also seen that the results obtained for $\alpha = 0.0, 0.1, 0.2, 0.4, 0.7, 0.9, 1.0$ are the same as for $\alpha = 0.5$. The view Figure 7.1 demonstrates how changing the parameter degrees will improve the suggested framework's durability.

	$\alpha = 0.0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1.0$
S_1	0.560	0.562	0.568	0.570	0.575	0.581	0.583	0.585	0.590	0.594	0.596
S_2	0.243	0.239	0.235	0.227	0.225	0.217	0,215	0.210	0.207	0.204	0.202
S_3	0.388	0.390	0.396	0.404	0.405	0.388	0.415	0.383	0.390	0.424	0.428
S_4	0.426	0.421	0.420	0.416	0.414	0.412	0.399	0.419	0.380	0.385	0.381
S_5	0.403	0.410	0.408	0.401	0.412	0.404	0.385	0.397	0.421	0.371	0.366

Table 7.1: Sensitivity Analysis

7.2. Comparative analysis

MCDM is the process of choosing and assessing options from a small or large pool according to pertinent factors. An extensive range of alternatives are evaluated using various criteria as part of MCDM. The goal of applying MCDM techniques to these kinds of challenges is to assist DMs in identifying the best and most desirable solution. Thus, researchers have introduced numerous MCDM techniques. In this section, we tried two MCDM approaches in the FFS environment. The suggested methods' validity and accuracy have been validated by contrasting them with established techniques.

A comparative analysis with IVFF-WASPAS [41], IVFF-SWARA [52], Pythagorean fuzzy entropy-SWARA-WASPAS [53], and spherical fuzzy CRITIC-WASPAS [2] will be conducted to confirm the robustness of the suggested technique.

- In [41], (IVFF-WASPAS) IVFFS was defined, and its basic features were examined. A new MCDM method with the WASPAS has been proposed by giving aggregation operators based on IVFFS.

- In [52], (IVFF-SWARA) a new technique is proposed to solve MCDM problems with SWARA and ARAS techniques based on IVFFSs.
- In [53], (Pythagorean fuzzy entropy-SWARA-WASPAS) a new MCDM technique is presented by combining entropy, SWARA, and WASPAS techniques under PFSs.
- In [2], (CRITIC-WASPAS) a new spherical fuzzy-type MCDM method using CRITIC and WASPAS approaches has been used to evaluate HW disposal alternatives.

The comparison approaches' computations were done with the same decision matrices and sub-criteria weights. The comparisons have led to Table 7.2 results. Table 7.2 illustrates the differences between the compared approaches. Incineration was the first alternative in all applicable methodologies (S_1). The outputs of the approach proposed in this study are similar to those of the IVFF-WASPAS method, IVFF-SWARA method, Pythagorean fuzzy entropy-SWARA-WASPAS, and spherical fuzzy CRITIC-WASPAS, with a few minor variations. The proposed ways help tackle MCGDM concerns more practically and wisely since they use IVFFs instead of traditional methodology to evaluate criteria and alternatives.

There may be variations in the results obtained when using different MCDM methods to solve a given problem. It makes perfect sense to have various outcomes because different procedures have different methodological consequences and goals (Table 7.2). It is evident from the findings that there have also been modest adjustments, which is quite understandable given the unique characteristics of each decision-making process. FFS can deliver precise and adaptable outcomes because of their structure and distinct membership levels. Consequently, the analysis verifies that the suggested approach's outcomes are accurate.

Method	S_1	S_2	S_3	S_4	S_5
IVFF-WASPAS [41]	1	1	5	3	6
IVFF-SWARA [52]	1	1	4	5	9
Pythagorean fuzzy entropy-SWARA-WASPAS [53]	1	1	5	3	7
Spherical fuzzy CRITIC-WASPAS [2]	1	1	3	6	8

Table 7.2: Ranking comparison

7.3. Superiority of suggested method

The FFS is the result of combining the FS, IFS, and PFS. PFS is determined by total squares equal to or less than one and member and nonmember satisfaction levels. Seldom does the DE provide a particular feature to the $\mathscr{M}$ and $\mathscr{N}$ so that the sum of the squares is more than 1. The PFS is, therefore, unable to appropriately handle this occurrence. One of the most comprehensive techniques for circumventing this constraint is FFS, which can manage inconsistent and partially unknown data, both common in real-world scenarios.

The current and sensitivity analyses suggest that the offered strategy's results overlap with the accessible approaches. The main advantage of the proposed approach over easily accessible DM solutions is that it contains additional information and addresses data uncertainty by taking $\mathscr{M}$ and $\mathscr{N}$ of criteria into consideration features. Information regarding the item can be studied more accurately and objectively. It is also a valuable tool in the DM process when dealing with inaccurate and imprecise data. As a result, the rationale for assigning a score value to one parameter does not affect the other values, resulting in the predicted information loss.

On the contrary, our proposed technique does not result in significant information loss. The desired methodology has an advantage over present methods in that it detects the level of discrimination and similarity between data, preventing judgments made for incorrect reasons. Merging incorrect and ambiguous information can aid with the DM process.

8. Implications

Medical waste is becoming a significant environmental concern due to the negative consequences of its unplanned disposal. Healthcare and medical facilities must be aware of the harmful consequences of medical waste and take necessary action to address it. This study provides a strategy to assist health workers in determining which waste disposal alternatives to maintain, given the critical need to dispose of medical waste and the treatment processes that are now available. The study has the following implications for healthcare management: The methods for analyzing MW disposal choices provide a rapid and reliable way to analyze potential alternatives. This could be very useful for practitioners and administrators in the healthcare industry. Healthcare facilities can handle the problem of selecting a waste disposal strategy by utilizing the helpful and straightforward methods offered. Within this framework, the proposed approaches offer a reliable and expedient way to preliminary evaluate medical waste disposal options. Since it prioritizes practical waste disposal options based on the essential components and resources accessible to that hospital unit, this helps healthcare administrators and practitioners.

This study advances our theoretical and practical knowledge of medical waste disposal planning. Medical waste is a significant source of environmental and health issues. Adopting medical devices, particularly by individuals and healthcare workers, has also increased medical waste. Globally, managing medical waste presents formidable obstacles, irrespective of a robust infrastructure. As a result, the study's findings offer a theoretical viewpoint on the issue of medical waste and its handling. This study's risk analysis links the key obstacles and intricacy of sustainable medical waste disposal planning. Academics can gain from the research's findings in two ways: (1) It offers guidance on managing medical waste disposal planning wisely. (2) The findings have the potential to inspire several ideas and investments while also helping to resolve the difficulties associated with disposing of medical waste. The current study evaluates produced medical waste based on multiple parameters, which aids in the rating and classifying disposal methods. In other words, this study will help identify the best and worst disposal methods, ensuring that medical waste—which poses a risk—is disposed of in the most suitable way possible. Governments must plan for the proper approach to dispose of medical waste in order to protect public health and minimize costs. Theoretical implications include

- suggestions on how to evolve the IVFFLS strategy,
- insights into using other methodologies for treating medical waste disposal,
- a detailed understanding of the research methods used.

An MCDM strategy was determined by considering aggregation operators and score-accuracy functions based on IVFFLS, so it offered a substantial contribution to the literature and shed light on medical waste disposal planning in a new context. Another practical application that combines the advantages of fuzzy and interval-valued techniques is the application of the IVFFLS approach. When evaluations are presented in linguistic values, fuzzy logic is used instead of typical numerical ratings to improve comprehension and interpretation. This kind of integration has been discussed in several studies in the literature on waste management, and several recommendations have been made for overcoming challenges and minimizing restrictions. This study's methodology offers a framework to generate outcomes that function in concert by taking more practical and accurate measures than earlier methods. The suggested framework helps businesses and regulators identify the significant obstacles that could arise while implementing an effective medical waste disposal planning system. The article primarily adds to the body of knowledge regarding identifying and analyzing obstacles to the implementation of the medical waste disposal system to address sustainability issues.

The following describes the theoretical and methodological consequences of the study: With the right mathematical software, the suggested approaches' adaptable structures allow them to be expanded to address any new DM challenge. All preference variables in the criterion are recorded based on the alternative assessment matrix. On the other hand, the score-accuracy functions of the proposed methods enable option ranking. This technique is dynamic and adaptable and may be applied to various DM situations since the user selects the parameter value based on the type of problem.

It is necessary to take into account the administrative consequences of incinerator technology. Investing much money in equipment and infrastructure is necessary to implement incineration as a medical waste disposal option. As a result, it is critical to thoroughly evaluate the operational aspects of incineration, including the cost of equipment and maintenance and the availability of adequate facilities. While pretending to be an incinerator for medical waste may come with a hefty initial cost, it can also yield long-term financial gains. The regulatory agency responsible for waste management and the environment must approve the implementation of incineration. In order to prevent fines or legal problems, the relevant permits must be obtained. Stakeholder participation, including local communities and government institutions, is necessary for implementing incineration. It is imperative to furnish these stakeholders with pertinent details regarding incineration's advantages and possible hazards, along with any safety and environmental preservation measures. Employees and other stakeholders will need to receive training and instruction in order to implement incineration. This covers the safe use of incinerator machinery as well as instruction on the advantages and restrictions of the technology. The incineration process can be continuously improved to make it more effective and efficient, but these advancements must be closely monitored. Therefore, it is critical to frequently assess and appraise the incineration process and pinpoint areas for improvement.

The study's conclusions have several significant ramifications for practitioners, academics, and policymakers in addition to its theoretical contribution. The research findings can support the conceptual premise that may aid sustainability and environmental managers in comprehending the significance of an efficient medical waste disposal system in emerging economies. This can help ensure sustainability and human well-being, identify the main obstacles to adoption, shape strategic decisions for successful medical waste disposal implementation, and maximize the financial value of efficient medical waste disposal practices in tangible and intangible forms. This study highlights the significance of efficient medical waste disposal in achieving operational excellence and sustainable development. The study's conclusions imply that, in order to maintain sustainability, medical waste disposal procedures must be implemented by the government and regulatory bodies. Therefore, the government and regulatory bodies must oversee sufficient resources to properly administer the system and actively participate in implementing medical waste disposal programs.

To evaluate from the perspective of Public Health, hospitals are responsible for the waste they produce. They must ensure that handling, treating, and disposing of that waste will not harm public health or the environment.

9. Conclusion

Preserving health, healing patients, and preserving lives are all achieved through healthcare operations. However, they also produce waste, 20% of which poses a danger of injury, infection, or exposure to chemicals or radiation. Waste management from health services is a complicated process. Even though hazardous medical waste poses risks, how to manage it is generally well-known and covered in manuals and other literature. Improper waste management can endanger patients, their families, medical waste workers, and the surrounding community. Furthermore, improper handling or disposal of such trash may contaminate or pollute the environment. This study uses the novel MCDM method for medical waste disposal planning to suggest solutions for the health system, human health, and environmental protection. To prioritize MW disposal methods, model the related uncertainty, and maximize their benefits, this work employs MCDM techniques based on IVFFLs. The planning of MW disposal is one pressing issue addressed by the suggested methodology. For this, a new decision-making methodology has been created. Therefore, the IVFF-entropy, IVFF-PIPCERIA combined IVFF-MARCOS procedures have been created. A unique fuzzy decision-making technique utilizing the entropy, PIPCERIA, and MARCOS approaches within the IVFF environment was given to evaluate the study framework. In assessing and determining the weights, it is crucial to use the DEs' role to calculate each difficulty's weight. As a result, each DE was asked to rank the significance of the MW treatment problem.
There are still several concerns with this study. First, there is a distinction between uncertainty and danger. Rather than favoring ambiguity, the main focus of this study is the effects of risk selection. Given that predicting the potential for an MW disposal service and its technology might be complicated, risk aversion is a crucial kind of uncertainty aversion. This study operationalized risk preference using prospect

theory. However, a more thorough assessment is required to detect potential problems with MW disposal. As a result, it could not identify the benefit and loss domains associated with MW disposal. Future studies should focus on merging specific MW disposal risk indicators with broad risk preference criteria.

Beyond the benefits of the proposed IVFF-based technique, its application in specific DM circumstances is restricted by its inability to evaluate the available options thoroughly. Building IVFFSs is easier when there are a lot of criteria and alternatives. To address these limitations, we would like to deepen our research in the following areas in our future work:

- Remanufacturing issues are less broadly applicable than the proposed solution. We aim to extend its application reach to include more intricate real-world disease management (DM) scenarios, including commercial, construction management, and medical.
- Extending the scope of outranking-based interval rough set theory methods—such as VIKOR, ELECTRE, DEMATEL, ANP, FMEA, BWM, and others—is another long-term objective.
- We aim to determine how various MCDM methods can be applied to the IVFF values.

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