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Research Article

NEW GENERALIZED DIFFERENCE SEQUENCE SPACES AND THEIR KÖTHE-TOEPLITZ DUALS

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Abstract: *We will define the sequence spaces $c_0(u,\Delta_v^m)_p$, $c(u,\Delta_v^m)_p$ and $\ell_\infty(u,\Delta_v^m)_p$ in this article. Furthermore, we give some topological properties and compute their Köthe-Toeplitz duals.*

Keywords: *Difference sequence spaces, α-, β- and γ- dual.*

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1. Introduction

We show w display the set of all sequences $x=(x_k)$, and c_0, ℓ_∞ and c, the linear spaces of null, bounded and convergent sequences with real terms, respectively, normed by $\|x\|_\infty = \sup_k |x_k|$. In all this study we use X instead of c_0, ℓ_∞ and c. The difference in sequence spaces

$$X(\Delta) = \{x = (x_k) : \Delta x \in X\}$$

first defined by Kızmaz [1]. Et and Çolak [2] generalized this.

Later, Et and Esi [3] widened the difference sequence spaces to the sequence spaces

$$X(\Delta_v^m) = \{x = (x_k) : (\Delta_v^m x_n) \in X\}$$

where $v = (v_n)$ be any fixed sequence of non-zero complex numbers and such that

$$(\Delta_v^m x_n) = (\Delta_v^{m-1} x_n - \Delta_v^{m-1} x_{n+1}),\ \Delta_v^m x_n = \sum_{i=0}^{m} (-1)^i \binom{m}{i} v_{n+i} x_{n+i}.$$

Bektaş, Et and Çolak [4] defined the sequence spaces $\Delta_v^m(X)$ for $X = \ell_\infty$, c and c_0, and worked out β - and γ - duals of the these.

Definition 1.1. Let X be a sequence space.

$$X^\alpha = \left\{a \in w : \sum_{n=1}^{\infty} |a_n x_n| < \infty, \text{for all } x \in X\right\}$$

$$X^\beta = \left\{a \in w : \sum_{n=1}^{\infty} a_n x_n \text{ converges}, \text{for all } x \in X\right\}$$

$$X^\gamma = \left\{a \in w : \sup_k \left|\sum_{n=1}^{k} a_n x_n\right| < \infty, \text{for all } x \in X\right\},$$

are called α-, β-, γ- dual spaces of X, respectively. $\emptyset \subset X^\alpha \subset X^\beta \subset X^\gamma$ is shown. Since $X \subset Y$, $Y^\eta \subset X^\eta$ for $\eta = \alpha, \beta, \gamma$. We have $X^{\alpha\alpha} = (X^\alpha)^\alpha$.

Et and Başarır [5] defined the sequence spaces

$$\Delta^m(X) = \{x = (x_n) : \Delta^m x \in X(p)\}$$

for $m \in \mathbb{N}$, where $X = \ell_\infty$, c or c_0.

Let U be the set of all sequences $u = (u_n)$ such that $u_n \neq 0$ and complex for all $n = 1,2,\dots$. Throughout the paper we write $w_n = 1/|u_n|$.

Malkowsky [6] defined the sequence spaces

$$X(u,\Delta) = \left\{x \in w: \left(u_n(x_n - x_{n+1})\right)_{n=1}^{\infty} \in X\right\}$$

where $u \in U$ and $X = \ell_\infty$, c or c_0.

After Asma and Çolak [7] defined the sequence spaces

$$X(u,\Delta,p) = \{x \in w: (u_n \Delta x_n)_{n=1}^{\infty} \in X(p)\}$$

where $u \in U$ and $X = \ell_\infty$, c or c_0.

Recently Bektaş, A. [8] defined the sequence spaces

$$X(u,\Delta^2,p) = \{x \in w: (u_n \Delta^2 x_n)_{n=1}^{\infty} \in X(p)\}$$

where $\Delta^2 x = (\Delta^2 x_n)_{n=1}^{\infty} = (\Delta x_n - \Delta x_{n+1})_{n=1}^{\infty}$.

Many studies have been carried out on dual and Köthe-Toeplitz dual etc. ([9], [10]).

2. Main Results

Let us consider $p = (p_n)$ as a sequence of strictly positive real numbers thought this study.

We define the sequence spaces as follows:

$$\boldsymbol{\ell_\infty(u,\Delta_v^m)_p = \{x \in w: (u_n \Delta_v^m x_n) \in \ell_\infty(p)\}},$$

$$\boldsymbol{c(u,\Delta_v^m)_p = \{x \in w: (u_n \Delta_v^m x_n) \in c(p)\}},$$

$$\boldsymbol{c_0(u,\Delta_v^m)_p = \{x \in w: (u_n \Delta_v^m x_n) \in c_0(p)\}}$$

for $\boldsymbol{p = (p_n)}$, $\boldsymbol{m} \in \mathbb{N}$ and $\boldsymbol{u \in U}$.

Theorem 2.1. $\boldsymbol{c_0(u,\Delta_v^m)_p}$, $\boldsymbol{c(u,\Delta_v^m)_p}$ and $\boldsymbol{\ell_\infty(u,\Delta_v^m)_p}$ are linear spaces.

Theorem 2.2. Let $\boldsymbol{p = (p_n)}$ be bounded and $\boldsymbol{M = max(1, H = sup_n p_n)}$. Then $\boldsymbol{\ell_\infty(u,\Delta_v^m)_p}$ and $\boldsymbol{c_0(u,\Delta_v^m)_p}$ are linear topological spaces by $\boldsymbol{g}$, defined by

$$\boldsymbol{g(x) = \sup_n |u_n \Delta_v^m x_n|^{p_n/M}}.$$

Furthermore $\boldsymbol{c(u,\Delta_v^m)_p}$ is paranormed by $\boldsymbol{g}$ if $\boldsymbol{inf_n p_n > 0}$.

Theorem 2.3. For every $\boldsymbol{p = (p_n)}$,

$$\textbf{(i) } \boldsymbol{\left[\ell_\infty(u,\Delta_v^m)_p\right]^\alpha = D_\alpha(u,p) = \bigcap_{S=2}^{\infty}\left\{a \in w: \sum_{n=1}^{\infty} |a_n||v_n|^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{1/p_i} w_i < \infty\right\}},$$

$$\textbf{(ii) } \boldsymbol{\left[\ell_\infty(u,\Delta_v^m)_p\right]^{\alpha\alpha} = D_{\alpha\alpha}(u,p) = \bigcup_{S=2}^{\infty}\left\{a \in w: \sup_{n \geq m+1} |a_n||v_n| \left[\sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{1/p_i} w_i\right]^{-1} < \infty\right\}}.$$

Proof. (i) Let $\boldsymbol{x \in \ell_\infty(u,\Delta_v^m)_p}$ and $\boldsymbol{a \in D_\alpha(u,p)}$. We choose $\boldsymbol{S > max\{1, sup_k |u_k \Delta_v^m x_k|^{p_k}\}}$. Since

$$\boldsymbol{\sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{1/p_i} w_i > \sum_{i=1}^{m} \binom{n-i-1}{m-1} S^{1/p_i} w_i}$$

for arbitrary $\boldsymbol{S > 1}$ $\boldsymbol{(n = 2m, 2m+1, \dots)}$ and $\boldsymbol{\left|u_i \Delta_v^{m-i} x_i\right| \leq M}$ $\boldsymbol{(1 \leq i \leq m)}$ for some constant $\boldsymbol{M}$, $\boldsymbol{a \in D_\alpha(u,p)}$ implies

$$\boldsymbol{\sum_{n=1}^{\infty} |a_n||v_n|^{-1} \sum_{i=1}^{m} \binom{n-i-1}{m-1} \left|\Delta_v^{m-i} x_i\right| < \infty}.$$

Since,

$$x_n = |v_n|^{-1}\left(\left|\sum_{i=1}^{n-m}(-1)^m\binom{n-i-1}{m-1}\Delta_v^m x_i + \sum_{i=1}^{m}(-1)^{m-i}\binom{n-i-1}{m-1}\Delta_v^{m-i}x_i\right|\right),$$

we can write

$$\sum_{n=1}^{\infty}|a_n x_n| = \sum_{n=1}^{\infty}|a_n||v_n|^{-1}\left(\left|\sum_{i=1}^{n-m}(-1)^m\binom{n-i-1}{m-1}\Delta_v^m x_i + \sum_{i=1}^{m}(-1)^{m-i}\binom{n-i-1}{m-1}\Delta_v^{m-i}x_i\right|\right)$$

$$\leq \sum_{n=1}^{\infty}|a_n||v_n|^{-1}\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}|\Delta_v^m x_i| + \sum_{n=1}^{\infty}|a_n||v_n|^{-1}\sum_{i=1}^{m}\binom{n-i-1}{m-1}|\Delta_v^{m-i}x_i|$$

$$\leq \sum_{n=1}^{\infty}|a_n||v_n|^{-1}\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i + \sum_{n=1}^{\infty}|a_n||v_n|^{-1}\sum_{i=1}^{m}\binom{n-i-1}{m-1}|\Delta_v^{m-i}x_i|$$

$< \infty$.

Therefore $a \in \left[\ell_\infty(u,\Delta_v^m)_p\right]^\alpha$.

Conversely, let $a \notin D_\alpha(u,p)$. Then for some integer $S > 1$, we have

$$\sum_{n=1}^{\infty}|a_n||v_n|^{-1}\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i = \infty.$$

If we define the sequence $x = (x_n)$ by

$$x_n = v_n^{-1}\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i \qquad (\mathbf{n = m+1, m+2, \ldots}),$$

then we obtain that $x \in \ell_\infty(u,\Delta_v^m)_p$ and $\sum_n|a_n x_n| = \infty$. So $a \notin \left[\ell_\infty(u,\Delta_v^m)_p\right]^\alpha$.

(ii) Let $a \in D_{\alpha\alpha}(u,p)$ and $x \in \left[\ell_\infty(u,\Delta_v^m)_p\right]^\alpha = D_\alpha(u,p)$, by part (i). Then for some $S > 1$, we can write

$$\sum_{n=1}^{\infty}|a_n x_n| = \sum_{n=m+1}^{\infty}|a_n||v_n|\left[\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i\right]^{-1}|x_n||v_n|^{-1}\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i$$

$$\leq \sup_{n\geq m+1}\left\{|a_n||v_n|\left[\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i\right]^{-1}\right\}\cdot\sum_{n=m+1}^{\infty}|x_n||v_n|^{-1}\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i.$$

Hence $\sum_{n=1}^{\infty}|a_n x_n| < \infty$, i.e., $a \in \left[\ell_\infty(u,\Delta_v^m)_p\right]^{\alpha\alpha}$.

Conversely, let $a \in \left[\ell_\infty(u,\Delta_v^m)_p\right]^{\alpha\alpha}$, but $a \notin D_{\alpha\alpha}(u,p)$. Hence for all integers $S > 1$, we can write

$$\sup_{n\geq m+1}|a_n||v_n|\left[\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}S^{1/p_i}\,w_i\right]^{-1} = \infty.$$

We recall that $\sum_{i=1}^{n-m}\binom{n-i-1}{m-1}y_i = 0$ $(\mathbf{n < m+1})$ for optionally y_i. Therefore, there is a strictly increasing sequence $(n(s))$ of integers $n(s) \geq m+1$ such that

$$|a_{n(s)}||v_{n(s)}|\left[\sum_{i=1}^{n(s)-m}\binom{n(s)-i-1}{m-1}S^{1/p_i}\,w_i\right]^{-1} > s^{m+1}\ \ (s = m+1, m+2, \ldots).$$

We define the sequence $x = (x_n)$ with

$$x_n = \begin{cases} |a_{n(s)}|^{-1}, & n = n(s) \\ 0, & n \neq n(s) \quad (n = m+1, m+1, \dots). \end{cases}$$

Then for all integers $S > m+1$, we can write

$$\sum_{n=1}^{\infty} |a_n||v_n|^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{1/p_i} w_i \leq \sum_{s=m+1}^{\infty} |a_{n(s)}|^{-1} |v_{n(s)}|^{-1} \sum_{i=1}^{n(s)-m} \binom{n(s)-i-1}{m-1} S^{1/p_i} w_i$$

$$\leq \sum_{s=m+1}^{S-1} |a_{n(s)}|^{-1} |v_{n(s)}|^{-1} \sum_{i=1}^{n(s)-m} \binom{n(s)-i-1}{m-1} S^{1/p_i} w_i$$

$$+ \sum_{s=S}^{\infty} |a_{n(s)}|^{-1} |v_{n(s)}|^{-1} \sum_{i=1}^{n(s)-m} \binom{n(s)-i-1}{m-1} S^{1/p_i} w_i.$$

Hence $x \in [\ell_\infty(u, \Delta_v^m)_p]^\alpha$ and $\sum_{n=1}^{\infty} |a_n x_n| = \sum_{S=1}^{\infty} 1 = \infty$. Thus $a \notin [\ell_\infty(u, \Delta_v^m)_p]^{\alpha\alpha}$.

Theorem 2.4. For every $p = (p_n)$ and $u \in U$,

(i) $$[c_0(u, \Delta_v^m)_p]^\alpha = M_\alpha(u, p) = \bigcup_{S=2}^{\infty} \left\{ a \in w : \sum_{n=1}^{\infty} |a_n| v_n^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{-1/p_i} w_i < \infty \right\},$$

(ii) $$[c_0(u, \Delta_v^m)_p]^{\alpha\alpha} = M_{\alpha\alpha}(u, p) = \bigcap_{S=2}^{\infty} \left\{ a \in w : \sup_{n \geq m+1} |a_n| \left[\sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{-1/p_i} w_i \right]^{-1} < \infty \right\}.$$

Proof. (i) Let $x \in c_0(u, \Delta_v^m)_p$ and $a \in M_\alpha(u, p)$. Then there is an integer $S > 1$ such that $|u_n \Delta_v^m x_n|^{p_n} \leq S^{-1}$. Since

$$\sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{-1/p_i} w_i > \sum_{i=1}^{m} \binom{n-i-1}{m-1} S^{-1/p_i} w_i$$

$a \in M_\alpha(u, p)$ implies

$$\sum_{n=1}^{\infty} |a_n||v_n|^{-1} \sum_{i=1}^{m} \binom{n-i-1}{m-1} |\Delta_v^{m-i} x_i| < \infty.$$

Then

$$\sum_{n=1}^{\infty} |a_n x_n| = \sum_{n=1}^{\infty} |a_n||v_n|^{-1} \left(\left| \sum_{i=1}^{n-m} (-1)^m \binom{n-i-1}{m-1} \Delta_v^m x_i + \sum_{i=1}^{m} (-1)^{m-i} \binom{n-i-1}{m-1} \Delta_v^{m-i} x_i \right| \right)$$

$$\leq \sum_{n=1}^{\infty} |a_n||v_n|^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} S^{-1/p_i} w_i + \sum_{n=1}^{\infty} |a_n||v_n|^{-1} \sum_{i=1}^{m} \binom{n-i-1}{m-1} |\Delta_v^{m-i} x_i|$$

$$< \infty.$$

Hence $a \in [c_0(u, \Delta_v^m)_p]^\alpha$.

Conversely, let $a \notin M_\alpha(u, p)$. Hence we can define a strictly increasing sequence $(n(s))$ of integers such that

$$M_\alpha(u, p) = \sum_{n=n(s)}^{n(s+1)-1} |a_n||v_n|^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} (s+1)^{-1/p_i} w_i > 1 \quad (s = 1, 2, \dots)$$

where $n(1) = 1$.

We define sequence $x = (x_n)$ such that

$$x_n = v_n^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} (s+1)^{-1/p_i}\, w_i.$$

Then $|u_n \Delta_v^m x_n|^{p_n} = \frac{1}{s+1} (n(s) \le n \le n(s+1)-1; s = 1, 2, \dots)$. Hence

$$\sum_{n=1}^{\infty} |a_n x_n| = \sum_{n=1}^{\infty} |a_n| |v_n|^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} (s+1)^{-1/p_i}\, w_i > 1$$

and $x \in c_0(u, \Delta_v^m)_p$. Hence $a \notin \left[c_0(u, \Delta_v^m)_p\right]^\alpha$. This is a contradiction. Therefore $a \in M_\alpha(u, p)$.

(ii) Proof is smilar to [11].

Theorem 2.5. For every $p = (p_n)$,

$$\left[c(u, \Delta_v^m)_p\right]^\alpha = \mathbf{D}_v^m(p)$$

where

$$\mathbf{D}_v^m(p) = M_\alpha(u, p) \cap \left\{ a \in w : \sum_{n=1}^{\infty} |a_n| |v_n^{-1}| \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} w_i < \infty \right\}.$$

Proof. Let $a \in \mathbf{D}_v^m(p)$ and $x \in c(u, \Delta_v^m)_p$. Then there is a complex number λ such that $|u_n \Delta_v^m x_n - \lambda|^{p_n} \to 0$ $(n \to \infty)$. We define $y = (y_n)$ with

$$y_n = x_n + \lambda\, v_n^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} u_i^{-1}.$$

Then $y \in c_0(u, \Delta_v^m)_p$ and

$$\begin{aligned} \sum_{n=1}^{\infty} |a_n x_n| &= \sum_{n=1}^{\infty} |a_n| \left| y_n - \lambda\, v_n^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} u_i^{-1} \right| \\ &\le \sum_{n=1}^{\infty} |a_n| |y_n| + |\lambda| \sum_{n=1}^{\infty} |a_n| |v_n^{-1}| \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} w_i \\ &< \infty. \end{aligned}$$

Hence $a \in \mathbf{D}_v^m(p)$.

Now let $a \in \left[c(u, \Delta_v^m)_p\right]^\alpha$. Since $\left[c(u, \Delta_v^m)_p\right]^\alpha \subset \left[c_0(u, \Delta_v^m)_p\right]^\alpha$ and $\left[c_0(u, \Delta_v^m)_p\right]^\alpha = M_\alpha(u, p)$ by Theorem 2.1 (i), then $a \in M_\alpha(u, p)$. If we put

$$x_n = v_n^{-1} \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} w_i,$$

then $x \in c(u, \Delta_v^m)_p$ and therefore

$$\sum_{n=1}^{\infty} |a_n| |v_n^{-1}| \sum_{i=1}^{n-m} \binom{n-i-1}{m-1} w_i < \infty.$$

Thus $a \in \mathbf{D}_v^m(p)$.

Theorem 2.6. For every $p = (p_n)$,

(i) $\left[\ell_\infty(u, \Delta_v^m)_p\right]^\beta = \mathbf{D}_v^\beta(p)$,

(ii) $\left[\ell_\infty(u, \Delta_v^m)_p\right]^\gamma = \mathbf{D}_v^\gamma(p)$, where

$$\mathbf{D}_v^{\beta}(\boldsymbol{p}) = \bigcap_{S=2}^{\infty}\left\{\boldsymbol{a} \in \boldsymbol{w}: \sum_{n=1}^{\infty} \boldsymbol{a}_n \boldsymbol{v}_n^{-1} \sum_{i=1}^{n-m} \binom{\boldsymbol{n}-\boldsymbol{i}-\mathbf{1}}{\boldsymbol{m}-\mathbf{1}} \boldsymbol{S}^{\mathbf{1}/p_i}\, \boldsymbol{w}_i \text{ converges and } \sum_{n=1}^{\infty} |\boldsymbol{R}_n| \sum_{i=1}^{n-m+1} \binom{\boldsymbol{n}-\boldsymbol{i}-\mathbf{1}}{\boldsymbol{m}-\mathbf{2}} \boldsymbol{S}^{\mathbf{1}/p_i}\, \boldsymbol{w}_i < \infty\right\},$$

$$\mathbf{D}_v^{\gamma}(\boldsymbol{p}) = \bigcap_{S=2}^{\infty}\left\{\boldsymbol{a} \in \boldsymbol{w}: \sup_n \left|\sum_{n=1}^{\infty} \boldsymbol{a}_n \boldsymbol{v}_n^{-1} \sum_{i=1}^{n-m} \binom{\boldsymbol{n}-\boldsymbol{i}-\mathbf{1}}{\boldsymbol{m}-\mathbf{1}} \boldsymbol{S}^{\mathbf{1}/p_i}\, \boldsymbol{w}_i\right| < \infty, \sum_{n=1}^{\infty} |\boldsymbol{R}_n| \sum_{i=1}^{n-m+1} \binom{\boldsymbol{n}-\boldsymbol{i}-\mathbf{1}}{\boldsymbol{m}-\mathbf{2}} \boldsymbol{S}^{\mathbf{1}/p_i}\, \boldsymbol{w}_i < \infty\right\},$$

and $\boldsymbol{R}_n = \sum_{i=n+1}^{\infty} \boldsymbol{v}_i^{-1} \boldsymbol{a}_i \quad (\boldsymbol{n} = \mathbf{1}, \mathbf{2}, \dots)$.

Proof. Proof is smilar to [11]. So we omitted it.

3. Conclusion

Numerous branches of mathematics use the theory of sequence space, such as the structural theory of topological vector spaces, summability theory, and function space theory. However, the convergence issues that arise from the subject place it under analysis rather than algebra.

Given that sequence convergence is a crucial concept in the foundational theory of mathematics, numerous convergence notions arise in areas such as summability theory, classical measure theory, approximation theory, and probability theory, with discussions focusing on the relationships between them.

Researchers in this field may explore the topological and geometric characteristics of these sequence spaces.

Ethical statement

The author declares that this document does not require ethics committee approval or any special permission. Our study does not cause any harm to the environment.

Conflict of interest

The author declares no potential conflicts of interest related to this article's research, authorship, and publication.

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