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# Selection of third-party logistics provider based on extended multimoora technique under double hierarchy linguistic single-valued neutrosophic set

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**Abstract** — The demand for third-party logistics (3PL) providers becomes an increasingly important issue for corporations seeking improved customer service and cost reduction. Currently, there is no way to select the appropriate method for selecting 3PL. Therefore, this paper develops a new extended multi-objective optimization ratio analysis plus full multiplicative form (MULTIMOORA) method under double hierarchy linguistic single-valued neutrosophic sets (DHLSVNSs). For this, we first develop a new mathematical tool, i.e., DHLSVNSs, by studying single-valued neutrosophic set (SVN) and double hierarchy linguistic term set (DHLTSs), which is very effective for solving uncertainty in decision-making problems. A list of Einstein aggregation operators and their fundamental aspects for DHLSVNSs are presented based on Einstein's norms, as aggregation operators play an essential role in decision-making. A step-by-step algorithm of the Extended DHLSVN-MULTIMOORA approach is designed to tackle ambiguous and uncertain data during decision-making problems. The algorithm developed for the suggested technique is illustrated with a numerical example relevant to 3PL. A comparison of the proposed methods with various existing methodologies is carried out to demonstrate the superiority of the suggested algorithms.

**Keywords:** *Double hierarchy linguistic single-valued neutroshopic set, multimoora technique, multi criteria group decision making problems (MCGDM), aggregation operators, third-party logistics provider*

## 1. Introduction

Logistics plays an important role in establishing an industry's supply chain. However, with the market becoming increasingly global, industries now see logistics as a critical area where they may reduce costs and raise the standard of their customer service [1]. Logistics outsourcing, often known as third-party logistics (3PL), is a growing trend in the global business sector [2]. According to [3,4], suppliers can provide enterprises with the necessary services, including professional logistics and transportation, warehousing, logistics information systems, product return services, and inventory management. As a result, 3PL plays an important part in the logistical activities between the outsourced firm, the marketplace, and the customers. The key advantages of logistics alliances are that they allow the outsourced firm to focus on its core competencies, increase efficiency, improve service, eliminate transportation costs, restructure supply chains, and build market credibility [5–7].

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Consequently, selecting a suitable 3PL provider who can meet various requirements is critical for an enterprise's growth and competency [8–10]. In the provider selection process, the outsourcing firm frequently faces difficulties working with many logistics suppliers. Analysts are faced with a challenging task in selecting suitable suppliers. To address this challenge, Multi-Attribute Group Decision Making (MAGDM) is a process that plays an important role in selecting the best solution. There are two key goals in this method. While the primary goal is to create an environment in which the value of some basic criteria can be easily assessed, the second goal is to analyze data that is often unclear or ambiguous. To manage these related data, researchers developed the Fuzzy Set (FS) [11] theory, which provides information for managing the imprecision and inaccuracy of information by assigning membership degrees to each element of a fixed set. However, this idea has limitations due to the lack of non-membership; therefore, Atanassov [12] extended FS by including non-membership and developing a new theory called intuitionistic fuzzy set (IFS). Many scholars have used this theory to solve various DM problems. However, it has been suggested that they cannot handle the ambiguous and contradictory information that occurs in reality. Therefore, Smarandache [13] developed a new theory of neutrosophic sets (NS), which describes uncertain data by considering three mutually independent functions, namely true, uncertain, and false lying in $]0^-, 1^+[$.

NS theory can represent unclear data better than FS, IFS, and uncertainty speculation because it is consistent with human instinct judgments and feelings. The NS theory deals with indeterminate information, but it isn't easy to enforce in practical situations. Thus Wang et al. [14] developed a special type of NS, namely a single-valued neutrosophic set (SVNS), to handle real-world problems easily. SVNS is a helpful tool for representing situations with incomplete, uncertain, and inconsistent information. Some scholars have studied SVNS and defined various aggregation operators (AO) for SVNS. Aggregating data from several sources into a single AO is important. As a result, Li et al. [15] introduced the innovative concept of generalized simplified neutrosophic Einstein AOs. For SVNSs, Liu [16] proposed AOs based on the Archimedean t-norm and t-conorm and applied them to decision-making problems. Ji et al. [17] concentrated on the Frank operations of SVNNs and created the SVN prioritized Bonferroni mean (SVNFNPBM) operator according to the Frank aggregation function. Nancy and Garg [18] established SVNN operations as Frank-weighted aggregation operators and suggested a decision-making framework. Biswas et al. [19, 20] utilized the TOPSIS method for decision-making problems under the SVN environment. Considering the neutrosophic set, Zhang et al. [21] developed the general cloud method and other related ideas, such as backward cloud generators, two aggregation operators, and NNC distance measure. Lu and Ye [22] introduced hybrid weighted arithmetic and geometric aggregation functions under SVN information and used these operators to build decision-making challenges. Baser and Uluçay [23] defined an effective Q-neutrosophic soft sets and Its Application in Decision Making. Baser and Uluçay [24] also defined the applications of neutrosophic soft set decision-making problems. Reseacher [25] designed a TOPSIS-Based Entropy Measure for N-Valued Neutrosophic Trapezoidal Numbers. Uluçay et al. [26] defined a prioritized aggregation operators for the Evaluation of renewable energy sources.

Later, Uluçay and Deli [27] invented a novel vikor method for generalized trapezoidal hesitant fuzzy numbers. Uluçay et al. [28] proposed a N-valued neutrosophic trapezoidal numbers with similarity measures. The knowledge evaluated in the neutrosophic environment is quantitative and is represented numerically. However, in practice, most unclear or ambiguous data examined by the decision-maker (DMs) have qualitative aspects, for example, extremely poor, poor, fair, slightly good, very good, outstanding. The linguistic variable [29] is essential to access information and process qualitative data in these situations. Therefore, Li et al. [30] considered three membership degrees, such as truth, indeterminacy, and falsity, in the form of linguistic variables and developed the linguistic neutrosophic sets (LNS). Since LNS is particularly suitable for representing more complex linguistic information predicted by humans. Later, Gou et al. [31,32] extended the LTS and developed a new theory, namely double hierarchy linguistic term set (DHLTS), for more robust modeling of expert expressions. In most real-world problems, getting the correct reflection of attributes is very difficult for DMs. Besides that, most DMs find it more suitable to conduct qualitative evaluations of attributes. DHLTS conveys appropriate data more conveniently in complex expressions than single LTSs. DHLTS consists of two components, the first and second hierarchy linguistic term sets, allowing more flexibility in describing uncertainty and ambiguity. Many researchers successfully applied this concept [33].

Saleem et al. [34] proposed double hierarchy hesitant linguistic Einstein aggregation operators to solve real word problems. Multi-attribute group decision-making (MAGDM) is a procedure in which a panel of decision experts assesses the most advantageous alternative that supports certain features. To solve the MAGDM problem, Brauers and Zavadskas developed MOORA and MULTIMOORA techniques [35]. The traditional MOORA technique uses precise data to assess the preferred alternative based on the relative significance of many criteria. Later, Brauers and Zavadskas [36] invented the MULTIMOORA technique by considering crisp set theory, which is based on three approaches: (1) ratio system approach, (2) reference point approach, and (3) full multiplicative form approach. The MULTIMOORA technique is one of the most important techniques for handling real-world problems more significantly. Thus, Brauers et al. [37] extended the MULTIMOORA method fuzzy set. In the context of FSs, Hafezalkotob et al. [38] perform a summary of the MULTIMOORA approach. Alkan et al. [39] used the fuzzy MULTIMOORA approach to rank renewable energy sources. Liang et al. [40] investigated the MULTIMOORA approach in picking mining methods. Fattahi and Khalilzadeh expanded the MULTIMOORA approach for risk evaluation in a fuzzy environment [41]. Based on the objective weighting technique, Dahooie [42] enhanced the fuzzy MULTIMOORA technique. Zhang et al. [43] integrated the suggested intuitionistic fuzzy MULTIMOORA method and applied it to energy storage technologies selection. To assess solid waste management strategies, Garg and Rani [44] presented a MULTIMOORA method involving aggregation operators under IFS. Later, Chen et al. [45] extended the MULTIMOORA method to linguistic evaluations. Zhang [46] considered the MULTIMOORA method to solve decision-making problems in a linguistic intuitionistic fuzzy environment. Ding and Zhong [15] introduced the MULTIMOORA approach using two-dimensional uncertain linguistic variables (TDULVs). Balezentis and Balezentis [47] suggested a 2-tuple linguistic fuzzy MULTIMOORA technique. Wei [48] proposed the 2-tuple linguistic intuitionistic FSs (2TLIFSs). Akram et al. [49] recently created the 2TLPF-MULTIMOORA technique.

In light of the above literature, it is analyzed that there is no application and detail about the combined study of SVNSs and DHLTSs for handling the uncertainty and fuzziness under the MULTIMOORA technique. So, Inspired by the above discussion in this study, we define a new theory, namely double hierarchy linguistic single-valued neutrosophic sets (DHLSVNSs), to evaluate decision-making problems more accurately. The main motivations for this work are as follows:

*i.* To develop a novel notion of double hierarchy linguistic single-valued neutrosophic sets (DHLSVNSs) by extending the SVNs to DHLTSs. DHLSVNSs is a more generalized version that effectively resolves ambiguity in decision-making problems. They are adaptable tools that allow DMs to provide assessments in the form of DHLSVNSs.

*ii.* To define new operational laws for DHLSVNSs based on Einstein t-norm and t-conorms.

*iii.* To develop a list of Einstein aggregation operators and discuss the related properties.

*iv.* To create an extended MULTIMOORA technique for solving decision-making problems. Because the MULTIMOORA approach has several standout qualities, including shorter computation times, more thorough mathematical calculations, simplicity, and consistency of outcomes. It comprises of three MOORA techniques utilize full multiplicative form, reference point, and ratio analysis. It uses aggregation algorithms to incorporate the subordinate ranks of the alternatives and more correctly reflect the results.

*v.* In order to show the applicability and dependability, we applied the provided methodology, a numerical case study selecting a third party logistic service provider.

*vi.* To demonstrate the stability and validity of our developed work, we compare our proposed technique to previous methods.

From literature the existence idea are very helpful to solve decision making problems, but this ideas only handle the decision making problems qualitatively. Therefore, the novelty of this paper is to developed a novel idea for solving decision making qualitatively. The main focus of this study is to find the best third party logistic service provider that can assist in real-life problems. This research study has contributed to the analysis of MCGDM under ambiguity in the following manner:

*i.* We design a new operation for DHLSVNSs to handle the decision issues more accurately.

*ii.* Einstein t-norm and t-conorm have great significance as they incorporate the properties of several others. Therefore, to aggregate the DM process, we define the aggregation operators and basic operation of DHLSVNSs based on the Einstein t-norm . We introduce a variety of aggregation operators such as the Einstein weighted averaging and geometric aggregation operator and thier basic properties.

*iii.* A new DHLSVN-Multimoora method is developed to handle complex decision making problems under DHLSVNSs.

*iv.* A novel distance measure and score function is proposed for finding the the ranking and distance between tow different double hierarchy linguistic neutrosophic numbers.

The summary of this article is as follows: The basic concepts related to SVNSs, LTSs and DHLTSs are given in Section 2. Section 3 includes the novel notion of DHLSVNSs and score function, which can help the DM process. Section 4 includes the distance measures and Einstein aggregation operators of DHLSVNSs. Section 5 presents a step-wise algorithm for Extended MULTIMOORA method under a double hierarchy linguistic single-valued neutrosophic context. Section 6 describes a numerical application related to third-party logistic selection. Section 7 compares the proposed method with existing techniques to demonstrate its applicability. Section 8 concludes this article. The list of abbreviations and symbols are given in Table 1.

**Table 1.** List of abbreviations and symbols

| **Description** | **Abbreviation** |
|---|---|
| Truth membership degree | $\mu(a)$ |
| Indeterminacy membership degree | $\eta(a)$ |
| Falsity membership degree | $\nu(a)$ |
| Linguistic single-valued neutrosophic number | LSVNNs |
| Linguistic truth membership degree | $\gimel^L_\mu(a)$ |
| Linguistic Indeterminacy membership degree | $\gimel^L_\eta(a)$ |
| Linguistic Falsity membership degree | $\gimel^L_\nu(a)$ |

**Table 1.** (Continued) List of abbreviations and symbols

| **Description** | **Abbreviation** |
|---|---|
| Single-valued neutrosophic set | SVNS |
| Neutrosophic set | NS |
| First hierarchy linguistic term | FHLT |
| Second hierarchy linguistic term | SHLT |
| Double hierarchy linguistic term sets | DHLTSs |
| Double hierarchy linguistic single-valued neutrosophic set | DHLSVNS |
| Double hierarchy linguistic single-valued neutrosophic numbers | DHLSVNNs |
| Double hierarchy linguistic single-valued neutrosophic Einstein weighted averaging | DHLSVNEWA |
| Double hierarchy linguistic single-valued neutrosophic Einstein arithmetic | DHLSVNEA |
| Double hierarchy linguistic single-valued neutrosophic Einstein weighted geometric | DHLSVNEWG |
| Double hierarchy linguistic single-valued neutrosophic Einstein geometric | DHLSVNEG |

## 2. Preliminaries

This section provides some basic notions to be required in the following sections.

**Definition 2.1.** [13] Let $X \neq \phi$. The structure $N = \{a, \langle \mu(a), \eta(a), \nu(a) | a \in X \rangle\}$ is called neutrosophic set (NS), where for each $a \in X$, $\mu(a), \eta(a), \nu(a) : X \longrightarrow \, ]0^-, 1^+[$ is the truth, indeterminacy and falsity membership degree respectively, with conditions $0^- \leq (\mu(a)) + (\eta(a)) + (\nu(a)) \leq 3^+$.

**Definition 2.2.** [14] For a non empty set $X$. The single-valued neutrosophic set (SVNS) is mathematically denoted by $V = \{a, \langle \mu(a), \eta(a), \nu(a) | a \in X \rangle\}$, where $a \in X$, $\mu(a), \eta(a), \nu(a) : X \longrightarrow [0, 1]$ represents the truth, indeterminacy and falsity membership degree respectively subject to the conditions $0 \leq (\mu(a)) + (\eta(a)) + (\nu(a)) \leq 3$,

**Definition 2.3.** [50] Let $\gimel = \{\gimel_\gamma | \gamma = 0, 1, \cdots, \tau\}$, is the linguistic term with odd cardinality and $\gimel^{[0,\tau]} = \{\gimel_\gamma | \gimel_0 \leq \gimel_\gamma \leq \gimel_\tau, \gamma \in [0, \tau]\}$ is the continuous linguistic term set. Then the structure $\gimel = \{a, \gimel^L_\mu(a), \gimel^L_\eta(a), \gimel^L_\nu(a) | a \in X\}$ is known as linguistic single-valued neutrosophic number (LSVNNs) , where for each $a \in X$, $\gimel^L_\mu(a), \gimel^L_\eta(a), \gimel^L_\nu(a) \in \gimel^{[0,\tau]}$ represent the truth, indeterminacy and falsity linguistic degree respectively, such that $0 \leq \mu(a) + \eta(a) + \nu(a) \leq 3\tau$. The tripled $\langle \gimel_\mu, \gimel_\eta, \gimel_\nu \rangle$ is said to LSVNNs and denoted by $\gimel = \langle \gimel_\mu, \gimel_\eta, \gimel_\nu \rangle$. If $\gimel_\mu, \gimel_\eta, \gimel_\nu \in \gimel$, then tripled $\langle \gimel_\mu, \gimel_\eta, \gimel_\nu \rangle$ are the original LSVNNs.

**Definition 2.4.** [31] Let $\gimel = \{\gimel_\gamma | \gamma = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$ be the first (FHLT) and $\daleth = \{\daleth_\varphi | \varphi = -\delta, \cdots, -1, 0, 1, \cdots, \delta\}$ be the second hierarchy linguistic term (SHLT) sets, then the double hierarchy linguistic term sets is symbolically denoted by

$$\gimel_\daleth = \left\{ \gimel_{\gamma\langle \daleth_\varphi \rangle} | \theta = -\tau, \cdots, -1, 0, 1, \cdots, \tau; \varphi = -\delta, \cdots, -1, 0, 1, \cdots, \delta \right\}$$

where $\gimel_\gamma$ is the first hierarchy and $\daleth_\varphi$ represent the second hierarchy linguistic terms, respectively.

## 3. Formation of Double Hierarchy Linguistic Single-Valued Neutrosophic Sets

This section explores the novel notion of double hierarchy linguistic single-valued neutrosophic sets on the base of [31, 32].

**Definition 3.1.** Let $\gimel = \left\{ \left\langle \gimel^L_\mu(a), \gimel^L_\eta(a), \gimel^L_\nu(a) | \mu, \eta, \nu = 0, 1, \cdots, \tau \right\rangle \right\}$ be the first hierarchy linguistic single-valued neutrosophic sets and

$$\daleth = \left\{ \left\langle \daleth^L_w(a), \daleth^L_x(a), \daleth^L_y(a) | w, x, y = 0, 1, \cdots, \delta \right\rangle \right\}$$

be the second hierarchy linguistic single-valued neutrosophic sets, then the double hierarchy linguistic single-valued neutrosophic sets (DHLSVNSs) is defined as:

$$\gimel\daleth = \left\{ \left\langle \gimel^L_{\mu\langle \daleth^L_w \rangle}, \gimel^L_{\eta\langle \daleth^L_x \rangle}, \gimel^L_{\nu\langle \daleth^L_y \rangle} \right\rangle | \mu, \eta, \nu \in [0, \tau] ; w, x, y \in [0, \delta] \right\} \tag{3.1}$$

Where $\gimel^L_\mu, \gimel^L_\eta, \gimel^L_\nu \in \gimel$ represents the truth, indeterminacy and falsity degree of first hierarchy linguistic term sets and $\daleth^L_w, \daleth^L_x, \daleth^L_y \in \daleth$ is the truth, indeterminacy and falsity degree of second hierarchy linguistic term sets, such that $0 \leq \mu + \eta + \nu \leq 3\tau$ and $0 \leq w + x + y \leq 3\delta$. Simply it can be represented as

$$\gimel\daleth = \left\langle \gimel_{\mu\langle \daleth_w \rangle}, \gimel_{\eta\langle \daleth_x \rangle}, \gimel_{\nu\langle \daleth_x \rangle} \right\rangle$$

**Definition 3.2.** Let $\gimel\daleth_i = \left\langle \gimel_{\mu_i\langle \daleth_{w_i} \rangle}, \gimel_{\eta_i\langle \daleth_{x_i} \rangle}, \gimel_{\nu_i\langle \daleth_{y_i} \rangle} \right\rangle (i \in N)$ be a DHLSVNSs. Then mathematically the score are denoted and defined by

$$S_C = \left( \left( 2 + \gimel_{\left(\frac{\mu_i}{\tau}\right) - \left(\frac{\eta_i}{\tau}\right) - \left(\frac{v_i}{\tau}\right)} \right) + \left( 2 + \gimel_{\left(\frac{w_i}{\delta}\right) - \left(\frac{x_i}{\delta}\right) - \left(\frac{y_i}{\delta}\right)} \right) \right) /2 \in [0, 1] \tag{3.2}$$

## 4. Einstein Operation

Since the inception of fuzzy set theory, the set theoretical operators have played an essential role. A variety of special operators have been incorporated in the general notions of the t-norms and t-conorms, which meet the needs of the conjunction and disjunction operators, accordingly. There are numerous t-norms and t-conorms types that can be employed to execute the corresponding intersections and unions. The Einstein product and Einstein sum are examples of t-norms and t-conorms, which are defined as follows.

**Definition 4.1.** [51] Let $a, b \in R$. Then, the family of Einstein t-norms are mathematically defined as

$$a \oplus_e b = \frac{a + b}{1 + ab} \tag{4.1}$$

**Definition 4.2.** [51] Let $a, b \in R$. Then, the family of Einstein t-conorms are mathematically defined as

$$a \otimes_e b = \frac{ab}{1 + (1 - a)(1 - b)} \tag{4.2}$$

for all $a, b \in [0, 1]^2$.

We introduce the Einstein operations for DHLSVNSs and examine some of their desirable characteristics.

**Definition 4.3.** Let $\beth\daleth_1 = \left\langle \beth_{\mu_1}\langle\daleth_{w_1}\rangle, \beth_{\eta_1}\langle\daleth_{x_1}\rangle, \beth_{\nu_1}\langle\daleth_{y_1}\rangle \right\rangle$ and $\beth\daleth_2 = \left\langle \beth_{\mu_2}\langle\daleth_{w_2}\rangle,\ \beth_{\eta_2}\langle\daleth_{x_2}\rangle, \beth_{\nu_2}\langle\daleth_{y_2}\rangle \right\rangle$ be the two double hierarchy linguistic single-valued neutrosophic sets. Then, Einstein's operational laws for (DHLSVNSs) are as follows:

*i.*

$$\beth\daleth_1 \oplus \beth\daleth_2 = \left( \begin{array}{c} \left( \beth_{\tau\left(\frac{\frac{\mu_1}{\tau}+\frac{\mu_2}{\tau}}{1+\frac{\mu_1}{\tau}\frac{\mu_2}{\tau}}\right)} \left\langle \daleth_{\delta\left(\frac{\frac{w_1}{\delta}+\frac{w_2}{\delta}}{1+\frac{w_1}{\delta}\frac{w_2}{\delta}}\right)} \right\rangle \right), \\ \left( \beth_{\tau\left(\frac{\frac{\eta_1}{\tau}\frac{\eta_2}{\tau}}{1+\left(1-\frac{\eta_1}{\tau}\right)\left(1-\frac{\eta_2}{\tau}\right)}\right)} \left\langle \daleth_{\delta\left(\frac{\frac{x_1}{\delta}\frac{x_2}{\delta}}{1+\left(1-\frac{x_1}{\delta}\right)\left(1-\frac{x_2}{\delta}\right)}\right)} \right\rangle \right), \\ \left( \beth_{\tau\left(\frac{\frac{\nu_1}{\tau}\frac{v_2}{\tau}}{1+\left(1-\frac{\nu_1}{\tau}\right)\left(1-\frac{v_2}{\tau}\right)}\right)} \left\langle \daleth_{\delta\left(\frac{\frac{y}{\delta}\frac{y_2}{\delta}}{1+\left(1-\frac{y}{\delta}\right)\left(1-\frac{y_2}{\delta}\right)}\right)} \right\rangle \right) \end{array} \right) \tag{4.3}$$

*ii.*

$$\beth\daleth_1 \otimes \beth\daleth_2 = \left( \begin{array}{c} \left( \beth_{\tau\left(\frac{\frac{\mu_1}{\tau}\frac{\mu_2}{\tau}}{1+\left(1-\frac{\mu_1}{\tau}\right)\left(1-\frac{\mu_2}{\tau}\right)}\right)} \left\langle \daleth_{\delta\left(\frac{\frac{w_1}{\delta}\frac{w_2}{\delta}}{1+\left(1-\frac{w_1}{\delta}\right)\left(1-\frac{w_2}{\delta}\right)}\right)} \right\rangle \right), \\ \left( \beth_{\tau\left(\frac{\frac{\eta_1}{\tau}+\frac{\eta_2}{\tau}}{1+\frac{\eta_1}{\tau}\frac{\eta_2}{\tau}}\right)} \left\langle \daleth_{\delta\left(\frac{\frac{x_1}{\delta}+\frac{x_2}{\delta}}{1+\frac{x_1}{\delta}\frac{x_2}{\delta}}\right)} \right\rangle \right), \\ \left( \beth_{\tau\left(\frac{\frac{v_1}{\tau}+\frac{v_2}{\tau}}{1+\frac{v_1}{\tau}\frac{v_2}{\tau}}\right)} \left\langle \daleth_{\delta\left(\frac{\frac{y_1}{\delta}+\frac{y_2}{\delta}}{1+\frac{y_1}{\delta}\frac{y_2}{\delta}}\right)} \right\rangle \right) \end{array} \right) \tag{4.4}$$

*iii.*

$$
k.\beth\daleth_1 = \left( \begin{array}{c}
\left( \beth_{\tau\left(\frac{\left(1+\frac{\mu_1}{\tau}\right)^k-\left(1-\frac{\mu_2}{\tau}\right)^k}{\left(1+\frac{\mu_1}{\tau}\right)^k+\left(1-\frac{\mu_2}{\tau}\right)^k}\right)} \left\langle \daleth_{\delta\left(\frac{\left(1+\frac{w_1}{\delta}\right)^k-\left(1-\frac{w_2}{\delta}\right)^k}{\left(1+\frac{w_1}{\delta}\right)^k+\left(1-\frac{w_2}{\delta}\right)^k}\right)} \right\rangle \right), \\
\left( \beth_{\tau\left(\frac{2\left(\frac{\eta_1}{\tau}\right)^k}{\left(2-\frac{\eta_1}{\tau}\right)^k+\left(\frac{\eta_1}{\tau}\right)^k}\right)} \left\langle \daleth_{\delta\left(\frac{2\left(\frac{x_1}{\delta}\right)^k}{\left(2-\frac{x_1}{\delta}\right)^k+\left(\frac{x_1}{\delta}\right)^k}\right)} \right\rangle \right), \\
\left( \beth_{\tau\left(\frac{2\left(\frac{v_1}{\tau}\right)^k}{\left(2-\frac{v_1}{\tau}\right)^k+\left(\frac{v_1}{\tau}\right)^k}\right)} \left\langle \daleth_{\delta\left(\frac{2\left(\frac{y_1}{\delta}\right)^k}{\left(2-\frac{y_1}{\delta}\right)^k+\left(\frac{y_1}{\delta}\right)^k}\right)} \right\rangle \right)
\end{array} \right) \tag{4.5}
$$

*iv.*

$$
\beth\daleth_1^k = \left( \begin{array}{c}
\left( \beth_{\tau\left(\frac{2\left(\frac{\mu_1}{\tau}\right)^k}{\left(2-\frac{\mu_1}{\tau}\right)^k+\left(\frac{\mu_1}{\tau}\right)^k}\right)} \left\langle \daleth_{\delta\left(\frac{2\left(\frac{w_1}{\delta}\right)^k}{\left(2-\frac{w_1}{\delta}\right)^k+\left(\frac{w_1}{\delta}\right)^k}\right)} \right\rangle \right), \\
\left( \beth_{\tau\left(\frac{\left(1+\frac{\eta_1}{\tau}\right)^k-\left(1-\frac{\eta_2}{\tau}\right)^k}{\left(1+\frac{\eta_1}{\tau}\right)^k+\left(1-\frac{\eta_2}{\tau}\right)^k}\right)} \left\langle \daleth_{\delta\left(\frac{\left(1+\frac{x_1}{\delta}\right)^k-\left(1-\frac{x_2}{\delta}\right)^k}{\left(1+\frac{x_1}{\delta}\right)^k+\left(1-\frac{x_2}{\delta}\right)^k}\right)} \right\rangle \right), \\
\left( \beth_{\tau\left(\frac{\left(1+\frac{v_1}{\tau}\right)^k-\left(1-\frac{v_2}{\tau}\right)^k}{\left(1+\frac{v_1}{\tau}\right)^k+\left(1-\frac{v_2}{\tau}\right)^k}\right)} \left\langle \daleth_{\delta\left(\frac{\left(1+\frac{y_1}{\delta}\right)^k-\left(1-\frac{y_2}{\delta}\right)^k}{\left(1+\frac{y_1}{\delta}\right)^k+\left(1-\frac{y_2}{\delta}\right)^k}\right)} \right\rangle \right)
\end{array} \right) \tag{4.6}
$$

### 4.1. Double Hierarchy Linguistic Single-Valued Neutrosophic Einstein Averaging Aggregation Information

This section devoted a list of Einstein weighted averaging aggregation operators and Einstein averaging aggregation operators for DHLSVNSs also describes its basic properties as follows:

**Definition 4.4.** Suppose we have a family $\beth\daleth_i = \left\langle \beth_{\mu_i}\langle\daleth_{w_i}\rangle, \beth_{\eta_i}\langle\daleth_{x_i}\rangle, \beth_{\nu_i}\langle\daleth_{y_i}\rangle \right\rangle$ $(i \in \mathbb{N})$ of single-valued neutrosophic double hierarchy linguistic numbers (SVNDHLNs) and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ represent weight vectors of given family restricted to $\omega_i > 0$, $\sum\limits_{i=1}^{n} \omega_i = 1$. Then based on above operational laws the double hierarchy linguistic single-valued neutrosophic Einstein weighted averaging (DHLSVNEWA) operator are defined as:

$$DHLSVNEWA(\gimel_{\daleth_1},\gimel_{\daleth_2},\cdots,\gimel_{\daleth_n}) = \omega_1.\gimel_{\daleth_1} \oplus \omega_2.\gimel_{\daleth_2},\cdots,\oplus\omega_n.\gimel_{\daleth_n}$$

$$= \left( \begin{array}{c} \left( \gimel_{\tau\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\tau}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\tau}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\tau}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\tau}\right)^{\omega_i}}\right)} \left\langle \daleth_{\delta\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\delta}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\delta}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\delta}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\delta}\right)^{\omega_i}}\right)} \right\rangle \right), \\ \left( \gimel_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{\eta_i}{\tau}\right)^{\omega_i}}{\left(2-\frac{\eta_1}{\tau}\right)^{\omega_i}+\left(\frac{\eta_1}{\tau}\right)^{\omega_i}}\right)} \left\langle \daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{\eta_i}{\delta}\right)^{\omega_i}}{\left(2-\frac{x_1}{\delta}\right)^{\omega_i}+\left(\frac{x_1}{\delta}\right)^{\omega_i}}\right)} \right\rangle \right), \\ \left( \gimel_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{v_1}{\tau}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(2-\frac{v_1}{\tau}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\frac{v_1}{\tau}\right)^{\omega_i}}\right)} \left\langle \daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{y_1}{\delta}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(2-\frac{y_1}{\delta}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\frac{y_1}{\delta}\right)^{\omega_i}}\right)} \right\rangle \right) \end{array} \right) \tag{4.7}$$

**Theorem 4.5.** Suppose we have a family $\gimel_{\daleth_i} = \left\langle \gimel_{\mu_i\langle\daleth_{w_i}\rangle}, \gimel_{\eta_i\langle\daleth_{x_i}\rangle}, \gimel_{\nu_i\langle\daleth_{y_i}\rangle} \right\rangle$ $(i \in \mathbb{N})$ of double hierarchy linguistic single-valued neutrosophic numbers and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ represent weight vectors of given family restricted to $\omega_i > 0$, $\sum_{i=1}^{n} \omega_i = 1$. Then, DHLSVNEWA operators satisfies the following properties as:

*i.* (Idempotency) Suppose for all $\gimel_{\daleth_i}(i \in \mathbb{N})$, is equal i.e $\gimel_{\daleth_i} = \gimel_{\daleth}$, then

$$DHLSVNEWA\left(\gimel_{\daleth_1},\gimel_{\daleth_2},\cdots,\gimel_{\daleth_n}\right) = \gimel_{\daleth}$$

*ii.* (Monotonicity) Consider $\gimel_{\daleth_i}$, $\gimel^*_{\daleth_i}$ be two sets of DHLEs, $\gimel_{\daleth_i} \leq \gimel^*_{\daleth_i}$ for all $i$; then:

$$DHLSVNEWA\left(\gimel_{\daleth_1},\gimel_{\daleth_2},\cdots,\gimel_{\daleth_n}\right) \leq DHLSVNEWA\left(\gimel^*_{\daleth_1},\gimel^*_{\daleth_2},\cdots,\gimel^*_{\daleth_3}\right)$$

*iii.* (Boundedness) Consider $\gimel^-_{\daleth} = \min_{1\leq i\leq n}\{\gimel_{\daleth_i}\}$, $\gimel^+_{\daleth} = \max_{1\leq i\leq n}\{\gimel_{\daleth_i}\}$. Then,

$$\gimel^-_{\daleth} \leq DHLSVNEWA\left(\gimel_{\daleth_1},\gimel_{\daleth_2},\cdots,\gimel_{\daleth_n}\right) \leq \gimel^+_{\daleth}$$

Proof. Given that $\gimel\daleth_i = \gimel\daleth$ for each $i$, then

$$
\begin{aligned}
& DHLSVNEWA(\gimel\daleth_1, \gimel\daleth_2, \cdots, \gimel\daleth_n) = \omega_1.\gimel\daleth_1 \oplus \omega_2.\gimel\daleth_2, \cdots, \oplus\omega_n.\gimel\daleth_n \\
= & \left( \begin{array}{c}
\left( \gimel_{\tau\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\tau}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\frac{\mu_i}{\tau}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\tau}\right)^{\omega_i} + \prod_{i=1}^{n}\left(1-\frac{\mu_i}{\tau}\right)^{\omega_i}}\right)} \left\langle \daleth_{\delta\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\delta}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\frac{\mu_i}{\delta}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\delta}\right)^{\omega_i} + \prod_{i=1}^{n}\left(1-\frac{\mu_i}{\delta}\right)^{\omega_i}}\right)} \right\rangle \right), \\
\left( \gimel_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{\eta_i}{\tau}\right)^{\omega_i}}{\left(2-\frac{\eta_1}{\tau}\right)^{\omega_i} + \left(\frac{\eta_1}{\tau}\right)^{\omega_i}}\right)} \left\langle \daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{\eta_i}{\delta}\right)^{\omega_i}}{\left(2-\frac{x_1}{\delta}\right)^{\omega_i} + \left(\frac{x_1}{\delta}\right)^{\omega_i}}\right)} \right\rangle \right), \\
\left( \gimel_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{v_1}{\tau}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(2-\frac{v_1}{\tau}\right)^{\omega_i} + \prod_{i=1}^{n}\left(\frac{v_1}{\tau}\right)^{\omega_i}}\right)} \left\langle \daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{y_1}{\delta}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(2-\frac{y_1}{\delta}\right)^{\omega_i} + \prod_{i=1}^{n}\left(\frac{y_1}{\delta}\right)^{\omega_i}}\right)} \right\rangle \right)
\end{array} \right)
\end{aligned}
$$

$$= \left( \begin{array}{c} \left( \daleth_{\tau\left(\frac{\left(1+\frac{\mu_i}{\tau}\right)^{\sum_{i=1}^{n}}-\left(1-\frac{\mu_i}{\tau}\right)^{\sum_{i=1}^{n}}}{\left(1+\frac{\mu_i}{\tau}\right)^{\sum_{i=1}^{n}}+\left(1-\frac{\mu_i}{\tau}\right)^{\sum_{i=1}^{n}}}\right)} \left\langle \neg_{\delta\left(\frac{\left(1+\frac{\mu_i}{\delta}\right)^{\sum_{i=1}^{n}}-\left(1-\frac{\mu_i}{\delta}\right)^{\sum_{i=1}^{n}}}{\left(1+\frac{\mu_i}{\delta}\right)^{\sum_{i=1}^{n}}+\left(1-\frac{\mu_i}{\delta}\right)^{\sum_{i=1}^{n}}}\right)} \right\rangle \right), \\ \left( \daleth_{\tau\left(\frac{2\left(\frac{\eta_i}{\tau}\right)^{\sum_{i=1}^{n}}}{\left(2-\frac{\eta_1}{\tau}\right)^{\sum_{i=1}^{n}}+\left(\frac{\eta_1}{\tau}\right)^{\sum_{i=1}^{n}}}\right)} \left\langle \neg_{\delta\left(\frac{2\left(\frac{\eta_i}{\delta}\right)^{\sum_{i=1}^{n}}}{\left(2-\frac{x_1}{\delta}\right)^{\sum_{i=1}^{n}}+\left(\frac{x_1}{\delta}\right)^{\sum_{i=1}^{n}}}\right)} \right\rangle \right), \\ \left( \daleth_{\tau\left(\frac{2\left(\frac{v_1}{\tau}\right)^{\sum_{i=1}^{n}}}{\left(2-\frac{v_1}{\tau}\right)^{\sum_{i=1}^{n}}+\left(\frac{v_1}{\tau}\right)^{\sum_{i=1}^{n}}}\right)} \left\langle \neg_{\delta\left(\frac{2\left(\frac{y_1}{\delta}\right)^{\sum_{i=1}^{n}}}{\left(2-\frac{y_1}{\delta}\right)^{\sum_{i=1}^{n}}+\left(\frac{y_1}{\delta}\right)^{\sum_{i=1}^{n}}}\right)} \right\rangle \right) \end{array} \right)$$

$$= \left\langle \daleth_{\mu\langle \neg_w \rangle}, \daleth_{\eta\langle \neg_x \rangle}, \daleth_{\nu\langle \neg_y \rangle} \right\rangle = \daleth_\neg$$

□

Proof. Since $\daleth_{\neg_i} \leq \daleth^*_{\neg_i}$ then, $\omega_i \daleth_{\neg_i} \leq \omega_i \daleth^*_{\neg_i}$, accordingly we deduce that $\oplus_{i=1}^{n} \omega_i . \daleth_{\neg_i} \leq \oplus_{i=1}^{n} \omega_i \daleth^*_{\neg_i}$. Hence, $DHLSVNEWA(\daleth_{\neg_1}, \daleth_{\neg_2}, \cdots, \daleth_{\neg_n}) = \oplus_{i=1}^{n} \omega_i \daleth_{\neg_i}$ and $DHLSVNEWA(\daleth^*_{\neg_1}, \daleth^*_{\neg_2}, \cdots, \daleth^*_{\neg_n}) = \oplus_{i=1}^{n} \omega_i \daleth^*_{\neg_i}$, we can generate

$$DHLSVNEWA(\daleth_{\neg_1}, \daleth_{\neg_2}, \cdots, \daleth_{\neg_n}) \leq DHLSVNEWA(\daleth^*_{\neg_1}, \daleth^*_{\neg_2}, \cdots, \daleth^*_{\neg_n})$$

□

Proof. Since $\daleth^-_{\neg} = \min_{1 \leq i \leq n} \{\daleth_{\neg_i}\}$ and $\daleth^+_{\neg} = \max_{1 \leq i \leq n} \{\daleth_{\neg_i}\}$, then according to the monotonicity properties

$$DHLSVNEWA(\daleth^-_{\neg_1}, \daleth^-_{\neg_2}, \cdots, \daleth^-_{\neg_n}) \leq DHLSVNEWA(\daleth_{\neg_1}, \daleth_{\neg_2}, \cdots, \daleth_{\neg_n}) \leq DHLSVNEWA(\daleth^+_{\neg_1}, \daleth^+_{\neg_2}, \cdots, \daleth^+_{\neg_n})$$

Furthermore, by mean of idempotency properties, we have

$$DHLSVNEWA(\beth_{\daleth_1}^{-},\beth_{\daleth_2}^{-},\cdots,\beth_{\daleth_n}^{-})=\beth_{\daleth_1}^{-},\quad DHLSVNEWA(\beth_{\daleth_1}^{+},\beth_{\daleth_2}^{+},\cdots,\beth_{\daleth_n}^{+})=\beth_{\daleth}^{+}$$

Accordingly, we can deduce that

$$\beth_{\daleth}^{-}\leq DHLSVNEWA\left(\beth_{\daleth_1},\beth_{\daleth_2},\cdots,\beth_{\daleth_n}\right)\leq\beth_{\daleth}^{+}$$

□

**Definition 4.6.** Suppose we have a family $\beth_{\daleth_i}=\left\langle\beth_{\mu_i\langle\daleth_{w_i}\rangle},\beth_{\eta_i\langle\daleth_{x_i}\rangle},\beth_{\nu_i\langle\daleth_{y_i}\rangle}\right\rangle$ $(i\in\mathbb{N})$ of double hierarchy linguistic single-valued neutrosophic numbers, then the double hierarchy linguistic single-valued neutrosophic Einstein arithmetic (DHLSVNEA) mean operators are as follows:

$$\begin{aligned}
&SVNDHLEA(\beth_{\daleth_1},\beth_{\daleth_2},\cdots,\beth_{\daleth_n})=\beth_{\daleth_1}\oplus\beth_{\daleth_2},\cdots,\oplus\beth_{\daleth_n}\\
&=\left(\begin{array}{c}
\left(\beth_{\tau\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\tau}\right)-\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\tau}\right)}{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\tau}\right)+\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\tau}\right)}\right)}\left\langle\daleth_{\delta\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\delta}\right)-\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\delta}\right)}{\prod_{i=1}^{n}\left(1+\frac{\mu_i}{\delta}\right)+\prod_{i=1}^{n}\left(1-\frac{\mu_i}{\delta}\right)}\right)}\right\rangle\right),\\
\left(\beth_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{\eta_i}{\tau}\right)}{\left(2-\frac{\eta_1}{\tau}\right)+\left(\frac{\eta_1}{\tau}\right)}\right)}\left\langle\daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{\eta_i}{\delta}\right)}{\left(2-\frac{x_1}{\delta}\right)+\left(\frac{x_1}{\delta}\right)}\right)}\right\rangle\right),\\
\left(\beth_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{v_1}{\tau}\right)}{\prod_{i=1}^{n}\left(2-\frac{v_1}{\tau}\right)+\prod_{i=1}^{n}\left(\frac{v_1}{\tau}\right)}\right)}\left\langle\daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{y_1}{\delta}\right)}{\prod_{i=1}^{n}\left(2-\frac{y_1}{\delta}\right)+\prod_{i=1}^{n}\left(\frac{y_1}{\delta}\right)}\right)}\right\rangle\right)
\end{array}\right)
\end{aligned}\tag{4.8}$$

**Theorem 4.7.** Suppose we have a family $\beth_{\daleth_i}=\left\langle\beth_{\mu_i\langle\daleth_{w_i}\rangle},\beth_{\eta_i\langle\daleth_{x_i}\rangle},\beth_{\nu_i\langle\daleth_{y_i}\rangle}\right\rangle$ $(i\in\mathbb{N})$ of double hierarchy linguistic single-valued neutrosophic numbers. Then, DHLSVNEA operators satisfies the following properties as:

*i.* (Idempotency) Suppose for all $i\in\mathbb{N}$, $\beth_{\daleth_i}=\beth_{\daleth}$. Then,

$$DHLSVNEA\left(\beth_{\daleth_1},\beth_{\daleth_2},\cdots,\beth_{\daleth_n}\right)=\beth_{\daleth}$$

*ii.* (Boundedness) Consider $\gimel_{\daleth}^{-} = \min_{1\leq i\leq n}\{\gimel_{\daleth_i}\}$ and $\gimel_{\daleth}^{+} = \max_{1\leq i\leq n}\{\gimel_{\daleth_i}\}$. Then,

$$\gimel_{\daleth}^{-} \leq DHLSVNEA\left(\gimel_{\daleth_1}, \gimel_{\daleth_2}, \cdots, \gimel_{\daleth_n}\right) \leq \gimel_{\daleth}^{+}$$

*iii.* (Monotonicity) Consider $\gimel_{\daleth_i}$, $\gimel^{*}_{\daleth_i}$ be two sets of DHLEs, $\gimel_{\daleth_i} \leq \gimel^{*}_{\daleth_i}$ for all $i$, then,

$$DHLSVNEA\left(\gimel_{\daleth_1}, \gimel_{\daleth_2}, \cdots, \gimel_{\daleth_n}\right) \leq DHLSVNEA\left(\gimel^{*}_{\daleth_1}, \gimel^{*}_{\daleth_2}, \cdots, \gimel^{*}_{\daleth_3}\right)$$

Proof. It is clear from Theorem 4.5. □

## 4.2. Double Hierarchy Linguistic Single-Valued Neutrosophic Einstein Geometric Aggregation Information

This section devoted a list of Einstein geometric aggregation operators such as double hierarchy linguistic single-valued neutrosophic Einstein Weighted geometric (DHLSVNEWG) and double hierarchy linguistic single-valued neutrosophic Einstein geometric (DHLSVNEG) operators also describes its basic properties as follows:

**Definition 4.8.** Suppose we have a family $\gimel_{\daleth_i} = \left\langle \gimel_{\mu_i}\langle\daleth_{w_i}\rangle, \gimel_{\eta_i}\langle\daleth_{x_i}\rangle, \gimel_{\nu_i}\langle\daleth_{y_i}\rangle \right\rangle (i \in \mathbb{N})$ of double hierarchy linguistic single-valued neutrosophic numbers (DHLSVNNs), then the double hierarchy linguistic single-valued neutrosophic Einstein geometric (DHLSVNEG) mean operators are as follows:

$$\begin{array}{c} DHLSVNEG(\gimel_{\daleth_1}, \gimel_{\daleth_2}, \cdots, \gimel_{\daleth_n}) = (\gimel_{\daleth_1}) \otimes (\gimel_{\daleth_2}), \cdots, \otimes (\gimel_{\daleth_n}) \\ \left( \begin{array}{c} \left( \gimel_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{\mu_i}{\tau}\right)}{\left(2-\frac{\mu_i}{\tau}\right)^{k}+\left(\frac{\mu_i}{\tau}\right)}\right)} \left\langle \daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{w_i}{\delta}\right)}{\left(2-\frac{w_i}{\delta}\right)+\left(\frac{w_i}{\delta}\right)}\right)} \right\rangle \right), \\ \left( \gimel_{\tau\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\eta_i}{\tau}\right)-\prod_{i=1}^{n}\left(1-\frac{\eta_i}{\tau}\right)}{\prod_{i=1}^{n}\left(1+\frac{\eta_i}{\tau}\right)+\prod_{i=1}^{n}\left(1-\frac{\eta_i}{\tau}\right)}\right)} \left\langle \daleth_{\delta\left(\frac{\prod_{i=1}^{n}\left(1+\frac{x_i}{\delta}\right)-\prod_{i=1}^{n}\left(1-\frac{x_i}{\delta}\right)}{\prod_{i=1}^{n}\left(1+\frac{x_i}{\delta}\right)+\prod_{i=1}^{n}\left(1-\frac{x_i}{\delta}\right)}\right)} \right\rangle \right), \\ \left( \gimel_{\tau\left(\frac{\prod_{i=1}^{n}\left(1+\frac{v_i}{\tau}\right)-\prod_{i=1}^{n}\left(1-\frac{v_i}{\tau}\right)}{\prod_{i=1}^{n}\left(1+\frac{v_i}{\tau}\right)+\prod_{i=1}^{n}\left(1-\frac{v_i}{\tau}\right)}\right)} \left\langle \daleth_{\delta\left(\frac{\prod_{i=1}^{n}\left(1+\frac{y_i}{\delta}\right)-\prod_{i=1}^{n}\left(1-\frac{y_i}{\delta}\right)}{\prod_{i=1}^{n}\left(1+\frac{y_i}{\delta}\right)+\prod_{i=1}^{n}\left(1-\frac{y_i}{\delta}\right)}\right)} \right\rangle \right) \end{array} \right) \end{array} \tag{4.9}$$

**Theorem 4.9.** Suppose we have a family $\beth_{\daleth_i} = \left\langle \beth_{\mu_i}\langle \daleth_{w_i}\rangle, \beth_{\eta_i}\langle \daleth_{x_i}\rangle, \beth_{\nu_i}\langle \daleth_{y_i}\rangle \right\rangle$ $(i \in \mathbb{N})$ of double hierarchy linguistic single-valued neutrosophic numbers. Then DHLSVNEG satisfies the following properties as:

*i.* (Idempotency) Suppose for all $i \in \mathbb{N}$, $\beth_{\daleth_i} = \beth_{\daleth}$. Then,

$$DHLSVNEG\left(\beth_{\daleth_1}, \beth_{\daleth_2}, \cdots, \beth_{\daleth_n}\right) = \beth_{\daleth}$$

*ii.* (Boundedness) Consider $\beth_{\daleth}^{-} = \min_{1\leq i\leq n}\{\beth_{\daleth_i}\}$, $\beth_{\daleth}^{+} = \max_{1\leq i\leq n}\{\beth_{\daleth_i}\}$. Then,

$$\beth_{\daleth}^{-} \leq DHLSVNEG\left(\beth_{\daleth_1}, \beth_{\daleth_2}, \cdots, \beth_{\daleth_n}\right) \leq \beth_{\daleth}^{+}$$

*iii.* (Monotonicity) Consider $\beth_{\daleth_i}$, $\beth^{*}_{\daleth_i}$ be two sets of DHLEs, $\beth_{\daleth_i} \leq \beth^{*}_{\daleth_i}$ for all $i$, then

$$DHLSVNEG\left(\beth_{\daleth_1}, \beth_{\daleth_2}, \cdots, \beth_{\daleth_n}\right) \leq DHLSVNEG\left(\beth^{*}_{\daleth_1}, \beth^{*}_{\daleth_2}, \cdots, \beth^{*}_{\daleth_3}\right)$$

Proof. It is clear from Theorem 4.5. □

**Definition 4.10.** Suppose we have a family $\beth_{\daleth_i} = \left\langle \beth_{\mu_i}\langle \daleth_{w_i}\rangle, \beth_{\eta_i}\langle \daleth_{x_i}\rangle, \beth_{\nu_i}\langle \daleth_{y_i}\rangle \right\rangle$ $(i \in \mathbb{N})$ of double hierarchy linguistic single-valued neutrosophic numbers and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ represent weight vectors of given family restricted to $\omega_i > 0$, $\sum\limits_{i=1}^{n} \omega_i = 1$. Then based on above operational laws the double hierarchy linguistic single-valued neutrosophic Einstein weighted geometric (DHLSVNEWG) operator are defined by

$$DHLSVNEWG(\beth_{\daleth_1}, \beth_{\daleth_2}, \cdots, \beth_{\daleth_n}) = (\beth_{\daleth_1})^{\omega_1} \otimes (\beth_{\daleth_2})^{\omega_2}, \cdots, \otimes (\beth_{\daleth_n})^{\omega_n}$$

$$\left(\begin{array}{c} \left(\beth_{\tau\left(\frac{2\prod_{i=1}^{n}\left(\frac{\mu_i}{\tau}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(2-\frac{\mu_i}{\tau}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\frac{\mu_i}{\tau}\right)^{\omega_i}}\right)}\left\langle \daleth_{\delta\left(\frac{2\prod_{i=1}^{n}\left(\frac{w_i}{\delta}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(2-\frac{w_i}{\delta}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\frac{w_i}{\delta}\right)^{\omega_i}}\right)} \right\rangle\right), \\ \left(\beth_{\tau\left(\frac{\prod_{i=1}^{n}\left(1+\frac{\eta_i}{\tau}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\frac{\eta_i}{\tau}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{\eta_i}{\tau}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\frac{\eta_i}{\tau}\right)^{\omega_i}}\right)}\left\langle \daleth_{\delta\left(\frac{\prod_{i=1}^{n}\left(1+\frac{x_i}{\delta}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\frac{x_i}{\delta}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{x_i}{\delta}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\frac{x_i}{\delta}\right)^{\omega_i}}\right)} \right\rangle\right), \\ \left(\beth_{\tau\left(\frac{\prod_{i=1}^{n}\left(1+\frac{v_i}{\tau}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\frac{v_i}{\tau}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{v_i}{\tau}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\frac{v_i}{\tau}\right)^{\omega_i}}\right)}\left\langle \daleth_{\delta\left(\frac{\prod_{i=1}^{n}\left(1+\frac{y_i}{\delta}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\frac{y_i}{\delta}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\frac{y_i}{\delta}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\frac{y_i}{\delta}\right)^{\omega_i}}\right)} \right\rangle\right) \end{array}\right) \tag{4.10}$$

**Theorem 4.11.** Suppose we have a family $\gimel\daleth_i = \left\langle \gimel_{\mu_i\langle \daleth_{w_i}\rangle}, \gimel_{\eta_i\langle \daleth_{x_i}\rangle}, \gimel_{\nu_i\langle \daleth_{y_i}\rangle} \right\rangle (i \in \mathbb{N})$ of double hierarchy linguistic single-valued neutrosophic numbers (DHLSVNNs) and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ represent weight vectors of given family restricted to $\omega_i > 0$, $\sum\limits_{i=1}^{n} \omega_i = 1$. Then DHLSVNEWG operator satisfies the following properties as:

*i.* (Idempotency) Suppose for all $i \in \mathbb{N}$, $\gimel\daleth_i = \gimel\daleth$. Then,

$$DHLSVNEWG\left(\gimel\daleth_1, \gimel\daleth_2, \cdots, \gimel\daleth_n\right) = \gimel\daleth$$

*ii.* (Boundedness) Consider $\gimel\daleth^- = \min_{1\leq i\leq n}\{\gimel\daleth_i\}$, $\gimel\daleth^+ = \max_{1\leq i\leq n}\{\gimel\daleth_i\}$. Then,

$$\gimel\daleth^- \leq DHLSVNEWG\left(\gimel\daleth_1, \gimel\daleth_2, \cdots, \gimel\daleth_n\right) \leq \gimel\daleth^+$$

*iii.* (Monotonicity) Consider $\gimel\daleth_i$, $\gimel\daleth^*_i$ be two sets of DHLEs, $\gimel\daleth_i \leq \gimel\daleth^*_i$ for all $i$, then

$$DHLSVNEWG\left(\gimel\daleth_1, \gimel\daleth_2, \cdots, \gimel\daleth_n\right) \leq DHLSVNEWG\left(\gimel\daleth^*_1, \gimel\daleth^*_2, \cdots, \gimel\daleth^*_3\right)$$

Proof. It is clear from Theorem 4.5. □

**Definition 4.12.** Let $\gimel\daleth_1 = \left\langle \gimel_{\mu_1\langle \daleth_{w_1}\rangle}, \gimel_{\eta_1\langle \daleth_{x_1}\rangle}, \gimel_{\nu_1\langle \daleth_{y_1}\rangle} \right\rangle$ and $\gimel\daleth_2 = \left\langle \gimel_{\mu_2\langle \daleth_{w_2}\rangle}, \gimel_{\eta_2\langle \daleth_{x_2}\rangle}, \gimel_{\nu_2\langle \daleth_{y_2}\rangle} \right\rangle$ be the two double hierarchy linguistic single-valued neutrosophic numbers (DHLSVNNs). Then, for any $\Delta > 0$, the distance of two DHLSVNNs $\gimel\daleth_1$ and $\gimel\daleth_2$ is mathematically defined by

$$d\left(\gimel\daleth_1, \gimel\daleth_2\right) = \frac{1}{6}\left[\begin{array}{c} \left|\gimel_{\left(\frac{\mu_1}{\tau}\right)} - \gimel_{\left(\frac{\mu_1}{\tau}\right)}\right|^{\Delta} + \left|\daleth_{\left(\frac{w_1}{\delta}\right)} - \daleth_{\left(\frac{w_2}{\delta}\right)}\right|^{\Delta} + \left|\gimel_{\left(\frac{\eta_1}{\tau}\right)} - \gimel_{\left(\frac{\eta_2}{\tau}\right)}\right|^{\Delta} \\ \left|\daleth_{\left(\frac{x_1}{\delta}\right)} - \daleth_{\left(\frac{x_1}{\delta}\right)}\right|^{\Delta} + \left|\gimel_{\left(\frac{\nu_1}{\tau}\right)} - \gimel_{\left(\frac{\nu_2}{\tau}\right)}\right|^{\Delta} + \left|\daleth_{\left(\frac{y_1}{\tau}\right)} - \daleth_{\left(\frac{y_1}{\tau}\right)}\right|^{\Delta} \end{array}\right]^{\frac{1}{\Delta}}$$

## 5. Extended DHLSVN-MULTIMOORA Technique

In this section, we present an extended version of the MULTIMOORA approach to handle MAGDM in the DHLSVN environment and evaluate the best choice in decision making. The DHLSVN-MULTIMOORA method consists of three methods: the double hierarchy single-valued neutrosophic ratio system (DHLSVNRS) approach, the double hierarchy linguistic single-valued neutrosophic reference point (DHLSVNRP) approach and the double hierarchy linguistic single-valued neutrosophic full multiplicative form (DHLSVNFMF) approach. The first four steps are the same in all three approaches. A decision matrix describes the values of alternatives supporting specific criteria in the MAGDM issue under an DHLSVNS context.

Suppose two sets $A = \{A_1, A_2, \cdots, A_m\}$ and $B = \{B_1, B_2, \cdots, B_n\}$ represent $m$ no of alternatives and $n$ no of criteria respectively. Let $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$ be the unknown weight vectors such that $\sum_{j=1}^{n} \omega_j = 1$ assign to corresponding criteria by a set $DM = \{DM_1, DM_2, \cdots, DM_g\}$ of decision makers. Our goal is to select the superior alternative among the possible alternatives that meet specific criteria using the extended DHLSVN-MULIMOORA approach, which is classified in the following section.

**Step 1.** Construction of DHLSVN decision matrix

Each DM examines the criteria for selecting alternatives. Decision makers (DMs) analyse the capabilities of alternatives that meet specific criteria and allocate the DHLSVNNs to each alternative that meets those criteria in the form of LT number. The DHLSVNNs decision matrix offered by $g$ no of DMs is as follows:

$$Q = [Q_{ij}^p]_{m\times n} = \begin{array}{c} A_1 \\ \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} B_1 & B_2 & \cdots & B_n \\ Q_{11}^p & Q_{12}^p & \cdots & Q_{1n}^p \\ Q_{21}^p & Q_{22}^p & \cdots & Q_{2n}^p \\ \vdots & \vdots & \ddots & \vdots \\ Q_{m1}^p & Q_{m2}^p & \cdots & Q_{mn}^p \end{pmatrix}$$

where each entry of the individual decision matrix is of the form $Q_{ij}^p = \left\langle \gimel^h_{\mu_{ij}\langle \daleth_{w_{ij}}\rangle}, \gimel^h_{\eta_{ij}\langle \daleth_{x_{ij}}\rangle}, \gimel^h_{\nu_{ij}\langle \daleth_{y_{ij}}\rangle} \right\rangle$, $(i \in \{1, 2, \cdots, m\})\,(j \in \{1, 2, \cdots, n\})$ and $(p \in \{1, 2, \cdots, g\})$.

**Step 2.** Normalization of DHLSVN decision matrix

If there are cost criteria in MAGDM problems, they must be normalized. The the following equation converts non-economical criteria to beneficial criteria:

$$N^c = \begin{cases} \gimel^h_{\mu_{ij}\langle \daleth_{w_{ij}}\rangle}, \gimel^h_{\eta_{ij}\langle \daleth_{x_{ij}}\rangle}, \gimel^h_{\nu_{ij}\langle \daleth_{y_{ij}}\rangle}, & \text{for benefit} \\ \gimel^h_{\nu_{ij}\langle \daleth_{y_{ij}}\rangle}, \gimel^h_{\eta_{ij}\langle \daleth_{x_{ij}}\rangle}, \gimel^h_{\mu_{ij}\langle \daleth_{w_{ij}}\rangle}, & \text{for cost} \end{cases}$$

**Step 3.** Construction of aggregated DHLSVN decision matrix

In the decision-making process, the aggregated DHLSVN decision matrix is created to determine the group decision of DMs by aggregating the individual judgement of DMs. By applying the double hierarchy linguistic single-valued neutrosophic Einstein weighted aggregation operator (4.7), and construct the double hierarchy linguistic single-valued neutrosophic aggregated matrix as follows:

$$Q = [Q_{ij}]_{m\times n} = \begin{array}{c} A_1 \\ \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} B_1 & B_2 & \cdots & B_n \\ Q_{11} & Q_{12} & \cdots & Q_{1n} \\ Q_{21} & Q_{22} & \cdots & Q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{m1} & Q_{m2} & \cdots & Q_{mn} \end{pmatrix}$$

**Step 4.** DHLSVN weighted aggregated decision matrix (DHLSVNWA)

By applying the weight vector of criteria and Definition 4.3, calculate the double hierarchy linguistic single-valued neutrosophic weighted aggregated decision matrix as follows:

$$\Re = [r_{ij}]_{m\times n} = \begin{array}{c} A_1 \\ \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} C_1 & C_2 & \cdots & C_n \\ r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix}$$

**Step 5.** DHLSVN Ratio system approach (DHLSVNRSP)

*i.* Calculate the Ratio $Y_i^+$ by using the double hierarchy linguistic single-valued neutrosophic Einstein arithmetic mean operators defined in (4.8).

*ii.* Calculate score value of $Y_i^+$ by Definition 3.2, of score function denoted by $Sc(Y_i^+)$.

*iii.* Arrange the score of $Y_i^+$ in increasing order. The maximum score of $Y_i^+$ will be the best alternatives.

**Step 6.** DHLSVN reference point approach (DHLSVNRP)

*i.* Again calculate the score of each entries of weighted aggregated decision matrix by using Definition 3.2.

*ii.* Calculate the reference point $(Q_j^*)$ of alternatives is as follows:

$$Q_j^* = \max_j (sc[Q_{ij}]_{m \times n})$$

*iii.* Calculate distance from each alternatives to reference point and weighted aggregated decision matrix by Definition 4.12.

*iv.* Rank the alternatives.

The conclusions are evaluated based on the values determined from the reference point. In this approach, we rank the alternatives based on maximum distance $\max_j \left(d\left(Q_{ij}, Q_j^*\right)\right)$in decreasing order and the best alternative has the lowest value that is $\min_i \left(\max_j \left(d\left(Q_{ij}, Q_j^*\right)\right)\right)$.

**Step 7.** DHLSVN full multiplicative approach

*i.* Utilizing (4.9), the defintion of double hierarchy linguistic single-valued neutrosophic Einstein geometric mean operator to calculate $A_i^+$.

*ii.* Calculate score of $A_i^+$ by Definition 3.2.

*iii.* The alternatives are ranked based on score value of $A_i^+$ in descending order. The alternative having maximum score value is the best results.

**Step 8.** Final Ranking

The final results of alternatives are determined from the ranking of the above three approaches. According to the ranking of all three approaches, alternatives are arranged in descending order, and select the best alternative. The graphical flowchart of the above-proposed method is provided in Figure 1.

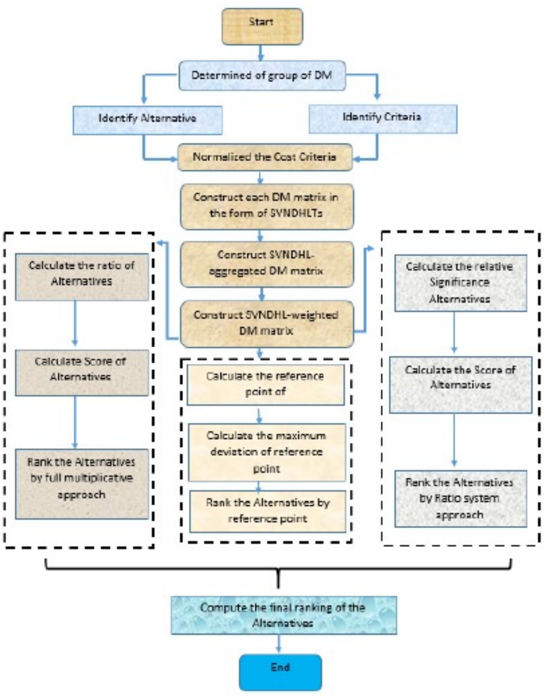


**Figure 1.** Graphical framework of proposed method

## 6. Application of Proposed Method

As with the expansion of the airport and the increase in air cargo business, the existing 3PL providers have been unable to meet the needs of the airport's transportation. This section provides a practical example concerning the selection of 3PL providers for the airport's transportation to validate the applicability and practicality of the developed methodology.

Therefore, we consider an illustrative case study in which an airport company wants to evaluate and select four (alternatives) potential 3PL providers, denoted as $\{A_1, A_2, A_3, A_4\}$ represent different airline companies. The four companies are passenger and cargo airlines, accounting for a major share of the air logistics market. For the selection of For 3PL providers, most research has focused on time rate, total assets, customer satisfaction, and personalized service. The definition of the most critical criteria of 3PL provider selection is shown as:

$B_1$: Total assets. All assets owned by a logistics enterprise
$B_2$: Time rate. Logistics delivery on time rate
$B_3$: Customer satisfaction. Matching degree of customer expectation and customer experience
$B_4$: Personalized service. Diversification degree in logistics products and services

On the based of above four defined criteria $B_j(j \in \{1,2,3,4\})$, we have four 3PL providers as alternatives $A_i(i \in \{1,2,3,4\})$ from which we will select the best 3PL. The following evaluation steps can solve the process of 3PL provider selection.

### 6.1. Evaluation Steps

In the process of the four 3PL provider selections, let the group of three expert $DM_1$, $DM_2$ and $DM_3$ are invited having weight vector $\omega_i = \{0.4, 0.5, 0.1\}$ to evaluate four alternatives based on the above criteria. The step-wise extended MULTIMOORA method within DHLSVNSs for selecting 3PL is as under:

**Step 1.** Construction of DHLSVN decision matrix

Construct the decision maker evaluation matrix in the from of DHLSVNSs , so the linguistic term set are denoted by $\gimel = \{\gimel_0 = \textit{medium}, \gimel_1 = \textit{low}, \gimel_2 = \textit{sightly low}, \gimel_3 = \textit{very low}, \gimel_4 = \textit{high}\ , \gimel_5 = \textit{slightly high}, \gimel_6 = \textit{very high}\}$ and $\daleth = \{\daleth_0 = \textit{right}, \daleth_1 = \textit{only right}, \daleth_2 = \textit{much}, \daleth_3 = \textit{very much}, \daleth_4 = \textit{little}\ , \daleth_5 = \textit{just little}\ , \daleth_6 = \textit{extermely little}\}$ are defined on the basis of following set as follows in Table 2-4

**Table 2.** Decision maker evaluation matrix $DM_1$

| $A_i$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | $\left\langle \gimel_{1\langle \daleth_1 \rangle}, \gimel_{3\langle \daleth_4 \rangle}, \gimel_{2\langle \daleth_1 \rangle} \right\rangle$ | $\left\langle \gimel_{4\langle \daleth_2 \rangle}, \gimel_{2\langle \daleth_1 \rangle}, \gimel_{0\langle \daleth_3 \rangle} \right\rangle$ | $\left\langle \gimel_{6\langle \daleth_1 \rangle}, \gimel_{2\langle \daleth_1 \rangle}, \gimel_{0\langle \daleth_2 \rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_3 \rangle}, \gimel_{5\langle \daleth_4 \rangle}, \gimel_{3\langle \daleth_0 \rangle} \right\rangle$ |
| $A_2$ | $\left\langle \gimel_{6\langle \daleth_3 \rangle}, \gimel_{6\langle \daleth_4 \rangle}, \gimel_{6\langle \daleth_2 \rangle} \right\rangle$ | $\left\langle \gimel_{5\langle \daleth_3 \rangle}, \gimel_{3\langle \daleth_2 \rangle}, \gimel_{1\langle \daleth_2 \rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_3 \rangle}, \gimel_{4\langle \daleth_2 \rangle}, \gimel_{3\langle \daleth_1 \rangle} \right\rangle$ | $\left\langle \gimel_{3\langle \daleth_4 \rangle}, \gimel_{1\langle \daleth_3 \rangle}, \gimel_{2\langle \daleth_2 \rangle} \right\rangle$ |
| $A_3$ | $\left\langle \gimel_{4\langle \daleth_2 \rangle}, \gimel_{2\langle \daleth_1 \rangle}, \gimel_{2\langle \daleth_3 \rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_4 \rangle}, \gimel_{4\langle \daleth_1 \rangle}, \gimel_{2\langle \daleth_1 \rangle} \right\rangle$ | $\left\langle \gimel_{3\langle \daleth_0 \rangle}, \gimel_{2\langle \daleth_0 \rangle}, \gimel_{1\langle \daleth_4 \rangle} \right\rangle$ | $\left\langle \gimel_{4\langle \daleth_3 \rangle}, \gimel_{2\langle \daleth_1 \rangle}, \gimel_{1\langle \daleth_0 \rangle} \right\rangle$ |
| $A_4$ | $\left\langle \gimel_{6\langle \daleth_4 \rangle}, \gimel_{0\langle \daleth_0 \rangle}, \gimel_{2\langle \daleth_1 \rangle} \right\rangle$ | $\left\langle \gimel_{4\langle \daleth_2 \rangle}, \gimel_{5\langle \daleth_3 \rangle}, \gimel_{1\langle \daleth_0 \rangle} \right\rangle$ | $\left\langle \gimel_{5\langle \daleth_4 \rangle}, \gimel_{2\langle \daleth_1 \rangle}, \gimel_{1\langle \daleth_3 \rangle} \right\rangle$ | $\left\langle \gimel_{0\langle \daleth_2 \rangle}, \gimel_{3\langle \daleth_4 \rangle}, \gimel_{0\langle \daleth_2 \rangle} \right\rangle$ |

**Table 3.** Decision maker evaluation matrix $DM_2$

| $A_i$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | $\left\langle \gimel_{0\langle \daleth_3\rangle}, \gimel_{4\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_1\rangle}, \gimel_{3\langle \daleth_2\rangle}, \gimel_{0\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_1\rangle}, \gimel_{2\langle \daleth_3\rangle}, \gimel_{0\langle \daleth_3\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_2\rangle}, \gimel_{4\langle \daleth_3\rangle}, \gimel_{1\langle \daleth_0\rangle} \right\rangle$ |
| $A_2$ | $\left\langle \gimel_{0\langle \daleth_1\rangle}, \gimel_{0\langle \daleth_2\rangle}, \gimel_{0\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_2\rangle}, \gimel_{2\langle \daleth_1\rangle}, \gimel_{4\langle \daleth_3\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_3\rangle}, \gimel_{2\langle \daleth_1\rangle}, \gimel_{1\langle \daleth_3\rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_3\rangle}, \gimel_{1\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_4\rangle} \right\rangle$ |
| $A_3$ | $\left\langle \gimel_{3\langle \daleth_1\rangle}, \gimel_{2\langle \daleth_2\rangle}, \gimel_{1\langle \daleth_4\rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_3\rangle}, \gimel_{3\langle \daleth_4\rangle}, \gimel_{2\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_1\rangle}, \gimel_{2\langle \daleth_1\rangle} \right\rangle$ | $\left\langle \gimel_{0\langle \daleth_3\rangle}, \gimel_{1\langle \daleth_6\rangle}, \gimel_{5\langle \daleth_1\rangle} \right\rangle$ |
| $A_4$ | $\left\langle \gimel_{5\langle \daleth_1\rangle}, \gimel_{3\langle \daleth_0\rangle}, \gimel_{3\langle \daleth_4\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_0\rangle}, \gimel_{2\langle \daleth_1\rangle}, \gimel_{2\langle \daleth_4\rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_1\rangle}, \gimel_{1\langle \daleth_3\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_2\rangle}, \gimel_{4\langle \daleth_1\rangle} \right\rangle$ |

**Table 4.** Decision maker evaluation matrix $DM_3$

| $A_i$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | $\left\langle \gimel_{2\langle \daleth_1\rangle}, \gimel_{2\langle \daleth_2\rangle}, \gimel_{4\langle \daleth_3\rangle} \right\rangle$ | $\left\langle \gimel_{0\langle \daleth_1\rangle}, \gimel_{1\langle \daleth_5\rangle}, \gimel_{2\langle \daleth_1\rangle} \right\rangle$ | $\left\langle \gimel_{1\langle \daleth_0\rangle}, \gimel_{2\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{0\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_2\rangle}, \gimel_{1\langle \daleth_2\rangle} \right\rangle$ |
| $A_2$ | $\left\langle \gimel_{3\langle \daleth_4\rangle}, \gimel_{1\langle \daleth_3\rangle}, \gimel_{2\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_2\rangle}, \gimel_{3\langle \daleth_2\rangle}, \gimel_{1\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{3\langle \daleth_0\rangle}, \gimel_{4\langle \daleth_0\rangle}, \gimel_{2\langle \daleth_1\rangle} \right\rangle$ | $\left\langle \gimel_{3\langle \daleth_1\rangle}, \gimel_{4\langle \daleth_1\rangle}, \gimel_{3\langle \daleth_0\rangle} \right\rangle$ |
| $A_3$ | $\left\langle \gimel_{2\langle \daleth_3\rangle}, \gimel_{3\langle \daleth_2\rangle}, \gimel_{1\langle \daleth_1\rangle} \right\rangle$ | $\left\langle \gimel_{3\langle \daleth_3\rangle}, \gimel_{2\langle \daleth_3\rangle}, \gimel_{3\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_1\rangle}, \gimel_{3\langle \daleth_1\rangle}, \gimel_{2\langle \daleth_0\rangle} \right\rangle$ | $\left\langle \gimel_{2\langle \daleth_3\rangle}, \gimel_{6\langle \daleth_2\rangle}, \gimel_{2\langle \daleth_0\rangle} \right\rangle$ |
| $A_4$ | $\left\langle \gimel_{1\langle \daleth_3\rangle}, \gimel_{4\langle \daleth_1\rangle}, \gimel_{3\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{3\langle \daleth_4\rangle}, \gimel_{4\langle \daleth_1\rangle}, \gimel_{1\langle \daleth_2\rangle} \right\rangle$ | $\left\langle \gimel_{4\langle \daleth_4\rangle}, \gimel_{2\langle \daleth_0\rangle}, \gimel_{1\langle \daleth_0\rangle} \right\rangle$ | $\left\langle \gimel_{3\langle \daleth_1\rangle}, \gimel_{4\langle \daleth_3\rangle}, \gimel_{4\langle \daleth_1\rangle} \right\rangle$ |

**Step 2.** Normalization of DHLSVN decision matrix

In this example all the criteria benefits; hence, here we skip the the normalization DHLSVN matrix.

**Step 3.** Construction of aggregated DHLSVN decision matrix

The aggregated DHLSVN decision matrix is constructed by using double hierarchy linguistic single-valued neutrosophic Einstein weighted averaging aggregation operator (4.7), and weight vector $\omega_i = \{0.4, 0.5, 0.1\}$ of decision maker in Table 5.

**Table 5.** Aggregated DHLSVN decision matrix

| $A_i$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | $\left\langle \gimel_{1.10\langle \daleth_{2.06}\rangle}, \gimel_{3.35\langle \daleth_{2.68}\rangle}, \gimel_{2.64\langle \daleth_{1.59}\rangle} \right\rangle$ | $\left\langle \gimel_{2.31\langle \daleth_{1.41}\rangle}, \gimel_{2.31\langle \daleth_{1.70}\rangle}, \gimel_{0.00\langle \daleth_{2.21}\rangle} \right\rangle$ | $\left\langle \gimel_{6.00\langle \daleth_{0.90}\rangle}, \gimel_{2.00\langle \daleth_{1.90}\rangle}, \gimel_{0.00\langle \daleth_{2.46}\rangle} \right\rangle$ | $\left\langle \gimel_{0.90\langle \daleth_{2.42}\rangle}, \gimel_{4.27\langle \daleth_{3.25}\rangle}, \gimel_{1.59\langle \daleth_{0.00}\rangle} \right\rangle$ |
| $A_2$ | $\left\langle \gimel_{6.00\langle \daleth_{2.19}\rangle}, \gimel_{0.00\langle \daleth_{2.79}\rangle}, \gimel_{0.00\langle \daleth_{2.00}\rangle} \right\rangle$ | $\left\langle \gimel_{3.21\langle \daleth_{2.42}\rangle}, \gimel_{2.46\langle \daleth_{1.42}\rangle}, \gimel_{2.10\langle \daleth_{2.46}\rangle} \right\rangle$ | $\left\langle \gimel_{1.62\langle \daleth_{2.74}\rangle}, \gimel_{2.88\langle \daleth_{0.00}\rangle}, \gimel_{1.70\langle \daleth_{1.77}\rangle} \right\rangle$ | $\left\langle \gimel_{2.52\langle \daleth_{3.27}\rangle}, \gimel_{1.16\langle \daleth_{2.21}\rangle}, \gimel_{2.56\langle \daleth_{0.00}\rangle} \right\rangle$ |
| $A_3$ | $\left\langle \gimel_{3.35\langle \daleth_{1.62}\rangle}, \gimel_{2.08\langle \daleth_{1.52}\rangle}, \gimel_{1.32\langle 3.16\rangle} \right\rangle$ | $\left\langle \gimel_{2.10\langle \daleth_{3.43}\rangle}, \gimel_{3.25\langle \daleth_{2.34}\rangle}, \gimel_{2.08\langle \daleth_{1.52}\rangle} \right\rangle$ | $\left\langle \gimel_{1.95\langle \daleth_{1.12}\rangle}, \gimel_{2.56\langle \daleth_{0.00}\rangle}, \gimel_{1.52\langle \daleth_{0.00}\rangle} \right\rangle$ | $\left\langle \gimel_{2.05\langle \daleth_{3.00}\rangle}, \gimel_{1.64\langle \daleth_{2.95}\rangle}, \gimel_{2.59\langle \daleth_{0.00}\rangle} \right\rangle$ |
| $A_4$ | $\left\langle \gimel_{6.00\langle \daleth_{2.58}\rangle}, \gimel_{0.00\langle \daleth_{0.00}\rangle}, \gimel_{2.56\langle \daleth_{2.24}\rangle} \right\rangle$ | $\left\langle \gimel_{2.58\langle \daleth_{1.29}\rangle}, \gimel_{3.20\langle \daleth_{1.59}\rangle}, \gimel_{1.42\langle \daleth_{0.00}\rangle} \right\rangle$ | $\left\langle \gimel_{3.75\langle \daleth_{3.11}\rangle}, \gimel_{2.46\langle \daleth_{0.00}\rangle}, \gimel_{1.00\langle \daleth_{0.00}\rangle} \right\rangle$ | $\left\langle \gimel_{0.82\langle \daleth_{1.90}\rangle}, \gimel_{3.09\langle \daleth_{2.79}\rangle}, \gimel_{0.00\langle \daleth_{1.32}\rangle} \right\rangle$ |

**Step 4.** Construction of weighted aggregated DHLSVN decision matrix

By considering weight vectors of criteria (0.2,0.3,0.4,0.1), and Definition 4.3, the DHLSVN weighted the aggregated matrix is determined in Table 6, as follows:

**Table 6.** DHLSVN weighted the aggregated matrix

| $A_i$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | $\left\langle \gimel_{0.22\langle \daleth_{0.42}\rangle}, \gimel_{5.21\langle \daleth_{4.99}\rangle}, \gimel_{4.98\langle \daleth_{4.57}\rangle} \right\rangle$ | $\left\langle \gimel_{0.72\langle \daleth_{0.43}\rangle}, \gimel_{4.32\langle \daleth_{3.97}\rangle}, \gimel_{0.00\langle \daleth_{4.27}\rangle} \right\rangle$ | $\left\langle \gimel_{6.00\langle \daleth_{0.36}\rangle}, \gimel_{3.59\langle \daleth_{3.52}\rangle}, \gimel_{0.00\langle \daleth_{3.91}\rangle} \right\rangle$ | $\left\langle \gimel_{0.09\langle \daleth_{0.25}\rangle}, \gimel_{5.74\langle \daleth_{5.59}\rangle}, \gimel_{5.27\langle \daleth_{0.00}\rangle} \right\rangle$ |
| $A_2$ | $\left\langle \gimel_{6.00\langle \daleth_{0.46}\rangle}, \gimel_{0.00\langle \daleth_{5.03}\rangle}, \gimel_{0.00\langle \daleth_{4.74}\rangle} \right\rangle$ | $\left\langle \gimel_{1.06\langle \daleth_{0.76}\rangle}, \gimel_{4.40\langle \daleth_{3.79}\rangle}, \gimel_{4.21\langle \daleth_{4.40}\rangle} \right\rangle$ | $\left\langle \gimel_{0.66\langle \daleth_{1.17}\rangle}, \gimel_{4.17\langle \daleth_{0.00}\rangle}, \gimel_{3.38\langle \daleth_{3.43}\rangle} \right\rangle$ | $\left\langle \gimel_{0.26\langle \daleth_{0.36}\rangle}, \gimel_{5.16\langle \daleth_{5.41}\rangle}, \gimel_{5.47\langle \daleth_{0.00}\rangle} \right\rangle$ |
| $A_3$ | $\left\langle \gimel_{0.75\langle \daleth_{0.33}\rangle}, \gimel_{4.78\langle \daleth_{4.54}\rangle}, \gimel_{4.44\langle \daleth_{5.15}\rangle} \right\rangle$ | $\left\langle \gimel_{0.65\langle \daleth_{1.15}\rangle}, \gimel_{4.78\langle \daleth_{4.34}\rangle}, \gimel_{4.20\langle \daleth_{3.86}\rangle} \right\rangle$ | $\left\langle \gimel_{0.80\langle \daleth_{0.45}\rangle}, \gimel_{3.97\langle \daleth_{0.00}\rangle}, \gimel_{3.24\langle \daleth_{0.00}\rangle} \right\rangle$ | $\left\langle \gimel_{0.21\langle \daleth_{0.32}\rangle}, \gimel_{5.28\langle \daleth_{5.54}\rangle}, \gimel_{5.47\langle \daleth_{0.00}\rangle} \right\rangle$ |
| $A_4$ | $\left\langle \gimel_{6.00\langle \daleth_{0.55}\rangle}, \gimel_{0.00\langle \daleth_{0.00}\rangle}, \gimel_{4.95\langle \daleth_{4.84}\rangle} \right\rangle$ | $\left\langle \gimel_{0.82\langle \daleth_{0.39}\rangle}, \gimel_{4.76\langle \daleth_{3.90}\rangle}, \gimel_{3.79\langle \daleth_{0.00}\rangle} \right\rangle$ | $\left\langle \gimel_{1.71\langle \daleth_{1.35}\rangle}, \gimel_{3.91\langle \daleth_{0.00}\rangle}, \gimel_{2.77\langle \daleth_{0.00}\rangle} \right\rangle$ | $\left\langle \gimel_{0.19\langle \daleth_{5.56}\rangle}, \gimel_{5.51\langle \daleth_{4.84}\rangle}, \gimel_{0.00\langle \daleth_{5.20}\rangle} \right\rangle$ |

**Step 5.** Ratio System Approach of SVNDHL (SVNDHLRSP)

*i.* In ratio system approach the $Y_i^+$ are calculated by using the Definition 4.6 of double hierarchy linguistic single-valued neutrosophic Einstein arithmetic (DHLSVNDEA) mean operators given in Table 6**.**

*ii.* According to definition the score calculate the score values of $Y_i^+$ given Table 7.

*iii.* According to score value $Y_i^+$ the alternatives are ranked in Table 7.

**Table 7.** Ranking of alternative based on ratio system approach

| $A_i$ | $Y_i^+$ | $Score\left(Y_i^+\right)$ | Ranking |
|---|---|---|---|
| $A_1$ | $\left\langle \daleth_{6.00\langle \neg_{1.24}\rangle}, \daleth_{1.74\langle \neg_{1.36}\rangle}, \daleth_{0.00\langle \neg_{0.00}\rangle} \right\rangle$ | 0.7814 | 3 |
| $A_2$ | $\left\langle \daleth_{6.00\langle \neg_{2.37}\rangle}, \daleth_{0.00\langle \neg_{0.00}\rangle}, \daleth_{0.00\langle \neg_{0.00}\rangle} \right\rangle$ | 0.8994 | 1 |
| $A_3$ | $\left\langle \daleth_{2.18\langle \neg_{1.96}\rangle}, \daleth_{1.75\langle \neg_{0.00}\rangle}, \daleth_{1.077\langle \neg_{0.00}\rangle} \right\rangle$ | 0.7030 | 4 |
| $A_4$ | $\left\langle \daleth_{6.00\langle \neg_{2.26}\rangle}, \daleth_{0.00\langle \neg_{0.00}\rangle}, \daleth_{0.00\langle \neg_{0.00}\rangle} \right\rangle$ | 0.8962 | 2 |

**Step 6.** DHLSVN reference point approach (DHLSVRP)

*i.* In this approach, the reference points $(Q_j^*)$ are calculated by evaluating the score of each entries of DHLSVN weighted the aggregated matrix in Table 8.

**Table 8.** Reference points

| $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|
| $\left\langle \daleth_{6.00\langle \neg_{0.46}\rangle}, \daleth_{0.00\langle \neg_{5.03}\rangle}, \daleth_{0.00\langle \neg_{4.74}\rangle} \right\rangle$ | $\left\langle \daleth_{0.82\langle \neg_{0.39}\rangle}, \daleth_{4.76\langle \neg_{3.90}\rangle}, \daleth_{3.79\langle \neg_{0.00}\rangle} \right\rangle$ | $\left\langle \daleth_{1.71\langle \neg_{1.35}\rangle}, \daleth_{3.91\langle \neg_{0.00}\rangle}, \daleth_{2.77\langle \neg_{0.00}\rangle} \right\rangle$ | $\left\langle \daleth_{0.26\langle \neg_{0.36}\rangle}, \daleth_{5.16\langle \neg_{5.41}\rangle}, \daleth_{5.47\langle \neg_{0.00}\rangle} \right\rangle$ |

*ii.* The distance from each alternatives to reference point and weighted aggregated decision matrix are calculated in Table 9.

**Table 9.** Distance of each alternatives and reference poin

| | $d\left(Q_{1j}, Q_j^*\right)$ | $d\left(Q_{2j}, Q_j^*\right)$ | $d\left(Q_{3j}, Q_j^*\right)$ | $d\left(Q_{4j}, Q_j^*\right)$ | $\max_j\left(d\left(Q_{1j}, Q_j^*\right)\right)$ | Ranking |
|---|---|---|---|---|---|---|
| $A_1$ | 0.450 | 0.242 | 0.439 | 0.034 | 0.450 | 4 |
| $A_2$ | 0.000 | 0.164 | 0.154 | 0.000 | 0.164 | 1 |
| $A_3$ | 0.430 | 0.157 | 0.065 | 0.009 | 0.430 | 3 |
| $A_4$ | 0.282 | 0.000 | 0.000 | 0.320 | 0.320 | 2 |

**Step 7.** DHLSVN full multiplicative approach

*i.* In this step the $A_i^+$ are computed by using the double hierarchy linguistic single-valued neutrosophic Einstein geometric mean operator equation in Table in Table 9

*ii.* The score of full multiplicative $A_i^+$ and ranking of alternatives are computed in Table 10.

**Table 10.** Full multiplicative

| *Alternative* | $A_i^+$ | $Sc\left(A_i^+\right)$ | Ranking |
|---|---|---|---|
| $A_1$ | $\left\langle \daleth_{6.00\langle \neg_{1.24}\rangle}, \daleth_{5.98\langle \neg_{5.97}\rangle}, \daleth_{5.41\langle \neg_{5.94}\rangle} \right\rangle$ | 0.220 | 3 |
| $A_2$ | $\left\langle \daleth_{6.00\langle \neg_{2.37}\rangle}, \daleth_{5.82\langle \neg_{5.87}\rangle}, \daleth_{5.69\langle \neg_{5.94}\rangle} \right\rangle$ | 0.251 | 2 |
| $A_3$ | $\left\langle \daleth_{2.18\langle \neg_{1.96}\rangle}, \daleth_{5.98\langle \neg_{5.86}\rangle}, \daleth_{5.95\langle \neg_{5.80}\rangle} \right\rangle$ | 0.126 | 4 |
| $A_4$ | $\left\langle \daleth_{6.00\langle \neg_{2.64}\rangle}, \daleth_{5.84\langle \neg_{4.68}\rangle}, \daleth_{5.90\langle \neg_{5.31}\rangle} \right\rangle$ | 0.291 | 1 |

**Step 8.** Final Ranking

The overall ranking of alternatives based on the above three approaches are given in Table 11.

**Table 11.** Overall ranking of alternatives

| *Alternative* | Ratio system | Reference point | Full multiplicative | Ranking |
|---|---|---|---|---|
| $A_1$ | 3 | 4 | 3 | 3 |
| $A_2$ | 1 | 1 | 2 | 1 |
| $A_3$ | 4 | 3 | 4 | 4 |
| $A_4$ | 2 | 2 | 1 | 2 |

The graphical ranking of alternatives based on Extended DHLSVN-MULTIMOORA Techniques are given in Figure 2:

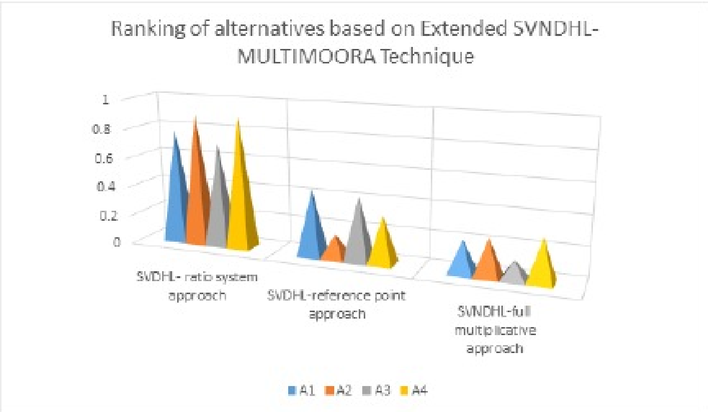


**Figure 2.** Graphically ranking of alternatives

## 7. Comparison Analysis

To verify the validity and significant effect of our developed strategy, we solve the problem by utilizing other approaches, including linguistic neutrosophic number weighted averaging (LNNWA) operator [50], TOPSIS method under linguistic neutrosophic number [52], generalized single valued neutrosophic linguistic weighted averaging (GSVNLWA) operator [53] and Single-valued neutrosophic linguistic TOPSIS [54]. The results are shown in Table 12.

**Table 12.** Comparative analysis

| **Existence Methods** | $\mathbf{A_1}$ | $\mathbf{A_2}$ | $\mathbf{A_3}$ | $\mathbf{A_4}$ |
|---|---|---|---|---|
| LNNWA [50] | 0.75 | 0.77 | 0.76 | 0.80 |
| LNN-TOPSIS [52] | 0.11 | 0.52 | 0.34 | 0.55 |
| GSVNLWA [53] | 0.73 | 0.78 | 0.76 | 0.79 |
| SVNL-Extended TOPSIS [54] | 0.86 | 0.91 | 0.88 | 0.92 |

To demonstrate the effectiveness of the proposed technique, the proposed approaches are compared with the existing operator LNNWA and LNN-TOPSIS in order to defend its dominance in DM problems. To achieve this, we first transform DHLSVN to LNN by second term equal to zero for comparison with the existing theory. By taking the same example, applying the existing LNNWA and LNN-TOPSIS method, the best result is $A_4$, which is similar to our proposed method, which shows the practicability of our proposed method.

To compare the proposed technique with the SVNL-TOPSIS and GSVNLWA operators we convert the DHLSVN to single-valued neutrosophic linguistic number and apply the existing method to same example the result are $A_4$, same to that of our method. Hence the same results indicate that our proposed method is an effective way to solve the decision-making problem.

## 8. Conclusion

In the current paper, the Extended DHLSVN-MULTIMOORA method consists of three parts: the ratio system approach, the reference point approach, and the complete multiplicative approach, which is developed to solve the MAGDM problem with vague information. DHLSVNSs is a more generalized tool that incorporates first and second hierarchy linguistic term sets with three mutually independent functions, namely true, uncertain, and false, to handle uncertain data more freely. Furthermore, the suggested research offered a list of new operation rules and Einstein aggregation operators by utilizing Einstein norms for DHLSVNSs to handle uncertainty in real-world decision-making problems. To handle multi-criteria group decision-making problems (MAGDM) A step-wise algorithm is given that is useful for DHLSVNSs. Finally, the proposed method is applied to third-party logistic service providers and also compared with other existing methods to show their effectiveness and applicability. The developed research has a variety of applications in real-world problems. In the future, many different MCGDM based on DHLSVNSs can be extended to various research areas, such as decision-making, medical diagnosis, pattern recognition, and image processing.

## Author Contributions

All the authors equally contributed to this work. They all read and approved the final version of the paper.

## Conflicts of Interest

All the authors declare no conflict of interest.

## Ethical Review and Approval

No approval from the Board of Ethics is required.

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