Erzincan Üniversitesi
Fen Bilimleri Enstitüsü Dergisi
2025, 18(2), 521-536
ISSN: 1307-9085, e-ISSN: 2149-4584
Araştırma Makalesi

Erzincan University
Journal of Science and Technology
2025, 18(2), 521-536
DOI: 10.18185/erzifbed.1592108
Research Article

An Encoding –Decoding Algorithm Based on k-Fermat and k-Mersenne Numbers

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Received: 27/11/2024, **Revised:** 08/02/2025, **Accepted:** 19/02/2025, **Published:** 31/08/2025

Abstract
In this study, we present an encoding/decoding algorithm using k-Fermat and k-Mersenne numbers. We use Fermat Q matrices and Mersenne R matrices, the terms of these matrices are composed of k-Fermat and k-Mersenne numbers, respectively, and look like Fibonacci Q matrices. In this method, which we will obtain by creating square matrices, we obtain different keys and messages. The purpose of this process is not only to increase the reliability of information security technology, but also to provide the ability to verify information at a high rate.

Keywords: coding/decoding algorithm, Fermat Q-matrix, Mersenne R-matrix, minesweeper

k-Fermat ve k -Mersenne Sayılarına Dayalı Kodlama-Kod Çözme Algoritması

Öz

Bu çalışmada k-Fermat ve k-Mersenne sayılarını kullanarak bir kodlama/kod çözme algoritması sunuyoruz. Fermat Q matrisleri ve Mersenne R matrislerini kullanıyoruz. Bu matrislerin terimleri sırasıyla k-Fermat ve k-Mersenne sayılarından oluşuyor ve bu matrisler Fibonacci Q matrislerine benziyor. Kare matrisler oluşturarak elde edeceğimiz bu yöntemde farklı anahtarlar ve mesajlar elde ediyoruz. Bu işlemin amacı yalnızca bilgi güvenliği teknolojisinin güvenilirliğini artırmak değil, aynı zamanda bilgilerin yüksek oranda doğrulanabilmesini sağlamaktır.

Anahtar Kelimeler: kodlama/kod çözme algoritması, Fermat -Q matris, Mersenne R-matris, minesweeper

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1. Introduction

The development of encoding-decoding model represents an important stage in communication. It is also a semiological conception. Fibonacci and Lucas numbers and their generalization have interesting applications to almost every field of art and science. Many researchers have introduced coding/decoding algorithms using Fibonacci and Lucas numbers [19-20]. Ucar et al., introduced Fibonacci Q matrices and R matrices [1]. Parasad introduced a new method for coding/decoding algorithms using Lucas p-numbers [2]. Diskaya et al. presented a new encryption algorithm based on Fibonacci polynomials and matrices [3], [21]. Jenan Shtayat et al. introduced an encoding/decoding algorithm using Padovan numbers [4]. Other studies using similar techniques are also discussed in [5-6].

k-Fermat numbers are defined as follows [7]

$$GF_n = k2^n + 1 \quad , n \geq 0. \tag{1}$$

The first few of these sequences are

$$GF_n = \{k + 1, 2k + 1, 4k + 1, 8k + 1, 16k + 1, 32k + 1, \dots\}$$

and Fermat Q_n matrices are defined as follows

$$Q_n = \begin{bmatrix} GF_{n+1} & GF_n \\ GF_n & GF_{n-1} \end{bmatrix}. \tag{2}$$

For $n = 2$ we get

$$Q_2 = \begin{bmatrix} GF_3 & GF_2 \\ GF_2 & GF_1 \end{bmatrix}.$$

namely

$$Q_2 = \begin{bmatrix} 8k + 1 & 4k + 1 \\ 4k + 1 & 2k + 1 \end{bmatrix}.$$

The elements of Q_2 are

$$q_1 = 8k + 1, q_2 = 4k + 1, q_3 = 4k + 1 \text{ and } q_4 = 2k + 1.$$

One can easily see that the determinant of Q_2 is $2k$. He and Shiue used a similar relationship as inspiration in his work called “On the Applications of the Linear Recurrence Relationships to

Pseudoprimes" in [8]. Now we introduce k-Mersenne numbers. k-Mersenne numbers are shown by GM_n and as GM_n satisfies the equation as follows [9]

$$GM_n = k2^n - 1 \quad , n \geq 0 \tag{3}$$

The first few terms of these sequences are

$$GM_n = \{k-1, 2k-1, 4k-1, 8k-1, 16k-1, 32k-1, \ldots\}$$

k-Mersenne R_n matrices are defined by as follows [10]

$$R_n = \begin{bmatrix} GM_{n+1} & GM_n \\ GM_n & GM_{n-1} \end{bmatrix}. \tag{4}$$

For $n = 2$ we get

$$R_2 = \begin{bmatrix} GM_3 & GM_2 \\ GM_2 & GM_1 \end{bmatrix}.$$

namely

$$R_2 = \begin{bmatrix} 8k-1 & 4k-1 \\ 4k-1 & 2k-1 \end{bmatrix}.$$

The elements of R_2 are

$$r_1 = 8k-1, r_2 = 4k-1, r_3 = 4k-1 \text{ and } r_4 = 2k-1.$$

One can easily see that determinant of R_2 is $-2k$ either.

Asci and Aydinyuz [16] generalized the AES-like cryptology on 2×2 matrices with the elements of k order Fibonacci polynomial series using a certain irreducible polynomial in the cryptology algorithm, which is called AES like cryptology on the k-order Fibonacci polynomial matrix. Anderson [18] introduced a number of keystream generators based on Fibonacci series. Karacam et al. [17] investigated the transmission of time and position variable cryptology in the Fibonacci and Lucas number series with music. In our study, we present a new encoding/decoding algorithm using k-Fermat and k-Mersenne numbers. Many researchers investigate k- Mersenne numbers and their properties [11-12]. We give a new coding-decoding system using Mersenne R matrices and Fermat Qmatrices. Our study consists of two parts. In the first part, we present a coding-decoding algorithm using Mersenne R matrices giving an example. We divide the message matrix into 2×2 block matrices. By mapping each letter in the alphabet to a different number, we obtain a codded matrix. With encoding and decoding operations, we can create a more secure password and decode the given password. In the second part, we present a coding-decoding algorithm using both Mersenne R matrices and Fermat Qmatrices, giving an example of the algorithm. Minesweeper is essentially an interesting way of looking at a potentially unconditioned (dry) subset of problems in Integer Programming and Linear Algebra. There are a lot of research about solving Minesweeper game. [13-15]. Uçar et al., used the encoding-decoding algorithm "Minesweeper Model" in [1]. We use this method, which is based on the principle of creating a stronger coding system by combining different number sequences of coding, was handled by Uçar and brought a different perspective to the coding algorithm.

2. A New Coding-Decoding Algorithm Using Mersenne R Matrix

In this part first, we give a character table which we use in coding/decoding. The first character is matched with n.

Table 1. Number values corresponding to letters.

A	B	C	D	E	F	G	H	I	J
n	$n+1$	$n+2$	$n+3$	$n+4$	$n+5$	$n+6$	$n+7$	$n+8$	$n+9$
K	L	M	N	O	P	Q	R	S	T
$n+10$	$n+11$	$n+12$	$n+13$	$n+14$	$n+15$	$n+16$	$n+17$	$n+18$	$n+19$
U	V	W	X	Y	Z	0	!	?	.
$n+20$	$n+21$	$n+22$	$n+23$	$n+24$	$n+25$	$n+26$	$n+27$	$n+28$	$n+29$

That the size of the message matrix M is $2m \times 2m$. We put " 0" between each word. The block matrices of M are shown by T_i. $1 \le i \le m^2$. The size of each block is 2×2. We use the matrices of the following forms:

$$T_i = \begin{bmatrix} t_1^i & t_2^i \\ t_3^i & t_4^i \end{bmatrix}, A_i = \begin{bmatrix} a_1^i & a_2^i \\ a_3^i & a_4^i \end{bmatrix}, Q_i = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix}, R_i = \begin{bmatrix} r_1 & r_2 \\ r_3 & r_4 \end{bmatrix}.$$

The number of the block matrices T_i is denoted by t. According to t, we choose n as follows.

$$n = \begin{cases} t & t \le 3 \\ \left[\!\left[\frac{t}{2}\right]\!\right] & otherwise \end{cases}$$

By choosing n as above, we write the Table 1 according to $mod30$ for n (Table1 can be extended depending on the massage content). Now we present the coding-decoding algorithm as in [1].

Mersenne Blocking Algorithm

First of all, in case of any minor of $(T_i)_{2\times 2}$ is zero we have to add zero sequence of the beginning of the T_i matrix. Our coding algorithm is based on six steps. We give the steps as follows:

(1) Divide the matrix M into block matrices T_i $(1 \le i \le m^2)$.
(2) We choose n.
(3) Determine t_j^i $(1 \le j \le m^2)$.
(4) We compute each $\det(T_i) \to w_i$.
(5) Construct the coded matrix P which is $[w_i, t_k^i]_{k=1,2,4}$.
(6) End of algorithm.

Decoding Algorithm

To decode the message, we take the coded matrix and apply an inversion process to it and we get the M matrix. Our decoding algorithm is based on eight steps. We give these steps as follows:

First, we know that det $(T_i) = w_i$.

(1) Compute R_n and determine r_j. $(1 \le j \le 4)$.
(2) Compute $r_1 t_1^i + r_3 t_2^i \to a_1^i$ $\quad (1 \le i \le m^2)$.
(3) Compute $r_2 t_1^i + r_4 t_2^i \to a_2^i$.
(4) Solve $(-k) \times w_i = a_1^i(r_2 x_i + r_4 t_4^i) - a_2^i(r_1 x_i + r_3 t_4^i)$ if i is even.
(5) Solve $-k2^{n-1} \times w_i = a_1^i(r_2 x_i + r_4 t_4^i) - a_2^i(r_1 x_i + r_3 t_4^i)$ if i is odd.
(6) Substitute for $x_i = t_3^i$.
(7) Construct T_i.
(8) And finally, we obtain M.

We use these steps above we present the following example. Utilizing Mersenne Q matrices we code and decode masseges.

2.1 An Example for Coding-Decoding Process

Considering the message "HOW OLD ARE YOU?" we get the following message matrix.

$$M = \begin{bmatrix} H & O & W & 0 \\ O & L & D & 0 \\ A & R & E & 0 \\ Y & O & U & ? \end{bmatrix}$$

Using Table 1, we obtain the coded matrix L as follows

$$L = \begin{bmatrix} 9 & 16 & 24 & 28 \\ 16 & 13 & 5 & 28 \\ 2 & 19 & 6 & 28 \\ 26 & 16 & 22 & 0 \end{bmatrix}.$$

Now we construct the block matrices T_i of L matrices.

(1) We can divide the L matrix of size 4×4 by the matrices T_i of size 2×2 since $m = 2$, we have four blocks.

$$T_1 = \begin{bmatrix} H & O \\ O & L \end{bmatrix}, \quad T_2 = \begin{bmatrix} W & 0 \\ D & 0 \end{bmatrix}, \quad T_3 = \begin{bmatrix} A & R \\ Y & O \end{bmatrix}, \quad T_4 = \begin{bmatrix} E & 0 \\ U & ? \end{bmatrix}$$

1. Since $t = 4 \geq 3$, we calculate $n = \left[\!\left[\frac{t}{2}\right]\!\right]$= 2. For n = 2, using the character Table 1. we get

H	O	W	0	O	L	D	0
9	16	24	28	16	13	5	28
A	*R*	*E*	0	*Y*	*O*	*U*	?
2	19	6	28	26	16	22	30

3. We have the elements of the blocks T_i $(1 \leq i \leq 4)$ matrix as follows

$t_1^1 = 9$	$t_2^1 = 16$	t_3^1 =16	$t_4^1 = 13$
t_2^1 =24	$t_2^2 = 28$	t_3^2 =5	$t_4^2 = 28$
$t_1^3 = 2$	$t_2^3 = 19$	$t_3^3 = 26$	$t_4^3 = 16$
$t_1^4 = 6$	$t_2^4 = 28$	$t_3^4 = 22$	t_4^4 =0

$w_1 = -139,\ w_2 = 532,\ w_3 = -462,\ w_4 = -616.$

4. At the end we have encoded matrix P as follows

$$P = \begin{bmatrix} -139 & 9 & 16 & 13 \\ 532 & 24 & 28 & 28 \\ -462 & 2 & 19 & 16 \\ -616 & 6 & 28 & 0 \end{bmatrix}.$$

Decoding algorithm:

(1) It is known that

$$R_2 = \begin{bmatrix} 8k-1 & 4k-1 \\ 4k-1 & 2k-1 \end{bmatrix}.$$

(2) The elements of R_2 are denoted by

$$r_1 = 8k-1, r_2 = 4k-1, r_3 = 4k-1 \text{ and } r_4 = 2k-1.$$

(3) We calculate the elements a_1^i and we composite the matrix E_i:

$$a_1^1 = 136k-25,\ \ a_1^2 = 304k-52, a_1^3 = 92k-21 \text{ and } a_1^4 = 160k-34.$$

(4) Now we calculate the elements a_2^i and we composite the matrix E_i:

$$a_2^1 = 68k-25,\ \ a_2^2 = 152k-52,\ \ a_2^3 = 46k-21 \text{ and } a_2^4 = 80k-34.$$

(5) We find the x_i values of the equations as follows

For $n = 2$ we use

$$-k2^{n-1} \times w_i = a_1^i\left(r_2 x_i + r_4 t_4^i\right) - a_2^i\left(r_1 x_i + r_3 t_4^i\right).$$

$i = 1,\ 278k = (136k-25)\big((4k-1)x_1 + (2k-1)13\big) - (68k-25)\big((8k-1)x_1 + (4k-1)13\big) \Rightarrow x_1 = 16.$

$i = 2, -1064k = (304k-52)\big((4k-1))x_2 + (2k-1)28\big) - (152k-52)((8k-1)x_2 + (4k-1)28) \Rightarrow x_2 = 5.$

$i = 3,\ 1386k = (92k-21)\big((4k-1)x_3 + (2k-1)16)\big) - (46k-21)\big((8k-1)x_3 + (4k-1)16)\big) \Rightarrow x_3 = 26.$

$$i = 4, -872k = (160k - 34)\big((4k-1)x_4 + (2k-1)0\big) - (80k - 34)((8k-1)x_4 + (4k - 1)0) \Rightarrow x_4 = 22.$$

(6) We rename x_i as follows

$$x_i = t_3^1 = 16,\ x_2 = t_3^2 = 5, x_3 = 26 \text{ and }, x_4 = 22.$$

(7) We construct the block matrices T_i :

$$T_1 = \begin{bmatrix} 9 & 16 \\ 16 & 13 \end{bmatrix},\quad T_2 = \begin{bmatrix} 24 & 28 \\ 5 & 28 \end{bmatrix},\ T_3 = \begin{bmatrix} 2 & 19 \\ 26 & 16 \end{bmatrix},\quad T_4 = \begin{bmatrix} 6 & 28 \\ 22 & 30 \end{bmatrix}.$$

(8) and finally, we obtain the message matrix L as follows

$$L = \begin{bmatrix} 9 & 16 & 24 & 28 \\ 16 & 13 & 5 & 28 \\ 2 & 19 & 6 & 28 \\ 26 & 16 & 22 & 30 \end{bmatrix}.$$

3. A Mixed Model: Minesweeper

In this part, we introduce Q_n for k-Fermat matrices and R_n for k-Mersenne matrices. Using these matrices, we present a new model for decoding. The main idea of this model blocking the message matrix M. We use k-Fermat matrices for odd indices and k-Mersenne matrices for even indices.

Coding Algorithm

We give the steps for coding as follows

(1) Divide the matrix L into blocks T_i $(1 \leq i \leq m^2)$.
(2) Choose n.
(3) Determine t_j^i $(1 \leq j \leq m^2)$.
(4) Compute $\det(T_i) = w_i$.
(5) Construct matrix F.
(6) End of algorithm.

Decoding Algorithm

We give the steps for decoding as follows

(1) Compute Q_n.

(2) Compute R_n.

(3) Compute $q_1t_1^i + q_3t_2^i \rightarrow a_1^i, i = 2l + 1$ for $0 \leq l \leq 2m$.
Compute $r_1t_1^i + r_3t_2^i \rightarrow a_1^i, i = 2l$ for $0 \leq l \leq 2m$.
Compute $q_2t_1^i + q_4t_2^i \rightarrow a_2^i, i = 2l + 1$ for $0 \leq l \leq 2m$.
Compute $r_2t_1^i + r_4t_2^i \rightarrow a_2^i, i = 2l$ for $0 \leq l \leq 2m$.

(4) Solve $(-k)2^{n-1} \times w_i = a_1^i\left(r_2t_3^i + r_4x_i\right) - a_2^i\left(r_1t_3^i + r_3x_i\right), i = 2p, 0 \leq p \leq 2m$.
Solve $k2^{n-1} \times w_i = a_1^i\left(q_2t_3^i + q_4x_i\right) - a_2^i\left(q_1t_3^i + q_3x_i\right), i = 2p + 1, 0 \leq p \leq 2m$.

(5) Substitute for $x_i = t_4^i$.

(6) Construct T_i.

(7) Construct L.

(8) End of the algorithm.

3.1 An Example for Coding-Decoding Process

Considering the massage "MIXED MODELLING FOR CRYPTOGRAPHY" we get the following message matrix.

$$M = \begin{bmatrix} M & I & X & E & D & 0 \\ M & O & D & E & L & L \\ I & N & G & 0 & F & O \\ R & 0 & C & R & Y & P \\ T & O & G & R & A & P \\ H & Y & 0 & 0 & 0 & 0 \end{bmatrix}_{6\times 6}$$

Coding Algorithm

We give the steps for coding as follows

(1) Divide the matrix M into blocks T_i $(1 \leq i \leq m^2)$

$$T_1 = \begin{bmatrix} M & I \\ M & O \end{bmatrix}, T_2 = \begin{bmatrix} X & E \\ D & E \end{bmatrix}, T_3 = \begin{bmatrix} D & 0 \\ L & L \end{bmatrix},$$

$$T_4 = \begin{bmatrix} I & N \\ R & 0 \end{bmatrix}, T_5 = \begin{bmatrix} G & 0 \\ C & R \end{bmatrix}, T_6 = \begin{bmatrix} F & O \\ Y & P \end{bmatrix}, T_7 = \begin{bmatrix} T & O \\ H & Y \end{bmatrix}, T_8 = \begin{bmatrix} G & R \\ 0 & 0 \end{bmatrix}, T_9 = \begin{bmatrix} A & P \\ 0 & 0 \end{bmatrix}$$

(2) Since $t = 9 > 3$, we calculate $n = \left[\!\left[\frac{t}{2}\right]\!\right] = 4$. For $n = 4$, we use the character table as follows for matrix M.

M	*I*	*X*	*E*	*D*	0	*M*	*O*	*D*	*E*	*L*	*L*
16	12	27	8	7	0	16	18	7	8	15	15
I	*N*	*G*	*0*	*F*	*O*	*R*	*0*	*C*	*R*	*Y*	*P*
12	17	10	0	9	18	21	0	6	21	28	19
T	O	G	R	A	P	H	Y	0	0	0	0
23	18	10	21	4	19	11	28	0	0	0	0

(3) We get the elements of the blocks T_i $(1 \le i \le 9)$ as follows

$t_1^1 = 16$	$t_2^1 = 12$	$t_3^1 = 16$	$t_4^1 = 18$
$t_1^2 = 27$	$t_2^2 = 8$	$t_3^2 = 7$	$t_4^2 = 8$
$t_1^3 = 7$	$t_2^3 = 0$	$t_3^3 = 15$	$t_4^3 = 15$
$t_1^4 = 12$	$t_2^4 = 17$	$t_3^4 = 21$	$t_4^4 = 0$
$t_1^5 = 10$	$t_2^5 = 0$	$t_3^5 = 6$	$t_4^5 = 21$
$t_1^6 = 9$	$t_2^6 = 18$	$t_3^6 = 28$	$t_4^6 = 19$
$t_1^7 = 23$	$t_2^7 = 18$	$t_3^7 = 11$	$t_4^7 = 28$
$t_1^8 = 10$	$t_2^8 = 21$	$t_3^8 = 0$	$t_4^8 = 0$
$t_1^9 = 4$	$t_2^9 = 19$	$t_3^9 = 0$	$t_4^9 = 0$

(4) Calculating the determinants w_i of blocks of T_i we find

$w_1 = 96$	$w_2 = 160$	$w_3 = 105$
$w_4 = -357$	$w_5 = 210$	$w_6 = -333$
$w_7 = 446$	$w_8 = 0$	$w_9 = 0$

(5) Now we can construct the matrix $F_i = \left[w_i, t_k^i\right]$ as follows for $k \in \{1,2,3\}$

$$\begin{bmatrix} 96 & 16 & 12 & 16 \\ 160 & 27 & 8 & 7 \\ 105 & 7 & 0 & 15 \\ -357 & 12 & 17 & 21 \\ 210 & 10 & 0 & 6 \\ -333 & 9 & 18 & 28 \\ 446 & 23 & 18 & 11 \\ 0 & 10 & 21 & 0 \\ 0 & 4 & 19 & 0 \end{bmatrix}$$

(6) End of the algorithm.

Decoding algorithm

(1) Let's find Q_4

$$Q_4 = \begin{bmatrix} GF_5 & GF_4 \\ GF_4 & GF_3 \end{bmatrix} = \begin{bmatrix} 32k+1 & 16k+1 \\ 16k+1 & 8k+1 \end{bmatrix}$$

(2) Now let's find $R_4 = \begin{bmatrix} GM_5 & GM_4 \\ GM_4 & GM_3 \end{bmatrix} = \begin{bmatrix} 32k-1 & 16k-1 \\ 16k-1 & 8k-1 \end{bmatrix}$

(3) If i is odd we use Fermat Q-matrices otherwise we use Mersenne R-matrices and we get as follows

$a_1^1 = 704k + 28$	$a_2^1 = 352k + 28$
$a_1^2 = 992k - 35$	$a_2^2 = 496k - 35$
$a_1^3 = 224k + 7$	$a_2^3 = 112k + 7$
$a_1^4 = 656k - 29$	$a_2^4 = 328k - 29$
$a_1^5 = 320k + 10$	$a_2^5 = 160k + 10$
$a_1^6 = 576k - 27$	$a_2^6 = 288k - 27$
$a_1^7 = 1024 + 41$	$a_2^7 = 512k + 41$
$a_1^8 = 656k - 31$	$a_2^8 = 328k - 31$
$a_1^9 = 432k + 23$	$a_2^9 = 216k + 23$

(4) For $n = 4$ we get following equations

$$768 = (704k + 28)\big((16k + 1)16 + (8k + 1)x_1\big) - (352k + 28)\big((32k + 1)16 + (16k + 1)x_1\big)$$

$$\Rightarrow x_1 = 18$$

$$-1280 = (992k - 35)\big((16k - 1)7 + (8k - 1)x_2\big) - (496k - 35)\big((32k - 1)7 + (16k - 1)x_2\big)$$

$$\Rightarrow x_2 = 8$$

$$840 = (224k + 7)\big((16k + 1)15 + (8k + 1)x_3\big) - (112k + 7)\big((32k + 1)15 + (16k + 1)x_3\big)$$

$$\Rightarrow x_3 = 15$$

$$-2856 = (656k - 29)\big((16k - 1)21 + (8k - 1)x_4\big) - (328k - 29)\big((32k - 1)21 + (16k - 1)x_4\big)$$

$$\Rightarrow x_4 = 0$$

$$1680 = (320k + 10)\big((16k + 1)6 + (8k + 1)x_5\big) - (160k + 10)\big((32k + 1)6 + (16k + 1)x_5\big)$$

$$\Rightarrow x_5 = 21$$

$$2664 = (576k - 27)\big((16k - 1)28 + (8k - 1)x_6\big) - (288k - 27)\big((32k - 1)28 + (16k - 1)x_6\big)$$

$$\Rightarrow x_6 = 19$$

$$3568 = (1024k + 41)\big((16k + 1)11 + (8k + 1)x_7\big) - (512k + 41)\big((32k + 1)11 + (16k + 1)x_7\big)$$

$$\Rightarrow x_7 = 28$$

$$0 = (656k - 31)(0 + (8k - 1)x_8) - (328k - 31)(0 + (8k + 1)x_8)$$

$$\Rightarrow x_8 = 0$$

$$0 = (432k + 23)(0 + (8k + 1)x_9) - (216k + 23)(0 + (16k + 1)x_9)$$

$$\Rightarrow x_9 = 0$$

(5) We rename x_i as follows

$$x_1 = t_4^1,\ x_2 = t_4^2, x_3 = t_4^3, x_4 = t_4^4, x_5 = t_4^5, x_6 = t_4^6, x_7 = t_4^7, x_8 = t_4^8, x_9 = t_4^9$$

(6) Now we can construct the block matrices T_i

$$T_1 = \begin{bmatrix} 16 & 12 \\ 16 & 18 \end{bmatrix}, T_2 = \begin{bmatrix} 28 & 8 \\ 7 & 8 \end{bmatrix}, T_3 = \begin{bmatrix} 7 & 0 \\ 15 & 15 \end{bmatrix},$$

$$T_4 = \begin{bmatrix} 12 & 17 \\ 4 & 0 \end{bmatrix}, T_5 = \begin{bmatrix} 10 & 0 \\ 6 & 21 \end{bmatrix}, T_6 = \begin{bmatrix} 9 & 18 \\ 28 & 19 \end{bmatrix}, T_7 = \begin{bmatrix} 23 & 18 \\ 11 & 28 \end{bmatrix}, T_8 = \begin{bmatrix} 10 & 21 \\ 0 & 0 \end{bmatrix}, \quad T_9 = \begin{bmatrix} 4 & 9 \\ 0 & 0 \end{bmatrix}$$

(7) At the end of the process we get L matrix combining the matrices T_i

(8) $$M = \begin{bmatrix} 16 & 12 & 27 & 8 & 7 & 0 \\ 16 & 18 & 7 & 8 & 15 & 15 \\ 12 & 17 & 10 & 0 & 9 & 18 \\ 4 & 0 & 6 & 21 & 28 & 19 \\ 23 & 18 & 10 & 21 & 4 & 9 \\ 11 & 28 & 0 & 0 & 0 & 0 \end{bmatrix}_{6\times 6}$$

Thus, the algorithm ends.

If we $k = 1$ and considering the massage “MIXED MODELLING FOR CRYPTOGRAPHY” in [5], we obtain the same results .

4. Conclusion

In this study, after giving a coding-decoding algorithm for Fermat Q and Mersenne R matrices, we first gave a coding-decoding algorithm for Mersenne R matrices. By using k-Fermat and k-Mersenne number sequences, it is aimed to both increase reliability and transfer information correctly by using two different sequences in addition to the single number sequence. By combining these two matrices, we constructed a new coding and decoding algorithm. We found more secure keys using Fermat Q matrices and Mersenne R matrices. Utilizing the “Minesweeper Model” we can make more reliable encryption.

Different security coding\decoding can be created by applying this work to all number sequences.

References

[1] Uçar, S., Taş, N., and Özgür, N. Y. (2019). A new application to coding theory via Fibonacci and Lucas numbers. *Mathematical Sciences and Applications E-Notes*, *7*(1), 62-70.

[2] Prasad, B. (2016). Coding theory on Lucas p numbers. *Discrete Mathematics, Algorithms and Applications*, *8*(04), 1650074.

[3] Dişkaya, O., Avaroğlu, E., Menken, H., & Emsal, A. (2022). A New Encryption Algorithm Based on Fibonacci Polynomials and Matrices. *Traitement du Signal*, *39*(5), 1453.

[4] Shtayat, J., and Al-Kateeb, A. (2019). An Encoding-Decoding algorithm based on Padovan numbers. *arXiv preprint arXiv:1907.02007*.

[5] Eser, E., Kuloglu, B., & Özkan, E. (2024). An Encoding-Decoding Algorithm Based on Fermat And Mersenne Numbers. *Applied Mathematics E-Notes*, *24*, 274-282.

[6] Wang, Z., Wang, L., Liu, S., & Wei, G. (2017). Encoding-decoding-based control and filtering of networked systems: Insights, developments and opportunities. *IEEE/CAA Journal of Automatica Sinica*, *5*(1), 3-18.

[7] Robinson, R. M. (1954). Mersenne and Fermat numbers. *Proceedings of the American Mathematical Society*, *5*(5), 842-846.

[8] He, T. X., Shiue, P. (2019). On the Applications of the Linear Recurrence Relationships to Pseudoprimes. *Journal of Mathematical Research with Applications*, *39*(6), 563.

[9] Kizilirmak, G.Ö., Taşci, D. (2022). On the Bi-Periodic Mersenne Sequence. *Fundamental journal of mathematics and applications (Online)*.

[10] Ochalik, P., Włoch, A. (2018). On generalized Mersenne numbers, their interpretations and matrix generators. *Annales Universitatis Mariae Curie-Skłodowska, sectio A–Mathematica*, *72* (1).

[11] Uysal, M., Kumarı, M., Kuloğlu, B., Prasad, K., & Özkan, E., (2025). On the Hyperbolic K-Mersenne and K-Mersenne-Lucas Octonions. Kragujevac Journal of Mathematics, 49, 5, 765-779.

[12] Kuloğlu, B., Eser, E., & Özkan, E., (2023). The r-circulant Matrices Associated with k-Fermat and k-Mersenne Numbers. Wseas Transactions on Mathematics, 22, 531-543.

[13] Kamenetsky, D., & Teo, C. H. (2007). Graphical Models for Minesweeper Project Report.

[14] Fowler, A., & Young, A. (2004). Minesweeper: A statistical and computational analysis.

[15] Becerra, D. J. (2015). *Algorithmic approaches to playing Minesweeper* (Doctoral dissertation).

[16] Asci, M., Aydinyuz, S. (2022). k-Order Fibonacci polynomials on AES-like cryptology. CMES-Computer Modeling in Engineering & Sciences, 131(1), 277-293.

[17] Karaçam, C., Algül, F.N., Tavit, D. (2021). Transmission of time and position variable cryptology in Fibonacci and Lucas number series with music. Journal of Mathematical Sciences and Modelling, 4(1): 38-50.

[18] Anderson, R. (1994). On Fibonacci keystream generators. In International Workshop on Fast Software Encryption, pp. 346-352.

[19] Koshy, T. (2019). Fibonacci and Lucas Numbers with Applications, Volume 2. John Wiley & Sons.

[20] Vajda, S. (2008). Fibonacci and Lucas numbers, and the golden section: theory and applications. Courier Corporation.

[21] Diskaya, O., Avaroglu, E., & Menken, H. (2020). The classical AES-like cryptology via the Fibonacci polynomial matrix. *Turkish Journal of Engineering, 4*(3), 123-128.