Sigma J Eng Nat Sci, Vol. 42, No. 6, pp. 1756−1762, December, 2024

Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2024.00000

Research Article

Analytical methodology for the design of load-bearing anisotropic panels for the wingbox of a light aircraft subject to geometric nonlinearity under compression

Oleg MITROFANOV[1], Maksim SHKURIN[1,*]

[1]*Department of Aircraft Design and Certification, Moscow Aviation Institute (National Research University), Moscow, 125993, Russia*

ARTICLE INFO

Article history
Received: 26 July 2023
Revised: 12 September 2023
Accepted: 08 January 2024

Keywords:
Geometrically Non-Linear Behavior, Anisotropic Material, Rectangular Panels, Longitudinal Compression

ABSTRACT

The subject of this research is the upper load-bearing anisotropic panels of the wingbox of the forward-swept wing of a light aircraft, exposed mainly to longitudinal compression. The panels are fixed along the long sides. The paper considers how to determine the minimum thickness of these panels at a postbuckling state under loads exceeding the operational level. Based on the analytical solution for geometrically nonlinear problems obtained by the Bubnov-Galerkin method, a technique for designing anisotropic panels at a postbuckling state is proposed. To determine the panel's thickness, the strength criteria for a monolayer of the composite material were used. The main result of the work is a nonlinear equation for the panel's minimum thickness. It incorporates the equivalence of the membrane stresses due to panel buckling and the limit stresses of the composite package monolayer.

Cite this article as: Mitrofanov O, Shkurin M. Analytical methodology for the design of load-bearing anisotropic panels for the wingbox of a light aircraft subject to geometric nonlinearity under compression. Sigma J Eng Nat Sci 2024;42(6):1756−1762.

INTRODUCTION

The use of composite materials in light aircraft reduces their structural weight. In works [1-2], it is rightly noted that the potential of composite panels in thin-walled structures prone to buckling is not fully utilized due to the lack of regulatory documents in the aerospace industry.

This paper considers the anisotropic upper panels of a forward-swept wing of low- and medium-payload-capacity aircraft, exposed mainly to compressive loads.

The panel's structural weight can be reduced by the use of rigid boundary conditions. Primarily, this is due to an increase in critical buckling stresses compared to the case of a hinged support. This is also true when designing panels for a postbuckling state using the methodology proposed in [3-4]. For this reason, in the current work, we consider problems associated with the design of rectangular anisotropic panels with the rigid boundary conditions applied along the long sides, and exposed to longitudinal compression at a postbuckling state. We also assume that the considered anisotropic panels are a part of the wingbox of the forward-swept wing, and for them, the loss of stability is acceptable under loads above the operational level.

***Corresponding author.**
*E-mail address: m.shkurin@mai.ru

This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkilic

Published by Yıldız Technical University Press, İstanbul, Turkey

The purpose of this work was to develop a methodology for determining the minimum thickness of anisotropic panels in a postbuckling state exposed to the design compressive loads. This methodology is based on an analytical solution to a geometrically nonlinear problem. The methodology allows for numerical implementation and is applicable in the early design stages.

Let's consider publications devoted to the design of composite panels. The article by Ni X., Prusty G., and Hellier A. [5] provides an extensive overview of the research related to the strength, stability, and bearing capacity of anisotropic panels, published from 2000 up to 2012. We also note the work of Gavva, L., and Firsanov, V. [6] which provides an overview of analytical methods and experimental approaches to the study of the stress-strain state of structurally anisotropic aircraft panels made of composite materials. Separately, we would like to mention the monograph by Falzon B. G., Aliabadi M. H. [7] which presents the results of experimental, analytical, and numerical studies of composite panels when considering problems of stability and postbuckling state.

The work by Qiao P. et al. [8] provides analytical solutions to stability problems for composite panels under biaxial loading for almost all possible types of boundary conditions. In other works, Qiao P. [9] and Qiao P. et al. [10] considered interesting studies on the identification of damage in composite panels. We also note the publications of Aliabadi M.H. et al. [11] and Aliabadi M.H. et al. [12] which present a new method for classifying and quantifying impacts for composite plates and the structure of a monitoring system (SHM) using neural networks (BNN).

Of undoubted interest are the publications by Tsai S.W. [13] and Kappel E. [14] which proposed the use of composite panels with the stacking family [±φ, ±ψ]. It is noted that the proposed stacking has advantages over traditional stacking when used in elements of aerospace composite structures.

We also note some analytical results obtained when considering geometrically nonlinear problems under shear [15–16].

From the above literature review, it follows that the design of composite panels is a complex and relevant problem, and the panel design for postbuckling remains insufficiently studied.

Returning to the objective of this work, we note that the implementation of rigid boundary conditions in analytical studies is an important task but leads to more cumbersome relations compared to the case of a hinged support.

MATERIALS AND METHODS

Basic Relations for Geometrically Nonlinear Anisotropic Panels

Below, we write down the initial relations for geometrically nonlinear anisotropic panels under compression. The condition of strain compatibility is as follows [17]:

$$L_1(\Phi) = L_2(W), \tag{1}$$

where

Φ is the Airy stress function Erie,

L_m are the following operators:

$$L_1(\Phi) = \frac{1}{E_y}\frac{\partial^4\Phi}{\partial x^4} - g_{31}\frac{\partial^4\Phi}{\partial x^3\partial y} + g_{22}\frac{\partial^4\Phi}{\partial x^2\partial y^2} - g_{13}\frac{\partial^4\Phi}{\partial y^3\partial x} + \frac{1}{E_x}\frac{\partial^4\Phi}{\partial y^4},$$

$$g_{31} = \frac{\eta_{y,xy}+\eta_{xy,y}}{G_{xy}},\ g_{22} = \frac{1}{G_{xy}} - \frac{\mu_{xy}}{E_y} - \frac{\mu_{yx}}{E_x},\ g_{13} = \frac{\eta_{x,xy}+\eta_{xy,x}}{G_{xy}},$$

$$L_2(W) = \left(\frac{\partial^2 W}{\partial x\partial y}\right)^2 - \left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right),$$

E_x, E_y, G_{xy}, μ_{xy} are the average characteristics of composite structure;

η_x, x_y, $\eta_{y,xy}$, $\eta_{xy,x}$, $\eta_{xy,y}$ are the anisotropic structure influence coefficients [17].

The geometrically nonlinear equilibrium equation [17] has the following form:

$$L_3(\Phi,W) - L_4(W), \tag{2}$$

where

$$L_3(F,W) = \frac{\partial^2\Phi}{\partial y^2}\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2\Phi}{\partial x^2}\frac{\partial^2 W}{\partial y^2} - 2\frac{\partial^2\Phi}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y},$$

$$L_4(W) = \frac{1}{\delta}\left[D_{11}\frac{\partial^4 W}{\partial x^4} + 2(D_{12}+2D_{66})\frac{\partial^4 W}{\partial x^2\partial y^2} + D_y\frac{\partial^4 W}{\partial y^4} + 4D_{16}\frac{\partial^4 W}{\partial x^3\partial y} + 4D_{26}\frac{\partial^4 W}{\partial x\partial y^3}\right];$$

D_{mn} (mn=11, 12, 22, 16, 26, 66) is the bending stiffnesses of a smooth anisotropic rectangular panel [17].

For the subsequent analytical solution of geometrically nonlinear problem by the Bubnov-Galerkin method, we will use the following equation:

$$\int_0^a\int_0^b \left[L_3(\Phi,W) - L_4(W)\right]W_k\,dxdy = 0, \tag{3}$$

where W_k is the deflection function

Methodology for Designing Anisotropic Panels for Post-Buckling State Under Strength Constraints

Let us consider a composite rectangular anisotropic panel under longitudinal compression (Figure 1). To approximate the deflection, considering the rigid support along the long sides, we use the following function:

$$W = f\cdot\sin^2\frac{\pi y}{b}\sin\frac{\pi(x-\alpha y)}{s}, \tag{4}$$

where f is the deflection amplitude, a is the tangent of wave inclination during buckling, s is the distance between nodal lines when buckling occurs.

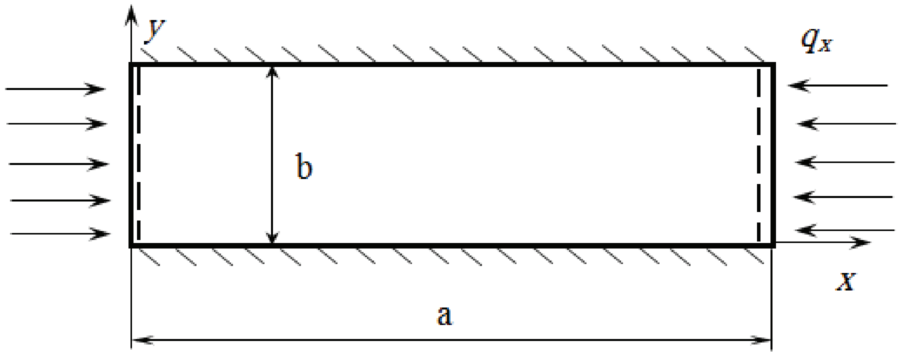

Figure 1. A rectangular anisotropic panel under longitudinal compression.

Note that in [16], the deflection type (4) was used in the analytical analysis of the postbuckling behavior of an anisotropic composite panel under shear. Now let's use equation (4) to model the postbuckling behavior of anisotropic composite panels under compression and develop a design methodology for the postbuckling state.

After substituting the deflection (4) into the strain compatibility equation (1), we can obtain an expression whose solution will be the following stress function:

$$\Phi = f^2\left\{A_1\cos\frac{2\pi(x-\alpha y)}{s} + A_2\cos\frac{2\pi y}{b} + A_3\cos\frac{4\pi y}{b} + A_4 cos\left(\frac{2\pi(x-\alpha y)}{s}-\frac{2\pi y}{b}\right) + A_5 cos\left(\frac{2\pi(x-\alpha y)}{s}+\frac{2\pi y}{b}\right)\right\} - \frac{p_x y^2}{2} - \frac{p_y x^2}{2} + p_{xy}xy \cdot \quad (5)$$

When considering particular solutions of the homogeneous equation with L1(Φ), we can determine the coefficients that account for anisotropic structure:

$$A_1 = \frac{s^2}{32b^2}\frac{1}{\frac{1}{E_y}+\frac{\alpha}{g_{31}}+\frac{\alpha^2}{g_{22}}+\frac{\alpha^3}{g_{13}}+\frac{\alpha^4}{E_x}}, A_2 = \frac{E_y}{32}\frac{b^2}{s^2}, A_3 = -\frac{E_y}{512}\frac{b^2}{s^2},$$

$$A_4 = -\frac{s^2}{64b^2}\frac{1}{\frac{1}{E}+\frac{1}{g}\frac{(\alpha b+s)}{b}+\frac{1}{g}\left[\frac{\alpha b+s}{b}\right]^2+\frac{1}{g}\left[\frac{\alpha b+s}{b}\right]^3+\frac{1}{E}\left[\frac{\alpha b+s}{b}\right]^4},$$

$$A_5 = -\frac{s^2}{64b^2}\frac{1}{\frac{1}{E}+\frac{1}{g}\frac{(\alpha b-s)}{b}+\frac{1}{g}\left[\frac{\alpha b-s}{b}\right]^2+\frac{1}{g}\left[\frac{\alpha b-s}{b}\right]^3+\frac{1}{E}\left[\frac{\alpha b-s}{b}\right]^4}.$$

Note also that the membrane stresses on the middle surface of an anisotropic plate are determined from the definition of the Airy function F (5):

$$\sigma_x = \frac{\partial^2\Phi}{\partial y^2},\ \sigma_y = \frac{\partial^2\Phi}{\partial x^2},\ \tau_{xy} = -\frac{\partial^2\Phi}{\partial x\partial y},$$

and, in particular, for the considered type of the boundary conditions and deflection (1) at px≠0, py≠0, pxy≠0, we can obtain

$$\sigma_x = \frac{\partial^2\Phi}{\partial y^2} = -f^2\Delta_{11} - p_x, \sigma_y = \frac{\partial^2\Phi}{\partial x^2} = -f^2\Delta_{22} - p_y,$$
$$\tau_{xy} = -\frac{\partial^2\Phi}{\partial x\partial y} = f^2\Delta_{33} - p_{xy} \quad (6)$$

where

$$\Delta_{11} = \pi^2\left\{-A_1\frac{4a^2}{s^2}\cos\frac{2\pi(x-\alpha y)}{s} - A_2\frac{4}{b^2}\cos\frac{2\pi y}{b} - A_3\frac{16}{b^2}\cos\frac{4\pi y}{b} - 4A_4\left(\frac{\alpha}{s}+\frac{1}{b}\right)^2 cos\left(\frac{2\pi(x-\alpha y)}{s}-\frac{2\pi y}{b}\right) - 4A_5\left(\frac{\alpha}{s}-\frac{1}{b}\right)^2 cos\left(\frac{2\pi(x-\alpha y)}{s}+\frac{2\pi y}{b}\right)\right\} \quad (7)$$

$$\Delta_{22} = \pi^2\left\{\left[-\frac{4A_1}{s^2}\cos\frac{2\pi(x-\alpha y)}{s}\right] + \frac{4A_4}{s^2}\cos\left(\frac{2\pi(x-\alpha y)}{s}-\frac{2\pi y}{b}\right) + \frac{4A_5}{s^2}\cos\left(\frac{2\pi(x-\alpha y)}{s}-\frac{2\pi y}{b}\right)\right\} \quad (8)$$

$$\Delta_{33} = \pi^2\left\{\left[-\frac{4A_1\alpha}{s^2}\cos\frac{2\pi(x-\alpha y)}{s}\right] + \frac{4A_4}{sb}\cos\left(\frac{2\pi(x-\alpha y)}{s}-\frac{2\pi y}{b}\right) + \frac{4A_4}{sb}\cos\left(\frac{2\pi(x-\alpha y)}{s}-\frac{2\pi y}{b}\right)\right\}. \quad (9)$$

After substituting expressions (1) and (5) into equation (4) and some cumbersome transformations, one can obtain the following non-linear relationship for the deflection amplitude (f≠0):

$$\frac{\pi^4}{16\delta}\left\{D_{11}\frac{3b}{s^3} + 2\left(D_{12}+2D_{66}\right)\frac{3\alpha^2b^2+4s^2}{s^3b} + \frac{D_{22}}{s^3b^3}\left(24s^2b^2\alpha^2+16s^4+3b^4\alpha^4\right) - \frac{3b\alpha D_{16}}{s^3} - \frac{3\alpha D_{26}}{bs^3}\left(4s^2+\alpha^2b^2\right)\right\}\frac{\pi^4f^2}{4sb}\left[2A_1+2A_2+2A_3+A_4+A_5\right] = \frac{3\pi^2b}{16s}\left(p_x+\frac{p_y\left(3\alpha^2b^2+4s^2\right)}{3b^2}+\alpha p_{xy}\right) \quad (10)$$

,

or, in a more compact form, we have

$$\overline{D}_{\alpha\beta}\delta^3 + f^2 B_{\alpha\beta}\delta = \left(q_x + \frac{q_y\left(3\alpha^2 b^2 + 4s^2\right)}{3b^2} + \alpha q_{xy} \right), \quad (11)$$

where $\overline{D}_{mn} = D_{mn}/\delta^3$ (m.n=11, 12, 22, 16, 26, 66); $q_x = p_x\delta$, $q_y = p_y\delta$, $q_{xy} = p_{xy}\delta$ are the loads acting on the panel in the general case,

$$\overline{D}_{\alpha\beta} = \overline{D}_{11}\left(\frac{\pi}{s}\right)^2 + \frac{2\pi^2\left(\overline{D}_{12} + 2\overline{D}_{33}\right)}{3}\left(\frac{4}{b^2} + \frac{3\alpha^2}{s^2}\right)$$
$$+ \overline{D}_{22}\pi^2\left\{\frac{24s^2b^2\alpha^2 + 16s^4 + 3b^4\alpha^4}{3b^4s^2}\right\}$$
$$-\overline{D}_{16}\frac{\alpha\pi^2}{s^2} - \overline{D}_{26}\frac{\alpha\pi^2}{s^2b^2}\left[4s^2 + \alpha^2 b^2\right],$$
$$B_{\alpha\beta} = \frac{4\pi^2}{3b^2}\left[2A_1 + 2A_2 + 2A_3 + A_4 + A_5\right]$$

Considering the longitudinal compression case ($p_x \neq 0$, $p_y = 0$, $p_{xy} = 0$) and introducing the designation $\gamma = (b/s)^2$, in the case of small deflections at $f \to 0$ (from equation (11)), we can use the system of equations

$$\frac{\partial q_x}{\partial \gamma} = 0, \frac{\partial q_x}{\partial \alpha} = 0, \quad (12)$$

to determine the critical parameters of wave formation and obtain the following expressions:

$$\overline{D}_{11} + 2\left(\overline{D}_{12} + 2\overline{D}_{66}\right)\alpha^2 + \frac{\overline{D}_{22}}{3}\left(\frac{16}{\gamma^2} + 3\alpha^4\right) - \alpha\overline{D}_{16} - \overline{D}_{26}\alpha^3 = 0, \quad (13)$$

$$4\left(\overline{D}_{12} + 2\overline{D}_{66}\right)\alpha\gamma + \overline{D}_{22}\left(16\alpha + 4\alpha^3\gamma\right) - \overline{D}_{26}\gamma - \overline{D}_{26}\left(4 + 3\gamma\alpha^2\right) = 0, \quad (14)$$

Note that the obtained expressions (13) and (14) can be reduced to a numerical solution for the critical values of parameters γ and α at buckling. The parameters γ and α do not depend on the panel thickness but are determined by the anisotropic panel structure.

Thus, to design load-bearing anisotropic panels with minimum thicknesses for the given flow qx, panel dimensions, and stacking, the following methodology can be used. First, using equations (13) and (14), numerically determine the critical parameters of wave formation. Second, calculate the ultimate stresses $\overline{\sigma}_x$ of the anisotropic structure. Third, using the limiting normal stress condition $\sigma_x = \overline{\sigma}_x$, solve equations (6) and (7) for the deflection amplitude *f*. Fourth, determine the potentially critical points of function $\Delta_{mm}(x, y)$, at which stresses can reach their absolute maximum values.

Now, we will present in more detail the procedure for calculating the ultimate strength stresses for the case of an anisotropic structure. We use the strength criteria for a monolayer of a composite package in the following form:

$$\left(\frac{\sigma_1^{(i)}}{\overline{\sigma}_1^{(i)}}\right)^2 - \frac{\sigma_1^{(i)}\sigma_2^{(i)}}{\left(\overline{\sigma}_1^{(i)}\overline{\sigma}_2^{(i)}\right)} + \left(\frac{\sigma_2^{(i)}}{\overline{\sigma}_2^{(i)}}\right)^2 + \left(\frac{\tau_{12}^{(i)}}{\overline{\tau}_{12}^{(i)}}\right)^2 \leq 1, \quad (15)$$

In the general case, the equations for estimating the stresses in the *i*-layer [18-19] are as follows:

$$\sigma_1^{(i)} = \sigma_x a_x^{(i)} + \sigma_y a_y^{(i)} + \tau_{xy} a_{xy}^{(i)},$$
$$\sigma_2^{(i)} = \sigma_x b_x^{(i)} + \sigma_y b_y^{(i)} + \tau_{xy} b_{xy}^{(i)}, \quad (16)$$
$$\tau_{12}^{(i)} = \sigma_x c_x^{(i)} + \sigma_y c_y^{(i)} + \tau_{xy} c_{xy}^{(i)},$$

Taking into account that the longitudinal compressive stresses act on the panel $\sigma_x = q_x/\delta$, we have

$$a_x^{(i)} = \frac{1}{E_x}\left[\begin{array}{l}\left(A_{11}^i\cos^2\phi_i + A_{21}^i\sin^2\phi_i + A_{31}^i\sin 2\phi_i\right)\\ -\mu_{yx}(A_{12}^i cos^2\phi_i + A_{22}^i\sin^2\phi_i + A_{32}^i\sin 2\phi_i)\end{array}\right]$$
$$+\frac{\eta_{xy,x}}{G_{xv}}\left[A_{13}^i\cos^2\varphi_i + A_{23}^i\sin^2\varphi_i + A_{33}^i\sin 2\varphi_i\right]$$

$$a_y^{(i)} = \frac{1}{E_y}\left[\begin{array}{l}-\mu_{xy}\left(A_{11}^i\cos^2\phi_i + A_{21}^i\sin^2\phi_i + A_{31}^i\sin 2\phi_i\right)+\\ +\left(A_{12}^i\sin^2\phi_i + A_{22}^i\cos^2\phi_i + A_{32}^i\sin 2\phi_i\right)\end{array}\right]$$
$$+\frac{\eta_{xy,y}}{G_{xy}}\left(A_{11}^i\cos^2\phi_i + A_{21}^i\sin^2\phi_i + A_{31}^i\sin 2\phi_i\right);$$

$$a_{xy}^{(i)} = \frac{1}{G_{xy}}\left[\eta_{x,xy}\left(A_{11}^i\cos^2\phi_i + A_{21}^i\sin^2\phi_i + A_{31}^i\sin 2\phi_i\right)\right.$$
$$+\eta_{y,xy}(A_{12}^i cos^2\phi_i + A_{22}^i\sin^2\phi_i + A_{32}^i\sin 2\phi_i)$$
$$\left.+\left(A_{13}^i\cos^2\phi_i + A_{23}^i\sin^2\phi_i + A_{33}^i\sin 2\phi_i\right)\right];$$

$$b_x^{(i)} = \frac{1}{E_x}\left[\left(A_{11}^i\sin^2\phi_i + A_{21}^i\cos^2\phi_i + A_{31}^i\sin 2\phi_i\right)\right.$$
$$-\mu_{yx}(A_{12}^i\sin^2\phi_i + A_{22}^i\cos^2\phi_i + A_{32}^i\sin 2\phi_i)$$
$$+\frac{\eta_{xy,x}E_x}{G_{xy}}\left(A_{13}^i\sin^2\phi_i + A_{23}^i\cos^2\phi_i + A_{33}^i\sin 2\phi_i\right);$$

$$b_y^{(i)} = \frac{1}{E_y}\left[-\mu_{xy}\left(A_{11}^i\sin^2\phi_i + A_{21}^i\cos^2\phi_i + A_{31}^i\sin 2\phi_i\right)\right.$$
$$+\left(A_{12}^i\sin^2\phi_i + A_{22}^i\cos^2\phi_i + A_{32}^i\sin 2\phi_i\right)$$
$$+E_y\frac{\eta_{xy,y}}{G_{xy}}\left(A_{13}^i\cos^2\phi_i + A_{23}^i\sin^2\phi_i + A_{33}^i\sin 2\phi_i\right);$$

$$b_{xy}^{(i)} = \frac{1}{G_{xy}}\left[\eta_{x,xy}\left(A_{11}^i\sin^2\varphi_i + A_{21}^i\cos^2\varphi_i + A_{31}^i\sin 2\varphi_i\right)\right.$$
$$-\eta_{y,xy}(A_{12}^i\sin^2\varphi_i + A_{22}^i\cos^2\varphi_i + A_{32}^i\sin 2\varphi_i)$$
$$\left.+\left(A_{13}^i\sin^2\varphi_i + A_{23}^i\cos^2\varphi_i + A_{33}^i\sin 2\varphi_i\right)\right];$$

$$c_x^{(i)} = \frac{1}{E_x}\left[\left(A_{21}^i - A_{11}^i\right)\frac{\sin 2\phi_i}{2} - \mu_{yx}\left(A_{22}^i - A_{21}^i\right)\frac{\sin 2\phi_i}{2} + \frac{\eta_{xy,x}}{G_{xy}}A_{33}^i \cos 2\phi_i\right];$$

$$c_y^{(i)} = \frac{1}{E_y}\left[-\mu_{yx}\left(A_{21}^i - A_{11}^i\right)\frac{\sin 2\phi_i}{2} + \left(A_{22}^i - A_{21}^i\right)\frac{\sin 2\phi_i}{2} + \frac{\eta_{xy,y}}{G_{xy}}A_{33}^i \cos 2\phi_i\right];$$

$$c_{xy}^{(i)} = \frac{1}{G_{xy}}\left[\eta_{x,xy}\left(A_{21}^i - A_{11}^i\right)\frac{\sin 2\phi_i}{2} + \eta_{y,xy}\left(A_{22}^i - A_{21}^i\right)\frac{\sin 2\phi_i}{2} + A_{33}^i \cos 2\phi_i\right];$$

$$A_{11}^{(i)} = \overline{E}_1^{(i)}\cos^4\phi_i + \overline{E}_2^{(i)}\sin^4\phi_i + 2(\overline{E}_1^{(i)}\mu_{12}^{(i)} + 2G_{12}^{(i)})\sin^2\phi_i\cos^2\phi_i,$$

$$A_{12}^{(i)} = A_{21}^{(i)} = \overline{E}_1^{(i)}\mu_{12}^{(i)} + [\overline{E}_1^{(i)} + \overline{E}_2^{(i)} - 2(\overline{E}_1^{(i)}\mu_{12}^{(i)} + 2G_{12}^{(i)})]\sin^2\phi_i\cos^2\phi_i,$$

$$A_{22}^{(i)} = \overline{E}_1^{(i)}\sin^4\phi_i + \overline{E}_2^{(i)}\cos^4\phi_i + 2(\overline{E}_1^{(i)}\mu_{12}^{(i)} + 2G_{12}^{(i)})\sin^2\phi_i\cos^2\phi_i,$$

$$A_{33}^{(i)} = [\overline{E}_1^{(i)} + \overline{E}_2^{(i)} - 2\overline{E}_1^{(i)}\mu_{12}^{(i)}]\sin^2\phi_i\cos^2\phi_i + G_{12}^{(i)}\cos^2 2\phi_i,$$

$$A_{13}^{(i)} = A_{31}^{(i)} = \left[\left(\overline{E}_1^{(i)}\cos^2\phi_i - \overline{E}_2^{(i)}\sin^2\phi_i\right) - \left(\overline{E}_1^{(i)}\mu_{12}^{(i)} + 2G_{12}^{(i)}\right)\cos 2\phi_i\right]\sin\phi_i\cos\phi_i;$$

$$A_{23}^{(i)} = A_{32}^{(i)} = \left[\left(\overline{E}_1^{(i)}\sin^2\phi_i - \overline{E}_2^{(i)}\cos^2\phi_i\right) - \left(\overline{E}_1^{(i)}\mu_{12}^{(i)} + 2G_{12}^{(i)}\right)\cos 2\phi_i\right]\sin\phi_i\cos\phi_i.$$

Let us carry out some transformations. First, we express the deflection amplitude f2 from the nonlinear equation (11) and substitute it into the equations for the total stresses (6), which we then substitute into the expressions for the stresses in the monolayer (16). Next, we substitute the result of the previous action into criterion (15) and obtain the following equation for the panel thickness:

$$\begin{aligned}
&\left(\overline{\sigma}_2^{(i)}\overline{\tau}_{12}^{(i)}\right)^2\left[\frac{\left(-\Delta_{11}a_x^{(i)} - \Delta_{22}a_y^{(i)} - \Delta_{33}a_{xy}^{(i)}\right)}{B_{\alpha\beta}}\left(\frac{\pi^2}{s^2}q_x - D_{\alpha\beta}\delta^3\right) + q_x a_x^{(i)}\right]^2 + \\
&+\overline{\sigma}_1^{(i)}\overline{\sigma}_2^{(i)}\left(\overline{\tau}_{12}^{(i)}\right)^2\left[\frac{\left(-\Delta_{11}a_x^{(i)} - \Delta_{22}a_y^{(i)} - \Delta_{33}a_{xy}^{(i)}\right)}{B_{\alpha\beta}}q_x b_x^{(i)}\right] + \left(\frac{\pi^2}{s^2}q_x - D_{\alpha\beta}\delta^3\right) + q_x a_x^{(i)}\Big] \times \\
&+\left(\overline{\sigma}_1^{(i)}\overline{\tau}_{12}^{(i)}\right)^2\left[\frac{\left(-\Delta_{11}b_x^{(i)} - \Delta_{22}b_y^{(i)} - \Delta_{33}b_{xy}^{(i)}\right)}{B_{\alpha\beta}}\left(\frac{\pi^2}{s^2}q_x - D_{\alpha\beta}\delta^3\right) + q_x b_x^{(i)}\right]^2 + \\
&+\left(\overline{\sigma}_1^{(i)}\overline{\sigma}_2^{(i)}\right)^2\left[\frac{\left(-\Delta_{11}c_x^{(i)} - \Delta_{22}c_y^{(i)} - \Delta_{33}c_{xy}^{(i)}\right)}{B_{\alpha\beta}}\left(\frac{\pi^2}{s^2}q_x - D_{\alpha\beta}\delta^3\right) + q_x c_x^{(i)}\right]^2 = \left[\delta\overline{\sigma}_1^{(i)}\overline{\sigma}_2^{(i)}\overline{\tau}_{12}^{(i)}\right]^2.
\end{aligned} \quad (17)$$

In practice, for aircraft anisotropic composite panels, the stacking of monolayers with the reinforcement of 0°/+45°/90° is used. In this case, the optimal design technique is reduced to considering three equations of the type (17) for the specified reinforcement angles. The differences between these equations will be due to coefficients $a_x^{(i)}, b_x^{(i)}$ and $c_x^{(i)}$. As a result, the maximum value should be chosen from the three resulting values of the total thicknesses. The critical parameters of wave formation during buckling are generally determined numerically using relations (13)-(14)

We also note that the final equation (17) includes the $\Delta_{mm}(x, y)$ terms, whose values determine the maximum stresses in the monolayers. For the considered deflection *W(x, y)* (4) and the stress function $\Delta_{mm}(x, y)$ (5), the maximum values in the general case should be determined numerically. Taking into account the structure of the stress function $\Delta_{mm}(x, y)$ to first approximation, to determine the potentially critical points, we can take

$$\cos 2\pi y/b \to 1, \cos 2\pi(x - \alpha y)/s \to 1. \quad (18)$$

Let us consider an example of the application of the above methodology to designing anisotropic panels for a postbuckling state and exposed to longitudinal compression. Note that the resulting expression (17) corresponds to the general case of combined loading when using Tsai's strength criterion. Further, as an example, we will consider anisotropic panels with 0°/+45° reinforcement under loading mainly by longitudinal compressive and shear flows using other strength criteria.

In the early stages of designing aircraft panels under static strength conditions, for a monolayer, the 1st strength theory $\sigma_1^{(i)} = \overline{\sigma}_1^{(i)}$ is typically used. Therefore, we use expression (17) for determining the thickness and the corresponding coefficients in the form

$$\left[\frac{\left(-\Delta_{11}a_x^{(i)} - \Delta_{22}a_y^{(i)} - \Delta_{33}a_{xy}^{(i)}\right)}{B_{\alpha\beta}}\left(\frac{\pi^2}{s^2}q_x - D_{\alpha\beta}\delta^3\right) + q_x a_x^{(i)}\right] = \left[\delta\overline{\sigma}_1^{(i)}\right], \quad (19)$$

where

$$a_x^{(0)} = \overline{E}_1^{(0)}\frac{1}{E_x}(1 - \mu_{12}^{(0)}\mu_{xy}), a_y^{(0)} = \overline{E}_1^{(0)}\frac{1}{E_y}(\mu_{12}^{(0)} - \mu_{yx}),$$

$$a_{xy}^{(0)} = \frac{\overline{E}_1^{(0)}}{G_{xy}}\left[\eta_{x,xy} + \eta_{y,xy}\mu_{12}^{(0)}\right]$$

$$a_x^{(45)} = \overline{E}_1^{(45)}\left[\frac{1}{2E_x}(1 - \mu_{yx})(1 + \mu_{12}^{(45)}) + \frac{\eta_{xy,x}(1 - \mu_{12}^{(45)})}{2G_{xy}}\right]$$

$$a_y^{(45)} = \overline{E}_1^{(45)}\left[\frac{1}{2E_y}(1 - \mu_{xy})(1 + \mu_{12}^{(45)}) + \frac{\eta_{xy,y}(1 - \mu_{12}^{(45)})}{2G_{xy}}\right]$$

$$a_{xy}^{(45)} = \frac{\overline{E}_1^{(45)}}{G_{xy}}\left[\frac{\eta_{x,xy}(1 + \mu_{12}^{(45)})}{2} + \frac{\eta_{y,xy}(1 + \mu_{12}^{(45)})}{2} + \frac{(1 - \mu_{12}^{(45)})}{2}\right].$$

Thus, it is necessary to solve equations (18) for δ (0) and δ (+45), and choose the maximum value of the thickness. Note that under uniaxial compression, the most likely cause of failure is the achievement of ultimate stresses in the layer with longitudinal reinforcement.

RESULTS AND DISCUSSION

In the general case, it is possible to determine the thickness of an isotropic panel of a given width, exposed to a given linear load (to a flow q=P/b, where b is the width of the panel), for the conditions of strength and stability and ensuring strength at a postbuckling state. That is, in the simplest case, when using the strength criterion, we have: $\delta = q/\overline{\sigma}$, where $\overline{\sigma}$ are the failure stresses. To design a panel for stability and the desired thickness, one can use the dependence $\delta^3 = qb^2/(kE)$, where k is the boundary condition factor and E is the modulus of elasticity of the material. In the most complex case of a panel designed for strength

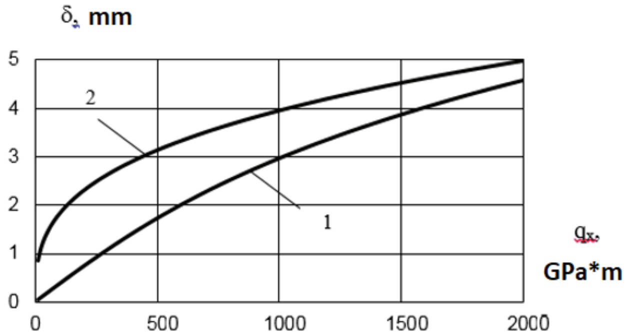

Figure 2. Orthotropic panel thickness as a function of compressive loads.

at postbuckling, we generally use more complex nonlinear relationships.

In this work, analytical relationships are obtained that allow one to determine the minimum thicknesses of anisotropic panels, considering geometric nonlinearity. The paper proposes a methodology that uses the postbuckling design procedure [3-4] for the design of load-bearing panels.

To illustrate this design methodology, let us consider Figure 2. Figure 2 shows an orthotropic panel thickness as a function of compressive loads, defined under the stability condition (curve No. 2), and under the strength condition at a postbuckling state (curve No. 1). The analysis takes into account the limit stresses. The panel dimensions are $a \times b = 400 \times 120$ mm. To calculate the orthotropic carbon fiber panel, the following properties were used: E1=125 GPa, E2=9 GPa, G12=5 GPa, $\bar{\sigma}_1$=1.4 GPa, μ21=0.28, compressive loads vary within qx=0...2000 GPa·m, laminate lay-up: $\bar{h}_0 = 0.3$, $\bar{h}_{\pm 45} = 0.7$, $\bar{\sigma}_{\bar{x}} = 0.529$ GPa.

We also note that the formally obtained expression (17) is identical to the equation obtained in [18, 19] for the case of panels with a hinged support. In the current paper, we consider a panel exposed to more stringent boundary conditions (the differences are in more complex coefficients $D_{\alpha\beta}$ and $B_{\alpha\beta}$) and having an anisotropic structure (hence the coefficients of anisotropic structure influence $\eta_{x,xy}$, $\eta_{y,xy}$, $\eta_{xy,x}$, $\eta_{xy,y}$). Note that the obtained analytical relationships can be used to design and analyze load-bearing panels with an anisotropic structure.

In addition, the panel thicknesses determined by the proposed methodology will have a minimum strength safety margin at η=1 with geometrically nonlinear behavior. The equality η=1 means that the absolute maximum stresses are equal to the limit stresses at potentially critical points, which are determined from equations (18).

CONCLUSION

Based on an analytical solution to a geometrically nonlinear problem for panels with anisotropic structure and rigid support along the long sides, this paper proposes a methodology for determining the optimal thicknesses obtained from the condition of reaching the maximum static strength at a postbuckling state of the panel. This methodology is part of the scientific and technical research related to the design of a perspective light aircraft.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] Degenhardt R, Teßmer J. Advances in computational stability analysis of composite aerospace structures. AIP Conf Proc 2010:1281. [CrossRef]

[2] Degenhardt R, Castro S, Arbelo M, Zimmerman R, Khakimova R, Kling A. Future structural stability design for composite space and airframe structures. Thin-Walled Structr 2014;81:29-38. [CrossRef]

[3] Mitrofanov OV. Applied methods for designing load-bearing panels from composite materials. Moscow, Russia: Sputnik+ Company; 2003.

[4] Mitrofanov O. Designing load-bearing panels of aircraft structures for the post-buckling state. Moscow: MAI;2020.

[5] Ni X, Prusty G, Hellier A. Buckling and post-buckling of isotropic and composite stiffened panels: a review on analysis and experiment (2000-2012). Int J of Maritime Eng 2021;157. [CrossRef]

[6] Gavva L, Firsanov V. Analytical review of account methods and experimental approaches to stress-strain state ınvestigation of structurally-anisotropic aircraft panels made from composite materials. IOP Conf Ser Mater Sci Eng 2020;927:012067. [CrossRef]

[7] Falzon BG, Aliabadi MH. Buckling and post-buckling structures II: Experimental, analytical and numerical studies. Comput Exp Methods Struct 2018;9:308. [CrossRef]

[8] Qiao P, Shan L. Explicit local buckling analysis of rotationally restrained composite plates under biaxial loading. Int J Struct Stab Dyn 2007;07:487-517. [CrossRef]
[9] Qiao P. Dynamics-based damage identification. Health Assess Eng Struct 2013:57-81. [CrossRef]
[10] Qiao P, Lu K, Lestari W. A combined static/dynamic technique for damage detection of laminated composite plates. Exp Mech 2007;48:17-35. [CrossRef]
[11] Yu H, Hami SA, Sharif KZ, Aliabadi MH. Structural health monitoring ımpact classification method based on bayesian neural network. Polymers 2022;14:3947. [CrossRef]
[12] Karmakov S, Aliabadi MH. Deep learning approach to ımpact classification in sensorized panels using self-attention. Sensors 2022;22:4370. [CrossRef]
[13] Tsai SW. Double-Double: New Family of Composite Laminates AIAA J 2021;59:1-11. [CrossRef]
[14] Kappel E. Double-Double laminates for aerospace applications - Finding the best laminates for given load sets. Compos Part C 2022;8:100244. [CrossRef]
[15] Mitrofanov O, Osman M. Assessment of the geometrically nonlinear behavior of thin cleavage type surface defects of the anisotropic structure under shear, J Phys: Conf Ser 2021:1925. [CrossRef]
[16] Mitrofanov OV. Evaluation of the postbuckling behavior of composite panels in shear, taking into account rigid support along the long sides and at initial deflection. Nat Tech Sci 2019;1:132-135. [CrossRef]
[17] Lekhnitskii S. Anisotropic Plates. 2nd ed. S.W. Tsai, T. Cheron (Translated Ed.). Yverdon, Switerland; Philadelphia: Gordon and Breach Science Publication; 1968.
[18] Mitrofanov O, Osman M. Design of thin orthotropic panels according to the post-buckling state under combined loading taking into account uniform heating, J Aerospace Systems 2022;5:29-36. [CrossRef]
[19] Mitrofanov O, Lebedevs I, Urbaha M. Design of thin composite skins of the anisotropic structure of bearing panels of aircraft structures in the post-buckling state under combined loading. Eng Rural Dev 2021:1145-1153. [CrossRef]