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Research Paper

Link Prediction and Maximum Flow in Transportation Network

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Abstract— This study conducted link prediction analysis and maximum flow analysis, which provide critical insights into alternative route inferences and traffic flow, based on real transportation network data. The dataset used in the analysis was specifically generated for this purpose. Data collection involved Bluetooth vehicle counting devices installed at 54 intersection points in the city center of Malatya, Turkey. The methodology leveraged approximately 50 million vehicle transition records to weight the transportation network graph. The Ford-Fulkerson method was utilized for the maximum flow analysis, while the Jaccard similarity metric was employed for the link prediction analysis. The graph construction and all analysis processes were carried out using the R programming language and the igraph graph library. The results of the analyses provided significant insights into alternative route corridors within the transportation network and the maximum traffic capacity of the roads. Consequently, the findings enabled the identification of critical points and potential congestion areas. The outcomes are expected to make a substantial contribution to enhancing the efficiency of the transportation network and improving traffic management strategies.

Keywords: Transportation network, Link prediction, Maximum flow

1. Introduction

Transportation networks constitute one of the cornerstones of the social, economic, and environmental aspects of urban life. An effective transportation system not only facilitates individuals' daily lives but also plays a critical role in ensuring the sustainable functioning of commerce, social interactions, and urban mobility. However, with increasing population and urbanization, traffic congestion and irregularities lead to significant time losses, environmental pollution, and substantial economic costs. This situation necessitates the efficient management of transportation networks and the early detection of issues in traffic flow, particularly in large cities.

In this study, a comprehensive analysis was conducted to uncover patterns that could contribute to the improvement of Malatya's transportation network. The most unique aspects of the study are the dataset used and the transportation network created. Specifically for this study, the intersections and road connections in the city center were modeled using graphs. The constructed transportation network graph consists of 54 intersections and 147 road connections. This uniquely designed network was weighted with data collected by Bluetooth vehicle counting devices installed at 54 intersection points.

The transportation network graph, created as both directed and weighted, was subjected to maximum flow and link prediction analyses. During the analysis process, the R programming language and the RStudio environment were employed, along with the igraph library, to perform maximum flow and link prediction analyses. The Ford-Fulkerson algorithm, a well-established method, was used to calculate bottleneck points and maximum flow values [1]. While maximum flow analysis determines the highest possible traffic capacity between specific routes in the transportation network, the Jaccard algorithm provides suggestions for opening alternative routes through similarity and difference measurements in the network [2].

This study is expected to make a significant contribution to the more effective management of urban transportation and the proactive prediction of traffic issues.

1.1. Related Works

The analysis of transportation networks has been a prominent research topic in traffic management and urban planning for many years. In particular, examining the relationships between nodes and edges within transportation networks through mathematical models offers an effective approach to optimizing traffic flow and identifying bottlenecks. In the literature, maximum flow algorithms are widely utilized in areas such as capacity analysis of transportation networks and the identification of critical connections. Additionally, algorithms based on similarity measurements provide significant contributions to the detection of bottlenecks and predictive modeling of connections within network structures. In this context, both theoretical and practical studies have developed innovative methods for improving transportation networks and addressing traffic problems.

Among the approaches aimed at enhancing the observability of traffic flow data, factorization methods stand out for their ability to identify necessary measurement data and flow dependencies within the network. These methods offer innovative tools to address the network observability problem [3]. Graph-based approaches have proven to be highly effective in solving transportation network problems and have also been successfully applied to a variety of other problem types [4].

For Maximum Flow:

This article presents a graph theory-based approach developed to optimize traffic load in a transportation network using real-world data comprising 438 million vehicle transitions. Maximum flow values and bottleneck points were determined using the Ford-Fulkerson algorithm, while the most influential intersection points were identified with the PageRank algorithm. Additionally, a novel algorithm was proposed to optimize traffic flow based on maximum demand capacity, aiming to improve the efficiency of the transportation network [1]. The maximum flow problem is a significant topic in optimization theory, aiming to find an appropriate flow that achieves the maximum possible rate between a source and a target in a flow network. This study proposes a new algorithmic approach based on "max-flow" to solve the maximum flow problem. Unlike traditional algorithms such as Ford-Fulkerson and Dinic, or methods relying on the Max-Flow Min-Cut Theorem, the proposed approach seeks to find the maximum flow between source and target nodes with fewer iterations and lower complexity. Additionally, an alternative method to solve transportation problems minimizing transportation costs was introduced [5]. Another study examines the historical context of the maximum flow problem, inspired by the 1955 RAND report by T.E. Harris and F.S. Ross and A.N. Tolstoï's 1930 work on transportation problems. These works motivated Ford and Fulkerson to develop their maximum flow problem framework, which was applied to the Soviet railway network [6]. An additional study introduces an algorithm capable of calculating maximum flows and minimum-cost flows in directed graphs within $m^{1+o}(1)$ time. The algorithm constructs flows using approximate undirected cycles and ensures high accuracy for flows that minimize general convex functions, providing near-linear time solutions to various problems [7]. For multimodal transportation networks, a novel connectivity criticality indicator based on network capacity was developed. A sensitivity-based solution algorithm was proposed to reduce the repeated resolutions required by typical network scanning methods. Numerical experiments revealed that the capacity-based indicator is more effective than traditional efficiency-based indicators, especially under disrupted network conditions [8]. Another paper investigates the multi-terminal maximum flow interdiction problem (MTNIP), which involves reducing maximum flow in a network by severing connections. Both an exact model (MTNIP-E) and a fast approximation model (MPNIM) were proposed. Results show that MPNIM operates significantly faster while providing results close to those of MTNIP-E in most cases [9]. A cross-docking strategy study focused on reducing goods' waiting times by optimizing material flow using the Ford-Fulkerson, Edmonds-Karp, and Dinic algorithms. Among these, the Dinic algorithm demonstrated better performance, solving the maximum flow problem more efficiently and identifying bottlenecks in the network [10]. Lastly, a max-flow multi-path routing algorithm designed to minimize delay, provide high data flow, and balance traffic load was presented. Based on the Ford-Fulkerson algorithm, this method identifies disjoint paths providing maximum flow and then cyclically distributes the traffic load across these paths. Simulations indicate that the algorithm outperforms multi-shortest-path methods [11].

For Link Prediction:

A novel mechanism based on link prediction has been developed for the evolution of transportation networks. Unlike traditional methods that typically consider only node degree or common neighbors, this model comprehensively evaluates connection probabilities based on factors such as node degree, distance, route, expected network structure, relatedness, population, and Gross Domestic Product (GDP). Numerical experiments conducted on China's high-speed rail, road, and civil aviation networks demonstrated that the proposed model outperforms other link prediction methods, offering superior optimization of network efficiency and more accurate prediction results [12]. Jaccard similarity scores have been applied to detect attacks in CAN bus networks. By measuring the proportion of common elements between two datasets, Jaccard similarity effectively distinguished between normal and attack data. Tested on four attack scenarios (DoS, fuzzy, replay, spoofing), the method demonstrated that

Jaccard similarity is a powerful tool for attack detection [13]. Link prediction is a significant research area with diverse applications. Graphs naturally represent interactions between different entities within a network, such as social networks, transportation networks, disease networks, and email/phone call networks. Link prediction has practical applications in analyzing and solving intriguing problems on these networks, including predicting disease spread, controlling privacy, detecting spam emails, or suggesting alternative routes based on current traffic patterns. These applications highlight the practical importance and potential of link prediction across various domains [14]. The concern for link prediction in networks has grown among researchers. A study comparing various link prediction methods for intercity transportation networks proposed a novel index to rank the results of different algorithms based on prediction accuracy and the number of existing links. Using simulated annealing, the optimal threshold value for link existence was determined. Experiments demonstrated that the proposed index performs well in intercity transportation networks [15]. An efficient rejection sampling method (ERS) with an "early stopping + condensation" strategy was proposed to calculate weighted Jaccard similarity in dense data. The statistical properties of ERS were analyzed, and experimental results showed that ERS is more efficient than existing methods [16]. In the "We the Media" networks, a multidimensional network model was proposed to predict connections between social users. Unlike traditional methods that rely solely on structural similarity, this study also considers users' public opinion characteristics. A link prediction algorithm incorporating these factors was developed. Experimental results revealed that the proposed algorithm significantly outperforms basic methods such as Common-Neighborhood-Driven, Jaccard, and SimRank. It was also found that the professional environment factor enhances prediction accuracy, while the social psychology factor reduces it [17]. Another study introduced a novel approach to solve the link prediction problem using line graphs in graph theory. While traditional methods adopt a graph classification approach to predict connections between two nodes, this study transforms the link prediction problem into a node classification problem in a line graph. Since each node in a line graph corresponds to an edge in the original graph, this approach prevents information loss and enables more efficient learning. Experiments conducted on fourteen datasets demonstrated that the proposed method provides faster training with fewer parameters and achieves higher accuracy than state-of-the-art methods [18]. The evolution of link prediction algorithms in complex networks is summarized in terms of physical approaches and methods (e.g., random walk and maximum likelihood-based methods). Their roles in applications such as network reconstruction, evaluating network evolution mechanisms, and classifying partially labeled networks are emphasized. Future challenges in the field are also highlighted [19]. Another study reviews similarity-based indices, probabilistic methods, dimensionality reduction approaches, learning-based models, and information-theoretic approaches. Applications in directed, temporal, bipartite, and heterogeneous networks are also examined. Experimental results are discussed, and application areas and future research directions are evaluated [20]. A new class of tests based on the Jaccard similarity index was proposed to test the homogeneity of two independent polynomial samples. Using separable statistics theory, the asymptotic powers of the tests were analyzed, and the most suitable similarity test in this class was identified [2]. To address the shortcomings of traditional recommendation systems, two new similarity models considering entire user rating vectors were developed. It was emphasized that approaches based solely on co-rated items are limited in identifying suitable neighbors in sparse datasets. Tests on the MovieLens dataset revealed that the proposed models provide more accurate and effective recommendations with lower computation times. In particular, the relevant Jaccard similarity model demonstrated superior performance compared to traditional similarity measures [21]. Jaccard indices have found wide application in measuring similarity between mathematical structures and datasets, inspiring various generalizations. This study explored advanced generalizations of the Jaccard index, particularly developing a coincidence index that accounts for intrinsic similarity levels. Additionally, generalizations measuring dependencies between sets, densities, and random variables were introduced, and an index was proposed to measure the chaining level of three structures. These methods play a significant role in similarity measurement between sets and the analysis of complex networks [22].

2. Materials and Methods

The analysis processes presented in the study consist of several stages. As illustrated in Figure 1, the analysis process is depicted in four stages. In Stage 1, the locations of Bluetooth devices installed in the city center of Malatya for collecting vehicle count data are specified. In Stage 2, approximately 50 million vehicle count records obtained from these Bluetooth devices are subjected to data preprocessing steps to be formatted for analysis. In Stage 3, a transportation network graph weighted with vehicle count data is constructed using transportation network data and vehicle count records. The constructed transportation network graph comprises 54 intersections and 147 road links. In Stage 4, a link prediction analysis, which aims to propose alternative route recommendations on the transportation network, and a maximum traffic capacity analysis between two different locations are conducted.

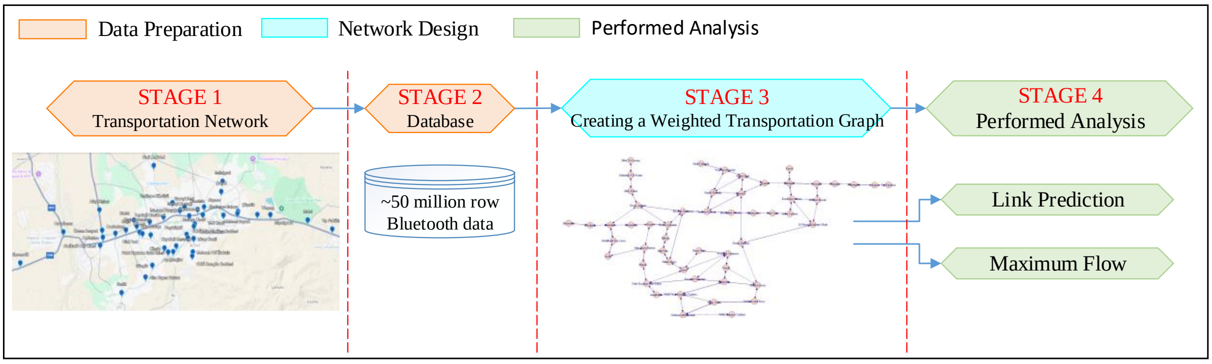

Figure 1. Graphical Summary of the Presented Study

The data used in this study are presented for the first time within this research. Images of Bluetooth devices used to generate vehicle count data are shown in Figure 2. The data include the number of vehicle passages at each intersection, time intervals, and other traffic parameters. Among the methods employed to optimize traffic flow, graph theory-based approaches hold a significant position. In this context, maximum flow analysis and link prediction methods are extensively utilized. Maximum flow analysis aims to maximize vehicle flow within the transportation network, while link prediction methods are developed to forecast future traffic trends.

Figure 2. Placement of Bluetooth Devices at Intersections

The Ford-Fulkerson algorithm was employed to solve the maximum flow problem. This algorithm identifies the maximum flow capacities between intersections and enables the detection of bottleneck points in traffic flow. The Jaccard metric was used in link prediction to infer alternative routes. Jaccard was applied as a similarity measure to assess the impact of each intersection on traffic flow and to understand traffic patterns. Both the modeling of the transportation network and the analysis processes were conducted using the R programming language and the igraph library. The results demonstrate that the algorithms employed are effective in enhancing the efficiency of the transportation network and optimizing traffic flow. This study provides a robust model for more efficient urban transportation planning and management.

2.1. Maximum Flow

Maximum Flow is a key problem in network optimization and flow theory. This problem aims to determine the maximum amount of flow that can be transferred from a source (starting node) to a sink (target node) within a flow network. A flow network comprises specific nodes (vertices) and directed edges (links) connecting these nodes. Each edge has a capacity representing the maximum amount of flow it can carry. The objective is to achieve the maximum flow from the source to the sink while adhering to these capacity constraints [10].

The Ford-Fulkerson algorithm is a greedy approximation algorithm designed to calculate the maximum flow value in a flow network. This algorithm was developed by L. R. Ford Jr. and D. R. Fulkerson in 1956 [10]. The basic idea of the algorithm is as follows: as long as there is a path from the source (starting node) to the sink (ending node), flow is sent along a path that has available capacity on all its edges. Such a path with available capacity is called an augmenting path. The algorithm's formula can be expressed as follows [11]:

$$\mathrm{c(V,E)} = \sum u \in \mathrm{S, u} \in \mathrm{T}\,|\mathrm{(u,v)} \in \mathrm{E\ c(u,v)}$$

$G(V, E)$ represents a finite directed graph, where edges (u, v) indicate connections between nodes, and $c(u, v)$ denotes the capacity value of the corresponding edge. Nodes s and t represent the source and sink points, respectively. The "cut" operation refers to the partitioning of the graph's nodes into two disjoint subsets S and T. Consequently, there are $2|V|-2$ possible cuts in the graph.. $c(V, E)$ provides the maximum capacity of the graph in terms of cuts [11].

The foundation of these algorithms is defined by the Max-Flow Min-Cut Theorem. This theorem states that the maximum flow in a network is equal to the capacity of the smallest cut (min-cut) that separates the source from the sink. In other words, the maximum amount of flow that can be sent from the source to the sink in a network is equivalent to the total capacity of the edges in the smallest cut that separates the source and sink [7].

The solution of maximum flow problems holds significant theoretical importance and has a wide range of practical applications. For example, it is used in optimizing load distribution in transportation networks, improving urban traffic flow, and ensuring efficient routing of data packets in communication networks. These applications highlight the importance and complexity of developing and implementing maximum flow algorithms.

2.2. Link prediction

Link prediction is a method aimed at predicting whether a future connection will form between two nodes in a network. This method is particularly used to analyze relationships and connections in various domains, such as social networks, biological networks, and transportation networks. Link prediction algorithms operate based on the structural properties of the existing network and similarity measures between nodes. For this purpose, various metrics are used to measure the similarity between nodes.

2.2.1. Jaccard Similarity Coefficient

The Jaccard Similarity Coefficient represents the ratio of common neighbors between two nodes to the total number of neighbors of those nodes. This metric indicates the extent of shared connections between two nodes. Jaccard similarity is calculated as the ratio of the intersection of two sets to the union of those sets [23].

The formula for Jaccard similarity is as follows:

$$\mathrm{J(A,B)} = \frac{|A \cap B|}{|A \cup B|}$$

Here:

- A and B are two sets (e.g., two different TFBS sets defined by distinct PWMs).
- $|A \cap B|$ is the number of elements in the intersection of sets A and B (shared elements).
- $|A \cup B|$ is the number of elements in the union of sets A and B (total elements).

A value of **1** indicates that the sets are completely identical.
A value of **0** indicates that the sets are entirely different.

The formula signifies the similarity between two sets as the ratio of the number of elements in their intersection to the number of elements in their union. This ratio ranges between 0 and 1; **1** implies the sets are identical, while **0** implies no similarity [23]. Jaccard similarity is widely used in social networks and knowledge-based systems to determine the likelihood of connection formation between nodes.

3. Experimetal Results

The study conducted a comprehensive analysis aimed at improving the transportation network of a city. A uniquely constructed transportation network graph was developed, comprising 54 vertices and 147 road connections. The visual representation of the transportation network graph is presented in Figure 3. The graph was designed as weighted and directed using data obtained from a custom-developed Bluetooth vehicle counting device.

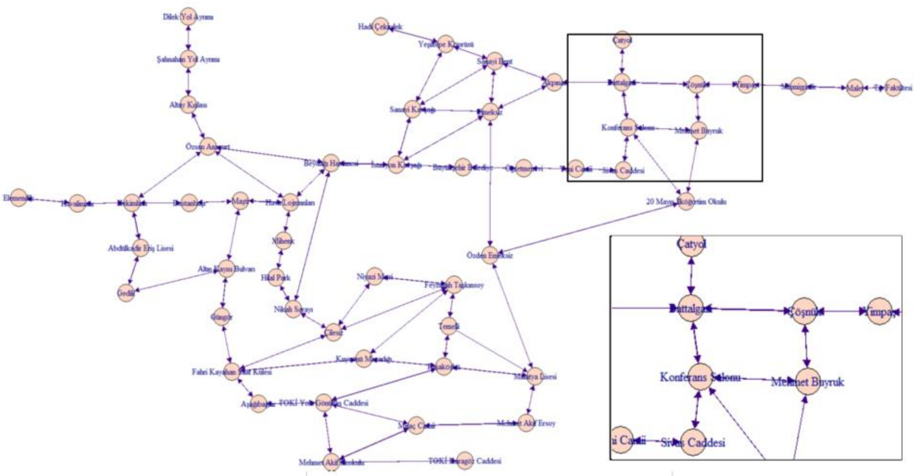

Figure 3. The Transportation Network Graph of Malatya City

Analyses have been conducted on the graph to determine the maximum flow capacity on specific routes and to predict alternative road connections.

Table 1. Comparison of the intersections' data for July and August

Intersection -1	Intersection -2	July	August
İstasyon Kavşağı	Yimpaş	13279	10703
Yimpaş	İstasyon Kavşağı	4459	3542
Güngör	Miraç Camii	7513	8437
Miraç Camii	Güngör	11489	12187
Gedik	İstasyon Kavşağı	15167	17163
İstasyon Kavşağı	Gedik	8488	11748
Elemendik	Tıp Fakültesi	13279	10703
Tıp Fakültesi	Elemendik	4459	3542
Maşti	Fahri Kayahan Saat Kulesi	23607	24351
Fahri Kayahan Saat Kulesi	Maşti	27024	28170
Gedik	Yimpaş	13279	10703
Yimpaş	Gedik	4459	3542
Altın Kayısı Bulvarı	Hadi Çekirdek	2350	2124
Hadi Çekirdek	Altın Kayısı Bulvarı	1140	996
Havalimanı	Aşağıbağlar	23607	24351
Aşağıbağlar	Havalimanı	27024	28170
Dilek Yol Ayrımı	Çöşnük	13279	10703
Çöşnük	Dilek Yol Ayrımı	4459	3542
Mihenk	Konferans Salonu	11989	9452
Konferans Salonu	Mihenk	4459	3542

3.1. Maximum Flow Analysis

In the constructed graph, the maximum vehicle carrying capacity for 10 different bidirectional routes was determined. Table 1 presents the maximum flow values between 20 distinct locations. For instance, based on July data, the maximum vehicle flow between İstasyon Intersection and Yimpaş Intersection was recorded as 13,279, whereas this value decreased to 10,703 in August. Conversely, when analyzing the reverse direction, the maximum number of vehicles traveling from Yimpaş Intersection to İstasyon Intersection was 4,459 in July, while it dropped to 3,542 in August. The analysis of the results reveals that maximum flow values may fluctuate across different months. These variations can be attributed to various physical factors or changes in people's preferences for alternative routes.

When the table is examined as a whole, a consistency among the values is observed. This consistency provides significant insights into the maximum vehicle capacities that the existing transportation network can handle on a monthly or seasonal basis, as represented on the transportation network graph.

3.2. Link Prediction Analysis

The identification of alternative routes plays a crucial role in improving the transportation network. In this study, the Jaccard similarity method was employed to determine alternative road routes based on existing vehicle passage profiles. Table 2 presents the Jaccard similarity results for specific intersection points. The dataset used in the analysis comprises the combined vehicle count data for July and August.

For instance, based on the vehicle count data, the Jaccard value between the Conference Hall intersection and the Akpınar intersection was calculated as 0.40. Similarly, the Jaccard value between the Conference Hall intersection and the Beydağı Hospital was found to be 0.20, and 0.33 with the Air Force Housing intersection. Interpreting these values, we can estimate that the probability of creating a direct road between the Conference Hall and Akpınar intersections is approximately 40%. This result indicates that vehicles passing through the Conference Hall intersection also frequently pass through the Akpınar intersection. Consequently, connecting these two intersections with a direct road could make a significant contribution to the transportation network.

Table 2. Total data of the intersections for July and August are the connection estimates

Intersection Names	Akpınar	Battalgazi	Beydağı Hastanesi	Elemendik	Hava Lojmanları	Hilal Park	Malet	Mehmet Buyruk	Miraç Camii	Niyazi Mısri	Özden Emeksiz
Altın Kayısı Bulvarı	0.00	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.33	0.00
Büyükşehir Belediye	0.00	0.00	0.14	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00
Çatyol	0.00	0.00	0.16	0.00	0.00	0.33	0.00	0.00	0.00	0.00	0.00
Çilesiz	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.50	0.00	0.00
Feyzullah Taşkınsoy	0.00	0.33	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00
Hadi Çekirdek	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00
Konferans Salonu	0.40	0.00	0.20	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.00
Mişmişpark	0.00	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.33	0.0
Nikah Sarayı	0.00	0.00	0.40	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.000

In Figure 4, the Jaccard values between all intersections are presented on a heat map. The dark red color at the intersection points indicates high Jaccard values, while the transition towards white signifies a decrease in similarity values.

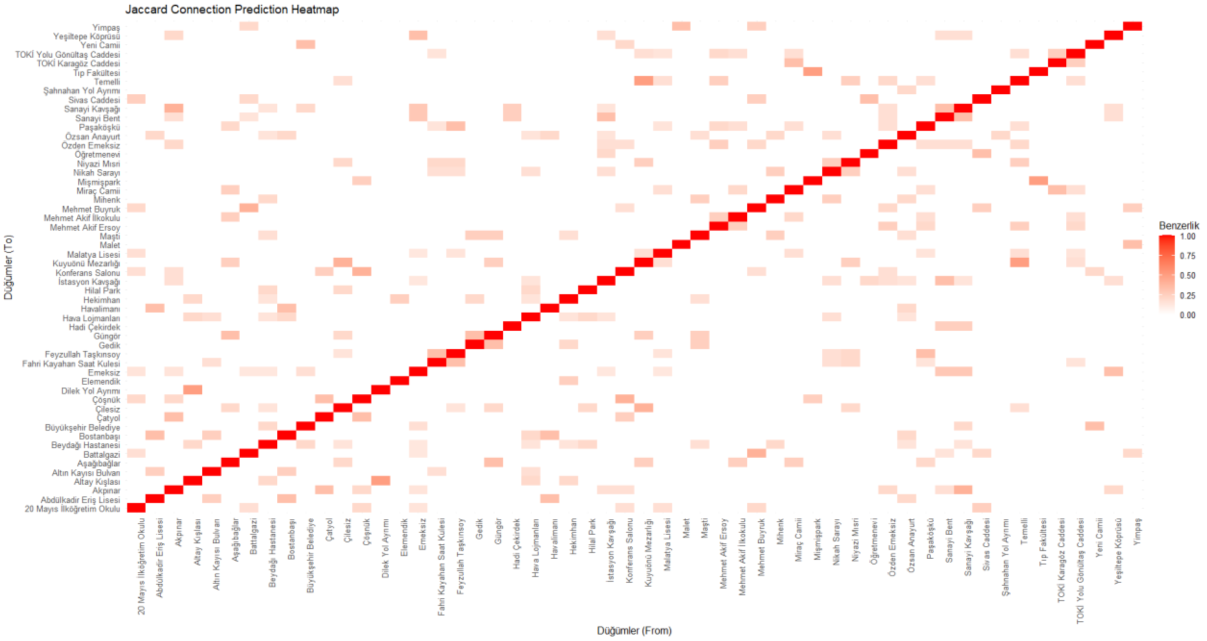

Figure 4. Jaccard similarity heat map of all intersections

4. Conclusion

In this study, graph-based analysis operations have been conducted to serve as a decision support system for improving the transportation network of a city. Decision support systems are critical tools for analyzing complex structures [24]. The research focuses on solving the maximum flow and link prediction problems within a transportation network designed based on real vehicle count data. The study specifically aims to identify link predictions and maximum traffic flow values within the network. The model developed using real traffic data has enabled the identification of critical nodes and connections in the transportation network. In this context, traffic density and connection dynamics within the network were analyzed. The Ford-Fulkerson algorithm was utilized for solving the maximum flow problem, while the Jaccard method was employed for link prediction.

The results obtained from the analysis provided insights into potential alternative routes. Moreover, the maximum flow values associated with specific routes have been highlighted as crucial metrics for managing traffic density. Opening alternative routes implies an increase in the maximum traffic capacity, indicating a direct relationship between the success of link prediction and maximum flow.

The findings emphasize the importance of data-driven approaches in transportation planning and offer critical guidance for infrastructure investments to decision-makers. Future studies aim to expand the scope of the analysis by incorporating larger datasets and graph-based methodologies.

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