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ON A NEW METHOD FOR CALCULATION OF THE NUMBER OF PRIME NUMBERS IN THE GIVEN INTERVAL

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ABSTRACT

Looking at the history of the development of the number theory, it becomes apparent that the solution to some of the fundamental problems of this theory depends on the number of primes in the given interval and rule for distribution of primes. Note that the methods of the theory of functions of complex variable was used for accurate assessment of the asymptotic distribution of prime numbers; inaccuracies in the results led to such a conclusion that it would be more useful to solve the problems of the number theory by utilizing the methods of the theory of functions of complex variable. In this work, exact number of primes in the given interval was found without using the methods of the theory of functions of complex variable; derived results will hugely contribute to the solution of several problems of the number theory. For describing the behavior of the distribution function of prime numbers it is necessary to investigate some auxiliary classes of natural numbers. The given paper is devoted to investigation of some subclasses of natural numbers, that allows to describe the distribution function of prime numbers [1], [2].

Keywords: *Prime number, Number theory, Complex analysis.*

BELİRLİ BİR ARALIKTAKİ ASAL SAYILARIN SAYISININ HESAPLANMASINDA YENİ BİR METOT ÜZERİNE

ÖZET

Sayı teorisinin tarihsel gelişimine bakıldığında bu teorinin belirli bir aralıktaki asal sayıların sayısına ve asal sayıların dağılım kuralına bağlı olan temel problemlerin bazılarına çözüm getirdiği görülmektedir. Kompleks fonsiyonlu değişkenler teorisinin metotları, asal sayıların asimptotik dağılımının doğru bir şekilde belirlenmesi için kullanıldığı; sonuçlardaki hataların kompleks fonsiyonlu değişkenler teorisinin metotlarını kullanarak sayı teorisinin problemlerini çözmenin daha faydalı olabileceğini gösterdiği dikkate alınmalıdır. Bu çalışmada belirli bir aralıktaki asal sayıların tam sayısına kompleks fonsiyonlu değişkenler teorisinin metotları kullanılmadan ulaşıldı. Elde edilen sonuçlar sayı teorisinin birtakım problemlerinin çözümüne büyük katkı sağlayacaktır. Asal sayıların dağılım fonksiyonunun davranışını tanımlamak için, doğal sayıların bazı yardımcı sınıflarının araştırılması gerekmektedir. Bu çalışma asal sayıların dağılım fonksiyonunun tanımlanmasına olanak sağlayan doğal sayıların bazı alt sınıflarının araştırılmasına yöneliktir.

Anahtar Kelimeler: *Asal sayılar, Sayı teorisi, Kompleks fonsiyonlu değişkenler teorisi.*

1. DENOTATION AND NECESSARY FACTS ([3], [4]).

Denote by N the set of all natural numbers, by $N(\leq n)$ the set of natural numbers not exceeding n.
It is obvious that the set $N(\geq 5)$ may be represented in the form

$$N(\geq 5) = \{5,6,7,...,n,...\} = \{6m-1, 6m, 6m+1, 6m+2, 6m+3, 6m+4\}_{m=1}^{\infty} =$$
$$= \{6m+\alpha, \alpha = -1,0,1,2,3,4\}_{m=1}^{\infty}. \quad (1.1)$$

From (1.1) it is seen that the natural numbers of the form $6m$, $6m+2$, $6m+3$ and $6m+4$ for any natural value of m are composite numbers; and only natural numbers $6m-1$ and $6m+1$ $(m \in N)$ may be also prime numbers.
Therefore we have [3]

Theorem 1.1. If n is a prime natural number, then it is necessary that n should have the form $n = 6m-1$ or $n = 6m+1$.
If the number $6m+1$ (or $6m-1$) is a composite number, then at least it has two cofactors, i.e.

$$6m \pm 1 = (6i+\alpha)(6j+\beta). \quad (1.2)$$
$$\alpha \quad and \quad \beta = -1,0,1,2,3,4. \quad (1.3)$$

From (1.2) we have

$$6m \pm 1 = 36ij + 6\beta i + 6\alpha j + \alpha\beta = 6m_0 + \alpha\beta. \quad (1.4)$$

By the calculations from (1.4) we get

$$\alpha\beta = \pm 1, \ , \quad (1.5)$$

where α and β, were determined in (1.3).

Taking into account (1.5), from (1.2) we get

$$6m+1 = (6i \pm 1)(6j \pm 1) \quad i, j \in N, \ i \geq j, \quad (1.6)$$
$$6m-1 = (6i \pm 1)(6j \mp 1) \quad i, j \in N, \ i \geq j. \quad (1.7)$$

From (1.6) we get

$$m = 6ij \pm (i+j), \quad (i \geq j), \quad (1.8)$$

from (1.7) we have

$$m = 6ij \pm (i-j), \quad (i \geq j). \quad (1.9)$$

Denote the set of natural numbers of the form (1.8) and (1.9) by M_1 and M_2, respectively. Then we have

Theorem 1.2. For the natural number of the form $6m+1$ (or $6m-1$), $m \in N$ to be a composite natural number, it is necessary and sufficient that $m \in M_1$ (or $m \in M_2$).
Indeed, if $m \in M_1$ then $m = 6ij \pm (i+j)$. Hence $6m+1 = 36ij \pm 6(i+j)+1 = (6i \pm 1)(6j \pm 1)$ and vice versa, if $6m+1$ is a composite number, then $6m+1 = (6i \pm 1)(6j \pm 1)$, hence it follows that $m = 6ij \pm (i+j) \in M_1$.
Denote by $H_1 = N \setminus M_1 (H_2 = N \setminus M_2)$, where $H_1 \cap M_1 \neq \varnothing$; $(H_2 \cap M_2) \neq \varnothing$.

Then we have

Theorem 1.3. For the natural number of the form $6m+1$ (or $6m-1$) to be a prime natural number, it is necessary and sufficient that $m \in H_1$ (or $m \in H_2$).

2. THE PROPERTY OF THE SET $M_1(\leq m)$.

By definition, if $n \in M_1(\leq m)$, then $n = 6ij \pm (i+j) \leq m$, $i, j \in N$.

Note that if $m = \max_{i,j}(6ij - i - j)$, then there exists a natural number ν for $i = j = \nu$, $6\nu^2 - 2\nu = m$, hence $\nu = \left[\dfrac{1+\sqrt{6m+1}}{6}\right]$ i.e. $1 \leq i \leq \nu_j$, and if $m = \max_{i,j}(6ij + i + j)$, then for $i = j = k$, $6k^2 + 2k = m$, hence $k = \left[\dfrac{-1+\sqrt{6m+1}}{6}\right]$ and $1 \leq j \leq k$.

Denote by

$$A_i(m) = \{6ij - i - j\} = \{(6i-1)j - i \leq m, \quad i \leq j, \ i, j \in N\}$$
$$B_j(m) = \{6ij + i + j\} = \{(6j+1)j + i \leq m, \quad i \leq j, \ i, j \in N\}$$
$$1 \leq i \leq \nu, \quad 1 \leq j \leq k$$

where $6i-1$ and $6j+1$ are only prime numbers.

Call $A_i(m)$ and $B_i(m)$ the subclasses with prime coefficients of the set $M_1(\leq m)$.

It should be noted that

$$M_1(\leq m) = \left(\bigcup_{i=1}^{\nu} A_i(m)\right) \cup \left(\bigcup_{j=1}^{k} B_j(m)\right).$$

Denote the set of prime coefficients of the subclasses $A_i(m)$ and $B_j(m)$, by $K_1(-)$ and $K_1(+)$:

$$K_1(-) = \{5, 11, 17, \ldots, 6\nu - 1\} = \{K_1^{(1)}(-), K_1^{(2)}(-), \ldots, K_1^{(\nu)}(-)\},$$

i.e.

$$K_1^{(1)}(-) = 5, \ K_1^{(2)}(-) = 11, \quad K_1^{(3)}(-) = 17, \ldots, K_1^{(\nu)}(-) = 6\nu - 1$$

respectively,

$$K_1(+) = \{7, 13, 19, \ldots, 6k+1\} = \{K_1^{(1)}(+), K_1^{(2)}(+), \ldots, K_1^{(k)}(+)\},$$

i.e.

$$K_1^{(1)}(+) = 7, \ K_1^{(2)}(+) = 13, \ldots, \quad K_1^{(k)}(+) = 6k+1,$$

where $6i-1$ and $6j+1$ are only prime numbers and the number of the elements of the set $K_2(-)$ equals

$$\gamma_2(-) = C_{\nu_0}^1 \cdot C_{k_0}^1.$$

Where $K_2(-)$ is the set with the elements of the form $6\tau - 1 (\tau \in N)$ being the product of two elements of the set $K_1(-)\cup K_1(+)$:

$$K_2(-)=\{5\cdot 7,5\cdot 13,...,11\cdot 7,11\cdot 13,...,(6\nu-1)(6k+1)\}=$$
$$=\left\{K_2^{(1)}(-),K_2^{(2)}(-),...,K_2^{(\gamma_2(-))}(-)\right\},$$

where

$$K_2^{(1)}(-)=5\cdot 7,K_2^{(2)}(-)=5\cdot 13,...$$

Here ν_0 is the number of prime elements of the set $K_1(-)$, k_0 is the number of prime elements of the set $K_1(+)$. Denote by $K_2(+)$ the set with the elements of the form $6\tau+1(\tau\in N)$ being the product of two elements of the set $K_1(-)\cup K_1(+)$:

$$K_2(+)=\{5\cdot 11,5\cdot 17,...,7\cdot 13,7\cdot 19,...\}=$$
$$=\left\{K_2^{(1)}(+),K_2^{(2)}(+),...,K_2^{(\gamma_2(+))}\right\},$$

where the number of the elements of the set $K_2(+)$ equals $\gamma_2(+)=C_{\nu_0}^2+C_{k_0}^2$

$$K_2(+)=5\cdot 11,K_2^{(2)}(+)=5\cdot 17,...,K_2^{(3)}(+)=7\cdot 13,...$$

The set $K_q(-)$ and $K_q(+)$, where $2\le i+j=q$, $1\le i\le\nu$, $1\le j\le k$ is determined in the same way.

Now we can calculate the number of the elements of subclasses of the set $M_1(\le m)$:

$$mesA_1(\le m)=mes\{5t-1\le m\}=\left[\frac{m+1}{5}\right]=$$
$$=\left[\frac{m+\frac{K_1^{(1)}(-)+1}{6}}{K_1^{(1)}(-)}\right]=\left[\frac{6m+K_1^{(1)}(-)+1}{6K_1^{1}(-)}\right] \qquad (2.1)$$

$$mesA_2(\le m)=mes\{11t-2\le m\}=\left[\frac{m+2}{11}\right]=$$
$$=\left[\frac{m+\frac{K_1^{(2)}(-)+1}{6}}{K_1^{(1)}(-)}\right]=\left[\frac{6m+K_1^{(2)}(-)+1}{6K_1^{2}(-)}\right] \qquad (2.2)$$

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$$mesA_\nu(\le m)=mes\{(6\nu-1)t-\nu\le m\}=\left[\frac{m+\nu}{6\nu-1}\right]=$$

$$=\left[\frac{6m+K_1^{(\nu)}(-)+1}{6K_1^{\nu}(-)}\right], \tag{2.3}$$

where $K_1^{(1)}(-)$, $K_1^{(2)}(-)$, ... , $K_1^{(\nu)}(-)$ are the elements of the set $K_1(-)$;

$$mesB_1(\le m)=mes\{7t+1\le m\}=\left[\frac{m-1}{7}\right]=$$

$$=\left[\frac{6m-K_1^{(1)}(-)+1}{6K_1^{\nu}(+)}\right] \tag{2.4}$$

$$mesB_2(\le m)=mes\{13t+2\le m\}=\left[\frac{m-2}{13}\right]=$$

$$=\left[\frac{6m-K_1^{(2)}(+)+1}{6K_1^{2}(+)}\right] \tag{2.5}$$

....

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$$mesB_k(\le m)=mes\{(6k+1)t+k\}=\left[\frac{m-k}{6k+1}\right]=$$

$$=\left[\frac{6m-K_1^{(k)}(+)+1}{6K_1^{(k)}(+)}\right], \tag{2.6}$$

where $K_1^{(1)}(+)$, $K_1^{(2)}(+)$,..., $K_1^{(k)}(+)$ are the elements of the set $K_1(+)$.

Let $m\in A_1(m)\cap A_2(m)$. Then $m=5t-1=11\tau-2$, $5t=11\tau-1$, $t=2\tau+\frac{\tau-1}{5}$, $\tau=5\tau_1+1$, $m=5\cdot 11\tau_1+9$ and $m=5\cdot 11\tau_0-46(\tau=\tau_1-1)$. Hence it is seen that the coefficient τ_0 represents the number of the form $6t+1$, i.e. $5\cdot 11$ is a natural number of the form $6t+1$.

The number of the elements of the set $A_1(m)\cap A_2(m)$ equals

$$\left[\frac{m+46}{5\cdot 11}\right]=\left[\frac{m+\frac{5\cdot 5\cdot 11+1}{6}}{5\cdot 11}\right]=\left[\frac{6m+5\cdot 5\cdot 11+1}{6\cdot 5\cdot 11}\right]. \tag{2.7}$$

And if $m\in A_1(m)\cap B_1(m)$, then

$$5t-1=7\tau+1,\quad 5t=7\tau+2,\quad t=\tau+2\frac{\tau+1}{5},\quad \tau=5\tau_1-1,\quad m=5\cdot 7\tau_1-6,$$

hence it is seen that the coefficient τ_1 represents the number of the form $6t-1$, i.e. 5.7 is a natural number of the form $6t-1$; the number of the elements of the set $A_1(m)\cap B_1(m)$ equals

$$\left[\frac{m+6}{5\cdot 7}\right]=\left[\frac{m+\frac{5\cdot 7+1}{6}}{5\cdot 7}\right]=\left[\frac{6m+5\cdot 7+1}{6\cdot 5\cdot 7}\right].$$

Similarly continuing (by induction), we find that the number of the elements of the set

$$s(m)=\left(\bigcap_{s_1=0}^{s} A_{i_{s_1}}(m)\right)\cap\left(\bigcap_{r_1=0}^{r} B_{j_{r_1}}(m)\right),\quad 2\le s_1+r_1\le q,\ 2\le q\le \nu+k,$$

we get

$$mesS(m)=\left[\frac{6m+a\prod_{s_1=0}^{s}(6s_1-1)\prod_{r_1=0}^{r}(6r_1+1)+1}{6\prod_{s_1=0}^{s}(6s_1-1)\prod_{r_1=0}^{r}(6r_1+1)}\right], \tag{2.8}$$

where

$$a=\begin{cases}1, & if\ \ s\ \ is\ an\ odd\ number;\\ 5 & if\ \ s\ \ is\ an\ even\ number,\end{cases}$$

$$\prod_{s_1=0}^{0}(-1)=1,\quad 1\le i_{s_1}\le \nu,\quad 1\le j_{r_1}\le k.$$

If we denote the number of the composite numbers of the form $6t+1(t\in N)$ by $P^{(+)}(6m+1)$, then from (2.1)-(2.8) we have

Theorem 2.1. For the given $m\in N$ the number of the elements of the set $M_1(\le m)$ (i.e. the number of the composite numbers of the form $6\tau+1(\tau\in N)$ that doesn't exceed $6m+1$) equals

$$P^{(+)}(6m+1)=\sum_{i=1}^{\nu_0}\left[\frac{6m+K_1^{(i)}(-)+1}{6K_1^{(i)}(-)}\right]+\sum_{j=1}^{k_0}\left[\frac{6m-K_1^{(j)}(+)+1}{6K_1^{(j)}(+)}\right]+$$
$$+\sum_{q=2}^{\nu_0+k_0}(-1)^{q-1}\left(\sum_{i=1}^{\gamma_q(-)}\left[\frac{6m+K_q^{(i)}(-)+1}{6K_q^{(i)}(-)}\right]+\sum_{j=1}^{\gamma_q(+)}\left[\frac{6m+5K_q^{(j)}(+)+1}{6K_q^{(j)}(+)}\right]\right), \tag{2.9}$$

where

$$\gamma_2(-)=C_{\nu_0}^1\cdot C_{k_0}^1,\gamma_3(-)=C_{\nu_0}^1C_{k_0}^2+C_{\nu_0}^3,...$$
$$\gamma_2(+)=C_{\nu_0}^2+C_{k_0}^2,\ \gamma_3(+)=C_{\nu_0}^2C_{k_0}^1+C_{k_0}^3,...$$

Denote by $\pi^{(+)}(6m+1)$ the number of prime numbers of the form $6\tau+1(\tau\in N)$, then by $H_1(\le m)=N(\le m)\setminus M_1(\le m)$ it holds

Theorem 2.2. For the given m, the number of prime numbers of the form $6\tau+1(\tau\in N)$ not exceeding

$6m+1$ (i.e. the number of the elements of the set $H_1(\le m)$) equals

$$\pi^{(+)}(6m+1) = m - P^{(+)},$$

where $P^{(+)}(6m+1)$ is determined in equality

$$\nu = \left[\frac{1+\sqrt{6m+1}}{6}\right], \quad k = \left[\frac{-1+\sqrt{6m+1}}{6}\right].$$

Example 1. Let $m = 50$, then $\nu = 3$, $k = 2$

$$K_1(-) = \{5,11,17\} = \{K_1^{(1)}(-), K_1^{(2)}(-), K_1^{(3)}(-)\}, \ \textit{i.e.}$$

$$K_1^{(1)}(-) = 5, \ K_1^{(2)}(-) = 11, \quad K_1^{(3)}(-) = 17.$$

$$K_1(+) = \{7,13\} = \{K_1^{(1)}(+), K_1^{(2)}(+)\}, \textit{i.e.}$$

$$K_1^{(1)}(+) = 7, \ K_1^{(2)}(-) = 13.$$

Then

$$\sum_{i=1}^{3}\left[\frac{m+\dfrac{K_1^{(i)}(-)+1}{6}}{K_1^{(i)}(-)}\right] + \sum_{j=1}^{2}\left[\frac{m-\dfrac{K_1^{(j)}(+)-1}{6}}{K_1^{(j)}(+)}\right] =$$

$$= \left[\frac{m+1}{5}\right] + \left[\frac{m+2}{11}\right] + \left[\frac{m+3}{17}\right] + \left[\frac{m-1}{7}\right] + \left[\frac{m-2}{13}\right]. \tag{2.10}$$

Since

$$K_2(-) = \{5\cdot 7, 5\cdot 13, 11\cdot 7, 11\cdot 13, 17\cdot 7, 17\cdot 13\}.$$

The elements $K_2(-)$ have the form $6\tau - 1$, and the number of the elements of the set $K_2(-)$ equals $\gamma_2(-) = C_3^1 \cdot C_2^1 = 6$, then

$$\sum_{i=1}^{\gamma_2(-)}\left[\frac{m+\dfrac{K_2^{(i)}(-)+1}{6}}{K_2^{(i)}(-)}\right] = \left[\frac{m+6}{5\cdot 7}\right] + \left[\frac{m+11}{5\cdot 13}\right] +$$

$$+ \left[\frac{m+13}{11\cdot 7}\right] + \left[\frac{m+24}{11\cdot 13}\right] + \left[\frac{m+20}{7\cdot 11}\right] + \left[\frac{m37}{13\cdot 17}\right]. \tag{2.11}$$

and

$$K_2(+) = \{5\cdot 11, 5\cdot 17, 11\cdot 17, 7\cdot 13\},$$

the elements $K_2(+)$ have the form $6\tau + 1$, and the number of the elements of the set $K_2(+)$ equals $\gamma_2(+) = C_\nu^2 + C_k^2 = 4$

$$\sum_{i=1}^{\gamma_2(+)}\left[\frac{m+\dfrac{K_2^{(i)}(+)+1}{6}}{K_2^{(i)}(+)}\right]=\left[\frac{m+46}{5\cdot 11}\right]+\left[\frac{m+71}{5\cdot 17}\right]+$$

$$+\left[\frac{m+156}{11\cdot 17}\right]+\left[\frac{m+76}{7\cdot 13}\right]. \qquad (2.12)$$

Further,

$$K_3(-)=\{5\cdot 11\cdot 17, 5\cdot 7\cdot 13, 11\cdot 7\cdot 13, 17\cdot 7\cdot 13\},$$

whose elements have the form $6\tau-1$, and the number of the elements $K_3(-)$ equals $\gamma_3(-)=C_{\nu_0}^3+C_{\nu_0}^1\cdot C_{k_0}^2=4$

$$\sum_{i=1}^{\gamma_3(-)}\left[\frac{m+\dfrac{K_3^{(i)}(-)+1}{6}}{K_3^{(i)}(-)}\right]=\left[\frac{m+156}{5\cdot 11\cdot 17}\right]+\left[\frac{m+76}{5\cdot 7\cdot 13}\right]+$$

$$+\left[\frac{m+167}{7\cdot 11\cdot 13}\right]+\left[\frac{m+258}{7\cdot 13\cdot 17}\right]. \qquad (2.13)$$

Since

$$K_3(+)=\{5\cdot 11\cdot 7, 5\cdot 17\cdot 7, 11\cdot 17\cdot 7, 5\cdot 11\cdot 13, 5\cdot 17\cdot 13, 11\cdot 17\cdot 13\}.$$

The elements $K_3(+)$ have the form $6\tau+1$, and the number of the elements equals $\gamma_3(+)=C_\nu^2\cdot C_k^1=6$

$$\sum_{j=1}^{\gamma_3(+)}\left[\frac{m+\dfrac{5K_3^{(j)}(+)+1}{6}}{K_3^{(j)}(+)}\right]=\left[\frac{m+321}{5\cdot 7\cdot 11}\right]+\left[\frac{m+496}{5\cdot 7\cdot 17}\right]+$$

$$+\left[\frac{m+1091}{7\cdot 11\cdot 13}\right]+\left[\frac{m+596}{5\cdot 11\cdot 13}\right]+\left[\frac{m+921}{5\cdot 13\cdot 17}\right]+\left[\frac{m+2026}{11\cdot 13\cdot 17}\right]. \qquad (2.14)$$

For

$$K_4(-)=\{5\cdot 11\cdot 17\cdot 7,\ 5\cdot 11\cdot 17\cdot 13\},$$

the elements $K_4(-)$ have the form $6\tau-1$, and the number of the elements equals $\gamma_4(-)=C_\nu^\nu\cdot C_k^1=2$

$$\sum_{i=1}^{\gamma_4(-)}\left[\frac{m+\dfrac{K_4^{(j)}(-)+1}{6}}{K_4^{(i)}(-)}\right]=\left[\frac{m+1091}{5\cdot 7\cdot 11\cdot 13}\right]+\left[\frac{m+2026}{5\cdot 11\cdot 13\cdot 17}\right], \qquad (2.15)$$

for

$$K_4(+)=\{5\cdot 11\cdot 7\cdot 13,\ 5\cdot 17\cdot 7\cdot 13, 11\cdot 17\cdot 7\cdot 13\},$$

the elements $K_4(+)$ are of the form $6\tau+1$, and the number of the elements equals $\gamma_4(+)=C_\nu^2\cdot C_k^k=3$

$$\sum_{j=1}^{\gamma_4(+)}\left[\frac{m+\dfrac{5K_4^{(j)}(+)+1}{6}}{K_4^{(j)}(+)}\right]=\left[\frac{m+4171}{5\cdot7\cdot11\cdot13}\right]+\left[\frac{m+6446}{5\cdot7\cdot13\cdot17}\right]+\left[\frac{m+1418}{7\cdot11\cdot13\cdot17}\right]. \quad (2.16)$$

$$K_5(-)=\{5\cdot11\cdot17\cdot7\cdot13\},\ \gamma_5(-)=C_\nu^\nu\cdot C_k^k=1$$

and the number of the elements equals

$$\left[\frac{m+14181}{5\cdot7\cdot11\cdot13\cdot17}\right] \quad (2.17)$$

and

$$K_5(+)\equiv\varnothing.$$

Thus, taking into account equalities (2.10)-(2.17), we get

$$P^{(+)}(301)=22 \quad (2.18)$$

and from (2.9) we have

$$\pi^{(+)}(6\cdot50+1)=\pi^{(+)}(306)=50-22=28. \quad (2.19)$$

3. PROPERTY OF THE SET $M_2(\le m)$.

By definition, if $n\in M_2(\le m)$ then

$$n=6it-i+t\le m,\quad i\le t,\ i,t\in N$$

or

$$n=6jt+j-t\le m,\quad j\le t,\ j,t\in N.$$

where $m=\max\limits_{i,t}\{6it-i+t\}$ or $m=\max\limits_{j,t}\{6jt+j-t\}$. Then there exists a natural number r, for $i=j=t=r$ we have $6r^2=m$, hence we get

$$r=\left[\frac{\sqrt{6m}}{6}\right],$$

where $1\le i\le r$, $1\le j\le r$.

Denote by

$$C_i(m)=\{(6i-1)t+i\le m,\quad i\le t,i,t\in N,\ 1\le i\le r\}$$

and

$$D_j(m)=\{(6j+1)t-i\le m,\quad j\le t,\ j,t\in N,\ 1\le j\le r\}$$

where $6i-1$ and $6j+1$ are only prime numbers.

Call $C_i(m)$ and $D_j(m)$ the subclasses with prime coefficients of the set $M_2(\le m)$. Obviously,

$$M_2(\leq m)=\left(\bigcup_{i=1}^{r}C_i(m)\right)\cup\left(\bigcup_{j=1}^{r}D_j(m)\right).$$

Denote by $K_1^{(1)}(-)$ and $K_2^{(1)}(+)$ the set of prime coefficients of the subclasses $C_i(m)$ and $D_j(m)$ and the set $M_2(\leq m)$:

$$K_1(-)=\{5,11,17,...,6r-1\},$$
$$K_1(+)=\{7,13,19,...,6r+1\},$$

where the elements of the set $K_1(-)\cup K_1(+)$ are only prime numbers.

As in the set $M_1(\leq m)$, here we also determine the set

$$K_2(-),\ K_2(+),\ K_3(-), K_3(+),..., K_q(-)\ and\ \ K_q(+),$$

and calculate the number of the elements of the subclasses $C_i(m)$ and $D_j(m)$

$$mes(C_1(\leq m))=mes(5t+1\leq m)=\left[\frac{m-1}{5}\right]=\left[\frac{6m-K_1^{(1)}(-)-1}{6K_1^{(1)}(-)}\right],$$

$$mes(C_2(\leq m))=mes(11t+2\leq m)=\left[\frac{m-2}{11}\right]=\left[\frac{6m-K_1^{(2)}(-)-1}{6K_1^{(2)}(-)}\right],$$

....

....

$$mes(D_1(\leq m))=mes(7t-1\leq m)=\left[\frac{m+1}{7}\right]=\left[\frac{6m+K_1^{(1)}(+)-1}{6m}\right],$$

$$mes(D_2(\leq m))=mes(13t-2\leq m)=\left[\frac{m+2}{13}\right]=\left[\frac{6m+K_1^{(2)}(+)-1}{6m}\right],$$

and etc.

If

$$R(m)=\left(\bigcap_{s_1=0}^{s}C_{i_{S_1}}(m)\right)\cap\left(\bigcap_{r_1=0}^{r}D_{jr_1}\right),$$
$$2\leq s_1+r_1=q\leq s+r$$

then

$$mesR(m)=\left[\frac{6m-1+b\prod_{s_1=0}^{s}(6s_1-1)\prod_{r_1=0}^{r}(6r_1+1)}{6\prod_{s_1=0}^{s}(6s_1-1)\prod_{r_1=0}^{r}(6r_1+1)}\right],$$

where

$$b = \begin{cases} 1, & if \quad s \text{ is an even number} \\ 5, & if \quad s \text{ is an odd number} \end{cases}$$

$$\prod_{0}^{0} (-1) = 1, \quad 1 \le i, \ j \le r,$$

where $K_q^{(1)}(+)$, $K_q^{(1)}(-)$, $K_q^{(2)}(+)$, $K_q^{(2)}(-)$,... is determined as in calculating the number of the elements of subclasses of the set $M_1(\le m)$.

Denote by $P^{(-)}(6m-1)$ the number of composite numbers of the form $6\tau - 1$ $(\tau \in N)$ not exceeding $6m-1$, then we have

Theorem 3.1. For the given $m \in N$, the number of the elements of the set $M_2(\le m)$ (i.e. the number of composite numbers of the form $6\tau - 1(\tau \in N)$ not exceeding $6m-1$) equals

$$P^{(-)}(6m-1) = \sum_{i=1}^{r} \left[\frac{6m + K_1^{(i)}(-) + 1}{6K_1^{(i)}(-)} \right] + \sum_{j=1}^{r} \left[\frac{6m - K_1^{(j)}(+) + 1}{6K_1^{(j)}(+)} \right] +$$
$$+ \sum_{q=2}^{r+r} (-1)^{q-1} \left(\sum_{i=1}^{\gamma_q(-)} \left[\frac{6m + 5K_q^{(i)}(-) + 1}{6K_q^{(i)}(-)} \right] + \sum_{j=1}^{\gamma_q(+)} \left[\frac{6m + K_q^{(j)}(+) - 1}{6K_q^{(j)}(+)} \right] \right) \quad (3.1)$$

where $\gamma_q(-)$ and $\gamma_q(+)$ is determined as in theorem 2.1.

Denote by $\pi^{(-)}(6m-1)$ the number of prime numbers of the form $6\tau - 1$ $(\tau \in N)$ not exceeding $6m-1$, then by

$$H_2(\le m) = N(\le m) \setminus M_2(\le m)$$

it holds

Theorem 3.2. For the given $m \in N$ the number of prime numbers of the form $6\tau - 1$ $(\tau \in N)$ not exceeding $6m-1$ (i.e. the number of the elements of the set $H_2(\le m)$) equals

$$\pi^{(-)}(6m-1) = m - P^{(-)}(6m-1),$$

where $P^{(-)}(6m-1)$ is determined in equality (3.1), $r = \left[\frac{\sqrt{6m}}{6} \right]$.

Example 2. Let $m = 50$, then $r = 2$ and

$$K(-) = \{5, 11\}, \quad K(+) = \{7, 13\}$$

i.e.

$$K_1^1(-) = 5, \ K_1^{(2)}(-) = 11, K_1^{(1)}(+) = 7, \ K_1^{(2)}(+) = 13.$$

Then from (3.1) we have

$$P^{(-)}(301)=\left[\frac{50-1}{5}\right]+\left[\frac{50-2}{11}\right]+\left[\frac{50+1}{7}\right]+\left[\frac{50+2}{13}\right]-$$

$$\left(-\left[\frac{50+29}{5\cdot 7}\right]+\left[\frac{50+54}{5\cdot 13}\right]+\left[\frac{50+64}{7\cdot 11}\right]+\left[\frac{50+119}{11\cdot 13}\right]+\right.$$

$$\left.+\left[\frac{50+9}{5\cdot 11}\right]+\left[\frac{50+15}{7\cdot 13}\right]\right)+\left(\left[\frac{50+327}{5\cdot 7\cdot 13}\right]+\left[\frac{50+834}{11\cdot 7\cdot 13}\right]+\right.$$

$$\left.+\left[\frac{50+64}{5\cdot 11\cdot 7}\right]+\left[\frac{50+119}{5\cdot 11\cdot 13}\right]\right)-\left[\frac{50+834}{5\cdot 7\cdot 11\cdot 13}\right]=$$

$$=(9+4+7+4)-(2+1+1+1+1)=24-6=18$$

$$P^{(-1)}(6m-1)=P^{(-)}(301)=18$$

i.e.

$$\pi^{(-)}(6m-1)=\pi^{(-)}(299)=50-18=32.$$

4. CALCULATION OF THE NUMBER OF PRIME NUMBERS NOT EXCEEDING $6m+1$.

Denote by $\pi(6m+1)$ the number of prime numbers not exceeding $6m+1$. Then from theorems 2.2 and 3.2 we have

Theorem 4.1. The number of prime numbers not exceeding $6m+1$ (except 2 and 3) equals

$$\pi(6m+1)=\pi^{(+)}(6m+1)+\pi^{(-)}(6m+1)=$$
$$=2m-\left(P^{(+)}(6m+1)+P^{(-)}(6m-1)\right)$$

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