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### STUDIES



### DOCUMENTATION



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*M. Rasim MUTUK - Sehavet MERSİNOĞLU*

# STUDIES

# THE EARTH'S INTERIOR AS INDICATED BY SEISMOLOGY

K. E. BULLEN *

## I. INTRODUCTION

During the present century it has come to be understood that Seismology is fruitful not only in applying physical methods to studying the characteristics of earthquakes, but in using recorded data from earthquakes to give quantitative knowledge of the Earth's internal structure.

The energy released at the source or «focus» in a large earthquake can reach the order of $10^{25}$ ergs. This figure is about $10^{5}$ times the energy released in the Bikini atom bomb explosion of 1946 July 24, and 100 to 1000 times the probable energy released in the hydrogen bomb explosions of 1954. Even earthquakes whose energy is appreciably less than the maximum can send waves right through the Earth with amplitudes sufficiently large to enable them to be recorded on emerging at points all over the outside surface. There are more than 600 seismological observatories, and the records or seismograms traced at these observatories provide the raw material from which inferences on the Earth's internal layering are made.

In order to use this material, a theory of seismic wave transmission is required. For this purpose, the Earth is treated to a first approximation as a perfectly elastic isotropic body, and the infinitesimal strain theory is used. This treatment requires the use of two-parameters to describe the elastic behaviour, in addition to the density r. It is convenient in the present context to select the two parameters as the incompressibility or bulk-modulus k, and the rigidity m. The incompressibility measures the resistance of the material to an applied symmetrical pressure, while the rigidity measures the resistance to an applied distortional stress. There are problems in which deviations from the first-approximation theory need to be considered, but the simple theory is adequate for the inferences to be made in what follows.

The terms «solid» and «fluid» will be used in connection with the Earth's deep interior, and, because of the high pressures involved, a word of explanation is necessary. A material in the Earth will be called solid when the stress-strain relations describing its behaviour are of the same mathematical form as the stress - strain relations that apply to perfectly elastic isotropic solids in the laboratory, the parameters k and m appearing as coefficients in these relations. Also, for a material to be called solid, it will be stipulated that both k and m must be greater than $10^{10}$ dyn/cm$^2$. When the term fluid is used, exactly the same description will apply except that it will be stipulated that m is less than $10^{10}$ dyn/cm$^2$. The value $10^{10}$ dyn/cm$^2$ is selected for convenience for the purpose of clear definition, and has no special physical significance. Since perfect elasticity has been postulated, the present theory takes no account of possible imperfections of elasticity in solid parts of the Earth, nor of viscosity in fluid parts. Such imperfections are not significant to the present context.

Through a solid medium as thus defined, the usual elasticity theory shows that two types of bodily waves can be transmitted. The first is the primary or P wave, which is dilatational or compressional, and has speed a given by

$$\alpha^2 = \frac{3k + 4\mu}{3\rho} . \tag{1}$$

The second is the secondary or S wave, which is rotational, and has speed b given by

$$\beta^2 = \frac{\mu}{\rho} . \tag{2}$$

For a fluid, m is relatively small, so that, approximately,

$$\alpha^2 = \frac{k}{\rho} ; \quad \beta = 0. \tag{3}$$

In addition, an earthquake gives rise to surface waves in which the motion of particles is mainly confined to the vicinity of the Earth's outer surface. These waves are important in giving information on the structure of the Earth's outermost 30 to 40 km. But since the present address is concerned mainly with the deeper interior, it will be the bodily waves which will be mainly considered.

## 2. DEVELOPMENT OF SEISMIC TRAVEL - TIME TABLES

Seismograms are in general very complicated in appearance. The bodily waves (as well as the surface waves) give rise to many trains of waves which are affected by reflections and refractions at internal surfaces of discontinuity and by reflections at the outside surface. Valuable information is yielded on the arrival-times of particular trains, and On their amplitudes and periods. For the most part, the more precise data for inferring properties of the Earth's deep interior has come from readings of arrival - times, and it will be . this part of the data that will be mainly used in the inferences to follow. (Data on amplitudes and periods, however, play a useful auxiliary role.)

In treating the arrival - time data, use is made of ray theory, in much the same way and with the same limitations as in optics. A «phase» of a seismogram is associated with a particular wave-train for which the corresponding ray has described a particular route, account being taken also of the P or S type' in different segments of the path, from the earthquake focus to the observatory.

One of the great labours of the present century has been the evolution of reliable seismological travel-time tables which give the travel - times of waves along rays for a large number of the phases observed on seismograms. Starting from crude beginnings, ZOPPRITZ of Göttingen derived useful tables in the first decade of the century, and these tables as adapted by TURNER of Oxford were used in preparing the International Seismological Summary for earthquakes occurring up to the end of 1929. By 1928 it was considered that enough data had been accumulated to make radical improvements in the tables. Comprehensive revisions were undertaken by JEFFREYS and myself and by GUTENBERG and RICHTER over a large part of the period 1930 to 1940, and resulted in tables in which the standard errors were reduced in many cases to less than one - tenth of those in the Z.-T. tables. The Z.-T. tables had already enabled a number of important inferences to be made on the Earth's interior, but the completion of newer tables in 1940 marked a stage in which it was possible to delineate the Earth's internal layering much more clearly.

The J.-B. tables have been used in compiling the I. S. S. for earthquakes since December 1936. (A preliminary version of the J.-B. tables was used in the I. S. S. for earthquakes over the period 1930-36.) The J.-B. tables relate to a standard model Earth which is spherically symmetrical and in which each internal spherical surface of equal velocity is required to enclose the same volume as the corresponding level surface of the actual Earth. Allowance for the Earth's ellipticity of figure is made by the use of separate tables. Allowances for other sources of deviation from spherical symmetry are not made since these are relatively slight and since the allowances are not yet sufficiently definitely known. Except in the outermost layers of the Earth, the deviation from symmetry (apart from that due to ellipticity) appears to be remarkably slight. The J.-B. tables give, for various phases, the travel-time T between a pair of points of the outside surface of the spherical Earth model in terms of the angular distance D subtended by the ray at the Earth's centre. (The tables also give the travel-times for rays starting at various assigned focal depths.)

## 3. P AND S VELOCITY DISTRIBUTIONS IN THE EARTH

By a mathematical process, involving the solution of an integral equation, it is possible to derive from the travel-time tables values of the P velocity a throughout the Earth, and of the S velocity b down to a depth of 2900 km. below the surface. The method formally fails for ranges of depth where the velocity v changes with the distance r from the centre in such a way as to violate the condition

$$\frac{dv}{dr} \leq \frac{v}{r} \,. \tag{4}$$

The condition (4), which requires the velocity gradient with respect to depth to be positive, or if negative to be fairly small, is satisfied throughout most of the Earth; but there are some limited ranges of depth in which there are possible complications arising from violation of (4). Even when (4) is satisfied, the data can fail to find the velocity distribution to good precision in ranges of depth where the velocity is increasing abnormally rapidly with increase of depth. For these reasons, the velocity distributions are better determined for some ranges of depth in the Earth than for others.

An early result of importance was the discovery of a discontinuity some tens of kilometres below the surface by A. MOHOROVICIC in a study of a Croatian earthquake in 1909. This work, supplemented by many further studies of «near» earthquakes, has led to a division of the Earth into the «crustal layers» above the MOHOROVICIC discontinuity, and the region below. The crustal layers appear to be 30 to 35 km. thick in most continental regions (thicker in the vicinity of some mountain ranges), and 5 to 10 km. thick below the main ocean floors. The P and S velocities vary much more rapidly in the crustal layers than in the sub-crustal region.

The existence of a discontinuity at much greater depth was established from seismic data by OLDHAM in 1906, and GUTENBERG in 1913 made a remarkably good calculation of its depth at 2900 km. below the surface. Using modern data, JEFFREYS has shown that this value is correct within less than 5 km. The part of the Earth down to the depth of 2900 km. is called the «mantle», and the part below, the «central core». The «mantle» is solid throughout except for the oceans and isolated packets of magma, since S as well as P velocities are everywhere transmitted. There are no authentic observations of S waves in the central core which suggests that the central core is mainly fluid. This conclusion is now well established in another way. Data on the tidal yielding of the solid outer part of the Earth, taken in conjunction with data on movements of the poles, lead to an estimate of the Earth's overall rigidity. From seismology, there is good knowledge of the rigidity throughout the mantle. It then becomes possible to estimate the average rigidity of the central core. A calculation of TAKEUCHI in 1950 implied that this average rigidity does not exceed $10^{10}$ dyn/cm$^{a}$. A similar calculation of MOLODENTSKY would permit a slightly higher rigidity, but still appreciably less than the value near the base of the mantle. It will be assumed here that m, is negligible in all parts of the central core except within 1250 km. of the centre. (The latter region, being less than 3 percent by volume of the whole core, could be solid without sensibly affecting TAKEUCHI's and MOLODENTSKY's calculations.)

A more recent discovery is of the exisfence of an «inner core» of radius near 1250 km., inside the central core. For want of a better name,; the part of the central core outside the inner core has been called the «outer core». The existence of the central core is established by the presence of a «shadow zone» for P waves from a given shallow - focus earthquake for 105° (D) ( 142°. But the shadow zone is not a complete shadow. Miss I. LEHMANN in 1936 postulated the presence of an inner core to explain certain small P readings in the shadow zone, and this was supported by work of GUTENBERG and RICHTER in 1938. Finally, JEFFREYS in 1939 showed that certain of the waves in the shadow zone had amplitudes which were irreconcilable with the alternative postulate that they were caused by diffraction round the boundary between the mantle and core. As will be shown, there is reason to believe that the inner core is solid.

Following the completion of the J.-B. tables in 1940, further detail was added to knowledge of the velocity distributions and the Earth was subdivided into eight regions according to the summary below. The velocities in the table are essentially those computed by JEFFREYS.

| *Region* | *Name* | *Range of depth (km.)* | *α (km/sec)* | *β (km/sec)* |
|---|---|---|---|---|
| A | Crustal layers | 0 — 33 | Widely variable | Widely variable |
| B | Mantle | 33 — 410 | 8.1 — 9.0 | 4.4 — 5.0 |
| C | | 410 — 1000 | 9.0 — 11.4 | 5.0 — 6.4 |
| D′ | | 1000 — 2700 | 11.4 — 13.6 | 6.4 — 7.3 |
| D″ | | 2700 — 2900 | 13.6 | 7.3 |
| E | Outer core | 2900 — 4980 | 8.1 — 10.4 | Assumed zero |
| F | | 4980 — 5120 | 10.4 — 9.5 | Not observed |
| G | Inner core | 5120 — 6370 | 11.2 — 11.3 | Not observed |

Boundaries between the regions in the above table are not precisely located except between A and B, and between D" and E. The formal solution of JEFFREYS shows C as a region where both P and S velocity gradients are appreciably steeper than in B and D, and gives abrupt changes of gradient at a depth near 410 km., but no discontinuity in the velocities or their gradients across the boundary between C and D'. GUTENBERG on the other hand finds no abrupt changes near 410 km., but abrupt reductions in gradient near 1000 km. GUTENBERG also considers that the P and S velocities decrease with depth within the outermost 100 km. of the region B so rapidly that the condition (4) is violated. The seismic data make it clear that there are abnormally large gradients somewhere inside the region including B and C, and because of this it is difficult to determine a precise solution for the velocities in B and C. Although the gradients obtained by JEFFREYS and GUTENBERG differ appreciably in the regions B and C, the velocities themselves agree within about 3 percent. Miss LEHMANN has lately shown that the boundary between B and C could, on the data so far available, be as high as a depth of 200 km. Miss LEHMANN thinks that changes in m may be principally responsible for the abnormalities in this part of the Earth.

In the regions D' and D", the velocities of JEFFREYS and GUTENBERG are in much closer agreement. In D', the velocities vary much more steadily than in the regions above, and BIRCH has presented a strong argument in favour of the region D' being close to chemical homogeneity. In D", the velocity gradients, both P and S, fall to nearly zero.

Between the outer core E and the inner core G, JEFFREYS finds evidence of a transition layer F characterised by a negative velocity gradient. The evidence for the existence of F rests on data from two deep-focus earthquakes under the Solomon Islands (1932 January 9) and the Celebes Sea (1934 June 29). The data do not enable the P velocity distribution in F to be determined in detail, and JEFFREYS has arbitrarily taken this velocity to be proportional to r. GUTENBERG has not found the region F, but states that his data do not preclude its existence.

Between F and G, there is a P velocity jump of 18 percent in the formal JEFFREYS solution. In GUTENBERG'S solution, there is no discontinuous jump in velocity between the outer and inner core, but there are rapid changes amounting to a little more than 10 percent spread over a range of depth of order 100 km. In the regions E and G, the velocities of the two authors are mainly in good agreement. The gradients in the outer core E are compatible with chemical homogeneity, while in the inner core G, the gradients are less than normal.

## 4. DERIVATION OF VALUES OF r, k AND m

From the discussion in Section 3, the values of the seismic velocities *a* and may be assumed known to greater or less accuracy in the Earth to a depth of 4980 km (it being here assumed that b, is negligible in the region E); the greatest uncertainties are in respect of the velocity gradients above D'. Inspection of (1), (2) and (3) then shows that k/r and m/r are likewise known down to 4980 km. If values of some third independent function of r, k and m were also known, it would be a matter of simple algebra to compute the distribution of values r, k and m throughout most of the Earth. No such function is, however, readily available, so that indirect means have to be sought.

It can be shown that in a chemically homogeneous region where also the compression corresponds to adiabatic conditions, the density gradient is given by

$$\frac{d\rho}{dr} = -\frac{Gm\rho}{r^2(k/\rho)}, \qquad (5)$$

where G is the constant of gravitation and m is the mass inside a sphere of radius r in the standard Earth previously defined. The equation (5) rests on the further assumption of hydrostatic stress in the Earth, but this is an adequate approximation for the purpose in hand.

In a preliminary attempt to find the Earth's density variation, the equation (5) was applied by me to the part of the mantle below the crustal layers, a conventional amount being subtracted from the mass of the whole Earth to allow for the crust. The equation (5) was used in conjunction with the equation

$$\frac{dm}{dr} = 4\pi r^2 \rho, \qquad (6)$$

and, on various grounds, a starting value of 3.32 $g/cm^3$ was postulated for the density $r_1$ at the top of the region B. By integrating (5) and (6) downwards, a formal density distribution for the mantle was derived. From this distribution it was possible to infer a value of the moment of inertia I of the central core, the moment of inertia of the whole Earth being well known. The result was

$$I = 0.57\ Ma^2, \qquad (7)$$

where M and a are the mass and radius of the central core. Now the coefficient 0.57 would imply a distribution of mass in the central core such that the density near the outside is appreciably greater than the density near the centre. And this is impossible for reasons of stability. It therefore became necessary to examine the various data and postulates on which this impossible result depended.

It transpired that the fault must lie either (a) in the assumption of complete chemical homogeneity in the mantle, or (b) in the assumption of the value of 3.32 $g/cm^3$ for $r_1$. The coefficient in (7) could be reduced to 0.40, the value for a central core of constant density, if $r_1$ were taken as high as 3.7 $g/cm^3$. As geological and geophysical opinion was strongly against a value as high as this, the explanation was taken to lie in the presence of marked deviations from chemical homogeneity in the mantle below the crust.

The calculation was then repeated with the modification that (5) was not used in the region C. This led to some formal indeterminacy, but it transpired that the necessary adjustment to the coefficient in (7) provided a powerful control on the allowable density distributions. The equation (5) was also used in the outer core E. It was not possible by this method to make any estimate of the density in the inner core beyond concluding that the density at the centre could not be less than 12.3 $g/cm^3$. An Earth model that has come to be called Model A was then constructed in which the central density was arbitrarily taken as 17.3 $g/cm^3$, 5 units more than the minimum allowable figure.

In Model A, the ranges of density in the regions B, C, D (including D' and D") and E are 3.32-3.64, 3.64-4.68, 4.68-5.69 and 9.4-11.5 $g/cm^3$. It will be noted that there is a sharp jump in density, in the ratio 1.65, across the boundary between the mantle and central core of Model A. The greatest proportional deviations between these figures and the densities in the actual Earth probably lie in the regions B and C; in part of B, the densities could conceivably be too low by 0.4 $g/cm^3$. Unless the equation (5) is seriously inadequate in the regions D' and E, the Model A densities are likely to be accurate within about 0.1 $g/cm^3$ in D' and 0.4 $g/cm^3$ in E.

With a formal density distribution determined in this way, it was a matter of simple computation to determine corresponding values of k and m. The ranges for k in the regions B, C, D and E in units of $10^{12}$ $dyn/cm^2$ are 1.2-1.7, 1.7-3.6, 3.6-6.5, and 6.2 -12.6; and for m, 0.6-0.9, 0.9-1.9, 1.9-3.0, and zero, respectively. The corresponding values of POISSON's ratio are 0.27-0.28, 0.28, 0.28-0.30, and 0.50, respectively. It will be noted that the rigidity rises at the base of the mantle to a value about four times that of steel at atmospheric pressure. Like the density, the rigidity changes considerably between the mantle and central core; in the case of the rigidity, there , is a reduction from 3.0 $x10^{12}$ $dyn/cm^2$ at the bottom of the mantle to a value of order $10^{10}$ $dyn/cm^2$ or less (on Takeuchi's calculation) at the top of the core.

The calculation also yielded values of the pressure p and the gravity - acceleration g for Model A. The pressures at depths of 410, 1000, 2900, 4980 and 6370 km. are 0.14, 0.39, 1.37, 3.27 and $3.64x10^{12}$ $dyn/cm^2$, respectively. The values of g lie within one percent of 990 $cm/sec^2$ down to a depth of 2500 km., increase to a maximum of nearly 1040 $cm/sec^2$ at 2900 km., and then fall steadily to zero at the centre.

## 5. PRESSURE AND COMPRESSIBILITY

Whereas the values of r and u change substantially across the boundary between the mantle and central core, the Model A values of the incompressibility k show relatively slight change, from 6.5 to 6.2 $x10^{12}$ $dyn/cm^2$. This difference of 5 percent is, moreover, within the uncertainties of the calculations and postulates underlying Model A. Further, the Model A values indicate no significant difference in the value (of order 3 units) of dk/dp on the two sides of the boundary.

These results led to the suggestion that for the range of representative atomic numbers Z for the materials likely to be present in the Earth below say a depth of 1000 km., the incompressibility k is essentially a smoothly-varying function of the pressure right through to the Earth's centre. Such a suggestion, moreover, received some support from high - pressure experimental data available at the time. In 1950, 1 set up a second Earth model, called Model B, with this suggestion adopted as the central hypothesis.

In constructing Model B, the restrictions imposed by the known mass and moment of inertia of the Earth were of course fitted. The equation (5) was used only for the regions D' and E. Work of BIRCH has since shown that this equation is likely to be fairly reliable in those regions. Values of k/r as derived from seismology were used in conjunction with the compressibility - pressure hypothesis in constructing Model B. It was found that this procedure was sufficient to determine the model almost uniquely.

Values of the density were computed outwards from the centre. It was then found that the rate of diminution of density with r found ' in the region D' could not (as in the case of Model A) be significantly increased in the regions above D' until a depth of about 100 km. was reached. (This last depth could, in the light of various uncertainties, be increased to 200 km., but not much more.) Thus Model B differs from Model A in having higher density values throughout most of the regions C and B. The difference comes about very largely as a result of removing the small discontinuity in k at the boundary between the mantle and core of Model A. Small though that discontinuity is, its removal entails an appreciable increase of mass in the regions B and C.

In most other respects, Model B resembles Model A fairly closely. This of course in to be expected since Model B was largely inspired by Model A. In addition, however, Model B has suggested a number of interesting new conclusions.

First, the compressibility - pressure hypothesis gives an interpretation of the near-zero velocity gradients in the region D" at the bottom of the mantle. In order to keep k smoothly varying through D", it is necessary to attribute the reduction in the seismic velocity gradients in D" to an increase in the density gradient with depth. A formal calculation gave a density gradient in D" equal to three times that in D', suggesting some accumulation of denser material at the base of the mantle.

In a similar way, the reduced P velocity gradient in the inner core G suggests that the density gradient in G is a little steeper than that in the outer core E. Hence there may be some significant departures from chemical homogeneity in the inner core.

If it is correct that there exists the region F of negative P velocity gradient, the compressibility - pressure hypothesis would require F to be a region of very steep density gradient, the total density increase through F being formally indicated as 3 $g/cm^3$ on using the velocity distribution set down by JEFFREYS.

The hypothesis also leads to an estimate of the density in the inner core. Using the JEFFREYS P velocity distribution, the indicated density at the Earth's pentre is close to 18 $g/cm^3$. If there is no region F of negative P velocity gradient; then the density at the centre could be as low as 141/2$g/cm^3$.

But perhaps the most interesting feature of the hypothesis is that it implies that the Earth's inner core is solid in the sense defined. By (1),

$$k + 4\mu/3 = \rho\alpha^2. \qquad (8)$$

Between F and G, $a^2$ increases by 39 percent on the JEFFREYS data, and (in effect) by 21 percent on the GUTENBERG data. The latter value of the increase in $a^2$ is close to the minimum possible needed to account for the observed P waves at distances less than 142° in the shadow zone. On stability grounds, the density r will not be less in the inner core G than the outer core E. Hence, by (8), either k or u must increase sharply between E and G. If the increase is solely in k the amount of increase must be at least 21 percent, in conflict with the compressibility-pressure hypothesis. The hypothesis therefore implies that the inner core is solid, with the rigidity m, equal to at least 21 percent of £ k. This value of m is at least 2 x $10^{12}$ $dyn/cm^2$, and on the JEFFREYS data would be nearly 4 x $10^{12}$ $dyn/cm^2$, and is therefore of the same order as the rigidity in the lower mantle D'.

Interesting support of the compressibility-pressure hypothesis has come from work in theoretical physics which adds some strength to the various conclusions just stated. FEYNMANN and others have studied properties of Thomas-Fermi - Dirac matter at pressures above $10^7$ atmospheres (i. e. $10^{13}$ $dyn/cm^2$), and data is available from work of

BRIDGMAN and others at pressures up to $10^5$ atmospheres. ELSASSER made an interpolation between the two sets of results, and in 1952 I adapted the interpolation to fit certain geophysical data at the intermediate pressure of $10^6$ atmospheres. This work showed that any jump in the incompressibility k for the materials expected to be present in the outer and inner cores of the Earth is very probably less than 5 percent and may be zero. And a jump of 5 percent is significantly less than that required if the inner core is to be fluid.

Various writers, for example JACOBS and SIMON, have provided plausible physical grounds for the existence of an inner core with the rigidity as above indicated. CHANDRASEKHAR finds that a sharp boundary such as an inner solid core would make it easier for the mode of convection currents, envisaged by ELSASSER and BULLARD in their theory of the Earth's magnetic field, to occur in the fluid outer core.

In an attempt to reach better precision, I have investigated the possibility of detecting S waves in the inner core from seismic records. The calculations show, however, that the main associated phase would be at best on the border of observability with present seismological resolving power. It is possible that with observations from earthquakes of magnitude at least 8 at distances between about 130° and 140°, enough evidence may be accumulated in the course of a few years to establish the value of the S velocity in the inner core.

In the light of all the available present evidence, it seems that continuity of dk/dp in the Earth below a depth of 1000 km. is well founded, and that continuity of k is a reasonably good first approximation. The evidence from physics indicates that there may be small sudden changes of k at various levels, but none approaching 21 percent inside the core. Thus Model B, like Model A, serves as a useful working model, fitting much quantitative data, but subject to modification, which may not be too great, as further seismological and physical detail emerges.

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