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ON THE CHINESE CHECKER SPHERE

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ABSTRACT

The object of the present paper is to study define the sphere in a particular metric on $\Re^3$ and determine the pieces of the sphere which is formed by these planes. It is used the distance between any two points at three dimensional Chinese Checker space.

AMS Subject Classification: *51K05, 51K99*

Keywords: *Chinese Checker plane, Chinese Checker space, Chinese Checker distance.*

ÇİN DAMA KÜRESİ ÜZERİNE

ÖZET

Bu çalışmada özel metrikli $\Re^3$ uzayında küreyi tanımlayarak, küreyi oluşturan parçalar belirlenmektedir. Bunu yaparken özel metrik olarak, üç boyutlu Çin Dama Uzayında iki nokta arasındaki uzaklığı veren Çin Dama metriği kullanılmaktadır.

Anahtar Kelimeler: *Çin Dama düzlemi, Çin Dama uzayı, Çin Dama uzaklığı.*

1. INTRODUCTION

In Chinese Checkers game, the style of movement is from southwest to northeast, from east to west and north and south. Krause, E. F. [5] keeping this rule in mind, asked the questions of how to develop a metric which would be similar to the movements made by playing Chinese Checkers. Chen, G. [3] has introduced the metric,

$$d_C(x,y) = d_L(x,y) + (\sqrt{2}-1)d_S(x,y),$$

where

$$d_L(x,y) = \max\{|x_1 - x_2|, |y_1 - y_2|\},$$

and

$$d_S(x,y) = \min\{|x_1 - x_2|, |y_1 - y_2|\},$$

for any two points $X = (x_1, y_1)$ and $Y = (x_2, y_2)$ in the analytical plane. The Chinese Checker plane geometry has been studied and improved up to now see $[1], [4], [6], [7]$. The above metric can be generalized and the Chinese Checker space of dimension three can be introduced using this metric in three dimensional analitical space by

$$d_C(P_1,P_2) = d_L(P_1,P_2) + \sqrt{2}-1)d_S(P_1,P_2)$$

where

$$d_L(P_1,P_2) = \max\{|x_1 - x_2|, |y_1 - y_2|, |z_1 - z_2|\},$$

and

$d_S(P_1,P_2)=\min\{|x_1-x_2|+|y_1-y_2|,|x_1-x_2|+|z_1-z_2|,|y_1-y_2|+|z_1-z_2|\}$,
instead of the well known Euclidean metric

$$d_E(P_1,P_2)=\left[(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2\right]^{1/2},$$

where $P_1=(x_1,y_1,z_1)$ and $P_2=(x_2,y_2,z_2)$. [4]

In this work ,the Chinese Checker sphere at the Chinese Checker space has been introduced and general sphere equation has been formulated. Throught this study we write CC instead of Chinese Checker fort he sake of short.

2. CHINESE CHECKER METRIC FOR THREE DIMENSIONAL SPACE

In [4], In three dimensinol CC space points, lines and planes are the same with in Euclidean case. It can be shown that if

$$d_C:\Re^3\times\Re^3\to[0,\infty)$$

$$d_C(P_1,P_2)=d_L(P_1,P_2)+\sqrt{2}-1)d_S(P_1,P_2), \quad (2.1)$$

where $\Re^3=(\Re^3,d_C)$ is a metric space.

3. CHINESE CHECKER SPHERE

In this work, we now define CC-sphere, proceeding according to use anologous Euclidean prescription.

$$C^2=\{X=(x,y,z):d_C(M,X)=r\},$$

that is

$$C^2=\{X=(x,y,z):d_L(M,X)+qd_S(M,X)=r\},$$

$$\max\{|x-a|,|y-b|,|z-c|\}+q\min\{|x-a|+|y-b|,|x-a|+|z-c|,|y-b|+|z-c|\}=r, \quad (3.1)$$

$r\geq 0$, $q=\sqrt{2}-1$ and $M=(a,b,c)$.

Theorem 3.1: The set C^2 given by Equation (3.1) is a CC-sphere.

Proof: Consider (3.1) one can see that the equations with absolute value expressions must be solve in all the possible cases of x, y, z. We give the prof for one case, for the other cases the prof similarly.

A) $|x-a|=|y-b|=|z-c|$

B) $|x-a|=|y-b|>|z-c|$

C) $|x-a|=|z-c|>|y-b|$

D) $|y-b|=|z-c|>|x-a|$

E) $|x-a|>|y-b|=|z-c|$

F) $|y-b|>|x-a|=|z-c|$

G) $|z-c|>|x-a|=|y-b|$

H) $|x-a|>|y-b|>|z-c|$

I) $|z-c|>|y-b|>|x-a|$

J) $|x-a|>|z-c|>|y-b|$

K) $|y-b|>|x-a|>|z-c|$

L) $|y-b|>|z-c|>|x-a|$

M) $|z-c|>|x-a|>|y-b|$

Let $|z-c| > |x-a| > |y-b|$.

We obtained for the subcases where (3. 1) has solution

$$|z-c| + q(|x-a| + |y-b|) = r$$

1) If $x \le a$, $y \le b$, $z \le c$ then, $c - z + q(a-x) + q(b-y) = r$

$x = y = 0 \Rightarrow (c-z) + qa + qb = r \Rightarrow z = -r + c + q(a+b)$

$x = z = 0 \Rightarrow c + qa + q(b-y) = r \Rightarrow y = -\frac{r}{q} + \frac{c}{q} + (a+b)$

$y = z = 0 \Rightarrow c + q(a-x) + qb = r \Rightarrow x = -\frac{r}{q} + \frac{c}{q} + (a+b)$

$y \le b, z \le c \Rightarrow \quad z - y < c - b$ in the domain

$x = 0 \Rightarrow (c-z) + qa + q(b-y) = r$

$z = 0 \Rightarrow y = -\frac{r}{q} + \frac{c}{q} + (a+b)$

$y = 0 \Rightarrow z = -r + c + q(a+b)$

we take remain of the line

$x \le a, z \le c \Rightarrow \quad z - x < c - a$ in the domain

$y = 0 \Rightarrow (c-z) + q(a-x) + qb = r$

$z = 0 \Rightarrow y = -\frac{r}{q} + \frac{c}{q} + (a+b)$

$x = 0 \Rightarrow z = -r + c + q(a+b)$

we take remain of the line

$x \le a, y \le b \Rightarrow \quad x - y < a - b$ in the domain

$z = 0 \Rightarrow c + q(a-x) + q(b-y) = r$

$x = 0 \Rightarrow y = -\frac{r}{q} + \frac{c}{q} + (a+b)$

$y = 0 \Rightarrow x = -\frac{r}{q} + \frac{c}{q} + (a+b),$

we take remain of the line

2) If $x \le a$, $y \ge b$, $z \ge c$ then, $(z-c) + q(a-x) + q(y-b) = r$

$x = y = 0 \Rightarrow (z-c) + qa - qb = r \Rightarrow z = r + c - q(a+b)$

$x = z = 0 \Rightarrow c + qa + q(b-y) = r \Rightarrow y = \frac{r}{q} + \frac{c}{q}(a-b)$

$y = z = 0 \Rightarrow c + q(a-x) - qb = r \Rightarrow x = -\frac{r}{q} - \frac{c}{q} + (a-b)$

$y \ge b, z \ge c \Rightarrow \quad z - y \succ c - b$ in the domain

$x = 0 \Rightarrow (z - c) + qa + q(y - b) = r$

$z = 0 \Rightarrow y = -\frac{r}{q} - \frac{c}{q} + (a - b)$

$x = 0 \Rightarrow z = r + c - q(a - b)$

we take remain of the line

$x \le a, z \ge c \Rightarrow \quad z + x \succ c + a$ in the domain

$y = 0 \Rightarrow (z - c) + q(a - x) - qb = r$

$z = 0 \Rightarrow x = -\frac{r}{q} - \frac{c}{q} + (a - b) = r$

$x = 0 \Rightarrow z = r + c - q(a - b)$

we take remain of the line

$x \le a, y \ge b \Rightarrow \quad x + y \prec b + a$ in the domain

$z = 0 \Rightarrow = c + q(a - x) + q(b - y) = r$

$x = 0 \Rightarrow y = \frac{r}{q} + \frac{c}{q} - (a - b)$

$y = 0 \Rightarrow x = -\frac{r}{q} - \frac{c}{q} + (a - b)$

we take remain of the line

3) If $x \le a$, $y \ge b$, $z \le c$ then , $(c - z) + q(a - x) + q(y - b) = r$

$x = y = 0 \Rightarrow (c - z) + qa - qb = r \Rightarrow z = -r + c + q(a - b)$

$x = z = 0 \Rightarrow c + qa + q(y - b) = r \Rightarrow y = \frac{r}{q} - \frac{c}{q} - (a - b)$

$y = z = 0 \Rightarrow c + q(a - x) - qb = r \Rightarrow x = -\frac{r}{q} + \frac{c}{q} + (a - b)$

$y \ge b, z \le c \Rightarrow \quad z + y \prec b + c$ in the domain

$x = 0 \Rightarrow (c - z) + qa + q(y - b) = r$

$z = 0 \Rightarrow y = \frac{r}{q} - \frac{c}{q} - (a - b)$

$y = 0 \Rightarrow z = -r + c - q(a - b)$

we take remain of the line

$x \le a, z \le c \Rightarrow \quad z - x < c - a$ in the domain

$y = 0 \Rightarrow c - z + q(a - x) + qb = r$

$z = 0 \Rightarrow y = -\frac{r}{q} + \frac{c}{q} + (a - b)$

$x = 0 \Rightarrow z = -r + c + q(a - b)$

we take remain of the line

$x \le a, y \ge b \Rightarrow \quad y + x < b + a$ in the domain

$z = 0 \Rightarrow c + q(a - x) + q(y - b) = r$

$$x = 0 \Rightarrow y = \frac{r}{q} - \frac{c}{q} - (a - b)$$

$$y = 0 \Rightarrow x = -\frac{r}{q} + \frac{c}{q} + (a + b)$$

we take remain of the line

4) If x≤a, y≤b, z≥c then (z-c)+q(a-x)+q(b-y)=r

x=y=0⇒(z-c)+qa-qb=r⇒ $z = r + c - q(a + b)$

x=z=0⇒c+qa+q(b-y)=r⇒ $y = -\frac{r}{q} - \frac{c}{q} + (a + b)$

y=z=0⇒c+q(a-x)+qb=r⇒ $x = -\frac{r}{q} - \frac{c}{q} + (a + b)$

y≤b, z≥c then z-y<c-b in the domain

x=0⇒(z-c)+qa+q(b-y)=r

z=0 ⇒ $y = -\frac{r}{q} - \frac{c}{q} + (a - b)$

y=0 ⇒ $z = r + c - q(a + b)$

we take remain of the line

x≤a, z≥c → z+x>c+a in the domain

y=0⇒(z-c)+q(a-x)+qb=r

z=0 için $x = -\frac{r}{q} - \frac{c}{q} + (a + b)$

x=0 için $z = -r + c + q(a - b)$

we take remain of the line

x≤a, y≤b ⇒ x-y<a-b in the domain

z=0⇒c+q(a-x)+q(b-y)=r

x=0 $y = -\frac{r}{q} - \frac{c}{q} + (a + b)$

y=0 $x = -\frac{r}{q} - \frac{c}{q} + (a + b)$

we take remain of the line

5)If x≥a, y≤b, z≤c then (c-z)+q(x-a)+q(b-y)=r

x=y=0⇒(c-z)-qa+qb=r⇒ $z = -r + c - q(a - b)$

x=z=0⇒c-qa+q(b-y)=r⇒ $y = -\frac{r}{q} + \frac{c}{q} - (a - b)$

y=z=0c+q(x-a)+qb=r$\Rightarrow x=\frac{r}{q}-\frac{c}{q}+(a-b)$

y≤b, z≤c ⇒ z-y<c-b in the domain

x=0⇒(c-z)-qa+q(b-y)=r

z=0 ⇒ $y=-\frac{r}{q}+\frac{c}{q}-(a-b)$

y=0 ⇒ $z=-r+c-q(a-b)$

we take remain of the line

x≥a, z≤c ⇒ z+x<c+a in the domain

y=0⇒ (c-z)+q(x-a)+qb=r

z=0 ⇒ $x=\frac{r}{q}-\frac{c}{q}+(a-b)$

x=0 ⇒ $z=-r+c-q(a-b)$

we take remain of the line

x≥a, y≤b ⇒ y+x<a+b in the domain

z=0⇒c+q(x-a)+q(b-y)=r

x=0 ⇒ $y=-\frac{r}{q}+\frac{c}{q}-(a-b)$

y=0 ⇒ $x=\frac{r}{q}-\frac{c}{q}+(a-b)$

we take remain of the line

6) If x≥a, y≥b, z≤c then (c-z)+q(x-a)+q(y-b)=r

x=y=0⇒(c-z)-qa-qb=r$\Rightarrow z=-r+c-q(a+b)$

x=z=0⇒c-qa+q(y-b)=r$\Rightarrow y=\frac{r}{q}-\frac{c}{q}+(a+b)$

y=z=0⇒c+q(x-a)-qb=r$\Rightarrow x=\frac{r}{q}-\frac{c}{q}+(a+b)$

y≥b, z≤c ⇒ z+y<c+b in the domain

x=0⇒ (c-z)-qa+q(y-b)=r

z=0 ⇒ $y=\frac{r}{q}-\frac{c}{q}+(a+b)$

y=0⇒ $z=-r+c-q(a+b)$

we take remain of the line

x≥a, z≤c ⇒ z+x<a+c in the domain

y=0$\Rightarrow$ (c-z)+q(x-a)-qb=r

z=0 için $x = \frac{r}{q} - \frac{c}{q} + (a+b)$

x=0 için $z = -r + c - q(a+b)$.

we take remain of the line

x≥a, y≥b iken x-y>a-b in the domain

z=0$\Rightarrow$c+q(x-a)+q(y-b)=r

x=0 $\Rightarrow$ $y = \frac{r}{q} - \frac{c}{q} + (a+b)$

y=0 $\Rightarrow$ $x = \frac{r}{q} - \frac{c}{q} + (a+b)$

we take remain of the line

7) If x≥a, y≤b, z≥c then (z-c)+q(x-a)+q(b-y)=r

x=y=0$\Rightarrow$(z-c)-qa+qb=r$\Rightarrow$ $z = r + c + q(a-b)$

x=z=0$\Rightarrow$c-qa+q(b-y)=r$\Rightarrow$ $y = -\frac{r}{q} - \frac{c}{q} - (a-b)$

y=z=0$\Rightarrow$ +q(x-a)+qb=r$\Rightarrow$ $x = \frac{r}{q} + \frac{c}{q} + (a-b)$

y≤b, z≥c $\Rightarrow$ y+z>b+c in the domain

x=0$\Rightarrow$ (z-c)-qa+q(b-y)=r

z=0 $\Rightarrow$ $y = -\frac{r}{q} - \frac{c}{q} - (a-b)$

y=0 $\Rightarrow$ $z = r + c + q(a-b)$

we take remain of the line

x≥a, z≥c $\Rightarrow$ z-x>c-a in the domain

y=0$\Rightarrow$ (z-c)+q(x-a)+qb=r

z=0 için $x = \frac{r}{q} + \frac{c}{q} + (a-b)$

x=0 için $z = r + c + q(a-b)$

we take remain of the line

$x \geq a$, y≤b $\Rightarrow$ x+y>a+b in the domain

z=0$\Rightarrow$c+q(x-a)+q(b-y)=r

x=0 $\Rightarrow$ $y = -\frac{r}{q} - \frac{c}{q} - (a-b)$

y=0 $\Rightarrow$ $x = \frac{r}{q} + \frac{c}{q} + (a - b)$

we take remain of the line

8)If $x \geq a$, $y \geq b$, $z \geq c$ then (z-c)+q(x-a)+q(y-b)=r

x=y=0$\Rightarrow$(z-c)-qa-qb=r$\Rightarrow$ $z = r + c + q(a + b)$

x=z=0$\Rightarrow$c-qa+q(y-b)=r$\Rightarrow$ $y = \frac{r}{q} + \frac{c}{q} + (a + b)$

y=z=0$\Rightarrow$c+q(x-a)-qb=r$\Rightarrow$ $x = \frac{r}{q} + \frac{c}{q} + (a + b)$

$y \geq b$, $z \geq c$ $\Rightarrow$ z-y>c-b in the domain

x=0 $\Rightarrow$(z-c)-qa+q(y-b)=r

z=0 $\Rightarrow$ $y = \frac{r}{q} + \frac{c}{q} + (a + b)$

y=0$\Rightarrow$ $z = r + c + q(a + b)1$

we take remain of the line

$x \geq a$, $z \geq c$ $\Rightarrow$ z-x>c-a in the domain

y=0$\Rightarrow$ (z-c)+q(x-a)-qb=r

z=0 $\Rightarrow$ $x = \frac{r}{q} + \frac{c}{q} + (a + b)$

x=0 $\Rightarrow$ $z = r + c + q(a + b)$

we take remain of the line

$x \geq a$, $y \geq b$ $\Rightarrow$ x-y>a-b in the domain

z=0$\Rightarrow$c+q(x-a)+q(y-b)=r

x=0 $\Rightarrow$ $y = \frac{r}{q} + \frac{c}{q} + (a + b)$

y=0 $\Rightarrow$ $x = \frac{r}{q} + \frac{c}{q} + (a + b)$

we take remain of the line

Example 3.1 CC-sphere with a radius r = 1 and centered at M = (0,0,0)

$C^2 = \{X = (x, y, z) : d_L(M, X) + q d_S(M, X) = 1\}$,

$$\max\{|x|, |y|, |z|\} + q \min\{|x| + |y|, |x| + |z|, |y| + |z|\} = 1$$

In thjs example the CC-sphere can be construted by using the theorem3.1. Graph of a CC- sphere can be easily drawn if it is represented by an equation of the form given in theorem 3.1. Graph of some of the CC-sphere are given figure.

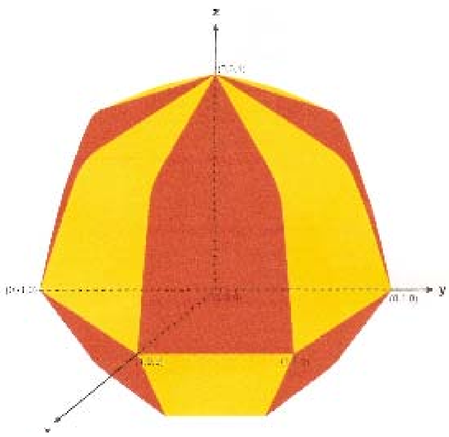

Figure 1. CC sphere with a radius of r = 1 and centered at M = (0,0,0) (scale model)

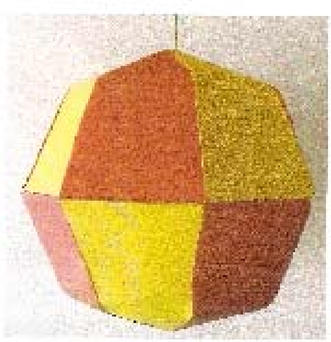

Figure2. Changed aspect of the CC-sphere. (scala model)

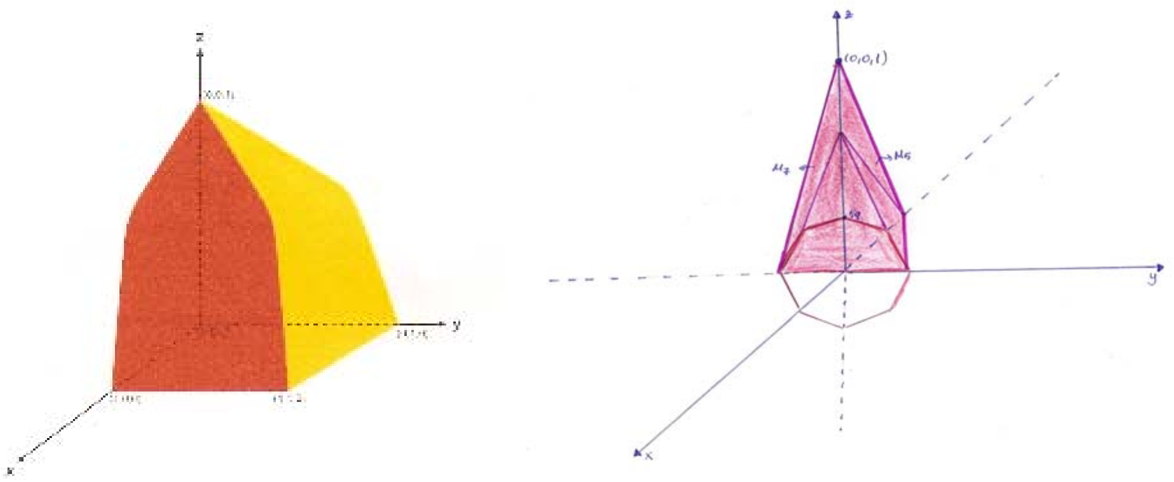

Figure 3. CC sphere with a radius of r = 1 and centered at M = (0,0,0). (the shape of CC- sphere one in eight)

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