Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2024.00145

Research Article

On solutions of time fractional order random HIV/AIDS modelling

Mehmet MERDAN[1,*], Selami ÖKTEM[1]

[1]*Department of Mathematical Engineering, Faculty of Engineering and Natural Science, Gümüşhane University, Gümüşhane, 29100, Türkiye*

ARTICLE INFO

Article history
Received: 02 October 2023
Revised: 25 November 2023
Accepted: 23 December 2023

Keywords:
Fractional HIV/AIDS Mode; Expected Value; Variance; Confidence Intervals

ABSTRACT

In this study, The fractional random HIV/AIDS model approximate analytical solutions were produced using the differential transformation method. The approximate analytical solution of the fractional order Random HIV/AIDS model was obtained with the help of the differential transformation method. For the fractional random HIV/AIDS model, which was created by choosing the initial conditions from the exponential, beta, and normal distributions, graphic simulations of the expected value, variance, and confidence intervals of the most commonly used probability characteristics were obtained with the help of the MATLAB package program. Results obtained are interpreted.

Cite this article as: Merdan M, Öktem S. On solutions of time fractional order random HIV/AIDS modelling. Sigma J Eng Nat Sci 2024;42(6):1899–1906.

INTRODUCTION

AIDS was first discovered in the USA in 1981. HIV (human immunodeficiency virus) factor causes AIDS by crashing the immune system in humans. The HIV virus, known to have emerged for the first time in 1960, was first seen in monkeys. In 2017, 940,000 HIV-related deaths and 1.8 million new cases of HIV infection were detected. As of the year 2022, over 37.7 million individuals have been infected with HIV [1]. There are two different variations of HIV. These are HIV-1, which is common worldwide, and HIV-2, which is more common in African countries. HIV-1 was first isolated in 1983 in Paris, France (Anderson, 1990). It is assumed that HIV-1 is transmitted to humans by a minimum of 4 zoonotic strains. It is estimated that this contamination may have occurred in the 1930s (±20 years) in the light of the molecular phylogenetic biological information available [2]. HIV-2 isolation was done by Clavel et al. in 1986 [3]. HIV-2 is less pathogenic than HIV-1. As a result of this situation, a longer prognosis is observed in the cases. While it is observed that immunodeficiency symptoms and AIDS occur later, the mother-infant transmission rate is much lower (2-7%) compared to HIV-1 (10-40%) [4].

HIV often compromised an individual's immune system, leading to the eventual development of acquired immune deficiency syndrome [5]. This was particularly true if the individual was unaware of their protection options and was not receiving active treatment for AIDS. AIDS continues to threaten our lives because no cure has yet been found. Thus, our greatest mathematical advantage in this field comes from knowing the dynamics of the epidemic, which is crucial [6]. In addition to understanding the dynamics of the epidemic, these studies also enable us

*Corresponding author.
*E-mail address: mehmetmerdan@gmail.com

This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkilic

to manage this process more efficiently and to make predictions about the disease [7].

In our study, fractional order random HIV/AIDS model was examined. Three distinct absolute continuous distributions were used to choose the initial conditions, and the probability characteristics of each were looked at. In our daily lives, stochastic differential equations are becoming more and more popular. They have applications in population dynamics, engineering, finance, and economics [8].

Functions for the expected values and variances of approximate analytical solutions of random equations are obtained.For the remainder of this investigation, part 3 of the fractional random HIV/AIDS model's solutions and behaviors, as well as the materials and methods from part 2 of the study, are both explored. The conclusion is presented in the final section.

MATERIALS AND METHODS

The concept of a fractional derivative was first introduced in the latter half of the 17th century and was developed by Leibniz and Newton. As a matter of fact, the concept of a fractional derivative predates that of an integer derivative. Due to the fact that the fractional derivative is really just an expanded version of the integer derivative. The Rieman-Liouville and Caputo definitions are the ones that are most frequently discussed in the literature [9].

For more than 300 years, the fractional derivative has drawn interest in mathematics. On this topic, scientists have conducted a number of investigations. Due to this circumstance, various definitions have occasionally emerged [10].

The fractional derivative has major advantages over the integer derivative in applied fields such as medicine, biology, and bioengineering. These advantages, which are supported by a number of models, not only improve the convenience of our daily lives but also present persuasive arguments for solutions to issues such as the spread of diseases [11]. Fractional differential equations in mathematical models have been more and more common in recent years due to their advantages. However, the problem's structure also influences the analytical solution of these equations [12].

Preliminaries

Definition 1. The Riemann-Liouville fractional derivative of order $0 < \alpha < 1$ of a function is defined as [13]

$$ {}^{RL}_{0}D^{\alpha}_{x} f(x) = \frac{1}{\Gamma(\alpha-1)} \frac{d}{dx} \int_0^x (x-t)^{-\alpha} f(t)dt \qquad (1) $$

Definition 2. The Riemann-Louville fractional integral of order $0 < \alpha < 1$ of a function is defined as [13]

$$ {}^{RL}_{0}I^{\alpha}_{x} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt \qquad (2) $$

where Γ(.) represents the Gamma function.

Definition 3 [14-15] The Caputo fractional derivative of order $\alpha > 0$ of a function $\omega \in C^{\mu}_{-1}(\mu \geq 1)$ in the sense of Caputo is defined as

$$ {}^{C}_{0}D^{\alpha}_{x} f(x) = \frac{1}{\Gamma(\mathrm{m}-\alpha)} \int_0^t (t-\vartheta)^{m-\alpha-1} \omega^m(\vartheta)d\vartheta \qquad (3) $$

where m - $1 < \alpha < m \in \mathbb{N}$. If $\alpha = m \in \mathbb{N}$, then

$$ D^{\alpha}_{t}\omega(t) = \frac{d^m\omega(t)}{dt^m}. $$

Definition 4: The Laplace transform of function is defined as

$$ \mathcal{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t)dt $$

for all real numbers $t \geq 0$ if the integral exists.

Definition 5. [14-15] The Laplace transform of the transform of the Caputo fractional derivative $D^{\alpha}_{t}\omega(t)$ is defined as

$$ \mathcal{L}[D^{\alpha}_{t}\omega(t)] = s^{\alpha}\mathcal{L}[\omega(t)](s) - \sum_{k=0}^{m-1} s^{\alpha-k-1}\omega^k(0^+) \qquad (4) $$

where $\alpha > 0$, m - $1 < \alpha < m$ and $\mathcal{L}$ denotes the Laplace transform operator.

DIFFERENTIAL TRANSFORM METHOD

The basic definitions of differential transform are introduced as follows. Let $u(t)$ be analytic in a domain D and let $t = t_0$ represent any point in D. The function $u(t)$ is then represented by one power series whose center is located at t_0. The differential transform of the k-th derivative of a function $u(t)$ is defined as follows:

$$ \mathcal{U}(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=t_0}, \forall t \in D $$

In (1), $u(t)$ is the original function and $U(k)$ is the transformed function[16]. The differential inverse transformation of $U(k)$ is defined as follows:

$$ u(t) = \sum_{k=0}^{\infty} \mathcal{U}(k)(t-t_i)^k, \forall t \in D $$

from (1) and (2), we obtain

$$ u(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=t_0}, \forall t \in D $$

The fundamental theorems of the one-dimensional differential transform[16-18] are:

Theorem 1. If $z(t) = u(t) \pm v(t)$, then $Z(k) = U(k) \pm V(k)$.

Theorem 2. If $z(t) = cv(t)$, then $Z(k) = cV(k)$, where c is constant.

Theorem 3. If $z(t) = \frac{dv(t)}{dt}$, then $Z(k) = (k+1)V(k+1)$.

Theorem 4. If $z(t) = \frac{d^n v(t)}{dt}$, then $\mathcal{Z}(k) = \frac{(k+n)!}{k!}\mathcal{V}(k+n)$.

Theorem 5. If $z(t) = u(t)v(t)$, then $\mathcal{Z}(k) = \sum_{r=0}^{k} \mathcal{U}(r)\mathcal{V}(k-r)$.

Theorem 6. If $z(t) = t^n$, then $\mathcal{Z}(k) = \delta(k-n) = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$.

In real applications, the function $u(t)$ is expressed by a finite series and (3) can be written as

$$u(t) = \sum_{k=0}^{N} \mathcal{U}(k)(t-t_i)^k, \forall t \in D$$

Equation (4) implies that $\sum_{k=N+1}^{\infty} \mathcal{U}(k)(t-t_i)^k$ is negligibly small.

Fractional Time Derived (Random) HIV/AIDS Model

This section presents the results of solving the random fractional order HIV/AIDS modeling using the differential transformation method (DTM) and provides examples of the variances, confidence intervals, and expected values of various probability distributions of these solutions [19-21]. There are now initial conditions that include randomly effective terms with various probability distributions. In recent years, the mean square calculation has been used to solve a few first-order random differential equations and models [22-32].

The deterministic form of the nonlinear HIV/AIDS model [19] is following:

$$\begin{aligned} D_t^{q_1} S(t) &= \mu - \beta S(t)I(t) - \mu S(t) \\ D_t^{q_2} I(t) &= \beta S(t)I(t) - \mu I(t) - \delta I(t) \end{aligned} \quad (5)$$

$$\begin{aligned} D_t^{q_3} A(t) &= \delta I(t) - \mu A(t) - dA(t) \\ S(t_0) &= S_0, I(t_0) = I_0, A(t_0) = A_0. \end{aligned} \quad (6)$$

In this model, those infected are denoted by $l(t)$. Those who are infected can transmit the disease to susceptible persons indicated by $S(t)$. Also, when infected persons in our model are sick, they are denoted by $A(t)$. The parameters of our model are μ, β, δ and d and take values in the range of $[0, \infty)$. The Table 1 below displays the values and descriptions for these parameters.

If DTM is applied with the fractional HIV/AIDS model given in (5) and the initial conditions given in (6),

Table 1. Parameter descriptions and values [19]

Parameter	Explanation	Value
μ	Birth and natural death rate	2
β	The rate of transmission of the infection	1.5
δ	Rate of infected individuals having AIDS	4
d	Death rate from AIDS disease	1

$$\begin{aligned} S(k+1) &= \frac{\Gamma(q_1(k+1))\left(\mu\delta(k) - \beta\sum(S(r)I(k-r), r=0..k) - \mu S(k)\right)}{\Gamma(q_1(k+1)+1)} \\ I(k+1) &= \frac{\Gamma(q_2(k+1))\left(\beta\sum(S(r)Y(k-r), r=0..k) - (\mu+\delta)I(k)\right)}{\Gamma(q_2(k+1)+1)} \\ A(k+1) &= \frac{\Gamma(q_3(k+1))\left(\delta I(k) - (\mu+d)A(k)\right)}{\Gamma(q_3(k+1)+1)} \end{aligned} \quad (7)$$

The fractional derivative of our model as $q_1 = \frac{9}{10}$, $q_2 = \frac{9}{10}$, $q_3 = \frac{9}{10}$

$$\begin{aligned} S(t) &= S_0 + 8.318033072t^{\frac{9}{10}} - 1.559631201t^{\frac{9}{10}}S_0\, I_0 \\ &\quad - 2.079508268t^{\frac{9}{10}}S_0 - 4.954320892t^{\frac{9}{5}} - 1.342089092\, t^{\frac{9}{5}}(S_0)^2\, I_0 \\ &\quad + 8.947260614\, t^{\frac{9}{5}} S_0\, I_0 - 7.157808491t^{\frac{9}{5}} I_0 \\ &\quad + 1.342089092\, t^{\frac{9}{5}} S_0\, (I_0)^2 + 2.385936164t^{\frac{9}{5}} S_0 \\ I(t) &= I_0 + 1.559631201\, t^{\frac{9}{10}} S_0\, I_0 - 6.238524804\, t^{\frac{9}{10}} I_0 \\ &\quad + 1.342089092\, t^{\frac{9}{5}} (S_0)^2\, I_0 - 12.52616486t^{\frac{9}{5}} S_0\, I_0 \\ &\quad + 28.63123396\, t^{\frac{9}{5}} I_0 - 1.342089092\, t^{\frac{9}{5}} S_0\, (I_0)^2 \\ A(t) &= A_0 + 4.159016536\, t^{\frac{9}{10}} I_0 - 3.119262402\, t^{\frac{9}{10}} A_0 \\ &\quad + 3.578904245\, t^{\frac{9}{5}} S_0\, I_0 - 21.47342548t^{\frac{9}{5}} I_0 + 5.368356368t^{\frac{9}{5}} A_0 \end{aligned} \quad (8)$$

is in the form above.

The initial conditions in our model are $S_0 \sim Exp(\lambda = 2)$ exponential distribution, $I_0 \sim N(\mu = 3, \sigma^2 = 9)$ normal distribution and $A_0 \sim \text{Beta}(k = 3, l = 3)$ beta distribution, chosen from three different absolute continuous distributions.

The parameters of the normally distributed random variable X are $X \sim N(\mu, \sigma^2)$. Using the moment generating function of the normal distribution, we get

$$M_X(t) = E[e^{tX}] = e^{\frac{1}{2}\sigma^2 t^2 + \mu t}$$

from (17), the 1st and 2nd moment of the random variable $X \sim N(\mu, \sigma^2)$ are,

$$E[X] = \mu, \quad E[X^2] = \sigma^2 + \mu^2,$$

is calculated as. If the basic properties of the expected value for the X and Y independent random variables are used, the expected value of equation (16) is

$$f(y) = \frac{1}{B(k,l)} y^{k-1}(1-y)^{l-1}, y \in (0,1), k > 0, l > 0$$

where Y is a random variable.

If there is a probability density function of the form, it has a standard Beta distribution and is denoted by Y~Beta(k, l). Expected value for this distribution

$$E(Y) = \frac{k}{k+l}$$

and variance

$$Var(Y) = \frac{kl}{(k+l)^2(k+l+1)}$$

is in the form above.

The moment generating function of the exponential distribution;

$$\mathrm{M(t)} = \frac{\lambda}{\lambda - t};\ t < \lambda$$

is indicated by $X\sim(\lambda)$. Expected value and variance[20]:

$$\mathrm{E(X)} = \frac{1}{\lambda}$$

$$Var(X) = \frac{1}{\lambda^2}.$$

$$\begin{aligned} E(S(t)) = {} & E(S_0) + 8.318033072t^{\frac{9}{10}} - 1.559631201t^{\frac{9}{10}}E(S_0)E(I_0) \\ & - 2.079508268t^{\frac{9}{10}}E(S_0) - 4.954320892t^{\frac{9}{5}} \\ & - 1.342089092\,t^{\frac{9}{5}}E((S_0)^2)\,E(I_0) + 8.947260614\,t^{\frac{9}{5}}E(S_0)\,E(I_0) \\ & - 7.157808491t^{\frac{9}{5}}E(I_0) + 1.342089092\,t^{\frac{9}{5}}E(S_0)\,E((I_0)^2) \\ & + 2.385936164t^{\frac{9}{5}}E(S_0) \end{aligned} \quad (9)$$

$$E(S(t)) = 2 - 5.198770670t^{\frac{9}{10}} + 78.32976332t^{\frac{9}{5}} \quad (10)$$

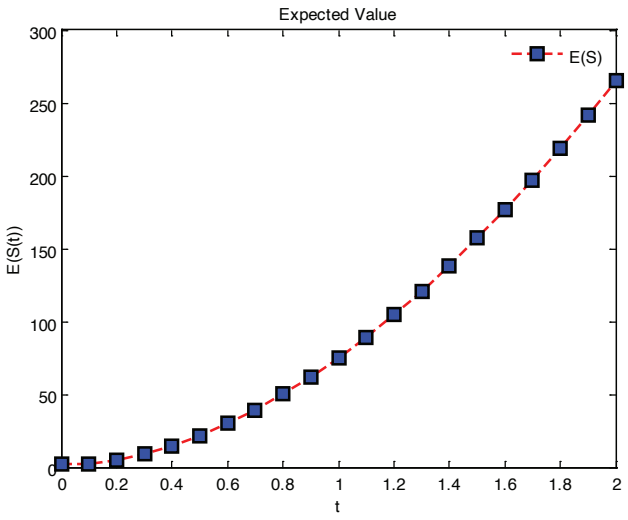

Figure 1. Time-dependent variation of the expected value of individuals susceptible to the virus.

The expectations can be compared to the deterministic results of equation (1) above (Figure 1) in a single graph (Figure 1). The following are the maximum and minimum values of the expected values of the random variables: $S(t)$ has a maximum value of 265.1 when $t = 2$ and a minimum value of 2 when $t = 0$.

$$\begin{aligned} E(I(t)) = {} & E(I_0) + 1.559631201\,t^{\frac{9}{10}}E(S_0)\,E(I_0) - 6.238524804\,t^{\frac{9}{10}}E(I_0) \\ & + 1.342089092\,t^{\frac{9}{5}}E(S_0)^2\,E(I_0) - 12.52616486t^{\frac{9}{5}}E(S_0)E(I_0) \\ & + 28.63123396\,t^{\frac{9}{5}}E(I_0) - 1.342089092\,t^{\frac{9}{5}}E(S_0)\,E((I_0)^2) \end{aligned} \quad (11)$$

$$E(I(t)) = 3 - 9.357787204t^{\frac{9}{10}} - 35.56536095t^{\frac{9}{5}} \quad (12)$$

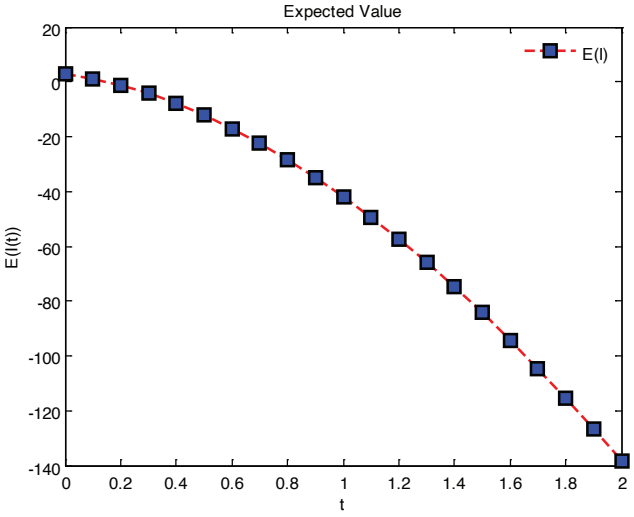

Figure 2. Time-dependent variation of the expected value of infected individuals.

For comparison with the deterministic outcomes of the equation above, expectations can be presented in a single graph (Figure 2). The following methods are used to determine the maximum and minimum values of the expected values of the random variables: At $t = 0$ and $t = 2$, respectively, $I(t)$ takes the maximum value of 3 and the minimum value of -138.3.

$$\begin{aligned} E(A(t)) = {} & E(A_0) + 4.159016536\,t^{\frac{9}{10}}E(I_0) - 3.119262402\,t^{\frac{9}{10}}E(A_0) \\ & + 3.578904245\,t^{\frac{9}{5}}E(S_0)\,E(I_0) - 21.47342548t^{\frac{9}{5}}E(I_0) \\ & + 5.368356368t^{\frac{9}{5}}E(A_0) \end{aligned} \quad (13)$$

$$E(A(t)) = \frac{2}{5} + 11.22934465t^{\frac{9}{10}} - 40.79950842t^{\frac{9}{5}} \quad (14)$$

The expectations can be presented in a single graph (Figure 3) for comparison with the deterministic outcomes of equation (1) above (Figure 3). The following methods

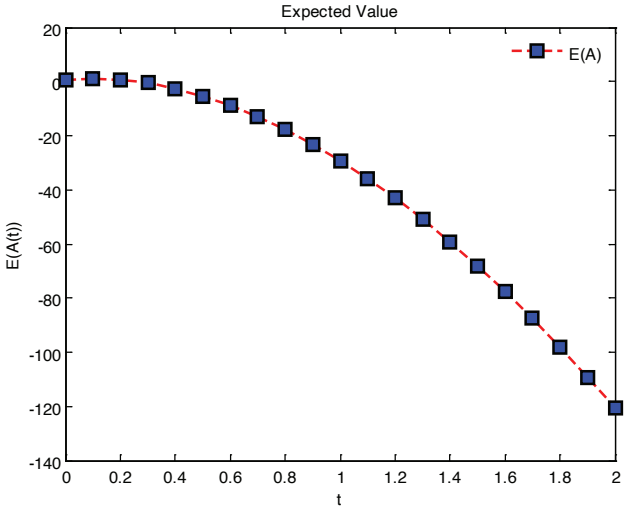

Figure 3. Time-dependent variation of the expected value of sick individuals.

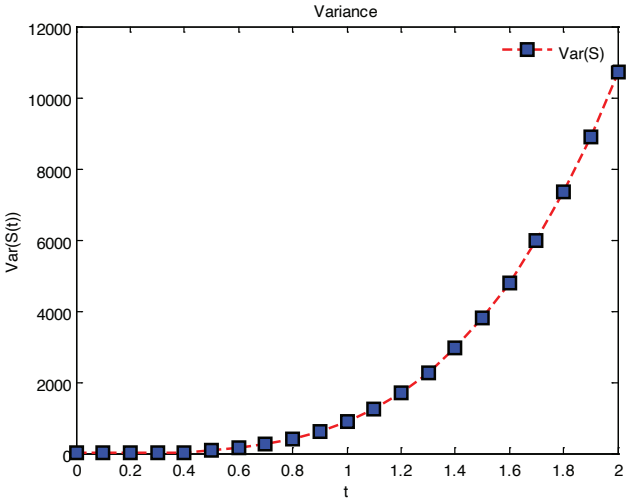

Figure 4. Time-dependent variation of the variance value of individuals susceptible to the virus.

are used to determine the maximum and minimum values of the expected values of the random variables: At $t = 0$ and $t = 2$, respectively, $A(t)$ takes the maximum value of 1.167 and the minimum value of -120.7.

In our model, variance values were calculated according to the values and distributions given above.

$$\begin{aligned} Var(S(t)) = Var(\, & S_0 + 8.318033072t^{\frac{9}{10}} - 1.559631201t^{\frac{9}{10}}S_0\, I_0 \\ & - 2.079508268t^{\frac{9}{10}}S_0 - 4.954320892t^{\frac{9}{5}} \\ & - 1.342089092\, t^{\frac{9}{5}}\,(S_0)^2\, I_0 + 8.947260614\, t^{\frac{9}{5}}\, S_0\, I_0 \\ & - 7.157808491t^{\frac{9}{5}}\, I_0 + 1.342089092\, t^{\frac{9}{5}}\, S_0\, (I_0)^2 \\ & + 2.385936164t^{\frac{9}{5}}\, S_0\,) \end{aligned} \tag{15}$$

$$\begin{aligned} Var(S(t)) = {} & Var(S_0) + \left(1.559631201t^{\frac{9}{10}}\right)^2 Var(S_0)Var(I_0) \\ & + (2.079508268t^{\frac{9}{10}})^2\, Var(S_0) + (1.342089092\, t^{\frac{9}{5}})^2 Var((S_0)^2)\, Var(I_0) \\ & + (8.947260614\, t^{\frac{9}{5}})^2 Var(S_0)\, Var(I_0) + (7.157808491t^{\frac{9}{5}})^2 Var(I_0) \\ & + (1.342089092t^{\frac{9}{5}})^2 Var(S_0)\, Var((I_0)^2) + (2.385936164t^{\frac{9}{5}})^2\, Var(S_0) \end{aligned} \tag{16}$$

$$Var(S(t)) = \frac{1}{4} + 6.554099996t^{\frac{9}{5}} + 881.7612031t^{\frac{18}{5}} \tag{17}$$

The $S(t)$ variances are shown above (Figure 4). The following is how the extreme variances of the random variables are obtained: Maximum max[Var(S(t))] = 10720 at time $t = 2$ and minimum min[Var(S(t))] = 0.25 at time $t = 0$.

$$\begin{aligned} Var(I(t)) = {} & Var(I_0 + 1.559631201\, t^{\frac{9}{10}}S_0\, I_0 - 6.238524804\, t^{\frac{9}{10}}\, I_0 \\ & + 1.342089092\, t^{\frac{9}{5}}\,(S_0)^2\, I_0 - 12.52616486t^{\frac{9}{5}}\, S_0\, I_0 \\ & + 28.63123396\, t^{\frac{9}{5}}\, I_0 - 1.342089092\, t^{\frac{9}{5}}\, S_0\, (I_0)^2) \end{aligned} \tag{18}$$

$$\begin{aligned} Var(I(t)) = {} & Var(I_0) + (1.559631201t^{\frac{9}{10}})^2\, Var(S_0)\, Var(I_0) \\ & + (6.238524804\, t^{\frac{9}{10}})^2\, Var(I_0) + (1.342089092\, t^{\frac{9}{5}})^2\, Var(S_0)^2\, Var(I_0) \\ & + (12.52616486t^{\frac{9}{5}})^2\, Var(S_0)Var(I_0) + (28.63123396\, t^{\frac{9}{5}})^2\, Var(I_0) \\ & + (1.342089092\, t^{\frac{9}{5}})^2\, Var(S_0)\, Var((I_0)^2) \end{aligned} \tag{19}$$

$$Var(I(t)) = 9 + 352.0970627t^{\frac{9}{5}} + 7969.873553t^{\frac{18}{5}} \tag{20}$$

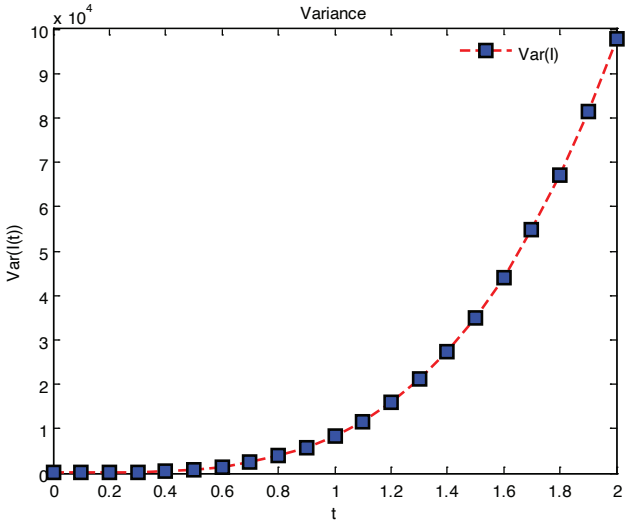

Figure 5. Time-dependent variation of the variance value of infected individuals.

The variances of $I(t)$ is given above (Figure 5). Extremum values of the variances of the random variables are obtained as follows: min[Var(I(t))] = 2 at $t = 0$ and max[Var(I(t))] = 9788 at $t = 2$.

$$\begin{aligned} Var(A(t)) = {} & Var(A_0 + 4.159016536\, t^{\frac{9}{10}}\, I_0 - 3.119262402\, t^{\frac{9}{10}}\, A_0 \\ & + 3.578904245\, t^{\frac{9}{5}}\, S_0\, I_0 - 21.47342548t^{\frac{9}{5}}\, I_0 \\ & + 5.368356368t^{\frac{9}{5}}\, A_0\,) \end{aligned} \tag{21}$$

$$Var(A(t)) = Var(A_0) + \left(4.159016536\, t^{\frac{9}{10}}\right)^2 Var(I_0) + (3.119262402\, t^{\frac{9}{10}})^2 Var(A_0) + (3.578904245\, t^{\frac{9}{5}})^2 Var(S_0)\, Var(I_0) + (21.47342548 t^{\frac{9}{5}})^2 Var(I_0) + (5.368356368 t^{\frac{9}{5}})^2 Var(A_0) \quad (22)$$

$$Var(A(t)) = \frac{1}{25} + 156.0659589 t^{\frac{9}{5}} + 4179.944036 t^{\frac{18}{5}} \quad (23)$$

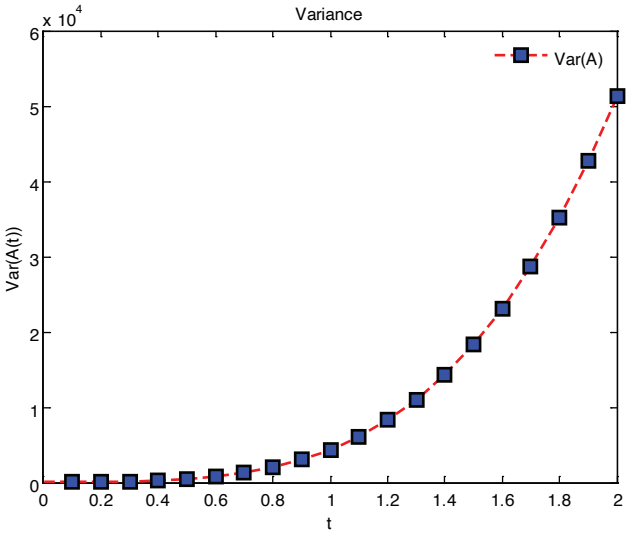

Figure 6. Time-dependent variation of the variance value of sick individuals.

The variances of $A(t)$ is given above (Figure 6). Extremum values of the variances of the random variables are obtained as follows: min[Var(I(t))] = 0.04 at t = 0 and max[Var(I(t))] = 51230 at t = 2.

Confidence intervals for random variable expected values,

$$(E(y(t)) - K.std(y(t)), E(y(t)) + K.std(y(t))$$

is equal, which can be determined using standard deviations. For K = 3, this formula yields a % 99 confidence interval(C.I.) for the expected value of a normally distributed random variable [21]. Figure 7 depicts a % 99 C.I. plotted with MATLAB (2013a).This popular rule, known as the three sigma rule, states that 99.73% of values for a normally distributed variable are within three standard deviations of the mean. Thus, using the appropriate parameters, we will compare the variations of the results for two continuous distributions with limited and unlimited support, respectively. Nearly all potential values for the random effects for both distributions will come from the same range if the parameters are chosen appropriately.

The confidence intervals of $S(t)$ are given in Figure 7. The extremum values of the confidence intervals are as follows: min(E(S(t)) - 3std(S)) = -45.48 at t = 2 and max(E(S(t))

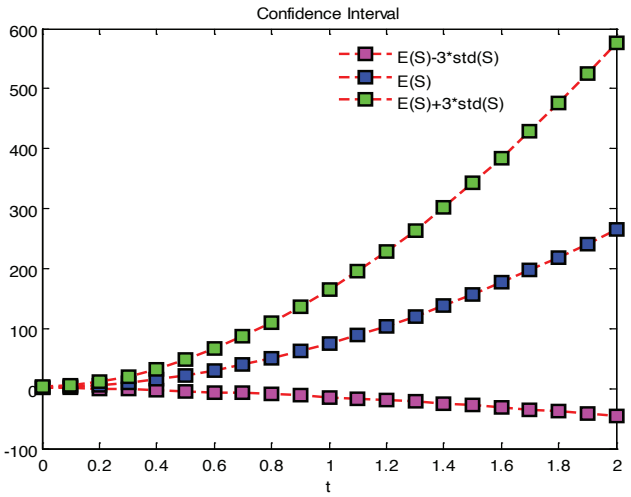

Figure 7. Time-dependent variation of % 99 C.I. intervals of virus-susceptible individuals.

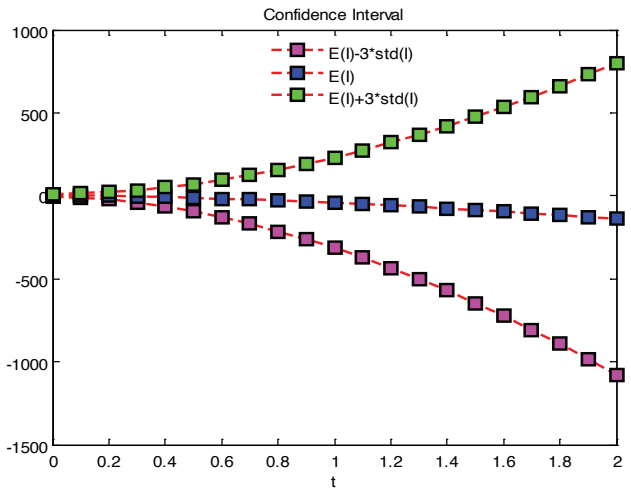

Figure 8. Time-dependent variation of %99 C.I. of infected individuals

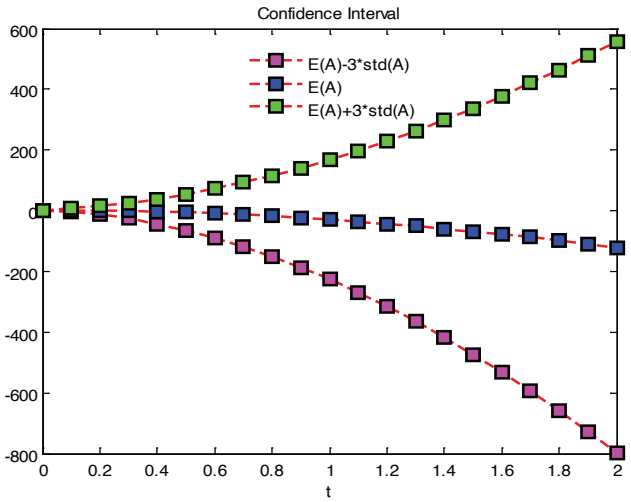

Figure 9. Time-dependent variation of % 99 C.I. of patients.

+ 3std(S)) = 575.6 at $t = 2$. Here, $K = 3$ gives an approximate %99 confidence interval.

The confidence intervals of $l(t)$ are given in Figure 8. The extremum values of the confidence intervals are as follows: min(E(I(t)) - 3std(I)) = -1077 at $t = 2$ and max(E(I(t)) + 3std(I)) = 800.2 at $t = 2$. Here, $K = 3$ gives an approximate %99 confidence interval.

The confidence intervals of $l(t)$ are given in Figure 9. The extremum values of the confidence intervals are as follows: min(E(A(t)) - 3std(A)) = -799.7 at $t = 2$ and max(E(A(t)) + 3std(A)) = 558.3 at $t = 2$. Here, $K = 3$ gives an approximate %99 confidence interval.

CONCLUSION

In this study, random fractional ordinary differential equations were solved using the fractional differential transformation method. The motivation of this study is to determine the probability characteristics of a randomized fractional-order HIV/AIDS Modelling under random effects. Normal, Beta, and exponential distributions are used to select the initial conditions or coefficients of random fractional ordinary differential equations. Expected value, variance, and confidence intervals from probability properties were found and graph for the analysis of random effect are presented accordingly. Examining standard deviations, variations, and confidence intervals for expected values reveals how virus transmission dynamics change over time. By selecting different probability distributions from many epidemic models in the literature, the behavior under random effects can be examined. We believe that this study will be an important component in mathematical modeling studies on the transmission of the HIV/AIDS virus. The results are guaranteed to be useful for the random differential equation system if the deterministic and random analysis results are comparable. The stochastic model can simulate the dynamics of disease transmission just as well as the deterministic model, but it can also yield information about the model's variability, including coefficients of variation and standard deviations. We observe that random expectations are also consistent with the deterministic model's results, which are comparable to the numerical results from the relevant study. Results for the expected values' confidence intervals are also provided by the random model. Any deterministic compartment disease model can be created using this method of creating random models from deterministic models. Using actual data for parameter variations rather than speculative values can improve the results. Research on the stability and ideal management of diverse mathematical models of other illnesses, like COVID-19 and tuberculosis, can also be utilized to explore the dynamics of disease dissemination.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] World Health Organization. HIV. 2022. Available at: https://www.who.int/news-room/factsheets/detail/hiv-aids Last Accessed Date: 19.02.2022.

[2] Worobey M, Gemmel M, Teuwen DE, Haselkorn T, Kunstman K, Bunce M. Direct evidence of extensive diversity of HIV-1 in Kinshasa by 1960. Nature 2008;455:661–664. [CrossRef]

[3] Clavel F, Guétard D, Brun-Vézinet F, Chamaret S, Rey MA, Santos-Ferreira MO, Montagnıer L. Isolation of a new human retrovirus from West African patients with AIDS. Science 1986;233:343–346. [CrossRef]

[4] Burgard M, Jasseron C, Matheron S, Damond F, Hamrene K, Blanche S. Mother-to-child transmission of HIV-2 infection from 1986 to 2007 in the ARNS French Perinatal Cohort EPF-CO1. Clin Infect Dis 2010;51:833–843. [CrossRef]

[5] Xuanpei Z, Wenshuang L, Fengying W, Xuerong M. Dynamics of an HIV/AIDS transmission model with protection awareness and fluctuations. Chaos Solit Fract 2023;169. [CrossRef]

[6] Vyambwera M. Mathematical modelling of the HIV/AIDS epidemic and the eff ect of public health education. Unpublished master's thesis. Cape Town: Western Cape University; 2014.

[7] Kirschner D. Using mathematics to understand HIV ımmune dynamics. Notices Am Math Soc 1996;43:191–202.

[8] Emvudu Y, Bongor D. Mathematical analysis of a HIV/AIDS model with treatment. IEJPAM 2012;4.

[9] Lüleci G. Solution of the initial boundary value problem for the fractional diffusion equation. Unpublished master's thesis. Kocaeli: Kocaeli Üniversitesi; 2019.

[10] Samko SG, Kilbas AA, Marichev OI. Fractional İntegrals and Derivatives Theory and Applications. Yverdon: Gordon ve Breach; 1993.
[11] Miller KS, Ross B. An Introduction to The Fractional Calculus and Fractional Differential Equations. London: John Wiley & Sons; 1993.
[12] İbiş B. Numerical solutions of differential-algebraic equations of fractional order. Unpublished doctoral thesis. İstanbul: Yıldız Technical University; 2011.
[13] Podlubny I. Fractional differential equations. San Diego, USA: Academic Press; 1998.
[14] Kilbas AA, Srivastava HM, Trujillo JJ. Theory and Applications of Fractional Differential Equations. Philadelphia, USA: Elsevier Science; 2006.
[15] Caputo M. Elasticita e Dissipazione. Bolonga: Zanichelli; 1969.
[16] Zhou J. Differential transformation and its applications for electrical circuits. Wuhan, China: Borneo Huazhong University Press; 1986.
[17] Pukhov GE. Computational structure for solving differential equations by Taylor transformations. Cybernt Syst Anal 1978;14:383–390. [CrossRef]
[18] Ayaz F. Solutions of the system of differential equations by differential transform method. Appl Math Comput 2004;147:547–567. [CrossRef]
[19] El-Metwally H, Sohaly MA, Elbaz IM. Stochastic global exponential stability of disease-free equilibrium of HIV/AIDS model. Eur Phys J Plus 2020;135:840. [CrossRef]
[20] Erbaş OS. Probability and Statistics. Ankara: Gazi Kitabevi; 2020.
[21] Feller W. An Introduction to Probability Theory and Its Applications. 3rd ed. New York, USA: John Wiley & Sons; 1968.
[22] Merdan M, Anac H, Bekiryazici Z, Kesemen T. Solving of some random partial differential equations by using differential transformation method and laplace-padé method. J Gumushane Univ Inst Sci Technol 2019;9:108–118.
[23] Merdan M, Şişman Ş. Analysıs of random discrete time logistic model. Sigma J Eng Nat Sci 2020;38:1269–1298.
[24] Merdan M, Altay Ö, Bekiryazici Z. Investigation of the behaviour of volterra ıntegral equations with random effects. J Gumushane Univ Inst Sci Technol 2020;10:205–216. [CrossRef]
[25] Bekiryazıcı Z, Kesemen T, Merdan M, Khaniyev T. Modeling disease transmission dynamics with random data and heavy tailed random effects: The Zika case. TWMS J App Eng Math 2023;13:1272–1286.
[26] Şengül S, Bekiryazıcı Z, Merdan M. Wong-Zakai approximation for stochastic models of smoking, Sigma J Eng Nat Sci 2023;41:958–968. [CrossRef]
[27] Merdan M, Atasoy N. On the solutions of fractional random ordinary differential equations with the Residual power series method, Alexandria Eng J 2023;70:169–177. [CrossRef]
[28] Jamil S, Farman M, Akgül A. Qualitative and quantitative analysis of a fractal fractional HIV/AIDS model, Alexandria Eng J 2023;76:167–177. [CrossRef]
[29] Xu C, Liu Z, Pang Y, Akgül A, Baleanu D. Dynamics of HIV-TB coinfection model using classical and Caputo piecewise operator: A dynamic approach with real data from South-East Asia, European and American regions. Chaos Solit Fract 2022;165:112879. [CrossRef]
[30] Farman M, Akgül A, Tekin MT, Akram MM, Ahmad A, Mahmoud EE, et al. Fractal fractional-order derivative for HIV/AIDS model with Mittag-Leffler kernel. Alexandria Eng J 2022;61:10965–10980. [CrossRef]
[31] Liu X, Ahmad S, Rahman M, Nadeem Y, Akgül A. Analysis of a TB and HIV co-infection model under Mittag-Leffler fractal-fractional derivative. Phys Script 2022;97:054011. [CrossRef]
[32] Ahmad S, Ullah A, Akgül A, De la Sen M. Study of HIV Disease and Its Association with Immune Cells under Nonsingular and Nonlocal Fractal-Fractional Operator, Complexity 2021;2021:1904067. [CrossRef]