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On $\alpha\omega$ -Homeomorphisms in Topological Spaces

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Abstract - A bijection $f:(X, \tau) \rightarrow (Y, \sigma)$ is called $\alpha\omega$ -homeomorphism if f and f^{-1} are $\alpha\omega$ -continuous. Also we introduce new class of maps, namely $\alpha\omega c$ -homeomorphisms which form a subclass of $\alpha\omega$ -homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all $\alpha\omega c$ -homeomorphisms forms a group under the operation composition of maps.

Keywords - $\alpha\omega$ -closed maps, $\alpha\omega^*$ -closed maps and $\alpha\omega$ -open maps, $\alpha\omega^*$ -open maps, $\alpha\omega$ -homeomorphism, $\alpha\omega c$ -homeomorphism.

1 introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. Levine [16] introduced the notion of generalized closed sets. After him different mathematicians worked and studied on different versions of generalized closed sets and related topological properties.

Generalized Homeomorphisms, $wg\alpha$ -homeomorphisms, $rg\alpha$ -homeomorphisms, rps -homeomorphisms and gs and sg homeomorphisms have been introduced and studied by Maki et al. [19], Sakthivel and Uma [25], Vadivel and Vairamanickam [30], Mary and Thangavelu [27], Devi et al. [9] respectively.

We give the definitions of some of them which are used in our present study. In this paper, we introduce the concept of $\alpha\omega$ -homeomorphism and study the relationship between homeomorphisms, $wg\alpha$ -homeomorphisms, $rg\alpha$ -homeomorphisms, rps -homeomorphisms, w -homeomorphisms, g -homeomorphisms and rwg -homeomorphisms. Also we introduce

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new class of maps $\alpha\omega$ -homeomorphisms which form a subclass of $\alpha\omega$ -homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all $\alpha\omega$ -homeomorphisms forms a group under the operation composition of maps.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) always denote topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure of A and the interior of A respectively. $X \setminus A$ or A^c denotes the complement of A in X .

We recall the following definitions and results.

Definition 2.1 A subset A of a topological space (X, τ) is called

- (i) semi-open set [17] if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.
- (ii) pre-open set [21] if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.
- (iii) α -open set [13] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
- (iv) semi-pre open set [2] ($=\beta$ -open[1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$) and a semi-pre closed set ($=\beta$ -closed) if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.
- (v) regular open set [28] if $A = \text{int}(\text{cl}(A))$ and a regular closed set if $A = \text{cl}(\text{int}(A))$.
- (vi) Regular semi open set [8] if there is a regular open set U such that $U \subseteq A \subseteq \text{cl}(U)$.
- (vii) Regular α -open set [31] (briefly, α -open) if there is a regular open set U such that $U \subseteq A \subseteq \alpha\text{cl}(U)$.

Definition 2.2 A subset A of a topological space (X, τ) is called

- (i) regular generalized α -closed set (briefly, $\text{rg}\alpha$ -closed)[31] if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular α -open in X .
- (ii) generalized closed set (briefly g -closed) [16] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (iii) generalized semi-closed set (briefly gs -closed)[4] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (iv) generalized semi pre regular closed (briefly $gspr$ -closed) set [24] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- (v) strongly generalized closed set [24] (briefly, g^* -closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- (vi) α -generalized closed set (briefly αg -closed)[20] if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (vii) ω -closed set [29] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- (viii) weakly generalized closed set (briefly, wg -closed)[23] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (ix) regular weakly generalized closed set (briefly, rwg -closed)[23] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- (x) semi weakly generalized closed set (briefly, swg -closed)[23] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .
- (xi) generalized pre closed (briefly gp -closed) set [18] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

- (xii) regular ω -closed (briefly $r\omega$ -closed) set [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X .
- (xiii) g^* -pre closed (briefly g^*p -closed) [32] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- (xiv) generalized regular closed (briefly gr -closed) set [7] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (xv) regular generalized weak (briefly rgw -closed) set [22] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X .
- (xvi) weak generalized regular- α closed (briefly $wgr\alpha$ -closed) set [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular α -open in X .
- (xvii) regular pre semi-closed (briefly rps -closed) set [26] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rg -open in X .
- (xviii) generalized pre regular weakly closed (briefly $gprw$ -closed) set [15] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X .
- (xix) α -generalized regular closed (briefly αgr -closed) set [33] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- (xx) R^* -closed set [12] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X .
- (xxi) generalized pre regular closed set (briefly gpr -closed) [11] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- (xxii) $\omega\alpha$ -closed set [6] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω -open in X .
- (xxiii) α regular ω -closed (briefly $\alpha r\omega$ -closed) set [37] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is rw -open in X .

The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3 A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- (i) regular-continuous (r -continuous) [3] if $f^{-1}(V)$ is r -closed in X for every closed subset V of Y .
- (ii) Completely-continuous [3] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y .
- (iii) strongly α -continuous [38] if $f^{-1}(V)$ is α -closed in X for every semi-closed subset V of Y .
- (iv) $\alpha r\omega$ -continuous [35] if $f^{-1}(V)$ is $\alpha r\omega$ -closed in X for every closed subset V of Y .
- (v) Strongly-continuous [28] if $f^{-1}(V)$ is Clopen (both open and closed) in X for every subset V of Y .
- (vi) α -continuous [13] if $f^{-1}(V)$ is α -closed in X for every closed subset V of Y .
- (vii) αg -continuous [20] if $f^{-1}(V)$ is αg -closed in X for every closed subset V of Y .
- (viii) wg -continuous [23] if $f^{-1}(V)$ is wg -closed in X for every closed subset V of Y .
- (ix) rwg -continuous [23] if $f^{-1}(V)$ is rwg -closed in X for every closed subset V of Y .
- (x) gs -continuous [4] if $f^{-1}(V)$ is gs -closed in X for every closed subset V of Y .
- (xi) gp -continuous [18] if $f^{-1}(V)$ is gp -closed in X for every closed subset V of Y .
- (xii) gpr -continuous [11] if $f^{-1}(V)$ is gpr -closed in X for every closed subset V of Y .
- (xiii) αgr -continuous [33] if $f^{-1}(V)$ is αgr -closed in X for every closed subset V of Y .
- (xiv) $\omega\alpha$ -continuous [6] if $f^{-1}(V)$ is $\omega\alpha$ -closed in X for every closed subset V of Y .
- (xv) $gspr$ -continuous [24] if $f^{-1}(V)$ is $gspr$ -closed in X for every closed subset V of Y .
- (xvi) g -continuous [6] if $f^{-1}(V)$ is g -closed in X for every closed subset V of Y .
- (xvii) ω -continuous [29] if $f^{-1}(V)$ is ω -closed in X for every closed subset V of Y .
- (xviii) $rg\alpha$ -continuous [31] if $f^{-1}(V)$ is $rg\alpha$ -closed in X for every closed subset V of Y .
- (xix) gr -continuous [7] if $f^{-1}(V)$ is gr -closed in X for every closed subset V of Y .

- (xx) g^*p -continuous [32] if $f^{-1}(V)$ is g^*p -closed in X for every closed subset V of Y .
- (xxi) rps -continuous [26] if $f^{-1}(V)$ is rps -closed in X for every closed subset V of Y .
- (xxii) R^* -continuous [12] if $f^{-1}(V)$ is R^* -closed in X for every closed subset V of Y .
- (xxiii) $gprw$ -continuous [15] if $f^{-1}(V)$ is $gprw$ -closed in X for every closed subset V of Y .
- (xxiv) $wgr\alpha$ -continuous [14] if $f^{-1}(V)$ is $wgr\alpha$ -closed in X for every closed subset V of Y .
- (xxv) swg -continuous [23] if $f^{-1}(V)$ is swg -closed in X for every closed subset V of Y .
- (xxvi) rw -continuous [5] if $f^{-1}(V)$ is rw -closed in X for every closed subset V of Y .
- (xxvii) rgw -continuous [22] if $f^{-1}(V)$ is rgw -closed in X for every closed subset V of Y .

Definition 2.4 A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- (i) α -irresolute [13] if $f^{-1}(V)$ is α -closed in X for every α -closed subset V of Y .
- (ii) irresolute [6] if $f^{-1}(V)$ is semi-closed in X for every semi-closed subset V of Y .
- (iii) contra ω -irresolute [29] if $f^{-1}(V)$ is ω -open in X for every ω -closed subset V of Y .
- (iv) contra irresolute [13] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y .
- (v) contra r -irresolute [3] if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y .
- (vi) rw^* -open (resp rw^* -closed) [5] map if $f(U)$ is rw -open (resp. rw -closed) in Y for every rw -open (resp. rw -closed) subset U of X .
- (vii) contra continuous [10] if $f^{-1}(V)$ is open in X for every closed subset V of Y .

Lemma 2.5 [37]

- i) Every closed (resp. regular-closed, α -closed) set is $\alpha\omega$ -closed set in X .
- ii) Every $\alpha\omega$ -closed set is αg -closed set
- iii) Every $\alpha\omega$ -closed set is αgr -closed (resp. $\omega\alpha$ -closed, gs -closed, $gspr$ -closed, wg -closed, rwg -closed, gp -closed, gpr -closed) set in X

Lemma 2.6 [37] If a subset A of a topological space X and

- i) If A is regular open and $\alpha\omega$ -closed then A is α -closed set in X
- ii) If A is open and αg -closed then A is $\alpha\omega$ -closed set in X
- iii) If A is open and gp -closed then A is $\alpha\omega$ -closed set in X
- iv) If A is regular open and gpr -closed then A is $\alpha\omega$ -closed set in X
- v) If A is open and wg -closed then A is $\alpha\omega$ -closed set in X
- vi) If A is regular open and rwg -closed then A is $\alpha\omega$ -closed set in X
- vii) If A is regular open and αgr -closed then A is $\alpha\omega$ -closed set in X
- viii) If A is ω -open and $\omega\alpha$ -closed then A is $\alpha\omega$ -closed set in X

Lemma 2.7 [37] If a subset A of a topological space X and

- i) If A is semi-open and sg -closed then it is $\alpha\omega$ -closed.
- ii) If A is semi-open and ω -closed then it is $\alpha\omega$ -closed.
- iii) A is $\alpha\omega$ -open iff $U \subseteq \alpha \text{int}(A)$, whenever U is rw -closed and $U \subseteq A$.

Definition 2.8 A topological space (X, τ) is called an α -space if every α -closed subset of X is closed in X .

Definition 2.9 A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- (i) **g -closed** [29] if $f(F)$ is g -closed in (Y, σ) for every closed set F of (X, τ) ,
- (ii) **w -closed** [22] if $f(F)$ is w -closed in (Y, σ) for every closed set F of (X, τ) ,
- (iii) **wg -closed** [23] if $f(F)$ is wg -closed in (Y, σ) for every closed set F of (X, τ) ,
- (iv) **rwg -closed** [23] if $f(F)$ is rwg -closed in (Y, σ) for every closed set F of (X, τ) ,
- (v) **rg -closed** [19] if $f(F)$ is rg -closed in (Y, σ) for every closed set F of (X, τ) ,

- (vi) **gpr-closed** [11] if $f(F)$ is gpr-closed in (Y, σ) for every closed set F of (X, τ) ,
- (vii) **regular closed** [31] if $f(F)$ is closed in (Y, σ) for every regular closed set F of (X, τ) .

Definition 2.10 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- (i) **g-open** [15] if $f(U)$ is g-open in (Y, σ) for every open set U of (X, τ) ,
- (ii) **w-open** [22] if $f(U)$ is w-open in (Y, σ) for every open set U of (X, τ) ,
- (iii) **wg-open** [23] if $f(U)$ is wg-open in (Y, σ) for every open set U of (X, τ) ,
- (iv) **rwg-open** [23] if $f(U)$ is rwg-open in (Y, σ) for every open set U of (X, τ) ,
- (v) **rg-open** [19] if $f(U)$ is rg-open in (Y, σ) for every open set U of (X, τ) ,
- (vi) **gpr-open** [11] if $f(U)$ is gpr-open in (Y, σ) for every open set U of (X, τ) ,
- (vii) **regular open** [31] if $f(U)$ is open in (Y, σ) for every regular open set U of (X, τ) .

Definition 2.11 A bijective function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) **generalized homeomorphism** (g-homeomorphism) [2] if both f and f^{-1} are g-continuous,
- (ii) **gc-homeomorphism** [2] if both f and f^{-1} are gc-irresolute,
- (iii) **rwg-homeomorphism** [16] if both f and f^{-1} are rwg-continuous,
- (iv) **w*-homeomorphism** [20] if both f and f^{-1} are w-irresolute,
- (v) **w-homeomorphism** [20] if both f and f^{-1} are w-continuous.
- (vi) **rps-homeomorphism** [27] if both f and f^{-1} are rps-continuous.
- (vii) **rg α -homeomorphism** [30] if both f and f^{-1} are rg α -continuous.
- (viii) **wgr α -homeomorphism** [25] if both f and f^{-1} are wgr α -continuous

3 $\alpha\omega$ -Homeomorphisms in Topological Spaces

We introduce the following definition.

Definition 3.1 A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called α **regular ω -homeomorphism** (briefly, $\alpha\omega$ -homeomorphism) if f and f^{-1} are $\alpha\omega$ -continuous.

We denote the family of all $\alpha\omega$ -homeomorphisms of a topological space (X, τ) onto itself by $\alpha\omega\text{-h}(X, \tau)$.

Example 3.2 Consider $X = Y = \{a, b, c, d\}$ with topologies $\tau = \sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ Let $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=c, f(b)=a, f(c)=b, f(d)=d$. Then f is $\alpha\omega$ -continuous and f^{-1} is $\alpha\omega$ -continuous. Therefore f is $\alpha\omega$ -homeomorphisms.

Theorem 3.3 Every homeomorphism is an $\alpha\omega$ -homeomorphism, but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a homeomorphism. Then f and f^{-1} are continuous and f is bijection. As every continuous function is $\alpha\omega$ -continuous, we have f and f^{-1} are $\alpha\omega$ -continuous. Therefore f is $\alpha\omega$ -homeomorphism.

The converse of the above theorem need not be true, as seen from the following example.

Example 3.4 Consider $X = Y = \{a, b, c, d\}$ with topologies $\tau = \sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and Define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=c, f(b)=a, f(c)=b, f(d)=d$. Then f is $\alpha\omega$ -homeomorphism but it is not homeomorphism, since the inverse image of closed $F = \{c, d\}$ in Y then $f^{-1}(F) = \{a, d\}$ which is not closed set in X .

Theorem 3.5 Every α -homeomorphism is an $\alpha\omega$ -homeomorphism but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a α -homeomorphism. Then f and f^{-1} are α -continuous and f is bijection. As every α -continuous function is $\alpha\omega$ -continuous, we have f and f^{-1} are $\alpha\omega$ -continuous. Therefore f is $\alpha\omega$ -homeomorphism.

The converse of the above theorem is not true in general as seen from the following example.

Example 3.6 Consider $X = Y = \{a, b, c, d\}$ with topologies $\tau = \sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=c, f(b)=a, f(c)=b, f(d)=d$. Then f is $\alpha\omega$ -homeomorphism but it is not α -homeomorphism, since the inverse image of closed $F=\{c, d\}$ in Y then $f^{-1}(F)=\{a, d\}$ which is not α -closed set in X .

Theorem 3.7 i) Every $\alpha\omega$ -homeomorphism is an αg -homeomorphism.

ii) Every $\alpha\omega$ -homeomorphism is an wg -homeomorphism (resp. gs -homeomorphism, rwg -homeomorphism, gp -homeomorphism, $gspr$ -homeomorphism, gpr -homeomorphism, $\omega\alpha$ -homeomorphism, αgr -homeomorphism)

Proof: i) Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a $\alpha\omega$ -homeomorphism. Then f and f^{-1} are $\alpha\omega$ -continuous and f is bijection. As every $\alpha\omega$ -continuous function is αg -continuous, we have f and f^{-1} are αg -continuous. Therefore f is αg -homeomorphism.

Similarly we can prove ii)

The converse of the above theorem is not true in general as seen from the following example.

Example 3.8 Consider $X = Y = \{a, b, c\}$ with topologies $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=a, f(c)=c$. Then this function is αg -homeomorphism, (resp. wg -homeomorphism, gs -homeomorphism, rwg -homeomorphism, gp -homeomorphism, $gspr$ -homeomorphism, gpr -homeomorphism, $\omega\alpha$ -homeomorphism, αgr -homeomorphism) but it is not $\alpha\omega$ -homeomorphism, since the closed set $F=\{b, c\}$ in Y , $f^{-1}(F)=\{a, c\}$ which is not $\alpha\omega$ -closed set in X .

Remark 3.9 The following examples shows that $\alpha\omega$ -homeomorphism are independent of pre-homeomorphism, β -homeomorphism, g -homeomorphism, ω -homeomorphism, rw -homeomorphism, swg -homeomorphism, rgw -homeomorphism, $wgr\alpha$ -homeomorphism, $rg\alpha$ -homeomorphism, $gprw$ -homeomorphism, g^*p -homeomorphism, gr -homeomorphism, R^* -homeomorphism, rps -homeomorphism, semi-homeomorphism.

Example 3.10 Let $X=Y=\{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$, Let map $f: X \rightarrow Y$ defined by $f(a)=b, f(b)=a, f(c)=c$. Then pre-homeomorphism, β -homeomorphism, g -homeomorphism, ω -homeomorphism, rw -homeomorphism, swg -homeomorphism, rgw -homeomorphism, $wgr\alpha$ -homeomorphism, $rg\alpha$ -homeomorphism, $gprw$ -homeomorphism, g^*p -homeomorphism, gr -homeomorphism, R^* -homeomorphism, rps -homeomorphism but it is not $\alpha\omega$ -homeomorphism, since the inverse image of the closed set $\{b, c\}$ in Y is $\{a, c\}$ which is not $\alpha\omega$ -closed set in X .

Example 3.11 $X=Y=\{a,b,c,d\}$, $\tau = \{X, \phi, \{a\},\{b\},\{a,b\},\{a,b,c\}\}$ $\sigma =\{Y, \phi, \{a\},\{b\},\{a,b\},\{a,b,c\}\}$ Let map $f: X \rightarrow Y$ defined by $f(a)=b, f(b)=a, f(c)=d, f(d)=c$ then $\alpha\omega$ -homeomorphism but not, g -homeomorphism, ω -homeomorphism, rw -homeomorphism, $gprw$ -homeomorphism, g^*p -homeomorphism, gr -homeomorphism, R^* -homeomorphism as closed set $F=\{d\}$ in X , then $f(F)=\{c\}$ in Y , which is not gr -closed (resp. g -closed, g^*p -closed, ω -closed, rw -closed, $gprw$ -closed, gr -closed, R^* -closed) set in Y .

Theorem 3.12 Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a bijective $\alpha\omega$ -continuous map. Then the following are equivalent.

- (i) f is a $\alpha\omega$ -open map,
- (ii) f is $\alpha\omega$ -homeomorphism,
- (iii) f is a $\alpha\omega$ -closed map.

Proof: Proof follows from theorem 3.39 in [36].

Remark 3.13 The composition of two $\alpha\omega$ -homeomorphism need not be a $\alpha\omega$ -homeomorphism in general as seen from the following example.

Example 3.14 Consider $X = Y = Z= \{a, b, c, d\}$ with topologies $\tau = \sigma = \mu = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=c, f(b)=a, f(c)=b, f(d)=d$. and $g : Y \rightarrow Z$ defined by $g(a) = b, g(b) = a, g(c) = d, g(d) = c$ then both f and g are $\alpha\omega$ -homeomorphisms but their composition $g \circ f : (X, \tau) \rightarrow (Z, \mu)$ is not $\alpha\omega$ -homeomorphism because for the open set $\{a, b\}$ of (X, τ) , $g \circ f (\{a, b\}) = g(f(\{a, b\})) = g(\{a, c\}) = \{a, d\}$, which is not $\alpha\omega$ -open in (z, μ) . Therefore $g \circ f$ is not $\alpha\omega$ -open and so $g \circ f$ is not $\alpha\omega$ -homeomorphism.

Definition 3.15 A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **$\alpha\omega c$ -homeomorphism** if both f and f^{-1} are $\alpha\omega$ -irresolute. We say that spaces (X, τ) and (Y, σ) are $\alpha\omega c$ -homeomorphic if there exists a $\alpha\omega c$ -homeomorphism from (X, τ) onto (Y, σ) .

We denote the family of all $\alpha\omega c$ -homeomorphisms of a topological space (X, τ) onto itself by $\alpha\omega c-h(X, \tau)$.

Theorem 3.16: Every $\alpha\omega c$ -homeomorphism is an $\alpha\omega$ -homeomorphism but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an $\alpha\omega c$ -homeomorphism. Then f and f^{-1} are $\alpha\omega$ -irresolute and f is bijection. By theorem 3.20 in [35] f and f^{-1} are $\alpha\omega$ -continuous. Therefore f is $\alpha\omega$ -homeomorphism.

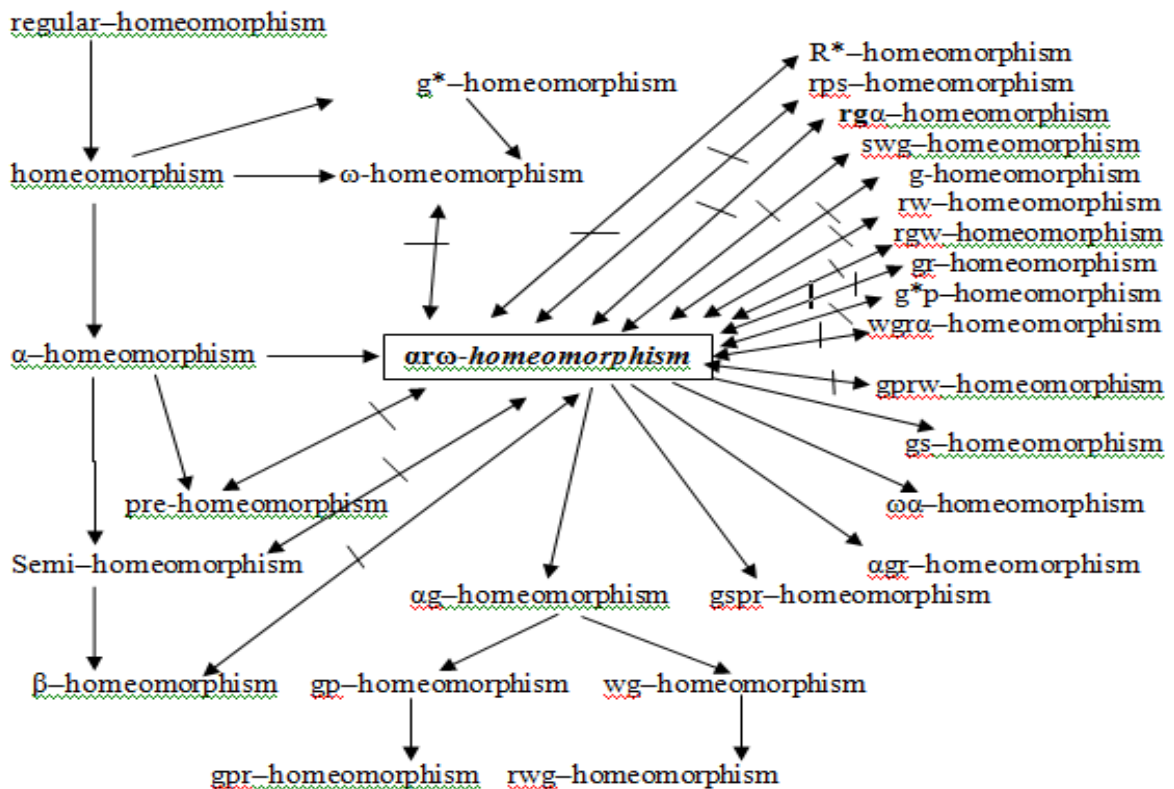
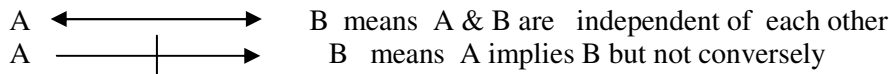
The converse of the above Theorem is not true in general as seen from the following example.

Example 3.17 Consider $X = Y = Z= \{a, b, c, d\}$ with topologies $\tau = \sigma = \mu = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=c, f(b)=a, f(c)=b, f(d)=d$. Then f is $\alpha\omega$ -homeomorphism but it is not $\alpha\omega c$ -homeomorphism, since f is not $\alpha\omega$ -irresolute.

Theorem 3.18 Every $\alpha\omega$ -homeomorphism is wg-homeomorphism (resp. α g-homeomorphism, gs-homeomorphism, rwg-homeomorphism, gp-homeomorphism, gspr-homeomorphism, gpr-homeomorphism, $\omega\alpha$ -homeomorphism, α gr-homeomorphism) but not conversely.

Proof: Proof follows from lemma 2.5 and 2.6.

Remark 3.19 From the above discussions and known results we have the following implications.



Theorem 3.20 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are $\alpha\omega$ -homeomorphisms, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is also $\alpha\omega$ -homeomorphism.

Proof: Let U be a $\alpha\omega$ -open set in (Z, η) . Since g is $\alpha\omega$ -irresolute, $g^{-1}(U)$ is $\alpha\omega$ -open in (Y, σ) . Since f is $\alpha\omega$ -irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is $\alpha\omega$ open set in (X, τ) . Therefore $g \circ f$ is $\alpha\omega$ -irresolute. Also for a $\alpha\omega$ -open set G in (X, τ) , we have $(g \circ f)(G) = g(f(G)) = g(W)$, where $W = f(G)$. By hypothesis, $f(G)$ is $\alpha\omega$ -open in (Y, σ) and so again by hypothesis, $g(f(G))$ is a $\alpha\omega$ -open set in (Z, η) . That is $(g \circ f)(G)$ is a $\alpha\omega$ -open set in (Z, η) and therefore $(g \circ f)^{-1}$ is $\alpha\omega$ -irresolute. Also $g \circ f$ is a bijection. Hence $g \circ f$ is $\alpha\omega$ -homeomorphism.

Theorem 3.21: The set $\alpha\omega$ -h (X, τ) is a group under the composition of maps.

Proof: Define a binary operation $*$: $\alpha\omega\text{-h}(X,\tau) \times \alpha\omega\text{-h}(X,\tau) \rightarrow \alpha\omega\text{-h}(X,\tau)$ by $f * g = g \circ f$ for all $f, g \in \alpha\omega\text{-h}(X,\tau)$ and \circ is the usual operation of composition of maps. Then by Lemma 2.8, $g \circ f \in \alpha\omega\text{-h}(X,\tau)$. We know that the composition of maps is associative and the identity map $I: (X,\tau) \rightarrow (X,\tau)$ belonging to $\alpha\omega\text{-h}(X,\tau)$ serves as the identity element. If $f \in \alpha\omega\text{-h}(X,\tau)$, then $f^{-1} \in \alpha\omega\text{-h}(X,\tau)$ such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of $\alpha\omega\text{-h}(X,\tau)$. Therefore $(\alpha\omega\text{-h}(X,\tau), \circ)$ is a group under the operation of composition of maps.

Theorem 3.22 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $\alpha\omega\text{-h}$ -homeomorphism. Then f induces an isomorphism from the group $\alpha\omega\text{-h}(X, \tau)$ onto the group $\alpha\omega\text{-h}(Y, \sigma)$.

Proof: Using the map f , we define a map $\Psi f : \alpha\omega\text{-h}(X,\tau) \rightarrow \alpha\omega\text{-h}(Y, \sigma)$ by $\Psi f (h) = f \circ h \circ f^{-1}$ for every $h \in \alpha\omega\text{-h}(X,\tau)$. Then Ψf is a bijection. Further, for all $h_1, h_2 \in \alpha\omega\text{-h}(X,\tau)$, $\Psi f (h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \Psi f (h_1) \circ \Psi f (h_2)$. Therefore Ψf is a homeomorphism and so it is an isomorphism induced by f .

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