Metaverse application development for teaching geometry in virtual reality environment

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Highlights	Abstract
• Metaverse applications have significant potential for teaching geometry. • Metaverse applications enhance students' motivation by offering diverse and effective learning experiences. • The widespread adoption of virtual reality-based Metaverse applications in educational settings depends on their user-friendliness. **Article Info:** Research Article **Keywords:** Metaverse, Application Development, Virtual Reality, Geometry Learning, Geometry Teaching	This study aims to explain the Metaverse application development process for teaching geometry in a virtual reality environment. The application was developed based on the instructional design model including analysis, design, development, implementation and evaluation phases. The target group was secondary school students. During the analysis phase, students' virtual reality experiences and their needs for using the Metaverse application academically were identified. Additionally, learning outcomes from the Secondary School Mathematics Course Curriculum were selected to determine the application content, and convenient educational software programs for a Metaverse environment were examined. During the design phase, Metaverse software that was suitable for the study's purpose was selected. Also, three-dimensional models of the objects and the environment for the application activities were designed through three-dimensional drawing programs. During the development phase, the activities' visualization and manipulation trials were conducted by transferring the three-dimensional designs into the Metaverse application. After necessary corrections, activities took their final form. During the implementation phase, 15 students in grade 10th used the Metaverse application. During the evaluation phase, participants stated their opinions about the application. Accordingly, suggestions for improving the Metaverse application in the field of teaching geometry and its widespread implementation were proposed.

1. Introduction

The term Metaverse originated in Vernor Vinge's novel "True Names" in 1981 and was later described in Neal Stephenson's "Snow Crash" in 1992 (Sun et al., 2022). The game software "Second Life", launched in 2003, was considered one of the first examples of Metaverse (Lamb, 2022) and the concept "Metaverse" was associated with three-dimensional virtual environments and virtual reality (VR) over time (Zyda, 2022). Published in 2011, the novel "Ready Player One" and the movie with the same name in 2018 depicted a kind of Metaverse called the OASIS (Kim, 2021). The metaverse concept refers to an environment that offers interaction and experience through digital avatars in virtual worlds (Kim, 2021).

According to Hwang and Chien (2022), the Metaverse is defined as shared, permanent and decentralized virtual environments that are supported by artificial intelligence. Multi-user support, database and Blockchain technology are required for these environments to be effective.

Despite not being a completely new concept, the Metaverse has been perceived as more of an idea related to the future (Sanglier Contreras et al., 2020). On the other hand, regardless of the flood of information from the Metaverse, Facebook and technology resources, it is still understood that many people do not have much idea about what exactly the Metaverse is and what it can be used for (Riva & Wiederhold, 2022). The Metaverse is an acronym that combines the words "meta" (transcendent) and "universe", and it refers to the next generation of the Internet. It can be stated that the Metaverse is a network of virtual environments or a virtual universe that can be experienced online. In this universe, users can interact with other people and elements in the same environment through avatars. The Metaverse can be defined as an environment where you can live digitally, a "digital habitat" (Recker et al., 2021).

The Metaverse is a technology that has the potential to change the way we live today and the way we connect with our family and friends (Mystakidis, 2022). It has clearly focused on the entertainment area and video games as its first application domain. However, the Metaverse has many benefits (Ibili et al., 2023). A virtual workplace can be built in the Metaverse, where communication can be established through video calls and colleagues can interact with each other beside carrying on a work (Chen, 2024). The Metaverse can be used in areas such as virtual counselling services, disease treatments, applications for individuals with disabilities, meditation and stress management, and the use of virtual and augmented reality for education (Radanliev, 2024; Wang et al., 2022). Furthermore, an educational institution can create a virtual version of its campus or a replica of the institution by creating a Metaverse application. This campus can be a virtual reflection of such physical spaces as classrooms, dining halls, teachers' rooms, etc. In this way, students, teachers and other staff can communicate and interact quickly and easily through video calls or conferences, as if they were physically present.

Metaverse applications can be used in teaching geometry due to the greater presence of visual elements and the difficulty of visualizing them in mind. Technological devices such as computers, mobile phones and tablets come first and foremost in teaching geometry as they attract students' attention (Karakuş, 2008). On the other hand, virtual reality technology, which is one of the newest advantages of the developing technology, is also expected to be used frequently in teaching geometry in the future (Domínguez Vázquez & Díaz Palencia, 2024).

Geometry is a discipline that studies the properties of geometric shapes and objects, and the relationships between them. The National Council of Teachers of Mathematics (NCTM) (2000) defines geometry as "the study of shape and space". It was emphasized that the use of three-dimensional geometric shapes in teaching geometry should be given importance, and students should be provided with opportunities for developing their spatial skills in the NCTM report (NCTM, 2000). In conventional teaching environments, three-dimensional objects have a static appearance as they are represented on a two-dimensional plane in textbooks or on the blackboard. Students, in turn, have difficulty in perceiving three-dimensional objects with such drawings and experience deficiencies in three-dimensional thinking skills in the following years (Accascina & Rogora, 2006). As a result, there are difficulties in learning subjects that require three-dimensional thinking and visual ability such as geometric objects and volume calculations (Accascina & Rogora, 2006; Topraklıkoğlu & Öztürk, 2021).

The difficulty in understanding abstract concepts significantly affects the learning process in disciplines based on abstract principles such as mathematics and geometry. Many studies have examined the difficulty in learning abstract concepts and made suggestions on how to facilitate learning by concretizing abstract concepts (Gadanidis, 2017; Tran et al., 2020; Uyen, et al., 2021; Veith et al., 2022). Teachers have difficulty in teaching abstract topics and theorems generally in mathematics classes due to a large number of such concepts in the content according to Tran et al. (2020). One of the main challenges in teaching abstract concepts is the difficulty students face in grasping and visualizing these ideas. Abstract concepts encountered in geometry subjects often do not have concrete physical representations that students can easily relate to (Nurrahmah et al., 2021; Yılmaz & Argün, 2018). This can hinder the learning process by

causing a disconnect between theoretical knowledge and the practical application of the concepts. To overcome this difficulty, researchers have emphasized the importance of using visual aids and digital technologies that facilitate concretization in teaching abstract concepts in teaching environments (Bakar, 2023). The use of computer-aided dynamic geometry software and digital visualization technologies helps students understand abstract concepts in a more relevant way and enables them to grasp abstract concepts (Bakar, 2023).

Efforts to overcome difficulties in teaching abstract concepts may be hampered by teachers' limited experience and insufficient willingness to use innovative instructional technologies and digital technologies such as virtual reality. If teachers lack the necessary skills, desire and self-confidence to use digital technologies and alternative approaches effectively in geometry teaching, this may hinder students' ability to grasp abstract concepts (Permatasari & Andayani, 2021). To eliminate this barrier, researchers have emphasized the importance of providing professional development opportunities for teachers to use innovative instructional technologies such as virtual reality (Gokbulut & Durnali, 2023; Mystakidis et al., 2021; Nelson & Ahn, 2018). Equipping teachers with the necessary skills and knowledge to use innovative instructional technologies such as virtual reality can facilitate the teaching and learning of abstract concepts, especially in mathematics and geometry (İbili, 2013).

One of the important reasons for difficulties and failures in teaching geometry is that students cannot fully perceive three-dimensional shapes represented in two-dimensional planes and have not developed sufficient visualization skills from an early age (Accascina & Rogora, 2006; Topraklıoğlu & Öztürk, 2021). In order to overcome this problem, it is necessary to adopt a student-centered approach by using technological tools in learning environments. In addition, in teaching geometry, it is recommended to focus on practical activities on concrete and three-dimensional structures instead of two-dimensional studies with paper and pencil (Çetintav, 2023; İbili & Şahin, 2015).

Clements (2003) stated that students' difficulties with three-dimensional shapes may be due to the ability to mentally rotate an object or to understand the spatial relationships between objects and suggested that students should benefit from concrete materials and digital tools while learning geometric concepts. Physical models, dynamic geometry software or virtual reality tools can help abstract concepts concretize. Furthermore, students need to develop conceptual understanding and procedural knowledge in tandem. To illustrate, while the volume calculation of a cube is a kind of procedural knowledge, understanding how and why this formula works requires the student's conceptual learning. Tasks including stages such as exploration, recognition, understanding features, and problem solving in a virtual reality environment can help students overcome their difficulties in visualizing three-dimensional objects.

2. Importance of the Metaverse in Teaching Environments

The Metaverse has the potential to transform learning with conventional teaching methods into innovative learning experiences. It offers students the opportunity to experience the information that can be obtained only from textbooks or two-dimensional screens through three-dimensional and interactive environments (Topraklıoğlu & Öztürk, 2023a, 2023b). Activities in the virtual world transform the students from passive recipients of information into active participants who engage in the process actively and learn by experience.

Active processing of the information can also result in improvement in students' cognitive capacities (Arslan & Ünal, 2022). According to Sweller's (2010) cognitive load theory, learning processes are related to students' cognitive capacities. Learning environments and processes should be designed in such a way that students' cognitive capacities can be used efficiently at its most. Sweller (2010) argues that educational materials and teaching strategies should balance the cognitive load. An effective learning environment should adjust the learner's intrinsic cognitive load in a way that is appropriate to the learner's current knowledge level. It should also minimize the external cognitive load and maximize the latent cognitive load. Besides, the virtual reality environment is closed to the physical outdoors and thanks to that advantage the external load that students have to process unnecessary information can be limited. Additionally, when

designing activities in the virtual environment, teachers can increase the latent load by providing information processing opportunities more, and students can be engaged more in the learning process.

Metaverse applications transform learning into a social experience by eliminating geographical restrictions, bringing students together from different parts of the World and creating a space where they can work collaboratively with their classmates and participate in interactive problem-solving activities. Vygotsky's socio-cultural learning theory argues that social interaction and cultural tools play a critical role in students' learning processes. Interactive learning environments enable students to understand knowledge not in an individual way but in collaboration with others (Bayrakçı, 2013). Zone of proximal development by Vygotsky refers to the tasks that the learner cannot accomplish alone but can accomplish together with a guide or a more competent person. Interactive learning environments, in that sense, provide students with a higher level of cognitive and skill development through social interaction and guidance on the basis of the zone of proximal development principle (Ünveren Kapanadze, 2019). By interacting with other students or the teacher in interactive learning environments, students can improve their ability to learn more complex topics and their problem-solving skills. Such environments encourage not only the transfer of knowledge but also the development of students' thinking processes (Gao, 2023).

Students can meet in the same virtual classroom, collaborate on group projects, and receive instant feedback from their teachers through the Metaverse. Such interactions can particularly improve their problem-solving, critical thinking, and creative thinking skills (Eşin & Özdemir, 2022; Park, 2022; Wu et al., 2023). The Metaverse offers a flexible teaching environment that can adapt to individual learning pace and level. While it may be difficult to adjust the pace of teaching for each student in conventional classrooms, in the Metaverse each student can experience learning at his/her own pace (Asmaroini et al., 2024; Onu et al., 2024; Zhao et al., 2022).

Siyaev and Jo (2021) aimed to investigate the use of Metaverse and mixed reality technologies for teaching the maintenance of Boeing 737 aircraft. It was concluded that the mixed reality teaching system significantly improved the aircraft maintenance teaching and the operations by providing a more effective and intuitive interaction with virtual objects in a mixed reality environment. Eşin and Özdemir (2022) found that teachers' opinions about the use of Metaverse in teaching mathematics were generally positive. In that sense, according to the teachers, Metaverse could be used mainly for concrete in geometry and that was useful. Indarta et al. (2022) found that the use of digital learning environments based on augmented reality and virtual reality has accelerated the adoption of Metaverse technology in the field of education. It is stated in the study that Metaverse is expected to penetrate into many spheres of life in the following 10-15 years. Yağcı and Şentürk (2023) concluded that the use of Metaverse in science education provides opportunities to perform difficult experiments and learn collaboratively. It was also determined in the study that virtual laboratory applications in the Metaverse environment can provide significant contributions to students by enabling learning through inquiry, providing instant feedback and structuring the knowledge in mind. Gao and Lee (2024) showed that the Metaverse based teaching approach in teaching youth cultural heritage related to the ancient city of Pingyao provided positive effects on learning motivation, perception of presence, student achievement, and learning satisfaction.

Since mathematics is a discipline that includes abstract concepts, it is the area where students have more difficulty than other courses (Burns & Hamm, 2011; Çifci, 2021). Students' better understanding of mathematical concepts and operations can be achieved through the use of Metaverse applications and virtual environments with high visual richness that help to concretize abstract concepts (Marini et al., 2022; Yılmaz & Coşkun Şimşek, 2023). Students, for example, can manipulate three-dimensional objects in the Metaverse, observe them from different angles and see the properties of the objects directly while learning geometry topics. In this way, students can understand geometric objects more easily and enrich their learning not only through formulas and visuals, but also through direct experience (González, 2018).

The widely accepted constructivist approach, which has formed the basis of curricula in recent years, focuses on students discovering knowledge through their own experiences and active participation instead of providing students with ready-made knowledge (Dikkartın Övez & Acar, 2022). The goal-based scenario approach, which is one of the constructivist strategies, has similarities with problem-based learning and

cooperative learning and finds its place in teaching processes. The approach aims to associate the skills to be acquired with daily life and is presented within the framework of a scenario (İnan & Kılıçer, 2022). In this way, students can actively participate in the learning process and produce concrete products at the end of the process. Through Metaverse applications, teachers can create goal-based scenarios for their students to collaboratively learn about real-life problems.

Another difficulty encountered by students in understanding three-dimensional objects is the failure in comprehending spatial relationships and structural features correctly (Hawes & Ansari, 2020). Lohman (1996) provides a theoretical framework that can be used to understand students' challenges in visualizing three-dimensional objects. When students cannot accurately visualize geometric shapes and their interrelations, they often have difficulty in making correct connections between such elements. In this case, more experience and practice are required for the students to visualize three-dimensional structures of the objects in their minds. The framework presented by Lohman (1996) emphasizes that although spatial skills are a developmental process, they can be improved through effective teaching methods. According to this framework, spatial skills can be developed better when students are given a variety of hands-on tasks through visualization techniques. According to Lohman (1996), difficulties in visualizing three-dimensional objects are directly related to the development of spatial skills and students' cognitive capacities. Through digital visualization tools such as properly structured virtual reality applications, students can experience spatial application experiences, and their geometry achievement can be increased by improving their mental visualization skills.

3. Advantages of Metaverse Applications Compared to Virtual Reality Applications

Conventional virtual reality applications, usually developed for a specific purpose, offer singular and limited experiences (Visconti et al., 2023). These applications, often by targeting a single user, focus on a specific scenario such as training, gaming or simulation and this experience is presented to the user within a restricted virtual space. On the other hand, Metaverse applications offer a much wider and continuous environment (Visconti et al., 2023). Users can move freely between various learning areas and explore them seamlessly in the Metaverse. As a result, teaching experiences can go beyond the limits of conventional virtual reality environments and become permanent in a wider virtual universe (Ball, 2022).

In conventional virtual reality applications, the user is mostly alone or interacts with a limited number of other users. This can make learning processes restrictive, particularly where social interactions are important. In contrast, the Metaverse provides an environment where users can constantly connect with each other, meet in virtual classrooms, or collaborate on projects. Students can come together from different geographical regions, work together on interactive course materials and have a richer learning experience by interacting with the teacher and other students (Çelik & Baturay, 2024).

Conventional virtual reality applications mostly have a fixed content and predetermined scenarios. It is not possible for users to contribute content to such applications or make changes in virtual environments. However, Metaverse applications allow users to create avatars, virtual spaces and digital materials. In educational environments, this allows teachers and students to customize virtual classrooms and to organize virtual spaces in a way that suits them through the share of projects or materials. This can enable students to participate in creative processes and to contribute to learning environments (Asmaroini et al., 2024; Waqar, 2024).

In conventional virtual reality applications, when users leave the application, the experience ends, and the environment remains passive until it reopens. Metaverse applications, on the other hand, extend this experience in a continuous way and offer a world where users can return at any time and continue interacting with other users. In educational context, this feature allows students and teachers to access virtual classrooms repeatedly, which ensures that the learning process is uninterrupted. For these reasons, Metaverse applications can enrich the learning process by providing a more interactive, accessible and user-centered experience compared to conventional virtual reality applications, especially in educational environments (Inceoglu & Ciloglugil, 2022; Kye et al., 2021; Phakamach et al., 2022).

4. Significance and Purpose of the Study

It is crucial for students, who live in a three-dimensional physical world with three-dimensional objects, to learn geometric objects in a three-dimensional environment in order to achieve the learning outcomes in a course such as geometry, which is abstract and where visualization is important. The use of three-dimensional modelling with virtual reality technology and Metaverse applications are current tools that can be used for teaching and learning geometry (Eşin & Özdemir, 2022; Su et al., 2022; Topraklıkoğlu & Öztürk, 2023a, 2023b). Virtual reality technology and Metaverse applications are among the rapidly developing innovative technologies (Alkhwaldi, 2024). Metaverse applications have been counted among promising technologies and the benefits of their use for teaching purposes have been mentioned in many studies (Lin et al., 2022; Onu et al., 2024; Talan & Kalinkala, 2022).

Metaverse applications used for instructional purposes enable students to concretize the subjects, concepts and situations learned, while providing opportunities to learn from each other through social learning and to reflect on the learned subject (Ritala et al., 2024; Villamil & King, 2024). Examining the effectiveness of Metaverse applications on learning is considered as a topic worth searching (MacCallum & Parsons, 2019; Topraklıkoğlu & Öztürk, 2023a, 2023b). Therefore, this study was decided and a Metaverse application development for teaching geometry in a virtual reality environment was aimed. In the following sections, the development process of Metaverse application for teaching geometry in a virtual reality environment is explained.

5. Metaverse Application Development Process for Teaching Geometry

A Metaverse application for teaching geometry in a virtual reality environment was developed in accordance with the analysis, design, development, implementation and evaluation (ADDIE) model (Werner & DeSimone, 2012). In this section, studies carried out during the stages of the ADDIE model are explained.

5.1. Analysis Phase

The first step of the development process of Metaverse application for teaching geometry is the analysis. In the analysis phase, before starting to develop the application, needs analysis, environment analysis, target group analysis and content analysis were carried out, literature on the subject was reviewed and pilot studies were conducted.

At the secondary school level, the mathematics course draws attention with low success rates (Barnett et al., 2018). Geometry subjects of the mathematics course include topics that are more difficult to learn since they require students to use spatial abilities (Akay, 2013; Fujita & Jones, 2007; Gökkurt et al., 2015). Area and volume calculations in three-dimensional solids, views of solids from different directions and questions related to the space diagonals of solids are the topics that require visualization and make essential real or virtual objects use. In the light of the literature review, it has been concluded that there are problems in teaching geometry topics and an application in which virtual or real objects can be manipulated is needed for teaching geometry (Hwang et al., 2009; Topraklıkoğlu & Öztürk, 2021). In that sense, the subject to be taught in this study was determined as "Solid Bodies" topic in the "Space Geometry" subject in 10th grade, which is stated in the Secondary School Mathematics Course Curriculum (Ministry of National Education [MoNE], 2018) and the learning outcome was determined as "Forms the length, area and volume relations of right prisms and right pyramids".

There are studies showing that teaching three-dimensional objects in a real three-dimensional environment produces more successful results compared to teaching them in two-dimensional environments (Gülburnu, 2013; İbili & Şahin, 2015; Topraklıkoğlu, 2018; Topraklıkoğlu & Öztürk, 2021). Teaching three-dimensional objects in a three-dimensional environment can be possible with real three-dimensional objects in the environment that we live in or in three-dimensional virtual environments. Although the use of real objects in teaching is very effective, it also has some limitations and difficulties such as lack of rich content, difficulty in preparing materials, difficulty in examining the material for all students simultaneously, and limited manipulation (İbili & Şahin, 2015; Pişkintunç et al., 2012). Providing three-dimensional

environments different from the real environment that we live in can be possible with such technologies as virtual reality headsets and holograms. Virtual reality technology is considered as a preferable option for teaching geometry as it is both cheaper and accessible more easily compared to hologram technology.

Upon the selection of the virtual reality environment as teaching environment, analyses were conducted to determine the most proper virtual reality environment for the target group. Among the students with previous experience in using cardboard and virtual reality headsets that are connected to the computer, there are also students who have no experience with any virtual reality device. Therefore, it was understood that students should be provided with a training on the use of the selected virtual reality headset prior to the teaching process.

Geometry subjects at the secondary school level require students to use their metacognitive skills and critical thinking skills (Mandacı Şahin & Kendir, 2013; Nasution & Sinaga, 2017). As a consequence, students should be in a flexible teaching process that will enable them to learn at their own pace and to carry out group work including both peer learning on the basis of social interaction and learning activities through discovery rather than merely lecturing (Asmaroini et al., 2024; Onu et al., 2024). Metaverse applications can involve all these elements in a virtual reality environment (Kim, 2021; Kye et al., 2021; Phakamach et al., 2022). Thus, developing a Metaverse application for teaching geometry in a virtual reality environment was determined. Since there are virtual reality headsets with different features and also different Metaverse applications, the virtual reality headset and Metaverse application appropriate for the students in the target group were selected.

5.1.1. Selecting a Suitable Device for Metaverse Application

While there are significant differences among virtual reality headsets, wireless and standalone headsets offer great advantages in terms of portability and user experience (Romoser et al., 2024; Trinidad-Fernández et al., 2023). Headsets called cardboard are developed to be used with mobile devices and are not suitable for long-term virtual reality experiences due to the limited processing power and low resolution. On the other hand, although wired virtual reality headsets provide high performance because of powerful processors, cable dependency limits the user experience by restricting user movements, especially in educational environments that require large space (Liu et al., 2024). However, Meta Quest 2, a wireless device, offers a more fluid experience in teaching environments by allowing users to move freely in virtual environments (Hagge, 2024). This feature makes the teaching process more effective in areas that require physical movement and interaction such as geometry (Nakov et al., 2023).

As a device that offers enhanced interaction and user experience, Meta Quest 2 differs from cardboard and the wired virtual reality headsets (Rockcastle et al., 2024). While cardboards are suitable for short-term experiences, they are not comfortable for long-term use in more detailed teaching environments. Interaction is also quite limited and offers mostly passive experiences. Meta Quest 2, on the other hand, allows users to interact with virtual objects through gesture recognition, haptic feedback and advanced control mechanisms. Furthermore, its wireless and lightweight design ensures the continuity of virtual teaching experiences by preserving users' freedom of movement. Accordingly, students can examine geometric objects from different angles by manipulating them with natural hand movements and learn concepts in three dimensions (Su et al., 2022).

In terms of easy installation and use, the standalone feature of Meta Quest 2 offers a significant advantage. Although cardboards have the advantage of simple installation, the device performance is not sufficient for long-term and intense virtual reality experiences (King & Salvo, 2024). Meanwhile, the use of wired virtual reality headsets can be difficult due to the complex installation requirements and the need for powerful hardware. Meta Quest 2, on the other hand, can be quickly installed without any requirement for additional hardware and it allows users to easily integrate into the educational environment merely by wearing the headset (Liu et al., 2024; Rockcastle et al., 2024).

High-resolution display and powerful graphics processor of Meta Quest 2 provide a much clearer and realistic visual experience compared to the limited resolutions of cardboard (Law et al., 2024; Rendevski, 2022). Meta Quest 2, a standalone headset, can meet the quality and speed requirements for teaching

applications by offering better performance than many wired virtual reality headsets because of its portability feature (Baldoni et al., 2024). This feature has the potential to enable students to interact in virtual education environments without reducing visual quality. On the basis of the above-mentioned features, Meta Quest 2 headset is agreed to be suitable for the Metaverse application developed in the study.

5.1.2. *Software Selection for Metaverse Application Development*

The selection of software for developing a Metaverse application is a strategic decision to be taken considering both instructional goals and technical requirements (Lee et al., 2022). In this selection, variables such as the content of the application to be developed, the characteristics of the target group, the capacity of the devices to be used and the budget should be taken into consideration (Topraklıkoğlu & Öztürk, 2023b). The use of Metaverse applications for teaching purposes is quite effective, particularly in subjects where abstract concepts need visualizing and getting interactive. However, when selecting the ideal software for developing such applications, the extent to which the tools that the software provides are compatible with teaching purposes should be taken into consideration. Application development software that encourages students' active participation, facilitates interaction and offers the possibility of customization based on requirements stand out in this context (Topraklıkoğlu & Öztürk, 2023b).

The design of Metaverse application must be compatible with the virtual reality headsets to be used for teaching. Accordingly, the virtual reality headsets in use in this study are Meta Quest 2 virtual reality headsets, which do not require a computer connection and have their own application store. Therefore, with an aim to implement the activities, applications in the Meta application store were examined. It was found that there was no specific Metaverse application development software available for teaching geometry. As a consequence, it was proposed that the most appropriate software for the Metaverse application development for teaching geometry could be the software that provides a virtual working environment, the three-dimensional drawing software or the architecture software. In the light of this, among the three-dimensional drawing applications available in the Meta application store, software called Arthur, ShapesXR and Gravity Sketch were found to be rated highly and useful. Arthur is Metaverse software that aims to provide a virtual working environment which enables users to create virtual office spaces, organize meetings with their teams and manage their projects through collaboration (Arthur Technologies, 2024). ShapesXR (ShapesXR, 2023) and Gravity Sketch (Gravity Sketch, 2019) are, on the other hand, three-dimensional drawing software. All these software were evaluated based on their usage features such as drawing geometric objects, image and sound quality, multi-user support, error rates within the application, user input and data storage, use of controllers and interface design. As a consequence, Gravity Sketch was determined to be the most suitable software to develop a Metaverse application for teaching geometry among these software. The software selection process was described in detail in the development phase.

Gravity Sketch software offers an effective development environment for Metaverse applications, particularly in areas such as geometry (Gravity Sketch, 2019). The main reasons for selecting this software are the lack of a ready-made teaching mathematics infrastructure and the high cost of developing custom software. Although Gravity Sketch is drawing-based software, it provided a suitable infrastructure to transform geometry topics into an interactive Metaverse application. The selection of the above-mentioned software at this point is a good example of how teachers can get the most out of existing software when they do not have the talent or budget to develop ready-made software. Device compatibility and easy accessibility of the application also played a role in the decision-making process and these features contributed to providing a smooth experience in the educational environment.

While selecting a development software for Metaverse applications, interaction opportunities offered by the environment are one of the most significant factors that enrich the teaching process. Gravity Sketch, with its structure allowing users to interact directly with three-dimensional objects, provided students with the opportunity to directly manipulate geometric shapes and examine them from different angles (Gravity Sketch, 2019). Such interactions can increase students' comprehension, especially in subjects that require visual support, such as geometry. In that sense, despite being a drawing program, Gravity Sketch was considered to have the capacity to support learning geometry in the Metaverse application as a functional tool in teaching.

5.2. Design Phase

The second step of the development process of Metaverse application for teaching geometry is the design phase. In the design phase, the implementation is assumed to be put into action benefiting from the conditions analyzed before and a draft is prepared on paper (Werner & DeSimone, 2012). Since the ADDIE model is a continuous circle, necessary changes are also made in the following design phase based on the changes to be made in the phases after the current design phase. What is experienced in the analysis phase is reflected into the design phase. In the design phase of this study, the teaching process design, the three-dimensional design of the activities and questions to be used in the teaching process in accordance with the course's learning outcomes determined in the analysis phase, the Metaverse application's virtual environment design, and the design of the orientation training process that will enable students to use the Metaverse application more easily was made.

The time allocated in the Secondary School Mathematics Course Curriculum (MoNE, 2018) for the learning outcomes of the "Solid Bodies" topic in the "Space Geometry" subject is 20 lesson hours. Since there were four Meta Quest 2 virtual reality headsets to be used with the Metaverse application for teaching geometry, 15 students were studied in groups of three. It was supposed that the use of the whole time allocated for the learning outcomes of the "Solid Bodies" topic in the Secondary School Mathematics Course Curriculum (MoNE, 2018) only for the implementation of the activities would not be appropriate. Therefore, 8 lesson hours out of 20 lesson hours were allocated for the activities in the Metaverse application for teaching geometry.

For the design of Metaverse application, a virtual environment in which teaching would take place was planned via Gravity Sketch software. In the design of the virtual environment, a three-dimensional environment was created to make the process take place in a closed physical structure similar to the classroom environment. This virtual environment was designed to be simple and realistic with walls covered with figures, and allowing freedom of movement without letting the activities overlap with each other.

In the analysis phase of Metaverse application for teaching geometry, it was detected that there were students who had not experienced Metaverse applications before. For this reason, prior to the teaching process, an orientation training about the selected virtual reality headset and the related Metaverse application was planned to make the students use the virtual reality headsets smoothly. In the design of orientation training, it was planned that the training would last one lesson hour for each group in which one out of four participants was the teacher. The content of this training was planned within the framework of learning the functions of the buttons on the virtual reality headsets, learning the functions of the buttons on the controllers for the operating system of the device and the Gravity Sketch software, learning how to move, draw, write, enlarge and reduce and many other manipulations in the Gravity Sketch software.

In the final step of the design phase, the evaluation process at the end was designed in order to get an idea about the effectiveness of Metaverse application for teaching geometry. In this design, it was aimed to prepare the interview questions to determine students' views on the Metaverse application using a semi-structured interview form. In this way, it was thought that students' experiences with virtual reality headsets, their experiences with the Metaverse application and their experiences in the learning geometry process would provide information about the effectiveness of the Metaverse application for teaching geometry.

5.3. Development Phase

The development phase of Metaverse application for teaching geometry started with the implementation of the plans put forward in the design phase. In the development phase, modelling of the activities and creation of the virtual reality environment were carried out.

It was first planned to use 3ds Max three-dimensional modelling software for the three-dimensional designs of the Metaverse application for teaching geometry in the design phase. As a result, three-dimensional models of the activities in the Metaverse application were designed via Tinkercad software.

The Metaverse application was first started to be designed in Arthur software. However, since the Metaverse is a recent study field, such kind of virtual reality-based Metaverse software has started to be developed in recent years too. Therefore, some software errors may occur in such software, and the solution of these errors cannot be found by users especially in free versions. Although the Metaverse application worked stably in the Arthur platform on the first trials, some software errors were encountered in the subsequent pilot implementations. Thus, ShapesXR and Gravity Sketch software were tried as a result of searching for a different Metaverse software. Consequently, Gravity Sketch software was selected for the Metaverse application development due to the difficult control and low capabilities of ShapesXR software for the environment design. Then, the activity models designed via Tinkercad software were integrated into Gravity Sketch software, where the necessary visual adjustments and environmental adjustments were made.

As a virtual environment in which teaching geometry was carried out, a Metaverse application was developed using Gravity Sketch software. Considered minimizing the processing speed, simple drawings were preferred for the Metaverse application development. However, some virtual surfaces were covered with figures in order to obtain the most realistic environment. On the walls of the virtual environment that was designed as a loft apartment, activities were as follows: an activity consisting of unit cubes was placed on one wall, an activity involving the area and volume calculations of rectangular prisms was placed on another wall, and an activity involving calculating the volume of a pyramid was placed on another wall, and an activity involving the calculation of cube expansions was placed on the mezzanine floor. Students can navigate between the locations of the activities by teleporting or dragging. The software's layers feature allowed the drawings of activities to be prepared on separate layers. Geometric objects in the activities were locked. In this way, changes made by each student group during the teaching process could take place without deforming the placement and shapes of the objects in the activities. Figure 1 shows an in-application screenshot taken from the teaching process in the Metaverse application developed through Gravity Sketch software.

Fig. 1. In-Application Screenshot of the Developed Metaverse Application

5.4. Implementation Phase

In the implementation phase, the Metaverse application for teaching geometry was introduced to the students and the activities were carried out as planned. During the implementation phase, a set of processes were undertaken to ensure the active participation of students in the teaching process, to solve the difficulties encountered during the process and to support the acquisition of the targeted learning outcomes.

The implementation phase started with the students' orientation training for the virtual reality headsets and the Metaverse application. In the orientation training, students learnt the physical and software functions of

the headset, experienced the use of the buttons on the controllers, and improved their basic manipulation skills both on the objects and on the avatar in Gravity Sketch software. The orientation training was completed in one lesson hour for each group and during the training, students' capability for using the headset with ease during the implementation was aimed at. Figure 2 shows a photograph taken during the students' orientation training for the virtual reality headsets and the Metaverse application.

Fig. 2. Photograph of Students at Orientation Training

Through the use of four virtual reality headsets, teaching activities were carried out by the use of the Metaverse application in groups consisting of one teacher (one of the researchers in this study) and three students. During the process, which lasted eight lesson hours, students completed the activities with group work. Students accessed different activity areas by using teleporting or dragging methods in the Metaverse application. In the activities, students were allowed to make their own discoveries instead of direct guidance. Students, for example, were asked to make volume calculations using unit cubes and they could, in turn, find out the basic formulae for volume calculations. Students in the groups carried out the activities by exchanging ideas with each other and by that means, learning occurred through social interaction. Each student was allowed to learn at his/her own pace within the groups, so that both quick learners and the ones who were in need of more time could get the maximum efficiency from the process. Figure 3 shows a photograph of students while using the developed Metaverse application.

Fig. 3. Photograph of Students Using the Developed Metaverse Application

Technical problems that occurred during the implementation or the points where students had difficulty in the Metaverse application's virtual reality environment were noted down and instant solutions were produced accordingly. To illustrate, for the controller problems regarding the device usage, battery replacement was provided, or re-pairing was performed. Devices that were about to run out of the battery were connected to power banks. All things considered, the implementation phase was completed as a process in which students both experienced the technology and acquired the geometry learning outcomes.

5.5. Evaluation Phase

In order to evaluate the effectiveness of the Metaverse application for teaching geometry, students' opinions were asked at the end of the implementation. Besides, the researcher who guided the students while using the Metaverse application made observations. The evaluation process revealed to which extent the teaching activities were successful and what the students thought about teaching geometry using the Metaverse. Feedback from the students is crucial for understanding the effectiveness of the teaching process and the appropriateness of the Metaverse application for teaching geometry. In the evaluation phase, a semi-structured interview form was applied. This form included questions investigating students' experiences in the Metaverse application, their thoughts about virtual reality headsets, and their learning experiences during the teaching geometry process. These questions are as follows; "What do you think about geometry activities in the Metaverse environment?", "What is your opinion about the contribution of the Metaverse activities you have experienced to your learning geometry?", "Have you experienced any difficulty or problems during the geometry activities in the Metaverse environment? If yes, can you explain? The interviews were designed to find out what the students thought about each activity, which activities were more useful, how they experienced the use of virtual reality headsets, and how the Metaverse application contributed to learning geometry. Through the answers given by the students to the questions in the interview forms, their opinions about the Metaverse application, virtual reality headsets and their experiences in the teaching geometry process were obtained. In addition, observations made by the researcher who guided the students while using the Metaverse application also provided clues about which aspects of the process should be improved.

The data obtained from the semi-structured interviews with the participants were transcribed. Then, the data were analyzed using the content analysis method. Content analysis is an analysis and interpretation process carried out by conceptualizing the collected data, organizing them in line with the emerging concepts and determining themes according to these arrangements (Silverman, 2018).

The codes, categories and themes generated from students' responses about what they think about geometry activities in a Metaverse environment were given in Table 1.

Table 1.
Codes, categories and themes generated from the answers to the first interview question

Number of Students	Codes	Categories	Theme
3	Understanding the subject	Usefulness in understanding the subject	Ease of use and advantages in education
7	Usefulness		
2	Providing learning retention	Facilitating learning	
4	Providing three-dimensional animation		
1	Providing clarity on difficult subjects		
2	Facilitating understanding		
1	Attractiveness	Enjoyment and attraction	
3	Enjoyment		

According to Table 1, it was concluded that students perceived the geometry activities in a Metaverse environment as a tool that facilitated learning and provided learning with fun in the lesson. They expressed such thoughts by saying, "*I think it was useful. Especially in subjects such as prisms that require three-dimensional thinking, it helps students to fit the subject better in their minds*" (P-10) and "*To be honest, it was very fun and logical*" (P-13).

The codes, categories and themes generated from the students' responses about what they think about the contribution of the Metaverse activities they have experienced to their geometry learning were given in Table 2.

Table 2.
Codes, categories and themes generated from the answers to the second interview question

Number of Students	Codes	Categories	Themes
7	Learning the subjects better	Making the subject easier to learn	Motivation and fun-oriented benefits
1	Ease of learning cube expansion		
1	Recognizing the critical points		
1	Understanding unsolved question types		
2	Better visualization of images	Assisting the visualization of three-dimensional objects	
2	Ability to visualize in a simpler way		
1	Learning the subject without distraction	Providing learning with fun and focus on the lesson	
2	Learning the subject in a more permanent and interesting way		

As can be seen in Table 2, it was concluded that the Metaverse activities experienced by the students contributed to their geometry learning by increasing their motivation for learning and enabling them to learn the subject matter. One of the students expressed his thought saying, "Since the virtual environment is three-dimensional, I can now solve the questions much more comfortably because I can see the crucial points that I miss in the questions given in two dimensions" (P-6).

The codes, categories and themes generated from the students' responses about what they think about the difficulties they have experienced during geometry activities in the Metaverse environment were given in Table 3.

Table 3.
Codes, categories and themes generated from the answers to the third interview question

Number of Students	Codes	Categories	Themes
12	Not having difficulties	Not having difficulties	Not having difficulties
1	Nausea	Headache problem	Disadvantages and negative aspects
2	Headache		
2	Difficulty in fitting the headsets	Difficulties caused by not fitting the headsets correctly	
1	Eye fatigue		
2	Blurred vision until getting used to.	Unaccustomed to headset use	
2	Difficulty in switching between activities		

Examining Table 3, it was deduced that there were students who did not experience any difficulty during geometry activities in the Metaverse environment, as well as students who had problems due to eye fatigue and inability to get used to the headset use. The students expressed these thoughts as "*I didn't have any problems*" (P-3), "*Sometimes I saw blurry, and it was a kind of headache towards the end. It was a little difficult to get used to it, but I got used to it anyway*" (P-4) and "*I had some difficulty only when changing places and switching between the activities, but if I were used to it, this would not be a problem, so I don't think it is a major problem*" (P-9).

The theme "ease of use and advantages in education" draws attention to the fact that Metaverse applications in a virtual reality environment stand out in terms of facilitating the teaching process. It can be stated that Metaverse applications in a virtual environment contribute to students' meaningful and permanent learning by concretizing abstract and perplexing subjects thanks to the advantages mentioned by the students under the theme of "ease of use and advantages in education". The importance of virtual reality technology can be noted particularly in terms of providing practicability in a realistic environment for the subjects where learning by doing is challenging. It can be mentioned that this situation enables both teachers and students to use their time efficiently and to make the teaching process more effective and memorable. In addition, it can be stated that students can better comprehend three-dimensional objects and strengthen their mental models thanks to Metaverse applications in the virtual environment in areas such as mathematics, science and geography, which require spatial ability, and the use of three-dimensional visuals is of great importance. While virtual reality technologies facilitate students' mental visualization of abstract concepts, it can be stated that virtual reality technologies take place as an innovative and transformative instructional technology in teaching environments.

The theme of "motivation and fun-oriented advantages" draws attention to the motivation increase and entertaining function of Metaverse applications in the virtual environment. It is considered vital to make the learning process fun in order to enable students to focus their attention better on the lessons and enjoy the learning process. Visually engaging virtual reality applications can facilitate both students' motivation and meaningful learning by active participation in the lesson.

The theme of "having no difficulty" emphasized that most of the students did not have difficulty during geometry activities in the Metaverse environment and that most students could use the Metaverse application in their teaching environments without any problems. Under the theme of "disadvantages and negative aspects", students emphasized the negative effects of using virtual reality technology on social life and learning processes. Blurred vision problem due to the failure in placing the virtual reality headset properly and the possibility of ergonomic discomfort in long-term use were mentioned by the students. However, such problems can be overcome by adapting the technology to the instructional environment in a planned and conscious way, applying customized equipment for individual users on the basis of the user feedback, and limiting the duration of use. Although the negative aspects cannot be completely eliminated before virtual reality headsets are further developed, lightened and redesigned in a way that does not cause other problems, a more conscious virtual reality use can be achieved by raising awareness of the emerging problems.

Based on the students' responses to the interview questions, it can be stated that the Metaverse application is an effective tool for teaching geometry. Students stated that three-dimensional visualizations and interactive processes made abstract geometric concepts more comprehensible. The time spent in the virtual environment increased the students' motivation by making learning fun. However, some students experienced physical discomfort such as headache and eye strain due to using the virtual reality headsets. Although it was difficult to get used to the virtual reality headsets as well as the use of the Metaverse application in the initial stages, the students adapted to it in a short time. The students highlighted that the use of Metaverse should be planned carefully in terms of duration and stated that short-term sessions were more efficient. In the light of the researcher's observations who guided the students while using the Metaverse application, it can be stated that the visualization and interaction features of the Metaverse application accelerated the students' learning process and helped them acquire the course's learning outcomes more easily. Furthermore, it can be stated that during the implementation of the application, students participated in the activities enthusiastically and they were highly into the process while carrying out tasks in the application.

6. Discussion, Results and Suggestions

The development process of Metaverse application for teaching geometry was discussed in this study. The results demonstrated that virtual reality and Metaverse technologies could be effective in teaching abstract subjects such as geometry. Studies in literature (Abraham et al., 2023; Camilleri, 2024; Johnson et. al., 2016; Kaddoura & Al Husseiny, 2023; Sarıtaş & Topraklıkoğlu, 2022; Topraklıkoğlu & Öztürk, 2023a) also demonstrated that Metaverse applications offer students different and effective learning experiences and increase students' motivation. For instance, Johnson et al. (2016) revealed that virtual reality-based teaching methods were effective in increasing student engagement. Similarly, the findings of this study also demonstrated that geometry lessons in a virtual reality environment increased students' interest and participation in the lessons. The finding of this study that Metaverse applications increase students' motivation to learn is in line with Gao and Lee's (2024) finding that Metaverse-based teaching approach had positive effects on learning motivation, perception of presence, student achievement and learning satisfaction, which is mentioned in their study aiming to investigate the effect of using Metaverse platform in education. Some students experienced difficulties in using Metaverse technology. However, students having difficulties also adapted to using virtual reality technology in a short time.

Zahabi and Abdul Razak (2020) stated that virtual reality environments might be complex for some students in the initial stages, but these difficulties could be overcome as the process of using technology proceeds. Participants of this study also expressed similar opinions to the above-mentioned statement of Zahabi and Abdul Razak (2020). The results of the study indicated that activities for learning geometry in the Metaverse application facilitate understanding abstract geometric concepts. This finding was also emphasized in the study of Hsu (2021), who stated that virtual reality-based educational materials could present abstract mathematical concepts to students in a more concrete and comprehensible way and could, in turn, increase the students' level of achievement.

Teachers aiming to use virtual reality devices in their lessons can make classroom management easier by conducting small group studies. Students newly accustomed to using virtual reality devices can adapt to using the technology more easily through peer-learning and collaborative work. It could also make it easier for teachers to respond to each student's questions and requests. It is crucial to be proficient at the application in use not only for solving technical problems experienced in using the Metaverse application during the teaching process but also for the capability to find immediate alternative solutions in case of a problem.

Metaverse applications, which are mostly used for games and entertainment purposes, and which find a place in application markets as games and auxiliary tools, are becoming more accessible as time goes by (Topraklıkoğlu & Öztürk, 2023b). However, it is noteworthy that educational applications are not yet available in application markets. In that case, teachers willing to use a Metaverse application only for a certain subject in their own lessons have to develop the subject-related application by themselves or have

it developed by the ones with software knowledge (Wu & Hao, 2023). Given this situation, the sustainability of Metaverse applications in terms of their use for teaching purposes is negatively affected. However, as virtual reality devices get more accessible and their use for educational purposes increases, Metaverse applications for educational purposes will also be more common, as in the case with the educational computer programs.

In conclusion, this study demonstrated that Metaverse applications for virtual reality-based geometry teaching had a significant potential in education and offered an alternative approach to conventional teaching methods. Students expressed their opinions on this issue by stating that thanks to the Metaverse application, they could easily see and better understand the relations that they did not understand in the drawings encountered in geometry lessons at school. This finding coincides with the findings of Accascina and Rogora (2006) as well as Topraklıkoğlu and Öztürk (2021) that the Metaverse application can be a solution to the problem of students' difficulty in perceiving three-dimensional objects when three-dimensional objects are shown in a two-dimensional plane in textbooks or on the blackboard. In addition, it also coincides with the finding of Eşin and Özdemir (2022) where the participants' opinions about the use of Metaverse in teaching mathematics were positive and where they expressed that Metaverse could be used mainly for concretization in geometry and that it can be useful. However, to achieve a more efficient and effective use of these technologies in classroom environments, teachers and students are supposed to get adapted to technology. In that sense, it is considered that more training and support should be provided (Aydın & Şahin, 2021). Additionally, in order for virtual reality-based Metaverse applications to become widespread in teaching environments and to attract a wider user base, it is important for virtual reality technology to become more user-friendly, more accessible, and more affordable. On the other hand, in order for the use of virtual reality technology to gain sustainability in teaching environments, educational administrators' and investors' support should increase, and teachers' access to virtual reality technologies and Metaverse applications should be provided.

As a suggestion, it can be stated that further studies should be conducted with students in different age groups and the studies should be centered on the implementation of Metaverse applications for teaching geometry through different teaching approaches. That is due to the fact that the advantages or the disadvantages that will be offered by the use of virtual reality devices for learning geometry at different ages as well as different education levels are unpredictable. Depending on the features of the Metaverse application, the control of the virtual reality devices can be accomplished with diverse difficulties. Accordingly, virtual reality device control can also be handled by elementary school students with various difficulty levels. On the other hand, born into digital technology, these children's experience with technological devices since their early childhood can result in an unexpectedly rapid adaptation to virtual reality technology as well. Taking both possibilities into account, it will gain more significance for teachers to plan appropriate technological and pedagogical content based on their students' level of readiness. On the other hand, customizing Metaverse applications to provide education will increase the impact of Metaverse and virtual reality technology in education. Giving in-service training to teachers on the use of virtual reality and Metaverse applications can also make educational processes more efficient.

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