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ESNEK BİRLEŞİMLERİN ÇAPRAZSIZ ÇELİK KOLONLARIN EFEKTİF UZUNLUK FAKTÖRÜ VE KRİTİK ELASTİK BURKULMA YÜKÜNE ETKİSİ

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ÖZET

Bu çalışmada, çelik kolonlar için efektif uzunluk faktörü (K-faktörü) ve kritik elastik burkulma yükü ifadeleri esnek çelik kiriş kolon birleşimlerinin birleşim karakterleri dikkate alınarak elde edilmiştir. Çelik kiriş kolon birleşimlerinin esnek olarak modellenmesi eşdeğer dönel yaylar ile sağlanmış ve analitik ifadeler rijit ve esnek birleşim durumları için ayrı ayrı elde edilmiştir. Kiriş kolon birleşimlerin moment-dönme karakterlerinin efektif uzunluk faktörü ve kritik elastik burkulma yükü üzerindeki etkileri nümerik olarak çalışılmış ve esnek birleşimlerin çelik kolon davranışı üzerine olan etkileri incelenmiştir.

Anahtar Kelimeler: *Efektif Uzunluk Faktörü, Kolon Burkulması, Esnek Birleşim*

THE EFFECT OF FLEXIBLE JOINTS ON THE EFFECTIVE LENGTH FACTOR AND CRITICAL ELASTIC BUCKLING LOAD OF UNBRACED STEEL COLUMNS

ABSTRACT

In this study, the effective length factor (K-factor) and critical elastic buckling load of steel columns are derived considering the effects of the flexible joint characteristics of steel beam-to-column connections. Steel beam-to-column connections are modeled flexibly by using equivalent rotational springs and analytical expressions are presented for rigid and flexible cases separately. The effects of the moment-rotation characteristics of connections on the effective length factor and critical elastic buckling load are numerically studied and the effect of flexible connections on the steel column behavior is examined.

Keywords: *Effective Length Factor, Column Buckling, Flexible Joint*

1. INTRODUCTION

In many specifications and design codes for steel structures, the effective column length factor is used for column buckling checks or takes part in second order analysis [1-3]. The effective length factor and buckling load derivation is rather simple, while the moment–rotation characteristics of the joints can be directly introduced through the boundary conditions. This applies within the elastic range, where the moment–rotation relation is linear and the joint rotational stiffness can be considered as constant, while the normal and shear deformation of the joint is proven to be negligible [4].

In last twenty years, numerous experimental and analytical models have been developed for analysis of steel frames to fulfill the requirements of realistic steel column behavior [5]. In classical theory of structures, connections of members in joints of linear systems are most often idealized as ideally pinned or absolutely rigid. Many worldwide investigations, based on experimental results and numerical simulations have pointed out that the great number of connections of members in joints of linear systems can be classified neither as ideally pinned nor as perfectly rigid [6]. The joint behavior in between these extremes is called semi-rigid and has gained interest to reflect the realistic behavior of steel frames [7]. The influence of semi-rigid connections on structural response not only changes the moment distribution in the beams and columns, but also increases the drifts on steel frames [8].

In this paper, the governing equations for determining column length factor for flexibly jointed and unbraced frames are derived in a similar way as they are for rigidly jointed unbraced frames. A practical linear stability analysis for establishing the effect of joint flexibility on the buckling load of columns in steel plane frames is presented herein.

2. EFFECTIVE LENGTH FACTOR AND BUCKLING LOADS

The effective length concept is one method of estimating the interaction effects of the total frame on a compression element being considered [9]. This concept uses K-factors to equate the strength of a framed compression element of length L to an equivalent pin-ended member of length KL subject to axial load only. Although the effective length factor concept is completely valid for ideal structures, its practical implementation involves several assumptions of idealized conditions [10].

In order to understand the significant role of flexible connections on the buckling load of the unbraced steel columns, analytical solutions of the linear buckling behavior of unbraced rigid steel columns is investigated on the first step. The model used for the determination of effective length factor (K-factor) for a framed column subjected to sidesway, is shown in Figure 1. The column in question is denoted by center column (Cc) with two restraining columns (Tc and Bc) and four restraining beams (TLb, TRb, BLb, BRb) which can be rigidly or flexibly connected to the beams at joints A and B.

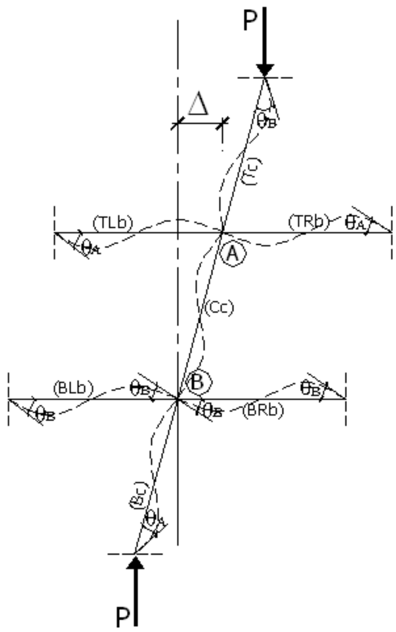

Figure 1. Unbraced Column Model

The assumptions used for the model are given below:

1- All members in rectangular multi-bay multi-storey frames are prismatic and behave elastically.
2- The axial forces in the beams are negligibly small.
3- All columns in a story buckle simultaneously.
4- Stability functions for all columns in the sub assemblage model are identical.
5- At a joint, the restraining moment provided by the beams are distributed among the columns in proportion to their stiffness.
6- At buckling, the rotations at the near and far ends of the beams are assumed to be equal in magnitude and direction (double curvature bending).

The slope-deflection equations for the column members are as follows:

For Top Column:

$$M_{A,Tc} = \left(\frac{EI}{L}\right)_{Tc}\left(s_{ii}\theta_A + s_{ij}\theta_B - \left(s_{ii}+s_{ij}\right)\frac{\Delta}{L_{Tc}}\right) \tag{1}$$

For Centre Column:

$$M_{A,Cc} = \left(\frac{EI}{L}\right)_{Cc}\left(s_{ii}\theta_A + s_{ij}\theta_B - (s_{ii}+s_{ij})\frac{\Delta}{L_{Cc}}\right) \quad (2)$$

$$M_{B,Cc} = \left(\frac{EI}{L}\right)_{Cc}\left(s_{ij}\theta_A + s_{ii}\theta_B - (s_{ii}+s_{ij})\frac{\Delta}{L_{Cc}}\right) \quad (3)$$

For Bottom Column:

$$M_{B,Bc} = \left(\frac{EI}{L}\right)_{Bc}\left(s_{ij}\theta_A + s_{ii}\theta_B - (s_{ii}+s_{ij})\frac{\Delta}{L_{Bc}}\right) \quad (4)$$

where; E: Modulus of elasticity, I: Moment of inertia, L: Member length , s_{ii} and s_{ij}: Stability functions and Δ: Lateral drift.

Since the second order effects can be neglected for beam members, the slope-deflection equations for the beam members are as follows:

For Top Left Beam:

$$M_{A,TLb} = \left(\frac{EI}{L}\right)_{TLb}(4\theta_A + 2\theta_A) \quad (5)$$

For Top Right Beam:

$$M_{A,TRb} = \left(\frac{EI}{L}\right)_{TRb}(4\theta_A + 2\theta_A) \quad (6)$$

For Bottom Left Beam:

$$M_{B,TLb} = \left(\frac{EI}{L}\right)_{TLb}(4\theta_B + 2\theta_B) \quad (7)$$

For Bottom Right Beam:

$$M_{B,TRb} = \left(\frac{EI}{L}\right)_{TRb}(4\theta_B + 2\theta_B) \quad (8)$$

Since we write the joint equations for joint A and joint B and substitute the moment equations in order to write the story sway equilibrium, we obtain:

$$\begin{bmatrix} s_{ii}+\dfrac{6}{G_A} & s_{ij} & -\left(s_{ii}+s_{ij}\right) \\ s_{ij} & s_{ii}+\dfrac{6}{G_B} & -\left(s_{ii}+s_{ij}\right) \\ -\dfrac{6}{G_A} & -\dfrac{6}{G_B} & (kL)^2_{Cc} \end{bmatrix}\begin{bmatrix} \theta_A \\ \theta_B \\ \dfrac{\Delta}{L_{Cc}} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

For non trivial solution we obtain:

$$\frac{G_A G_B (\pi/K)^2-36}{6(G_A+G_B)}-\frac{(\pi/K)}{\tan(\pi/K)}=0 \tag{10}$$

The ratio of column stiffnesses meeting at joints to beam stiffnesses meeting at joints is called relative stiffness factors and can be written separately for each joint as below:

$$G_A=\frac{\sum_A\left(\frac{I}{L}\right)_c}{\sum_A\left(\frac{I}{L}\right)_b}=\frac{\text{sum of column stiffnesses meeting at joint A}}{\text{sum of beam stiffnesses meeting at joint A}} \tag{11}$$

$$G_B=\frac{\sum_B\left(\frac{I}{L}\right)_c}{\sum_B\left(\frac{I}{L}\right)_b}=\frac{\text{sum of column stiffnesses meeting at joint B}}{\text{sum of beam stiffnesses meeting at joint B}} \tag{12}$$

In Figure 2, equation (10) is expressed as in a nomograph form like in many national codes and specifications. In order to determine the K factor using nomograph, G_A and G_B at the ends of the column in question are determined and a straight line joining the appropriate points on the G_A and G_B scales is then drawn. The value of the effective length factor K is obtained as the intersection of the straight line with the K scale.

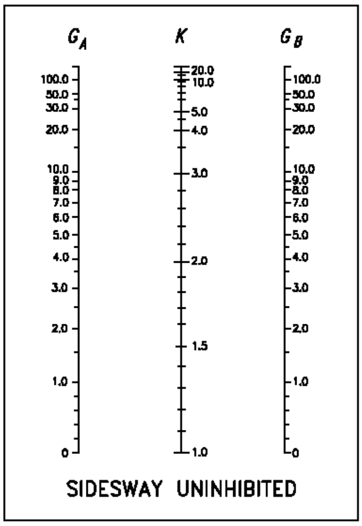

Figure 2. Nomograph for Sidesway Uninhibited Columns [11]

After determining the K factor from the nomograph, the effective length can be calculated as follows:

$$L_k = K\,L \qquad (13)$$

Also, critical elastic buckling load of unbraced steel columns can be calculated by using effective length of column:

$$P_{cr} = \frac{\pi^2 EI}{L_k^2} \qquad (14)$$

where, EI is the flexural rigidity of column.

3. MODIFICATIONS FOR PINNED END CONDITIONS

In using the effective length factor, the engineer should be aware of the assumptions used in the development of these charts. If the actual conditions do not accord with the assumptions used, the K values obtained may be erroneous. In this study, some simple modifications to the nomographs to relax some assumptions used in their development are discussed. These modifications are introduced to enhance the versatility of the charts in determining K factors for framed columns.

If the far end of the restraining beam is hinged, it can be shown readily by using the slope-deflection equation that the moment-rotation relationship at the frame end is given below:

$$M = \frac{3EI}{L}\theta \tag{15}$$

For an unbraced frame (sidesway uninhibited case), the assumption used was that the rotations at the framed end and the far end are equal:

$$M = \frac{6EI}{L}\theta \tag{16}$$

Comparison of the equations for pinned case, it can be seen that the effect of the hinge can be taken into account by decreasing the beam stiffness by a factor of 0.5 for an unbraced frame.

4. MODIFICATIONS FOR FLEXIBLE BEAM END CONNECTIONS

There is a large amount of work dealing with rigidly jointed frameworks [12-15]. However, with the presence of steel beam-to-column connections it is necessary to modify the equations slightly to take account of the connection flexibility which will have a noticeable influence on the strength of the frame. The influence of connection flexibility can easily be adopted to slope deflection equations as shown in Figure 3.

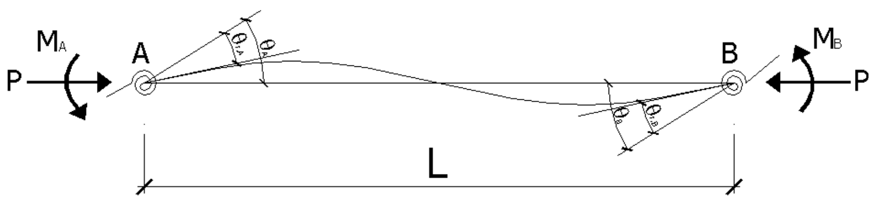

Figure 3. Beam Element with Flexible Connections

The beam-to-column connections are modeled by rotational springs. The presence of rotational springs introduces relative rotations of $\theta_{r,A}$ and $\theta_{r,B}$ at the ends of the beam [16]. The relative rotations can be written as below:

$$\theta_{r,A} = \frac{M_A}{R_{k,A}} \tag{17}$$

$$\theta_{r,B} = \frac{M_B}{R_{k,B}} \tag{18}$$

where $R_{k,A}$ and $R_{k,B}$ are elastic stiffness of flexible connections at the ends of beam

If the ends of the beam are not rigidly but flexibly connected to other members and the second order effects for beam elements are neglected, the slope-deflection equations can be written as below [17]:

$$M_A = \left(\frac{EI}{L}\right)\left(4\left(\theta_A - \frac{M_A}{R_{k,A}}\right) + 2\left(\theta_B - \frac{M_B}{R_{k,B}}\right)\right) \tag{19}$$

$$M_B = \left(\frac{EI}{L}\right)\left(2\left(\theta_A - \frac{M_A}{R_{k,A}}\right) + 4\left(\theta_B - \frac{M_B}{R_{k,B}}\right)\right) \tag{20}$$

If the equations are solved;

$$M_A = \left(\frac{EI}{LR^*}\right)\left(\left(4 + \frac{12EI}{LR_{k,B}}\right)\theta_A + 2\theta_B\right) \tag{21}$$

$$M_B = \left(\frac{EI}{LR^*}\right)\left(2\theta_A + \left(4 + \frac{12EI}{LR_{k,A}}\right)\theta_B\right) \tag{22}$$

$$R^* = \left(1 + \frac{4EI}{LR_{k,A}}\right)\left(1 + \frac{4EI}{LR_{k,B}}\right) - \left(\frac{EI}{2}\right)^2\left(\frac{4}{R_{k,A}R_{k,B}}\right) \tag{23}$$

Assuming that the beam ends have equal initial stiffness $R_{k,A}=R_{k,B}=R_k$ and we have for an unbraced frame $\theta_B = \theta_A$, so the equations becomes:

$$M = \frac{6EI}{L}\left(\frac{1}{1 + 6\,EI/LR_k}\right)\theta_A \tag{24}$$

Upon comparing equations of rigid and flexible connections; it can be seen that the effect of connection flexibility can be taken account by modifying the beam stiffness of unbraced frame by a factor of $1/(1+6EI/LR_k)$.

5. BEAM-TO-COLUMN CONNECTION MODELING

In conventional analysis and design of steel frames, beam-to-column connections are assumed absolutely rigid or ideally pinned in order to simplify the tedious analysis. The rigid joint assumption implies that slope continuity exists between adjoining members, and that fully moment is transferred from the beam to the column. On the other hand, the assumption of ideally pinned connections implies that beams will behave as simply supported members and that the columns will carry no moments from beams. Although these assumptions simplify the analysis and design procedures, the validity is questionable. The experimental observations shows that all connections used in practice possess stiffness that fall between the extreme cases of fully rigid and ideally pinned [18].

5.1. Nonlinear Moment-Rotation Behavior

The deformation behavior of a beam-to-column connection can be described by a nonlinear moment-rotation relationship. If a moment, M, is applied to a connection, it rotates by an amount θ_r. The rotation represents the change in the angle between the beam and the column from its original configuration as seen in Figure 4.

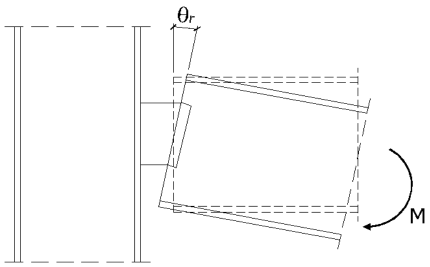

Figure 4. Moment-Rotation of Beam-to-Column Element

5.2. Flexible Connection Modeling

Many experimental studies on steel beam-to-column connections have been performed, and a huge moment-rotation data has been collected [19]. Researchers have developed several beam-to-column connection models; linear, polynomial, power, and exponential models [20]. In this study, the three-parameter power model is adopted. The power model contains three parameters: initial connection stiffness (R_{ki}), ultimate connection moment capacity (M_u), and shape parameter (n) [21]. The power model may be written as:

$$m = \frac{\theta}{\left(1+\theta^n\right)^{1/n}} \quad \text{for} \quad \theta > 0 \quad \text{and} \quad m > 0 \qquad (25)$$

where $m=M/M_u$, M_u: ultimate moment capacity of the connections, and $\theta= \theta_r/\theta_0$, θ_0:reference plastic rotation.

When connection is loaded, the connection tangent stiffness (R_{kt}) at an arbitrary rotation can be derived by simply differentiating equation above as:

$$R_{kt} = \frac{dM}{d|\theta_r|} = \frac{M_u}{\theta_0 (1+\theta)^{1+1/n}} \quad (26)$$

Also, when the connection is not loaded, the tangent stiffness is equal to the initial connection stiffness.

6. NUMERICAL STUDY

In order to demonstrate the differences in evaluating the effective length factor (K-Factor) and critical elastic buckling load of an unbraced steel column with rigid and semi-rigid beam-to-column connections, a numerical example is investigated. The modulus of elasticity is assumed as 205000 MPa and all beams are W18x55, columns are W18x76 which is given in Figure 5.

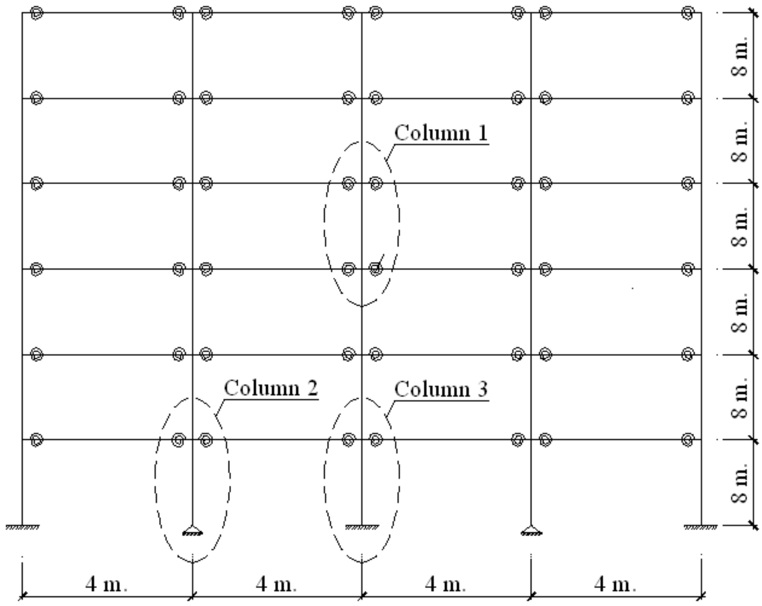

Figure 5. Geometry of Unbraced Steel Frame

The unbraced steel frame is based with both pinned and fixed supports. The beam-to-column connections are modeled as rotational springs physically tied to the ends of the beam elements. The initial connection stiffness is adopted as the elastic spring constant for the flexible beam-to-column connection, which can be obtained by a first order elastic analysis considering moment-rotation relationship of connections. In this study, two of most common semi-rigid connection types; double web angles, top and seat angles with double web angles are used for semi-rigid connections with three parameter power model.

Also, all beam-to-column connections are formed by using L 6x4x3/4 profiles as shown in Figure 6.

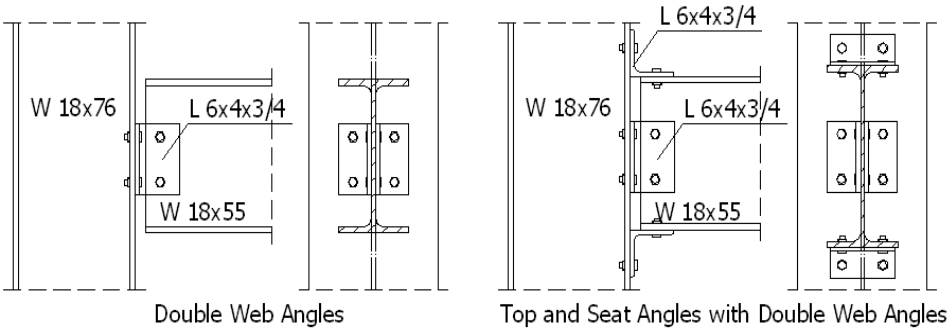

Figure 6. Beam-to-Column Connection Models

The moment-rotation curves of double web angles, top and seat angles with double web angles are given in the Figure 7.

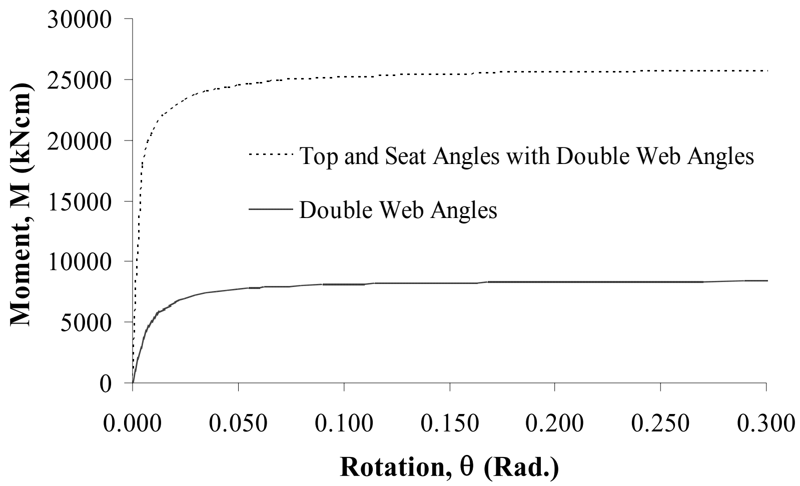

Figure 7. Moment-Rotation Curves of Beam-to-Column Connections

The effective length factor (K-Factor) and critical buckling load for pinned, double web angles, top and seat angles with double web angles and rigid connection assumptions are given in Table 1.

Table 1. Effective Length Factors and Buckling Loads for Various Types of Connections

Beam-to-Column Connection Type	Effective Length Factor (K-Factor)			P_{cr} (kN)		
	Column1	Column2	Column3	Column1	Column2	Column3
Hinge Assumption	1.556	2.081	1.433	7229	4041	8523
Double Web Angles	1.556	2.049	1.413	7655	4169	8766
Top & Seat Angles with Double Web Angles	1.439	1.993	1.377	8452	4406	9230
Rigid Assumption	1.298	1.888	1.307	10388	4910	10245

7. CONCLUSIONS

Based on the realistic beam-to-column modeling, the study shows that joint flexibility is a very important parameter that needs to be incorporated into the stability analysis of frames with flexible beam-to-column connections. Numerical results for simple elastically unbraced frames with semi-rigid connections reveal the effect of joint flexibility on the buckling load. It is found that assuming flexible connections in steel columns always leads to a reduction of their buckling load, which is proven to be significant in the numerical case.

The conclusions of this study dealing with the effect of flexible joints on the buckling load of unbraced steel columns with various beam-to-column connection configurations can be summarized as follows:

1. A method for determining the effect of joint flexibility on the buckling load of steel plane frames is proposed. This technique is very simple and can be employed by the average engineer for the determination of the buckling load of framed columns with the aid of a calculation spreadsheet or easy software.
2. Through the analysis of simple steel columns; it is found that the combined effect of joint flexibility on the buckling load is always unfavorable and in most cases very significant.
3. Code specifications can be extended to include the effect of joint flexibility on the buckling load of framed compression columns.
4. Analytical solutions of the linear buckling behavior of unbraced steel columns can serve as a powerful tool for understanding the significant role of flexible connections on the buckling load of the unbraced steel columns.

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REFERENCES

[1] L.R.F.D., Load and Resistance Factor Design Specification for Structural Steel Buildings, American Institute of Steel Construction Inc., Chicago, (1999).

[2] Eurocode 3 Design of Steel Structures, Part 1.1: General Rules and Rules for Buildings, CEN Document (1994).

[3] TS 648, Çelik Yapıların Hesap ve Yapım Kuralları, T.S.E. (1980).

[4] Chen, W. F., Goto, Y. and Liew, R., "Stability Design of Semi Rigid Frames", Wiley, New York, 35-70 (1993).

[5] Kishi, N. and Chen, W.F., "Moment–Rotation Relations of Semi-Rigid Connections with Angles", ***Journal of Structural Engineering***, ASCE, 116 (7): 1813–1834 (1990).

[6] Faella, C., Piluso, V. and Rizzano, G., "Structural Steel Semi Rigid Connections", CRC Press, Florida, 1-50 (2000).

[7] Hasan, R., Kishi, R. and Chen, W.F., "A New Nonlinear Connection Classification System", ***Journal of Constructional Steel Research***, 47: 119-140 (1998).

[8] Seçer, M. ve Bozdağ, Ö., "Yarı-Rjit Birleşimlerin Üç Boyutlu Yapı Davranışına Etkisi", ***6th International Congress on Advances in Civil Engineering***, Bogazici University, Istanbul, (2004).

[9] Chen, W.F. and Kim, S.E., "LRFD Steel Design Using Advanced Analysis", CRC Press, New York, 250-350 (1997).

[10] Gantes C.J. and Mageirou, G.E., "Improved Stiffness Distribution Factors for Evaluation of Effective Buckling Lengths in Multi-Story Sway Frames", ***Engineering Structures***, 27: 113-124 (2005).

[11] Chen, W.F. and Lui, E.M., "Stability Design of Steel Frames", Boca Raton Florida, CRC Press, 66-76 (1991).

[12] Timoshenko, S.P. and Gere, J., "Theory of Elastic Stability", Mc-Graw Hill, New York, 10-50 (1961).

[13] Livesley, R.K., "Matrix Methods of Structural Analysis", Pergamon Press, Great Britain, 30-50 (1975).

[14] Bazant, Z.P. and Cedolin, L., "Stability of Structures: Elastic, Inelastic, Fracture and Damage Theories", Oxford University Press, New York, 5-20 (1991).

[15] Chan, S.L., "Non-linear Behavior and Design of Steel Structures", ***Journal of Constructional Steel Research***, 57: 1217-1231 (2001).

[16] Lorenz, R.F., Kato, B. and Chen, W.F., "Semi Rigid Connections in Steel Frames", McGraw-Hill, New York, 100-150 (1993).

[17] Hadianfard, M.A. and Razani, R., "Effects of Semi-Rigid Behavior of Connections in the Reliability of Steel Frames", ***Structural Safety***, 25: 123-138 (2003).

[18] Liew, J.Y.R., Yu, C.H., Ng, Y.H. and Shanmugam, N.E., "Testing of Semi-Rigid Unbraced Frames for Calibration of Second-order Inelastic Analysis"**, *Journal of Constructional Steel Research***, 41: 159-195 (1997).

[19] Chen, W.F. and Kishi, N., "Semi-Rigid Steel Beam-to-Column Connections: Data Base and Modeling", ***Journal of Structural Engineering***, ASCE, 115: 105-119 (1989).

[20] Chan, S.L. and Chui, P.P.T., "Static and Cyclic Analysis of Semi Rigid Steel Frames", 200-300 Elsevier Science, (2000).

[21] Lee, S.S. and Moon, T.S., "Moment–Rotation Model of Semi-Rigid Connections with Angles", ***Engineering Structures***, 24: 227-237 (2002).