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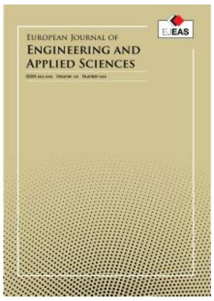

Research Article

A Liénard Oscillator Circuit with a Memristive Bridge Rectifier

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Abstract: A novel Liénard oscillator design incorporating a bridge rectifier with an R-L-C output filter and negative resistance converter is presented. The bridge rectifier exhibits memristive behavior and provides the even nonlinear resistance required for Liénard oscillation in the periodic steady state. The circuit's mathematical model, including state-space equations and analysis of its nonlinear characteristics, is developed. LTSpice simulations demonstrate that at higher frequencies, the memristive bridge functions as a nonlinear resistor, enabling the circuit to operate as a Liénard oscillator. The simulation results show periodic waveforms and limit cycles characteristic of Liénard systems, though with notable deviations from ideal sinusoidal behavior due to the Schottky diodes' threshold effects and second harmonic generation. While the proposed oscillator has more state variables than traditional two-variable Liénard systems, it successfully achieves sustained oscillations using readily available components: Schottky diodes, passive elements, and an operational amplifier. The circuit's behavior is analyzed through voltage-current characteristics, limit cycles, and phase portraits, confirming its operation as a Liénard-type oscillator. This work opens new possibilities for implementing Liénard oscillators using semiconductor elements and suggests directions for future research in parametric analysis and analytical solutions.

Keywords: *Liénard Oscillator, Schottky Diode Bridge Rectifier, R-L-C Filter, Limit Cycle, Circuit Dynamics.*

Memristif Köprü Doğrultuculu Bir Liénard Osilatör Devresi

Öz. *Bu çalışmada* köprü doğrultucu ve R-L-C çıkış filtresi ile negatif direnç dönüştürücü içeren yeni bir Liénard osilatör tasarımı önerilmiştir. Köprü doğrultucu, memristif davranış sergilemekte ve periyodik kararlı durumda Liénard salınımı için gerekli olan çift doğrusal olmayan direnci sağlamaktadır. Devrenin matematiksel modeli, durum-uzay denklemleri ve doğrusal olmayan özelliklerinin analizi dahil olmak üzere geliştirilmiştir. LTSpice simülasyonları, daha yüksek frekanslarda memristif köprünün doğrusal olmayan bir direnç olarak işlev gördüğünü ve devrenin bir Liénard osilatörü olarak çalışmasını sağladığını göstermektedir. Simülasyon sonuçları, Liénard sistemlerine özgü periyodik dalga formları ve limit döngüleri göstermektedir, ancak Shottky diyotlarının eşik etkileri ve ikinci harmonik üretimi nedeniyle ideal sinüzoidal davranıştan kayda değer sapmalar görülmektedir. Önerilen osilatör, geleneksel iki değişkenli Liénard sistemlerinden daha fazla durum değişkenine sahip olmasına rağmen, kolayca temin edilebilen bileşenler kullanarak (Schottky diyotları, pasif elemanlar ve bir işlemsel yükselteç) sürekli salınımları başarıyla elde etmektedir. Devrenin davranışı, gerilim-akım karakteristikleri, limit döngüleri ve faz portreleri aracılığıyla analiz edilmiş, Liénard tipi bir osilatör olarak çalıştığı doğrulanmıştır. Bu çalışma, Liénard osilatörlerinin yarı iletken elemanlar kullanılarak uygulanması için yeni olanaklar sunmakta ve parametrik analiz ve analitik çözümler konusunda gelecekteki araştırmalar için farklı bakış açıları önermektedir.

Anahtar kelimeler: *Liénard Osilatörü, Schottky Diyot Köprü Doğrultucu, R-L-C Filtresi, Limit Döngüsü, Devre Dinamikleri*

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1. Introduction

The Van Der Pol Oscillator (VDPO), first created in 1920 with a triode vacuum tube, is a well-known and extensively studied oscillator [1]. The stable oscillations discovered in the triode circuit are known as relaxation oscillations, a concept introduced by Van der Pol [2]. This circuit also demonstrates frequency de-multiplication, an example of deterministic chaos [3]. The discovery of this oscillator significantly contributed to the development of nonlinear oscillator theory [4]. French physicist Alfred-Marie Liénard introduced equations to model oscillating circuits [5]. The Van der Pol Oscillator is described by the Van der Pol equation, which is a special case of the Liénard equations. Many studies have been conducted on the Liénard Oscillator and Liénard equations, which have been applied across diverse fields such as electronic oscillators, radio engineering, chemical reactions, optoelectronic systems, lasers, predator-prey studies, population studies, biological studies, and vibration modeling [6-9]. The dynamics of coupled Van der Pol oscillators are commonly studied [10-12]. The synchronization of such oscillators is an important research area in natural sciences and engineering [13-19]. Van der Pol or Liénard's oscillators are used to examine chaos phenomenon [20-24].

Although semiconductor circuit elements were not available when these oscillators were first developed, modern Liénard Oscillators can be built using various semiconductor components [25–27]. For instance, an optoelectronic integrated circuit with a resonant tunneling diode and an optical communications laser diode operates as a voltage-controlled oscillator as described by Liénard's equation [25]. The nonlinear memristor element can also be used in Liénard systems, exhibiting complex behaviors such as hidden attractors and mixed-mode oscillations [11]. A reverse-parallel Schottky diode array-based VDPO has been developed, with its differential equation and waveforms examined through simulations using Simulink and LTspice programs [27]. In [28], a Liénard Oscillator which makes use of a Chua diode combining of a JFET and Schottky diode bridge has been made, and it was shown to behave as a Liénard Oscillator analytically and with simulations. In [29], it is shown that a cheap microcontroller such as Arduino Nano Klon V3.0 can be used to make a Liénard Oscillator.

Memristors are newly found nonlinear circuit elements [30-32]. Nonlinear resistors with a state-variable dependency are also called memristive systems [33]. Memristors have specific features known as the three fingerprints [33, 34]. Memristor can be used in oscillators [35, 36]. Most of the memristor-based oscillators studies are of chaotic nature [35, 37, 38]. Since it has been claimed that the memristor-based Liénard systems or oscillators show very rich dynamics [39], it is imperative to examine them. For example, in [40], it has been shown that a Liénard Oscillator with a memristor emulator with a cubic characteristic shows mixed-mode oscillations with simulations and experiments. In [41], it has been shown that, in some of its operation region, a rectifier with an R-L-C circuit at its output behaves as if a memristive system. Corinto and Ascoli [41] demonstrated that the electronic system they designed using only passive components behaved like a memristor. They showed through PSpice simulations that an RLC filter circuit powered by a full-wave rectifier exhibited memristive properties and should have a pinched hysteresis curve with zero-crossing. Inspired by this study, other researchers used an RC filter instead of an RLC filter to obtain a hysteresis curve with fewer circuit elements, and they used such an emulator in the construction of a chaos circuit [42]. The memristive circuit given in [41] can also provide an even memristance function in the periodic steady state. In the literature, this property has not been used to make a Liénard Oscillator yet. In this study, a rectifier with an R-L-C circuit at its output has been used to make a Liénard Oscillator for the first time in the literature. Its analytical model has been given. The simulations and the experimental results have been used to prove that the rectifier-based oscillator circuit operates as a Liénard oscillator in the steady-state. In [43], it has been shown that two anti-parallel memristors sold in the market [44] can be employed to make a Liénard Oscillator. since it provides an even memristance function in the periodic steady state. A rectifier is shown to behave as a time-variant nonlinear resistor or as a memristor in some of its operation region, its state-space equations are given, and simulation and experimental results of proof of concept are given in [41]. The bridge rectifier has also been used to make chaotic generators [42]. Employing the memristive bridge is cheaper than using two memristors in an application [45]. Such a circuit can also provide an even memristance function in the periodic steady state. To the best of our knowledge, such a bridge rectifier has not been used to make a non-chaotic oscillator or a Liénard Oscillator yet.

In this study, the nonlinear circuit element required for the Liénard Oscillator is made by connecting a rectifier-based nonlinear resistor, a negative resistance converter, an inductor, and a capacitor in parallel. This Liénard Oscillator is implemented with off-the shelves components. The circuit uses a diode bridge rectifier, 4 LTI resistors, 2 LTI inductors, 2 LTI capacitors, and an op-amp. In this study, it is also to be shown that the diode bridge with an R-L-C filter has a nonlinear resistance function, which is an even function in the steady-state, and allows the circuit to operate as a Liénard Oscillator. The state-space equations of the Liénard Oscillator are given, and its simulations are made in LTSpice design program.

This study is organized in the following order. In the second section, basic information on Liénard Equation and Liénard Systems is given, the new Liénard Oscillator circuit topology is introduced, its operation principles are explained, and its dynamic model is given. In the third section, the simulation results of the circuit obtained with a SPICE based circuit design program are presented, and it is proven that this oscillator is a Liénard-like oscillator using circuit simulations. The paper concludes with the last section.

2. Generic Liénard Oscillator Circuit and the Bridge Rectifier-Based Liénard Oscillator

In this section, the generic Liénard Oscillator is firstly summarized, and then the Liénard Oscillator circuit proposed in this study is introduced. A Liénard Oscillator is made of a nonlinear resistor that may take negative and positive values, a capacitor, and an inductor. The structure of its nonlinear resistor and the new oscillator are explained in the following subsections. The circuit of the generic Liénard Oscillator circuit is shown in Figure 1 [27].

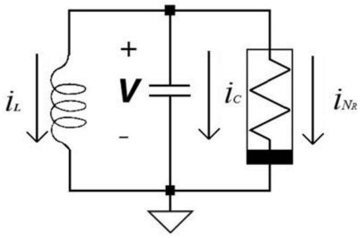

Figure 1. *The generic Liénard Oscillator [27]*

2.1. The Negative Resistance Converter Circuit

Negative impedance or negative resistance converter refers to circuits used to create negative resistance. The op-amp-based negative resistance converter employing three resistors, R_1, R_2, and R_3, used in this study is shown in Figure 2. Based on the theoretical perspective:

The input current of the negative resistance converter circuit is given as

$$i_{Neg} = g_N v(t) = -\frac{R_2}{R_1 R_3} v(t) \tag{1}$$

If $R_3 = R_2$, this current turns into

$$i_{Neg} = g_N v(t) = -\frac{v(t)}{R_1} \tag{2}$$

The conductance of the negative resistance converter is given as

$$g_N = v(t)/i_{Neg} = -\frac{R_2}{R_1 R_3} = -\frac{1}{R_1} \tag{3}$$

The negative resistance converter is a key to sustaining oscillations by compensating for losses in the LC tank circuit so that it is appropriate to use that an operational amplifier. It should be added to the circuit in parallel with the LC tank circuit for compensation of losses.

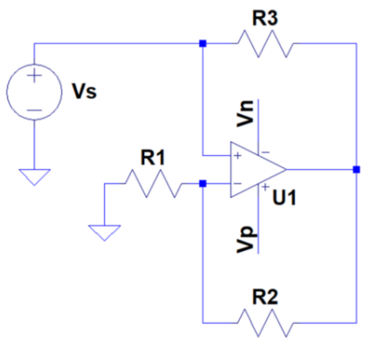

Figure 2. *A schematic for an op-amp based negative resistance converter circuit proposed and used in the Liénard Oscillator*

2.2. Memristive-Model of the Rectifier with R-L-C Load

In this section, the memristive model of a rectifier presented in [41] is summarized. The rectifier circuit is with R-L-C filter shown in Figure 3. Such a circuit behaves as a memristive system [41], and its equations are given as follows. In this circuit, V_g is the source voltage and I_g is the instantaneous rectifier input current. In Figure 3, all the diodes, D1-D4, are chosen as 1N5817, a Schottky diode, which has practically the voltage drop of ~0.45V.

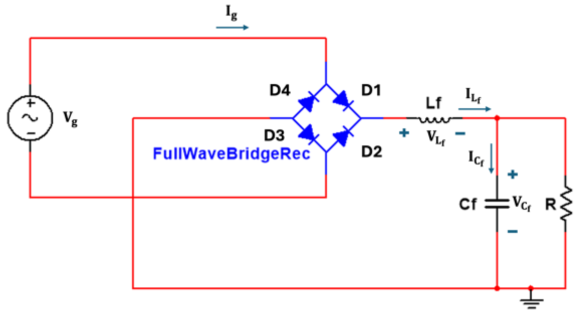

Figure 3. *The Rectifier circuit with RLC filter*

The source current, which is the same as the rectifier current, can be expressed as:

$$I_g = (I_{L_f} + 2I_S).\tanh\left(\frac{V_g}{2nV_T}\right) \tag{4}$$

In this equation, I_{L_f} represents the inductor current, I_s is the leakage current of the diode, V_T is the thermal voltage of the diodes, n is the ideality factor of the diode, and V_g is the source voltage.

I_g can be expressed as follows by applying the Taylor series expansion of the hyperbolic tangent function in the equation above.

$$I_g(t) = G(V_{C_f}, I_{L_f}, V_g, t)V_g(t) \tag{5}$$

where $(V_{C_f}, I_{L_f}, V_g, t)$ represents the equivalent conductance of the diode bridge, and it is defined as:

$$G(V_{C_f}, I_{L_f}, V_g, t) = (I_{L_f} + 2I_S)\left(\frac{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m+1)!}{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m)!}\right) \tag{6}$$

The state variables of the rectifier are the inductor current I_{L_f} and the capacitor voltage V_{C_f}. The rate of change of them are given respectively as

$$\frac{dI_{L_f}}{dt} = \frac{V_g}{L_f} - \frac{V_{C_f}}{L_f} - \frac{2.n.V_T}{L_f}\ln\left(\frac{I_{L_f}+2I_S}{2.I_S.\exp(-V_g/2nV_T).\cosh(V_g/2nV_T)}\right) \tag{7}$$

and

$$\frac{dV_{C_f}}{dt} = \frac{I_{L_f}}{C_f} - \frac{V_{C_f}}{RC_f} \quad (8)$$

2.3. The Nonlinear Resistor of the Liénard Oscillator Circuit

A nonlinear resistor must be used in the Liénard Oscillator. The Liénard Oscillator examined in this study is made of the memristive rectifier with an R-L-C load behaving as a nonlinear resistor, whose nonlinear resistance is an even function of voltage, and the negative resistor converter circuit as shown in Figure 4.

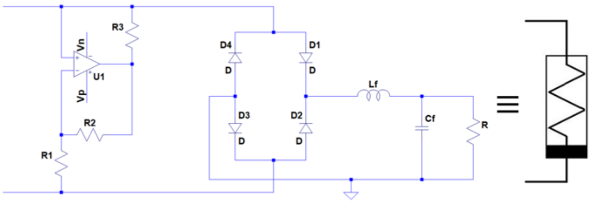

***Figure 4**. The nonlinear resistor circuit of the Liénard Oscillator consisting of the bridge rectifier with the R-L-C load and the opamp-based negative resistance converter*

Since the bridge rectifier and the negative resistance converter are connected in parallel, the terminal equation of the Liénard Oscillator is found as

$$I_{NR} = I_g + I_{neg} = \left(G\left(V_{C_f}, I_{L_f}, V_g, t\right) - g_N\right)V_g \quad (9)$$

$$= \left(\left(I_{L_f} + 2I_S\right)\frac{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m+1)!}{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m)!} - g_N\right)V_g = g_{eq}V_g \quad (10)$$

where g_{eq} *is* the equivalent resistance of the nonlinear resistor of the Liénard Oscillator and equal to

$$\left(\left(\left(I_{L_f} + 2I_S\right)\frac{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m+1)!}{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m)!}\right) - g_N\right).$$

The resistance of the bridge rectifier can be found as

$$R\left(V_{C_f}, I_{L_f}, V_g, t\right) = \frac{1}{g_{eq}} = \frac{1}{\left(\left(\left(I_{L_f} + 2I_S\right)\frac{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m+1)!}{\sum_{m=0}^{\infty}\left(\frac{V_g}{2nV_T}\right)^{2m}/(2m)!}\right) - g_N\right)} \quad (11)$$

$R\left(V_{C_f}, I_{L_f}, V_g, t\right)$ is a two variable function and it is an even function with respect to Vg:

$$R\left(V_{C_f}, I_{L_f}, V_g, t\right) = R\left(V_{C_f}, I_{L_f}, -V_g, t\right) \quad (12)$$

In this circuit, it is always true; $I_{L_f} \geq 0$.

2.4. The New Liénard Oscillator Circuit and Its State-space Model

By incorporating this nonlinear resistance shown in Figure 4 into the generic Liénard Oscillator shown in Figure 1, the Liénard Oscillator proposed in this study is obtained, which is shown in Figure 5. The bridge rectifier is used to achieve the even resistance function and the half-wave symmetry needed in the voltage of a Liénard Oscillator.

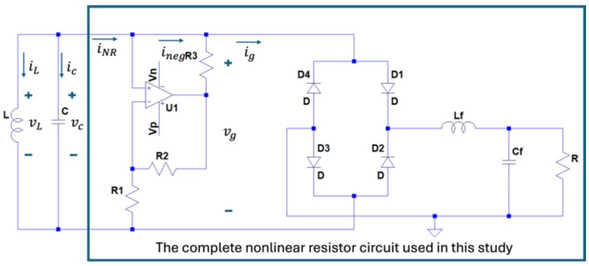

***Figure 5**. The bridge rectifier-based Liénard Oscillator rectifier*

If Kirchoff's Current Law is used for this oscillator:

$$i_L + i_C + i_{NR} = 0 \quad (13)$$

$$i_L + C\frac{dv_C}{dt} + \left(G\left(V_{C_f}, I_{L_f}, v_C, t\right) - g_N\right)v_C = 0 \quad (14)$$

$$i_L + C\frac{dv_L}{dt} + \left(G\left(V_{C_f}, I_{L_f}, v_C, t\right) - g_N\right)v_C = 0 \quad (15)$$

$$i_L + LC\frac{d}{dt}\frac{di_L}{dt} + \left(\left(\left(I_{L_f} + 2I_S\right)\frac{\sum_{m=0}^{\infty}\left(\frac{v_C}{2nV_T}\right)^{2m}/(2m+1)!}{\sum_{m=0}^{\infty}\left(\frac{v_C}{2nV_T}\right)^{2m}/(2m)!}\right) - g_N\right)v_C = 0 \quad (16)$$

The following state-space equations describe the oscillator and its dynamics:

$$i_L + LC\frac{d^2 i_L}{dt^2} + \left(\left(\left(I_{L_f} + 2I_S\right)\frac{\sum_{m=0}^{\infty}\left(\frac{v_C}{2nV_T}\right)^{2m}/(2m+1)!}{\sum_{m=0}^{\infty}\left(\frac{v_C}{2nV_T}\right)^{2m}/(2m)!}\right) - g_N\right)v_C = 0 \quad (17)$$

$$\frac{di_L}{dt} = \frac{v_C}{L} \tag{18}$$

$$\frac{dI_{L_f}}{dt} = \frac{v_C}{L_f} - \frac{V_{C_f}}{L_f} - \frac{2nV_T}{L_f}\ln\left(\frac{I_{L_f}+2I_S}{2.I_S.\exp\left(-v_C/2nV_T\right).\cosh\left(v_C/2nV_T\right)}\right) \tag{19}$$

and

$$\frac{dV_{C_f}}{dt} = \frac{I_{L_f}}{C_f} - \frac{V_{C_f}}{RC_f} \tag{20}$$

Liénard's equation is expressed as

$$\frac{d^2x}{dt^2} + f(x)\frac{dx}{dt} + g(x) = 0 \tag{21}$$

where x(t) is the state variable of the Liénard Oscillator, $f(x)$ is an even function, and $g(x)$ is an odd function.

Considering the periodic steady state, $v_C(t) = -v_C(t + T/2)$ and $I_L(t) = I_L(t + T/2)$, where v_C is the capacitor voltage and I_L is the inductor current. In the positive half-period ($0 < t < T/2$), if the bridge rectifier conductance is $G(V_{C_f}, I_{L_f}, v_C, t)$, then in the negative half-period ($T/2 > t > T$), it becomes $G(V_{C_f}, I_{L_f}, -v_C, t)$ as stated below.

$$G(V_{C_f}, I_{L_f}, v_C(t + T/2), t) = G(V_{C_f}, I_{L_f}, -v_C(t), t) \tag{22}$$

Therefore, in the steady state, the resistance of the rectifier circuit is an even function, and this proves that the oscillator operates as an extended Liénard Oscillator.

At high frequencies, since the bridge rectifier with R-L-C load behaves as a nonlinear resistor with an even resistance function, the circuit behaves as a Liénard Oscillator for high frequencies. However, at low frequencies, it is going to show more complex behavior. Perhaps, it is going to behave as an extended Liénard Oscillator. Its simulation is to be carried out in the next section.

3. Simulation of the New Liénard Oscillator

The Van der Pol equation does not have any solutions [46, 47]. However, it has some approximate solutions [48, 49]. That is why simulations are commonly used to study it. The bridge rectifier-based Liénard Oscillator proposed in this study shown in Figure 5 is simulated with the LTSpice circuit design program in this section. The circuit diagram of the Liénard Oscillator is shown in Figure 6. The parameters used in the simulation are given in Table 1. The values of the resistors R_2 and R_3 are taken as equal to obtain "-R_1" resistance value. The use of Schottky diodes is motivated by their common preference in the high-speed oscillator design, owing to their low threshold voltages and fast switching capabilities. The 1N5817 Schottky diodes and the Opamp LM741 are employed in this work. The simulation results of the circuit are given in Figures 7 to 15.

Table 1 Parameters of the Liénard Oscillator Circuit and the Characteristics of Schottky Diodes

Parameter	Value
C	100 µF
L	25 mH
R_1	2 kΩ
R_2	8 kΩ
R_3	8 kΩ
Is (Saturation Current)	31.7 µA
Rs (Series Resistance)	0.051 Ω
n (Ideality Factor)	1.373
V_T (Thermal Voltage)	26 mV @ Room Temperature.

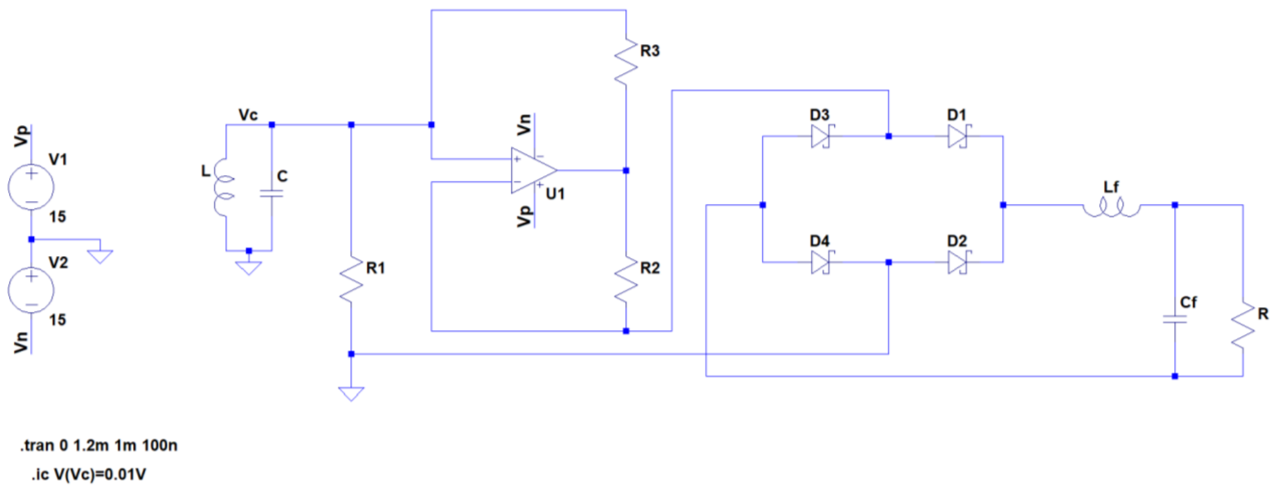

***Figure 6**. LTSpice modeling of rectifier based Liénard Oscillator circuit*

The voltage waveform of the circuit in the periodic steady state, as shown in Figure 7, resembles an almost sinusoidal wave. However, the peaks of the waveform are sharper compared to those of an ideal sinusoidal wave, highlighting the nonlinear nature of the oscillator. The circuit oscillates at roughly 100 Hz (~100.7Hz) and the voltage has an amplitude of 2.6 V.

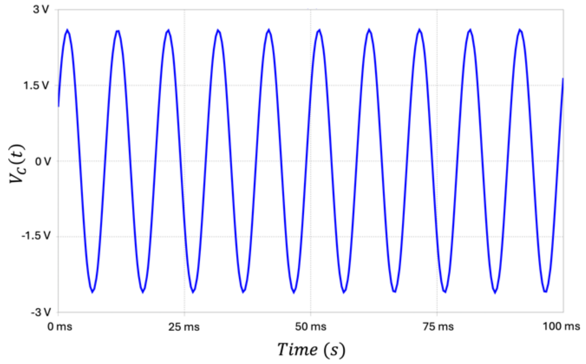

Figure 7. *The oscillator voltage in the periodic steady state.*

The current of the circuit capacitor at the periodic steady state is not a perfect sinusoidal waveform as expected from the voltage observed earlier in Figure 7 and the current is shown in Figure 8.

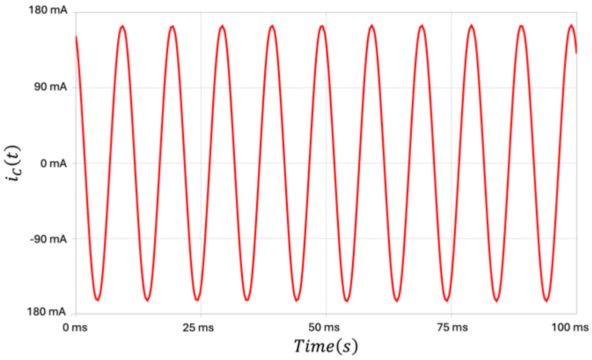

Figure 8. *The current of the circuit capacitor at periodic steady state*

Figure 9 illustrates the impact of the Schottky diodes' threshold voltage on the bridge rectifier's input current in the periodic steady state. This effect manifests as zero-crossing distortion, which is evident when the current transitions through zero. The distortion occurs because Schottky diodes require a small forward voltage to conduct and create a non-linear response. This characteristic results in a slight deviation from ideal behavior, where the current waveform exhibits a small flat or distorted region as it changes polarity, rather than smoothly transitioning through zero.

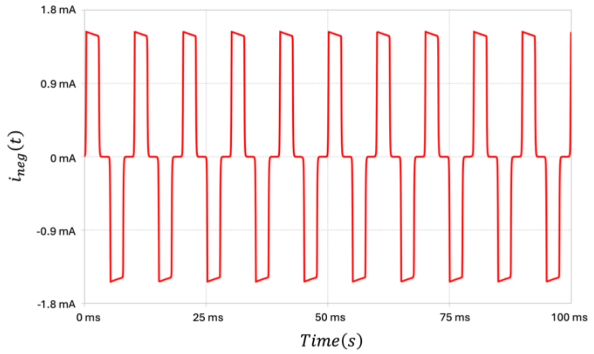

***Figure 9**. The input current of the bridge rectifier in the periodic steady state.*

It can be seen in Figure 10 that a large harmonic content does exist in the equivalent nonlinear resistor current in the periodic steady state. The diode's ability to produce harmonic distortion contributes to the complex non-linear behavior essential for chaotic circuits. Due to this feature, such a circuit can also be used as a Chua diode to build a chaotic oscillator circuit such as the one presented in [20].

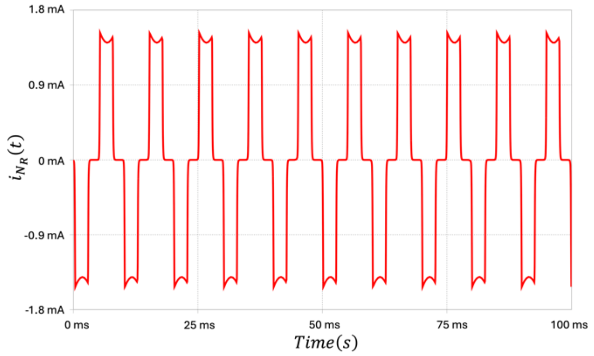

Figure 10. *The current of the nonlinear resistor in the periodic steady state.*

The waveform of the inductor current, as depicted in Figure 11, deviates from an ideal sinusoidal shape in the periodic steady state. Notably, the signal exhibits a similar characteristic to the capacitor current shown in Figure 8.

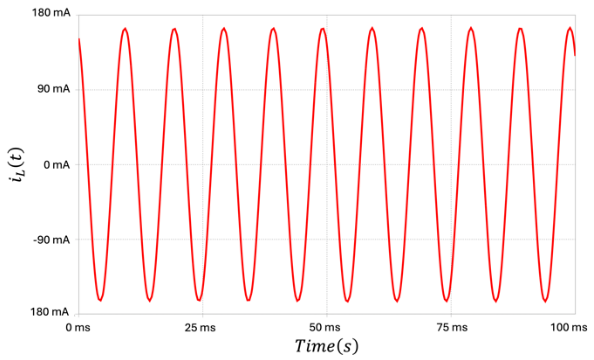

Figure 11. *The current of the circuit inductor in the periodic steady state.*

The limit cycle of the capacitor is shown in Figure 12, which resembles an ellipse.

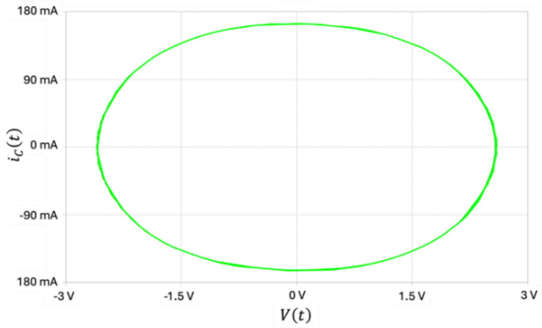

***Figure 12**. Limit cycle of the circuit capacitor*

Figure 13 illustrates the memristive rectifier's voltage-current relationship in the periodic steady state. The v-i curve of the rectifier has an odd function symmetry, and it is confined to the first and the third quadrants. The graph reveals the impact of the Schottky diode thresholds near the origin and the memristive behavior of the hysteresis curve of the bridge rectifier with the R-L-C load.

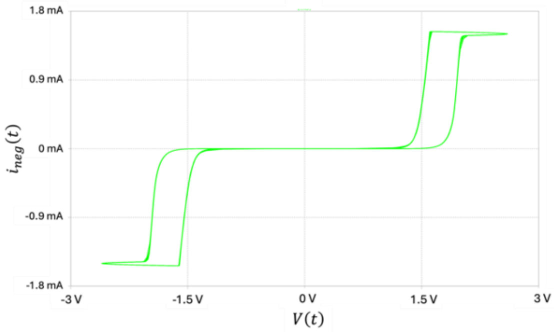

***Figure 13**. The V-I characteristic of the memristor based rectifier in the periodic steady state.*

The v-i characteristic of the circuit's nonlinear resistor, as illustrated in Figure 14, demonstrates also an odd symmetry and shows a negative resistance feature, which is confined to the second and fourth quadrants. Furthermore, the graph appears to capture the influence of the diodes' threshold voltage on the Chua diode's v-i characteristic. This effect is discernible in the subtle nuances of the curve shown in Figure 14, adding another layer of complexity to the nonlinear behavior of the circuit.

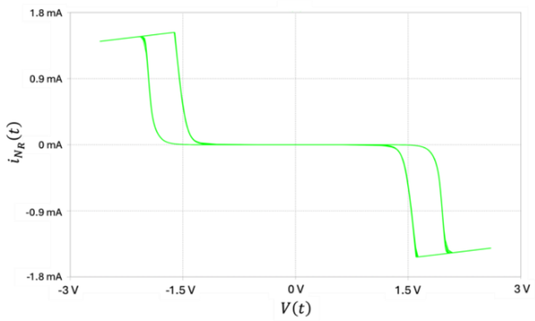

***Figure 14**. The V-I characteristics of the nonlinear resistor in the circuit in the periodic steady state.*

The limit cycle of the inductor in Figure 15 resembles an ellipse as the limit cycle of the capacitor also does as in Figure 12.

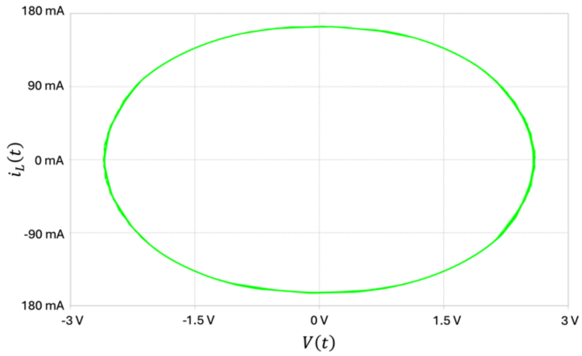

***Figure 15.** Limit cycle of the circuit inductor*

4. Conclusion

This study presents the development of a Liénard Oscillator incorporating a bridge rectifier and an R-L-C output filter. The rectifier circuit exhibits memristive characteristics, and the system's mathematical model has been formulated. LTSpice simulations were employed to analyze the oscillator's behavior, yielding limit cycles, current, and voltage waveforms. The simulation results demonstrate that the proposed circuit functions as a Liénard Oscillator, evidenced by its periodic waveforms and limit cycles.

At low oscillation frequencies such as 100 Hz, the memristive bridge acts as a nonlinear resistor, enabling the circuit to operate as a Liénard Oscillator. The memristive bridge rectifier is shown to provide the even nonlinear resistance necessary for a Liénard Oscillator in the periodic steady state. However, it's noted that this oscillator possesses more than two state variables, unlike the traditional two-variable Liénard system. The nonlinear capacitive junction currents may also need to be included in the oscillator model for accuracy in high frequency operation and we suggest it as future work.

This design opens possibilities for future variants using different semiconductor elements. While most Liénard oscillators lack exact analytical solutions, future research could also explore series or approximate solutions for this system. Additionally, a parametric study examining how circuit elements affect oscillator frequency could expand on the current simulations, though such investigations were beyond the scope of this work due to space limitations.

Author Contribution

Reşat Mutlu (RM) literature review and modeling, RM, Meltem Apaydın Üstün (MAÜ), Arif Kıvanç Üstün (AKÜ) data collection and preparation of the manuscript, MAÜ, AKÜ simulations, RM, MAÜ, AKÜ design, review and editing.

Declaration of Competing Interest

The authors declared no conflicts of interest with respect to the research, authorship, and/or publication of this article.

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