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THE STATICAL ANALYSIS OF SEMI-RIGID FRAMES BY DIFFERENT CONNECTION TYPES

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Abstract

In this study, elastic supported frames are analyzed by using a computer program. The connection flexibility is modeled by linear elastic rotational and lateral springs. Having the same geometry and cross-sections, the statical analysis is examined for five different spring combinations. Response characteristics of ten different two-storey frames are compared with reference to rotations and displacements of their joints. The study indicates that rotations and displacements of connections increase as the spring coefficients decrease.

1. Introduction

For conventional analysis and design of a steel-framed structure, the actual behavior of beam-to-column connections is simplified to the two idealized extremes of either rigid-joint or pinned joint behavior [1]. This assumption does not represent the actual behavior of a frame. Faults occurred during construction of a structure or later make a behavior of beam-to-column connection seem to be a behavior of semi-rigid connection.

Semi-rigid frames are frames for which the beam-to-column joints are neither pinned nor rigid [2]. Semi-rigid frames have been studied in a few decades [3-8]. Semi-rigid frames in most of these studies were represented only by rotational springs. In fact, frame elements in a structure have a semi-rigid behavior through the axial direction of themselves, indeed. In this study, the behaviors of beam-to-column connection are designed by discussing this lateral behavior. The supplemental effect of lateral springs will be discussed.

The behavior of two-storey frame will be examined by using two different connection models. In the first model, linear elastic rotational springs which represent flexible connection behavior are located at the ends of beams. In the second model, lateral springs are located at the ends of beams for the analysis indeed. The statical analysis is examined on these two semi-rigid models to obtain the rotations and displacements of joints and the moment values of spans.

Key Words: Semi-Rigid, Statical Analysis, Steel Frame

2. SEMI-RIGID FRAME MODELS

For the present study, two semi-rigid models with different connection type are used. The first semi-rigid model is shown in (Figure 2.1). The frame model includes a beam with moment of inertia I_b and length L, and two columns with moment of inertia I_c, and, length H. The modulus of elasticity E is the same in all frame elements.

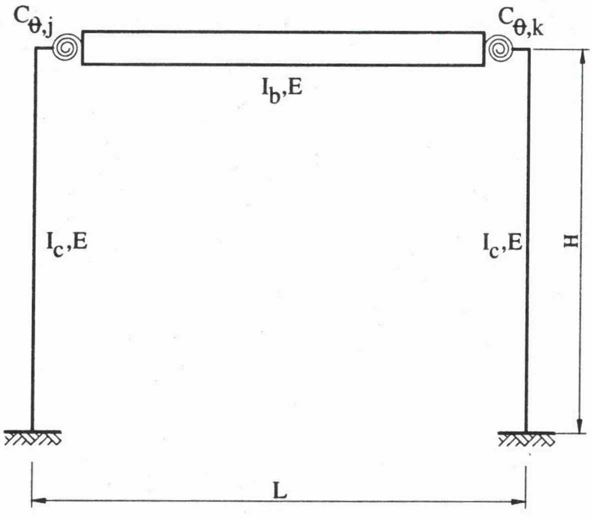


Figure 2.1 The first semi-rigid model.

The connections are represented by rotational springs near beam-to-column joints. Presence of rotational springs will introduce relative rotations of $\overline{\Phi}_j$ and $\overline{\Phi}_k$ at the ends of the beam [9]. Denoting the stiffness of connections at the ends of beam $C_{\theta,j}$ and $C_{\theta,k}$, respectively, the relative rotation between the joint and the beam end can be given by Eq.(1).

$$M_{jf} = C_{\theta,j} x \overline{\Phi}_j \quad ; \quad M_{kf} = C_{\theta,k} x \overline{\Phi}_k \qquad (1)$$

where M_{jf} and M_{kf} are flexural moments, respectively, at j and k ends of a frame element, $\overline{\Phi}_j$ and $\overline{\Phi}_k$ are rotations occurred by rotational springs.

If we denote the joint rotation at the ends of the beam by ϕ_j and ϕ_k respectively, the slope-deflection equations for the beam modified for presence of connections can be given by [10].

$$M_A = \frac{EI}{L}\left[s_{ii}\left(\theta_j - \frac{M_A}{C_{\theta,j}}\right) + s_{ij}\left(\theta_k - \frac{M_B}{C_{\theta,k}}\right)\right] \tag{2}$$

$$M_B = \frac{EI}{L}\left[s_{ij}\left(\theta_j - \frac{M_A}{C_{\theta,j}}\right) + s_{jj}\left(\theta_k - \frac{M_B}{C_{\theta,k}}\right)\right] \tag{3}$$

The beam stiffness matrix can be written as follows [2]:

$$[K_{bf}] = \begin{bmatrix} \frac{A_bE}{L} & 0 & 0 & -\frac{A_bE}{L} & 0 & 0 \\ 0 & -\frac{EI_b}{L^3}\Psi_1 & \frac{EI_b}{L^2}\Psi_2 & 0 & -\frac{12EI_b}{L^3}\Psi_1 & \frac{EI_b}{L^2}\Psi_3 \\ 0 & \frac{6EI_b}{L^2}\gamma_2 & \frac{EI_b}{L}\Psi_4 & 0 & -\frac{EI_b}{L^2}\Psi_2 & \frac{2EI_b}{L}\Psi_5 \\ -\frac{A_bE}{L} & 0 & 0 & \frac{A_bE}{L} & 0 & 0 \\ 0 & -\frac{EI_b}{L^3}\Psi_1 & -\frac{6EI_b}{L^2}\Psi_2 & 0 & \frac{12EI_b}{L^3}\Psi_1 & -\frac{EI_b}{L^2}\Psi_3 \\ 0 & \frac{EI_b}{L^2}\Psi_3 & \frac{2EI_b}{L}\Psi_5 & 0 & -\frac{6EI_b}{L^2}\gamma_3 & \frac{EI_b}{h}\Psi_5 \end{bmatrix} \tag{4}$$

$$\Psi_1 = s_{ii} + 2\times s_{ij} + s_{jj} \ ; \ \Psi_2 = s_{ii} + s_{ij} \ ; \ \Psi_3 = s_{ij} + s_{jj} \ ; \ \Psi_4 = s_{ii} \ ; \ \Psi_5 = s_{jj} \tag{5}$$

$$s_{ii} = \frac{4 + \frac{12\times EI}{L\times C_{\theta,k}}}{C^*} \ ; \ s_{jj} = \frac{4 + \frac{12\times EI}{L\times Cj}}{C^*} \ ; \ s_{ij} = s_{ji} = \frac{2}{C^*} \tag{6}$$

$$C^* = \left(1 + \frac{4\times EI}{L\times C_{\theta,j}}\right)\times\left(1 + \frac{4\times EI}{L\times C_{\theta,k}}\right) - \left(\frac{EI}{L}\right)^2 \times \frac{4}{C_{\theta,j}\times C_{\theta,k}} \tag{7}$$

The column stiffness matrix can be written as follows:

$$[K_{cf}]=\begin{bmatrix} \frac{12EI_c}{h^3} & 0 & -\frac{6EI_c}{h^2} & -\frac{12EI_c}{h^3} & 0 & -\frac{6EI_c}{h^2} \\ 0 & \frac{A_cE}{h} & 0 & 0 & -\frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2} & 0 & \frac{4EI_c}{h} & \frac{6EI_c}{h^2} & 0 & \frac{2EI_c}{h} \\ -\frac{12EI_c}{h^3} & 0 & \frac{6EI_c}{h^2} & \frac{12EI_c}{h^3} & 0 & \frac{6EI_c}{h^2} \\ 0 & -\frac{A_cE}{h} & 0 & 0 & \frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2} & 0 & \frac{2EI_c}{h} & \frac{6EI_c}{h^2} & 0 & \frac{4EI_c}{h} \end{bmatrix} \quad (8)$$

The second semi-rigid frame model is shown in (Figure 2.2). The model includes a beam with moment of inertia I_b and length L, and two columns with moment of inertia I_c, and length H. The modulus of elasticity E is the same in all frame elements.

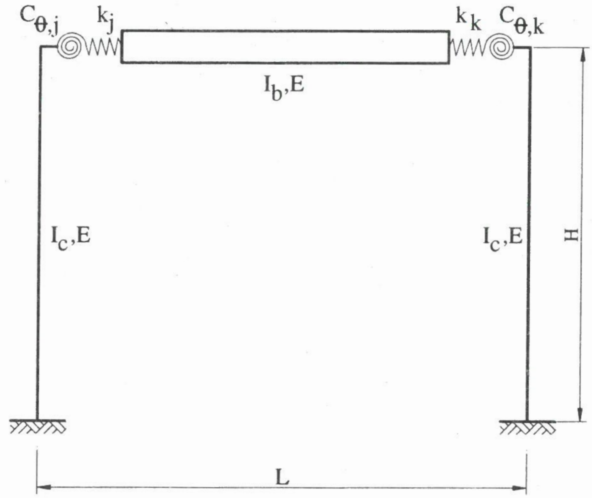


Figure 2.2 The second semi-rigid model.

The connections are represented by rotational and lateral springs near beam-to-column joints. Presence of lateral springs will introduce relative displacements of $\bar{\delta}_j$ and $\bar{\delta}_k$ at the ends of the beam k_j and k_k, respectively; the lateral total displacements can be given by Eqs. (9) and (10).[11]

$$\delta_s = \delta_f + \Sigma\bar{\delta} \quad (9)$$

$$\delta_s = P\left[\frac{1}{AE} + \frac{(k_j + k_k)}{k_k x k_j}\right] \tag{10}$$

The combined semi-rigid behavior is represented by lateral and rotational springs. The beam stiffness matrix can be written as follows:

$$[K_{bf}]=\begin{bmatrix} \frac{A_bE}{L}+R_b & 0 & 0 & -\frac{A_bE}{L}-R_b & 0 & 0 \\ 0 & -\frac{EI_b}{L^3}\Psi_1 & \frac{EI_b}{L^2}\Psi_2 & 0 & -\frac{12EI_b}{L^3}\Psi_1 & \frac{EI_b}{L^2}\Psi_3 \\ 0 & \frac{6EI_b}{L^2}\gamma_2 & \frac{EI_b}{L}\Psi_4 & 0 & -\frac{EI_b}{L^2}\Psi_2 & \frac{2EI_b}{L}\Psi_5 \\ -\frac{A_bE}{L}-R_b & 0 & 0 & \frac{A_bE}{L}+R_b & 0 & 0 \\ 0 & -\frac{EI_b}{L^3}\Psi_1 & -\frac{6EI_b}{L^2}\Psi_2 & 0 & \frac{12EI_b}{L^3}\Psi_1 & -\frac{EI_b}{L^2}\Psi_3 \\ 0 & \frac{EI_b}{L^2}\Psi_3 & \frac{2EI_b}{L}\Psi_5 & 0 & -\frac{6EI_b}{L^2}\gamma_3 & \frac{EI_b}{h}\Psi_5 \end{bmatrix}$$

(11)

where; R_b

$$R_b = \frac{A_bE\cdot(k_j \times k_k)}{L\cdot(k_j \times k_k) + A_bE\cdot(k_j + k_k)} \tag{12}$$

The structure stiffness matrix is obtained by assembling the column and beam stiffness matrices described above according to conventional stiffness matrix analysis procedure [12].

3. STATICAL ANALYSIS AND NUMERICAL STUDIES

The primary objective of the present study is to investigate the statistical characteristics of semi-rigid frames and how connection flexibility influences them. Response characteristics of ten different two-storey frames are compared with reference to rotations and displacements of their joints. In the present study, two-story semi-rigid frames having five different spring coefficients were studied. The semi-rigid models for the present analysis are given in (Figure 3.1) and (Figure 3.3). All frames have the same geometry, cross-section and material property to compare the influence of connection flexibility and connection type on statical characteristics. First, the values were determined by using a computer program written in MATLAB 6.5 editor [13].

All frame elements are designed by using steel sections. The modulus of elasticity E is $2.1x10^8$ kN/m^2 for all elements. The length of beam elements are chosen 5 meters and the length of column elements are chosen 3 meters. The beam sections are chosen as (W14x159) with cross-section area 0.0301 m^2 . The column sections are chosen as (W14x211) with cross-section area 0.0400 m^2. Therefore, the moment of inertia of each beam is $7.9084x10^{-4}$ m^4 and the moment of inertia of each column is $11.0718x10^{-4}$ m^4. The displacements and rotations of the first model are given in (Table 3.1) and (Table 3.2), respectively.

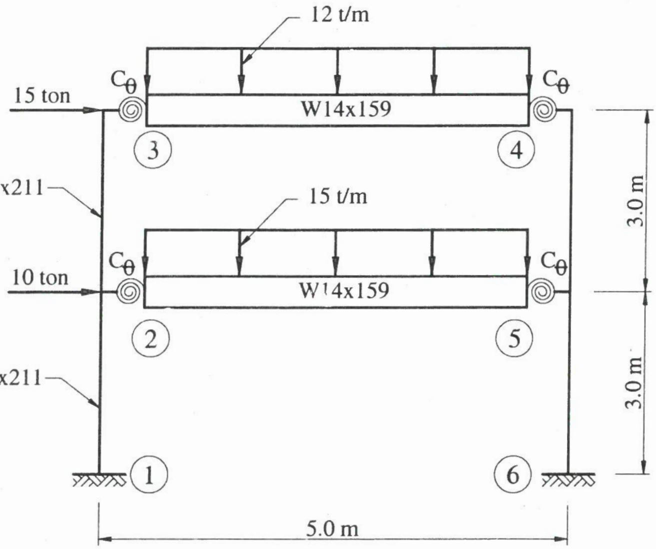


Figure 3.1 The first semi-rigid model for the present analysis.

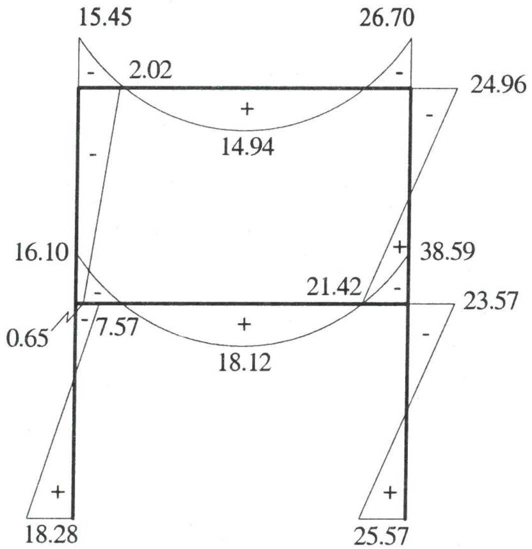
Table 3.1 The Displacements Of The First Model.

JOINTS	δ_X (m.)				
	$C_\theta = 2.10^4$ kN.m/rd.	$C_\theta = 1.10^5$ kN.m/rd.	$C_\theta = 5.10^5$ kN.m/rd.	$C_\theta = 1.10^6$ kN.m/rd.	$C_\theta = 1.10^{20}$ kN.m/rd.
1	0.00	0.00	0.00	0.00	0.00
2	0.03316	0.02651	0.02377	0.02336	0.02292
3	0.06288	0.04556	0.03875	0.03773	0.03668
4	0.06204	0.04497	0.03829	0.03730	0.03627
5	0.03311	0.02633	0.02352	0.02310	0.02265
6	0.00	0.00	0.00	0.00	0.00

Table 3.2 The Rotations Of The First Model.

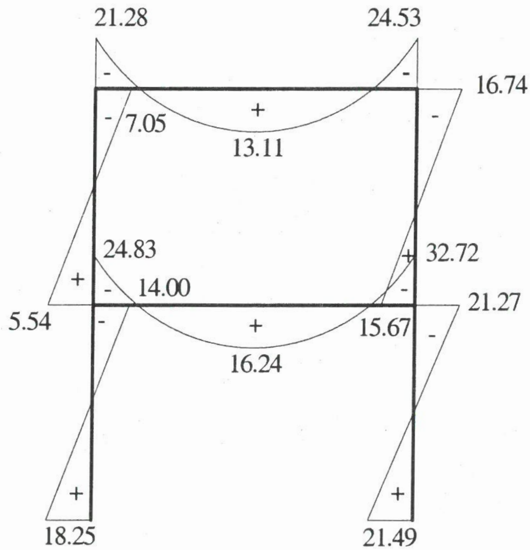
JOINTS	Θ (rd.)				
	$C_\theta= 2.10^4$ kN.m/rd.	$C_\theta= 1.10^5$ kN.m/rd.	$C_\theta= 5.10^5$ kN.m/rd.	$C_\theta= 1.10^6$ kN.m/rd.	$C_\theta= 1.10^{20}$ kN.m/rd.
1	0.00	0.00	0.00	0.00	0.00
2	0.01164	0.00461	0.00116	0.00060	0.00
3	0.00873	0.00298	0.00070	0.00036	0.00
4	-0.00167	-0.00093	-0.00026	-0.00014	0.00
5	0.00216	0.00023	-0.00001	-0.00001	0.00
6	0.00	0.00	0.00	0.00	0.00

The bending – moment diagrams of the first semi-rigid model, are presented in (Figure 3.2-a, b, c, d, e) for five different spring coefficients, respectively.



(a)

Figure 3.2-a Moment values for $C_\theta = 20000$ kN.m/rd.



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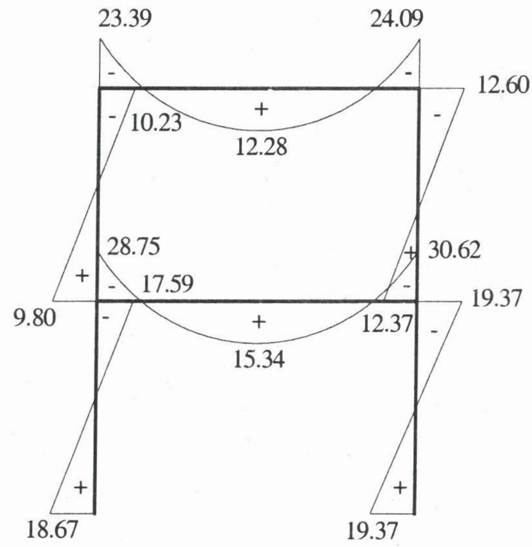
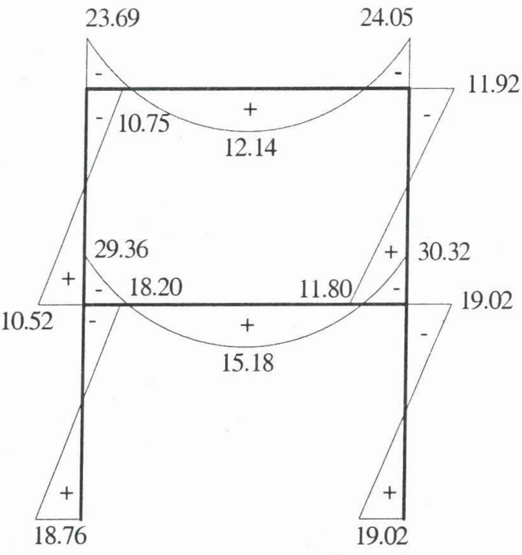
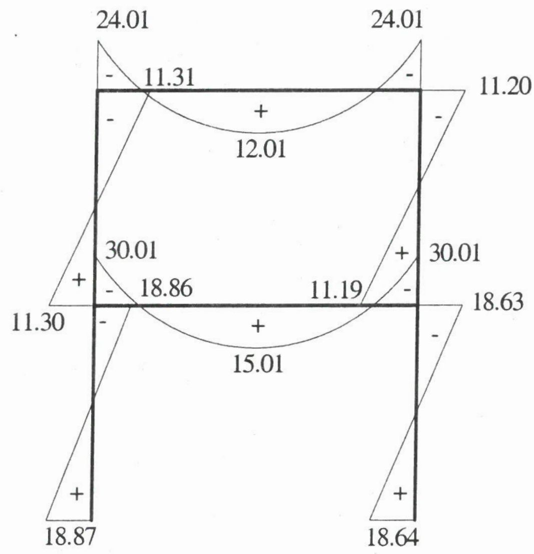


(c)

Figure 3.2 -b Moment values for $C_\theta = 100000$ kN.m/rd.
-c Moment values for $C_\theta = 500000$ kN.m/rd.



(d)



(e)

Figure 3.2 -**d** Moment values for $C_\theta = 1000000$ kN.m/rd.
-**e** Moment values for $C_\theta = 10^{20}$ kN.m/rd.

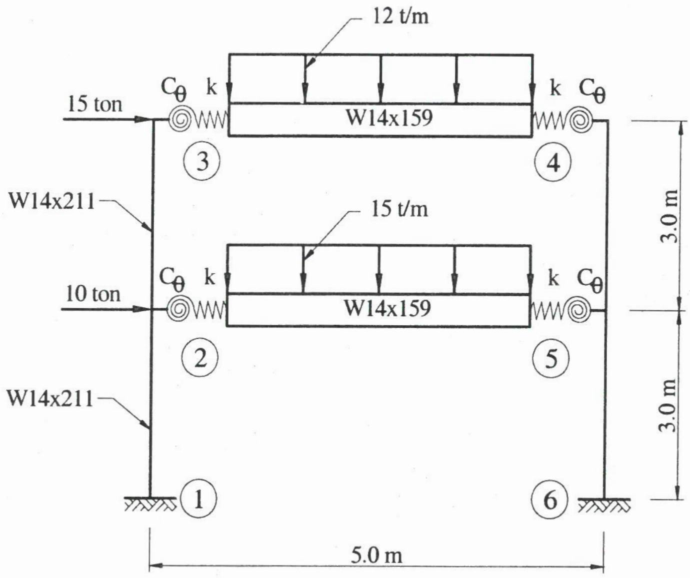


Figure 3.3 The second semi-rigid model for the present analysis.

The displacements and rotations of the second model are given in (Table 3.3) and (Table 3.4), respectively.

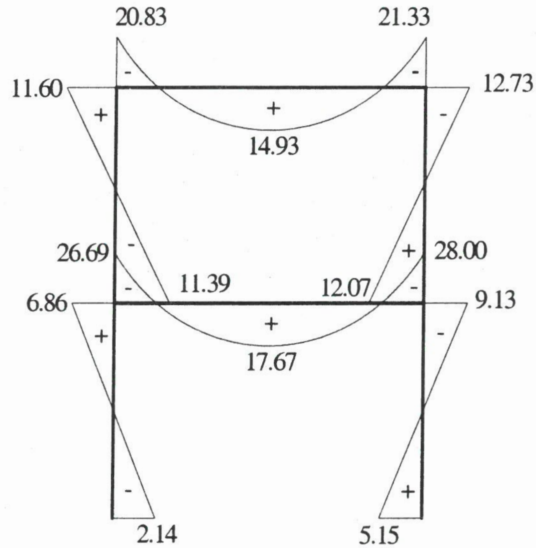
Table 3.3 The Displacements Of The Second Model.

JOINTS	δ_X (m.)				
	$C_\theta = 2.10^4$ kN.m/rd. $k = 2.10^4$ kN/m	$C_\theta = 1.10^5$ kN.m/rd. $k = 1.10^5$ kN/m	$C_\theta = 5.10^5$ kN.m/rd. $k = 5.10^5$ kN/m	$C_\theta = 1.10^6$ kN.m/rd. $k = 1.10^6$ kN/m	$C_\theta = 1.10^{20}$ kN.m/rd. $k = 1.10^{20}$ kN/m
1	0.00	0.00	0.00	0.00	0.00
2	0.00223	0.00056	0.00015	0.00008	0.00
3	0.00400	0.00097	0.00025	0.00013	0.00
4	0.00320	0.00051	0.00005	0.00002	0.00
5	0.00219	0.00041	0.00005	0.00001	0.00
6	0.00	0.00	0.00	0.00	0.00

Table 3.4 The Rotations Of The Second Model.

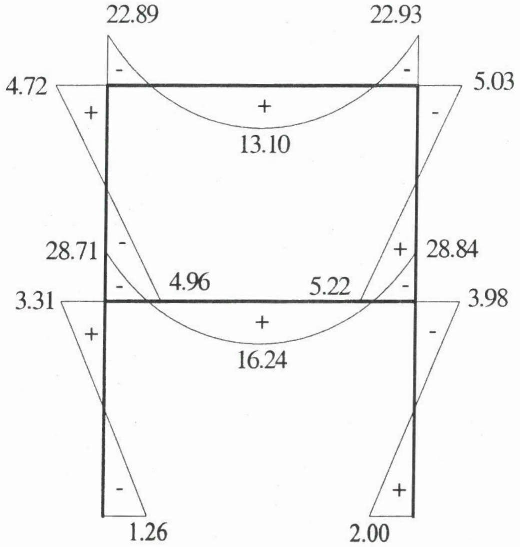
JOINTS	θ (rd.)				
	C_θ= 2.10^4 kN.m/rd. k= 2.10^4 kN/m	C_θ= 1.10^5 kN.m/rd. k= 1.10^5 kN/m	C_θ= 5.10^5 kN.m/rd. k= 5.10^5 kN/m	C_θ= 1.10^6 kN.m/rd. k= 1.10^6 kN/m	C_θ= 1.10^{20} kN.m/rd. k= 1.10^{20} kN/m
1	0.00	0.00	0.00	0.00	0.00
2	0.00514	0.00223	0.00059	0.00031	0.00
3	0.00536	0.00197	0.00048	0.00025	0.00
4	-0.00504	-0.00194	-0.00048	-0.00025	0.00
5	-0.00433	-0.00215	-0.00058	-0.00030	0.00
6	0.00	0.00	0.00	0.00	0.00

The bending – moment diagrams of the second semi-rigid model, are presented in (Figure 3.4-a, b, c, d, e) for five different spring coefficients, respectively.



(a)

Figure 3.4-a Moment values for C_θ = 20000 kN.m/rd., k=20000 kN/m.



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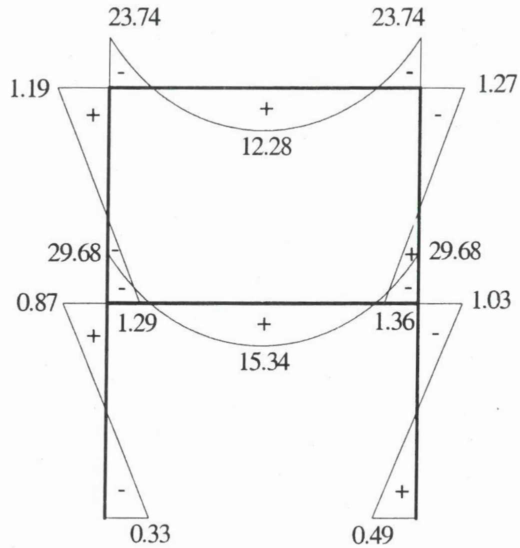
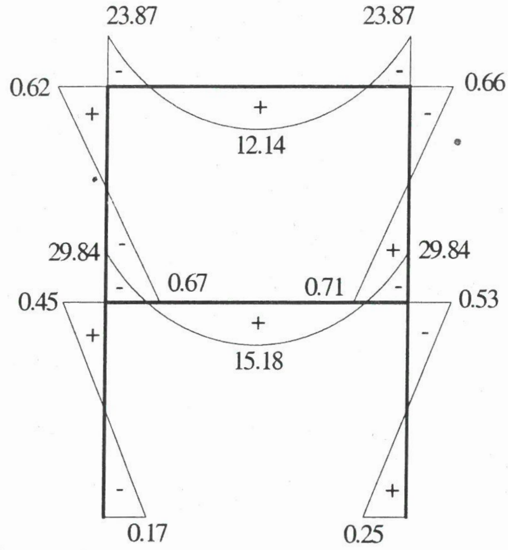


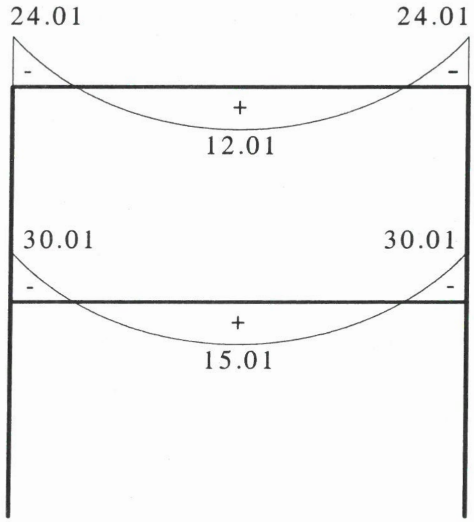
(c)

Figure 3.4 -**b** Moment values for C_θ = 100000 kN.m/rd., k=100000 kN/m.

-**c** Moment values for C_θ = 500000 kN.m/rd., k=500000 kN/m.



(d)



(e)

Figure 3.4 -d Moment values for C_0 = 1000000 kN.m/rd., k=1000000 kN/m.
-e Moment values for $C_0 = 10^{20}$ kN.m/rd. k= 10^{20} kN/m.

4. CONCLUSION

In this stüdy, two semi-rigid frame models were used. The connection flexibility was modeled by linear elastic rotational and lateral springs. A computer program was written to perform statical analysis for five spring coefficient values of each semi-rigid frame model.

In the first model, rotational springs were located at the ends of beam. Five different spring coefficients were used for the connection flexibility. In the second model, rotational and lateral springs were located at the ends of the beam. Indeed, five different coefficients were used for representing the connection flexibility. The statical characteristics were determined for each connection type. Statical properties were investigated with reference to rotations and displacements of their joints.

The study indicates that connection models have influences on the statical characteristics of frames. The type of the linear elastic connection springs affects the behavior and the lateral rigidity of semi-rigid frames.

It is obvious that the displacements and rotations of joints of a structure depend on the properties and types of joints. Two semi-rigid frame models have different connection types to each other. These differences have an influence on the displacements and rotations. For the first model under the same statical loads, higher displacements values were obtained. The additional springs located laterally increased the lateral rigidity of semi-rigid model.
For ultimate values of spring coefficients, displacement values occurred only for the first semi-rigid model. At this peak value, the lateral rigidity reached to the highest value. Also, the rotations of joints were affected by increase of spring coefficients. As the spring coefficient increases, the rotations decrease. For both semi-rigid models, every moment value of span decreases as spring coefficients increase. The spring types don't have any supplemental effects on the moment values of span.

Using lateral and rotational springs with the ultimate values; any moment values occurred on the columns because joints (2), (3), (4) and (5) behaved as fixed supported and there weren't any loads on the spans of the columns. For designing a structure well, an engineer has to determine the real behavior of a structure. To represent this behavior, engineers should know every displacement on each direction. A frame element behaves not only on the rotational direction but also, on the axial direction, too. As it can be seen from the results of this study, determining axial direction of a frame has an important influence on the behavior.

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FARKLI BAĞLANTI TİPLERİ İLE YARI-RİJİT ÇERÇEVELERİN STATİK ANALİZİ

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Bu çalışmada, bir bilgisayar programı kullanılarak elastik mesnetlenmiş çerçevelerin analizi yapılmıştır. Bağlantı esnekliği, doğrusal elastik dönel ve yanal yaylar ile modellenmiştir. Aynı geometri ve kesit alanına sahip beş farklı yay kombinasyonu için statik analiz yapılmıştır. İki katlı, on farklı çerçevenin; tepki karakteristikleri, düğüm noktalarının dönme ve deplasmanlarına bağlı olarak karşılaştırılmıştır. Çalışma, yay katsayıları azalırken; bağlantıların dönme ve deplasmanlarının, arttığını işaret etmektedir.

Anahtar Kelimeler: Çelik Çerçeve, Statik Analiz, Yarı-Rijit

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