CHAOS Theory and Applications in Applied Sciences and Engineering

e-ISSN: 2687-4539
RESEARCH ARTICLE
Vol.7 / No.1 / 2025 / pp.78-86
https://doi.org/10.51537/chaos.1603833

Emergence of Coexistence in the Cultural Dissemination of the Axelrod Model with Mass Media

Sadan Josué López Morales [*,1], **Ricardo Armando González Silva** [*,2] **and Mario Ignacio González Silva** [*,3]
[*]Centro Universitario de Los Lagos, Universidad de Guadalajara, C.P. 44100, Guadalajara, Jalisco, México.

ABSTRACT The Axelrod Model (AM) is widely recognized as a valuable tool for simulating cultural dissemination within societies. This study examines the impact of an intelligent mass media system on monoculture formation in AM and its effects on cultural dynamics. Agent-based simulations were conducted on a structured $G_{L,L}$ lattice network, employing an experimental design to observe effects of variations in key model parameters q, ϵ, and C. Results reveal dynamic transitions between monocultural and n-cultural states, contingent on the interplay of parameters. The experimental design enabled a novel visualization of the data, identifying robust patterns of coexistence, where both absorbing states coexist under specific conditions. Additionally, a relationship was established between the largest cultural region size ($S_{\max}$), the numbers of final cultural regions (n_R), and mass media information proportion in the lattice (ρ) using a least-squares approximation. This study confirms that mass media can be a critical force in cultural stability, not only promoting monocultural states but also facilitating cultural diversity under certain conditions.

KEYWORDS
Cultural dissemination
Agent-based modeling
Mass media
Coexistence
Emergence

INTRODUCTION

Cultural dissemination is the process of spreading and adopting ideas, beliefs, skills, values, and behaviors among members of a society. The first simulation model for this process is the AM (Axelrod 1997). The AM uses agent-based models (ABMs) to represent individuals in a society; ABMs are computational frameworks used to simulate interactions of individual agents representing entities such as people, organizations, or even biological organisms within an environment (Railsback and Grimm 2019). Each agent in these models is designed with specific characteristics, rules, and behaviors, allowing it to act autonomously and interact with other agents according to predefined protocols. In AM, an agent's culture consists of attributes such as beliefs, values, and behaviors. The model then simulates interactions between these agents under two key assumptions:

- Individuals are likelier to interact with others with similar cultural attributes.
- The interactions tend to increase the number of shared attributes, reinforcing cultural similarity and increasing the likelihood of future interaction.

Manuscript received: 18 December 2024,
Revised: 12 February 2025,
Accepted: 18 February 2025.

[1]sadan.lopez8247@alumnos.udg.mx
[2]ricardo.gsilva@academicos.udg.mx (**Corresponding author**)
[3]mario.gonzalezs@academicos.udg.mx

Several extensions of AM explore factors that influence cultural dissemination, such as noise effects and the role of network topology in cultural order (Klemm *et al.* 2003a,b). Other studies investigate the homogenizing effects of global feedback, demonstrating that strong feedback accelerates monoculture, while weaker feedback maintains limited diversity (Shibanai *et al.* 2001). The propaganda strategies in AM are explored and highlight the importance of timing and intensity to achieve effective influence (Carletti *et al.* 2006). There is research on AM that addresses mechanisms that sustain cultural diversity, emphasizing how limited confidence and metric traits affect system stability (Flache and Macy 2006). Another application of AM is in opinion dynamics, developing experiments that examine how tolerance and connectivity also form robust groups Jacobmeier (2005).

Moreover, the coevolution of cultural groups and networks is discussed in (Centola *et al.* 2007), underscoring how lattice network dynamics determine the stability of multiculturalism or convergence to a completely homogeneous society. The dual role of mass media is studied in (González-Avella *et al.* 2005), where strong media influence promotes cultural fragmentation, while weak influence drives cultural homogenization. Besides, Rodríguez *et al.* (2009) reveals how the strategic use of media can manipulate public perception to foster either resistance or cultural convergence. Studies on cultural dissemination have evolved significantly since the introduction of AM. Numerous studies have refined and expanded the theoretical framework to address various complex social phenomena. One key extension examined the impact of mass media by creating a type of super-agent that interacts with all agents in the system, demonstrating that mass media can drive cultural convergence in diverse societies or promote cultural diversity in moderately diverse contexts due to the interaction between mass media and local influences (Rodríguez and Moreno 2010).

Other research explored the role of cultural tolerance, as examined in (Gracia-Lázaro *et al.* 2011), which showed that tolerant traits have an evolutionary advantage due to their ability to facilitate cultural overlap and enhance social interactions. Similarly, in (Crokidakis 2012), it was highlighted how mass media influence can suppress critical phenomena and accelerate consensus formation in opinion dynamics. An application AM papper with intercultural interactions reveals how reciprocal media influence among social groups can lead to structured diversity, balancing global homogenizing effects with local heterogeneity (González-Avella *et al.* 2012). In another paper they identified ordered states akin to chimera states, demonstrating that one population can achieve coherent order while others remain disordered (González-Avella *et al.* 2014).

Further studies investigated hierarchical structures of cultural traits, revealing that high ultrametricity and significant cultural trait variation are essential for preserving diversity in complex systems (Stivala *et al.* 2014). The dynamics of innovation diffusion were also analyzed, showing that different network topologies influence the innovations spread, with highly connected random graphs producing S-shaped adoption curves characteristic of real-world systems (Tilles and Fontanari 2015). The interaction between cultural diversity and cooperation was studied (Stivala *et al.* 2016), showing that multicultural states enhance cooperation while monocultural states suppress it. In Hernández *et al.* (2018) examined mechanisms to preserve cultural diversity, developing robust cultural communities that avoid monocultural states while maintaining fragmented but cohesive groups.

The impact and evolution of social networks on cultural dissemination were explored in (Raducha and Gubiec 2017), showing how different rewiring mechanisms shape the network topology and cultural configurations. Additionally, the dynamics of minority groups in globalized societies were analyzed, demonstrating how certain structural conditions enable minority groups to thrive and coexist despite dominant mass media trends (Cosenza *et al.* 2020). In 2021, polarization in AM was studied, revealing disruptive effects on cultural dynamics and polarizing traits that inhibit traditional transitions between monocultural and multicultural states, with an emphasis on ideological divisions and network structures (Gracia-Lázaro *et al.* 2021). In 2024, competition among multiple mass media influences was studied, uncovering how weaker media can dominate under specific conditions and how alternative ordered states can emerge from agent interactions, providing comprehensive insights into the balance between local, global, rand structural interactions Alvarez-Llamoza *et al.* (2024)).

In our research, the cultural dissemination model described in Rodríguez *et al.* (2009) was replicated, focusing on the quantitative indices of direct and indirect influence of mass media. A new analytical approach was introduced, specifically identifying configurations where coexisting states emerge in the model (an equivalent idea like Campos Cantón (2025)). This new approach uses data science, presenting a graphical, descriptive, and categorized analysis of the different percentages of field information within the lattice network. Numerical results describe the relationships between dependent and independent variables, highlighting the ranges value where cultural diversity promotes coexisting states.

The article is divided into six sections; The first section provides the structural and functional ABM of AM with mass media. The second section describes the various metrics used to evaluate the model's stability, such as absorption states and the generation of cultural regions within it. The third section shows an experimental design divided into three different experiments: low, moderate, and high cultural diversity, aiming to observe the transitions occurring in each case. The fourth section shows the numerical results obtained and its descriptive analysis, followed by the last two sections dedicated to discussion and conclusions.

ABM AXELROD'S MODEL WITH MASS MEDIA

Agents' Characteristics and Groups

In this model, agents represent individuals with distinct cultural characteristics or features. A cultural $features$ vector defines each agent and their interaction is influenced by the degree of similarity to other agents in the system. In the following, let

$$[q] = \{0, 1, \cdots, q-1\}, q \in \mathbb{N}$$

The structure and grouping of agents are described as follows.

Agent Characteristics : Each agent i has *nominal feature vector* of dimension F, denoted by

$$\vec{\sigma}_i = \langle \sigma_{i1}, \sigma_{i2}, \cdots, \sigma_{iF} \rangle \in [q]^F$$

where σ_{if} is the value of *trait* f for agent i and can take any value from a finite set $[q]$, such that $\sigma_{if} \in [q]$ represent the diversity of possible cultural configurations in the model. These traits may symbolize various aspects of culture, such as beliefs, preferences, or social behaviors.

External Vector Field (VF) and External Conditions : To incorporate the influence of external factors on agents, the model introduces an external vector field (VF), which represents a constant source of cultural influence analogous to mass media (M). This VF acts on all agents, with the aim of inducing the adoption of certain cultural traits within the system (equation 6). It is characterized by

$$\vec{\sigma}_M = \langle \sigma_{M1}, \sigma_{M2}, \ldots, \sigma_{MF} \rangle$$

and is designed to share at least one trait with each agent on the network, sharing a minimal connection and interaction with all individuals (González-Avella *et al.* 2005).

Agent Groups : Based on the similarity of characteristics, agents can be grouped into two main categories:

Group A: Consists of agents with high cultural similarity, increasing their interaction likelihood and fostering homogenization. Frequent interaction among Group A agents facilitates the transmission of cultural traits through local networks of influence. This group is characterized by a high probability of interaction with culturally similar agents, making them *bridges* of cultural influence toward other agents in the system.

Group B: This group consists of agents with few or no common traits with other agents in their neighborhood. The low similarity in traits reduces their probabilities of interaction and adoption of new cultural characteristics, allowing them to maintain a distinct cultural identity or form isolated subcultures within the system. For Group B agents, the probability of direct interaction with others is low. However, if they are positioned near Group A agents, they may adopt certain cultural traits through an *indirect interaction*, mediated by the interactions of Group A agents with others.

Environment and External Conditions

The proposed model simulates a society structured in the $G_{L,L}$ lattice (of dimension $L \times L$), where each site on the grid represents a culturally characterized agent. This spatial environment allows agents to interact primarily with their four closest neighbors, promoting the diffusion of cultural traits based on similarity in characteristics.

The effect of VF on the agents is quantified and regulated by two fundamental parameters:

Effective Traits ϵ: This parameter represents an additional quantity of traits that VF effectively shares with each agent. The effective traits act as universal elements (such as language or widely understood symbols) that facilitate interaction and the potential influence of the VF, even when there is no match in an agent's nominal cultural traits. Thus ϵ strengthens the probability of interaction between VF and agents, allowing all agents a chance of being influenced.

Confidence Value C: This parameter measures the credibility or acceptance of the VF information. The confidence C is incorporated as an additional probability factor (it is explained how to be used in the cultural features update part) for an agent to directly adopt a trait from the VF. High values of C increase the VF influence and promote greater cultural homogenization within the system, in contrast, low values reduce its impact, allowing for the coexistence of diverse cultural configurations.

Dynamics and Agent Update

The model defines the interaction rules that shape cultural evolution over time. These rules dictate how agents modify their cultural traits based on interactions with neighbors and external influences. These rules are based on cultural similarity and the influence of local interactions with other agents, as well as external influence represented by the VF. Below, the main internal processes and the agent update mechanism are detailed.

Nominal probability of interaction: The interaction between two agents i and j is conditioned by the degree of overlap or similarity between their features, defined as the number of shared traits. This similarity is measured by the *overlap* $l(i, j)$, which is expressed as:

$$l(i,j) = \sum_{f=1}^{F} \delta_{\sigma_{if},\sigma_{jf}}. \tag{1}$$

where δ is the Kronecker function, which is 1 if $\sigma_{if} = \sigma_{jf}$ and 0 otherwise. The *nominal probability of interaction* between two agents i and j is given by:

$$p(i,j) = \frac{l(i,j)}{F}. \tag{2}$$

Thus, the probability that two agents interact and influence one another increases with the traits shared.

Extended Probability of Interaction: To model the interaction between agents and VF, and the probability of agents adopting external traits, an extended probability of interaction is defined. This probability incorporates both agents' nominal cultural traits and shared effective traits. It is expressed as:

$$p(i,M) = \frac{l(i,M) + \epsilon}{F + \epsilon} = \frac{l(i,M)/F + \epsilon/F}{1 + \epsilon/F}. \tag{3}$$

Note that: $p(i, M) \geq l(i, M)/F$. The equality holds if $\epsilon = 0$ or $l(i, M) = F$

This extended probability $p(i, M)$ regulates the effective interaction between each agent and VF, increasing the likelihood that agents adopt traits from VF even when overlap $l(i, M)$ is low. The effect of ϵ on interaction probability enables VF to exert a broader influence on the system (Rodríguez *et al.* 2009).

It should be noted that equations 2 and 3 determine whether an interaction is generated between the selected pair of agents.

Interaction between Agent Groups: As mentioned earlier in the model, agents are divided into two main groups based on their relationship with the external VF:

Group A: This group includes agents who share at least one trait with the VF. These agents can interact directly with the VF and receive its influence directly.

Group B: Agents in this group do not share any traits with the VF, so they cannot interact directly with it. However, they can be indirectly influenced through their interactions with agents in Group A.

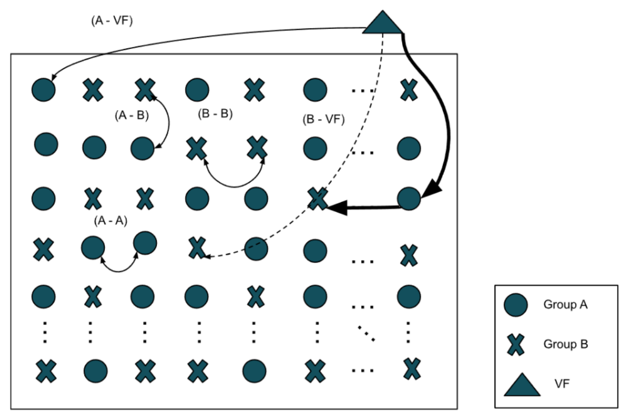


Figure 1 This is a representation of agents and dynamic interactions in the lattice network with VF is included (Rodríguez *et al.* 2009). Different lines represent the interactions types. Thin lines represent interaction between agents in group A with the VF, within group A, and within group B. The indirect interaction from the VF to agents in group B mediated by agents in group A, is depicted using thick lines. Dashed lines represent direct interaction between the VF and agents in group B.

The interactions in the model can be classified as follows (Figure 1):

- **Interactions between Group A Agents and the VF (A–VF)**:Agents in Group A, exhibiting characteristics similar to those of VF, have the possibility to interact directly with it and adopt its traits. These interactions are unidirectional and synchronous (all updates occur at the same time step), allowing the VF to directly influence Group A, spreading its cultural characteristics. This relationship is fundamental, as it turns Group A agents into a channel through which VF traits may potentially spread throughout society.

- **Interactions among Group A Agents (A–A)**: Group A agents frequently interact with each other due to their cultural similarities, facilitating the propagation of traits adopted from VF within the group. These interactions are unidirectional and synchronous, consolidating and strengthening the VF's influence on Group A, as traits adopted from the VF by one agent can quickly be shared with other agents in the same group. Thus, Group A agents tend to form homogeneous groups that are culturally aligned with the VF.

- **Interactions among Group B Agents (B–B)**: Group B agents, who do not share traits with the VF, interact with each other based on their own cultural characteristics. These interactions are unidirectional and synchronous, reinforcing this separate identity, as these agents are not directly exposed to the VF's traits and, therefore, are less susceptible to its influence. This interaction leads to the formation of subcultures within the lattice network.

- **Interactions between Group A and Group B Agents (A–B)**: Although Group B agents do not interact directly with the VF, they can be indirectly influenced through their interactions with Group A agents. When a Group A agent, who has already adopted a trait from the VF, interacts with a Group B agent, there is a possibility that the latter adopts this trait. This interaction is unidirectional and synchronous, as traits can potentially be shared both ways. In this way, Group A acts as a bridge, allowing the influence of the VF to propagate to Group B, albeit indirectly and less frequently. This chain of interactions (A–VF followed by A–B) enables VF traits to permeate the entire system, even reaching culturally distant agents in Group B.

- **Interaction between Group B and VF (B–VF)**: There is no direct interaction between the VF and Group B agents due to the absence of shared traits. In the model, this lack of direct interaction means that Group B agents are "isolated" from VF's direct influence. However, the model incorporates effective features (ϵ) to simulate a minimal unidirectional connection that the VF maintains with all agents, even those in Group B.

Cultural features Update : The agents update dynamics are carried out through an iterative process, using the following steps:

1. **Active Agent Selection**: In each iteration, an agent i is randomly selected to act as the *active* agent in the interaction.

2. **Interaction Agent Selection**: For each active agent randomly selects a nearby neighbor (one of the four closest neighbors) or the VF, for interaction fase.

3. **Determine the overlap** $l(i,s)$: Calculate the overlap, where $s = j$ for a neighboring agent or $s = M$ for the VF. When $s = M$, agent i interacts with the VF using the extended probability (3). On the other hand, if $s = j$ and $0 < l(i,j) < F$, agents i and j interact with the nominal probability (2). In both cases, if $l(i,s) < F$, interaction is required; otherwise, when $l(i,s) = F$, interaction is not necessary because they already share all cultural traits.

4. **Trait Copying Dynamics**: If the interaction is valid, active agent i proceeds to copy a trait from j agent with certain probability; as follows. Randomly chose a position trait f, such that $\sigma_{if} \neq \sigma_{jf}$. If the interaction agent is a neighbor j, agent i copies a trait σ_{jf}; if the interaction agent is the VF, agent i copies the trait σ_{Mf} of the VF, anyone of these cases are with probability p'. The probability p' of adopting the copied trait depends of C confidence value. This probability p' is responsible for regulating whether the selected feature is copied or not according to the following rule:

$$p' = p'(i,s) = \begin{cases} C, & \text{if } \sigma_{sf} = \sigma_{Mf} \text{ or } \sigma_{jf} = \sigma_{Mf}. \\ 1-C, & \text{if } \sigma_{if} = \sigma_{Mf}. \\ 1, & \text{if } \sigma_{if} \neq \sigma_{Mf} \neq \sigma_{jf}. \end{cases} \tag{4}$$

5. **Copy Probability Between Agents**: The total probability that the active agent i copies a trait from the interaction agent s (where s can be a neighbor j or the VF) is given by:

$$P(i,s) = \begin{cases} \frac{1}{5} \cdot p(i,j) \cdot p'; & s = j. \\ \frac{1}{5} \cdot p(i,M) \cdot p'; & s = M. \end{cases} \tag{5}$$

   Where the factor $\frac{1}{5}$ represents the probability of selecting an interaction agent among the five possible options (four neighbors and the VF), and p' is the additional probability of copying or discarding the trait based on the confidence value C.

With all these statements and looking $G_{L,L}$ as subset of $\mathbb{R}^{F+1}$ and $[q]^F \subset \mathbb{R}^{F+1}$; the mass media M defined as a vector field VF has the form:

$$\begin{aligned} VF : G_{L,L} &\to [q]^F, \\ VF(i) &= \sigma_i'. \end{aligned} \tag{6}$$

Where σ_i' is like σ_i but with f-position changed with the Trait Copying Dynamics rules (when i interact with VF).

MAIN ANALYSIS GUIDELINES

Absorbing States

When the system evolution dynamic stops or it undergoes no further changes we said the model reach an *absorbing state*. The absorbing states of the model can be:

- **Monocultural States**: Represent a class of absorbing states characterized by complete cultural homogeneity within the lattice. In these states $l(i,s) = F$.
- **n-cultural States**: This absorbing state is organized into multiple homogeneous cultural regions, distinct from each other. In these states $l(i,s) = 0$ between agents of diferent cultural region and $l(i,s) = F$ between agents on the same cultural region.

Cultural regions and the bigger cultural region (S_{max})

A *path* w in a lattice network is an ordered sequence of vertices starting from an initial vertex to an endpoint such that each consecutive pair of vertices in the sequence is connected by an edge.

For a σ cultural feature, we define a w_σ-path as a set of nodes (agents) such that $\sigma_i = \sigma$ for each agent (node) in a path w. Also we define a *σ cultural region*, as

$$R_\sigma = \{i : \sigma_i = \sigma \quad \text{and} \quad \forall\, i,\, j \text{ exists a } w_\sigma \text{ path}\}$$

Now, define, $S_{\max}$ as the biggest cultural region size, that is

$$S_{max} = max_\sigma\{|R_\sigma|\}$$

S_{max} value is used to quantify the final cultural structure of the system by evaluating homogeneity or fragmentation of the network. This value corresponds to the number of agents belonging to the largest cultural domain, meaning the largest set of agents sharing exactly the same cultural trait vector. Additionally, in the context of absorbing states, we can define

- $S_{\max} \approx L^2$. Monocultural state.
- $S_{\max} \ll L^2$. n-cultural state.

Analysis of S_{max} as a function of initial cultural diversity q and other model parameters, such as confidence C and effective traits ϵ, allows studying the transition between monocultural and n-cultural states.

Percentage of VF Information (ρ)

VF information percentage ρ measures how much of the VF external information has been adopted by agents at the dynamics ends. This parameter is defined as the average overlap between agents' cultural traits and VF, expressed as a percentage:

$$\rho = 100 \times \frac{1}{L^2 F} \sum_{i=1}^{L^2} \sum_{f=1}^{F} \delta_{\sigma_{if}, \sigma_{Mf}}, \tag{7}$$

Analysis of ρ evaluates VF effectiveness in culturally homogenizing the model and complements metrics such as S_{max} and number of cultures to characterize global model dynamics.

EXPERIMENTAL DESIGN

The experimental design was carried out with the following parameters and characteristics.

- There are L^2 agents where $L = 30$.
- Each agent i has a vector of nominal characteristics $\vec{\sigma}_i$ with fixed value $F = 4$ features, and each one take values in $[q]$.
- Two parameters are included ϵ and C.
  - The ϵ parameter takes values 0.01, 0.1, 0.5, and 1, it maximizes the amount of information.
  - The confidence C parameter will range from $C = 0$ to 1, with increments of 0.05, it represent confidence of mass media information.
- The simulation parameter q, will take values from 1 to 39.
- The simulation was divided into three experiments. In each experiment, ϵ and C take the full range of values, and q is divided into three value ranges, as follows:
  - Experiment 1: $q \in \{1, \cdots, 13\}$
  - Experiment 2: $q \in \{14, \cdots, 26\}$
  - Experiment 3: $q \in \{27, \cdots, 39\}$

Once experiments were completed, each one generated a database. The complete database contains approximately 108,000,000 data values.

RESULTS AND ANALYSIS

An analysis of the results obtained for each proposed experiment is presented below. The following two sections include figures showing different absorbing states and categorizing them based on the metrics described in them.

The following figure (Figure 2) were developed to identify simulation results for $S_{\max}$, cultural number regions and their relationship with confidence (in mass media) and information percentage.

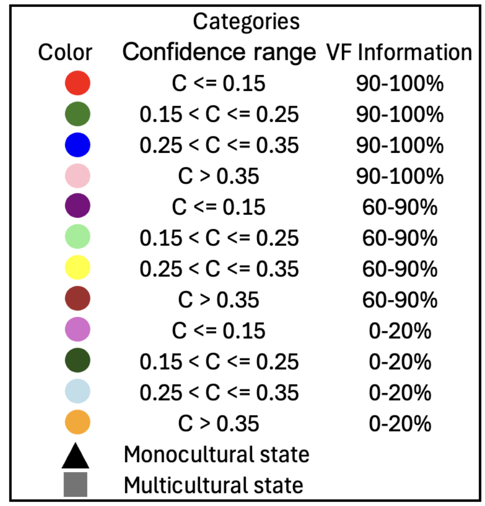


Figure 2 Filled squares represent a n-cultural state, filled triangles represent a monocultural state. The different colors are assigned to various categories depending on the percentage of the VF information (equation 7) distributed in the lattice and the different ranges of values of C

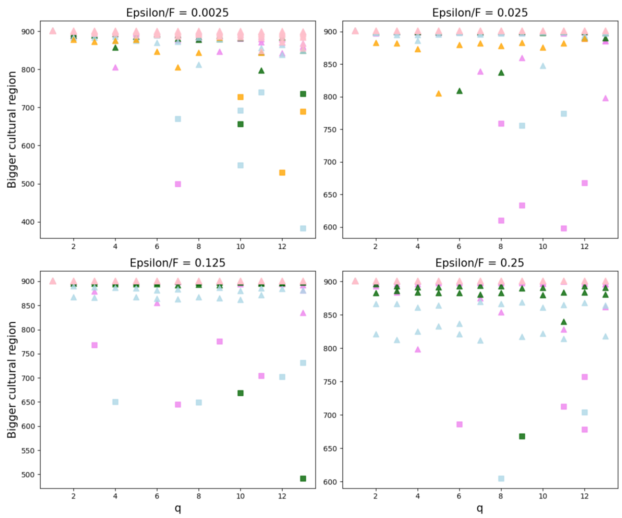
Bigger cultural region S_{max} results

Different absorbing states have been found and we show them for each experiment, we report two different views of each one to analyze accurately its relations with the parameters and the information measure of VF

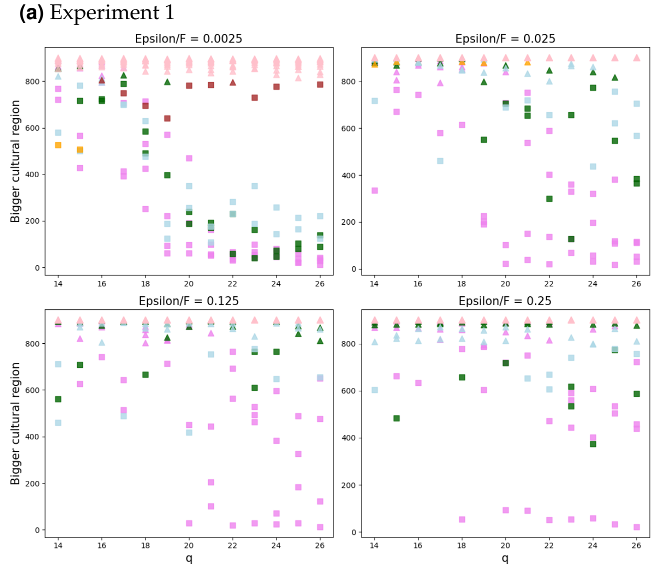
Experiment 1 ($1 \leq q \leq 13$) reveals a dominant monoculture pattern, independent of parameter C (Figure 3a). Monoculture is observed in scenarios with both low levels of information provided by VF (between 0% and 20%, represented by pink, green, light blue, and yellow colors) and high VF information levels (over 90%, represented by pastel pink). This generalized pattern suggests that the presence of monoculture does not significantly depend on the level of VF information. Additionally, a few isolated cases of n-cultural states were identified for $C < 0.35$, which coincide with very low levels of VF information (0% to 20%). These cases are exceptional and do not show a clear trend or defined effect of the parameter ϵ/F, which does not appear to significantly influence the observed absorbing state.

The range of q covers the entire spectrum analyzed (from 1 to 13) without detecting substantial changes in the observed patterns. This reinforces the conclusion that neither q nor ϵ/F exerts a considerable impact on the emergence of coexistence in the system, where states of monoculture or n-culturalism are observed under the given parameters.

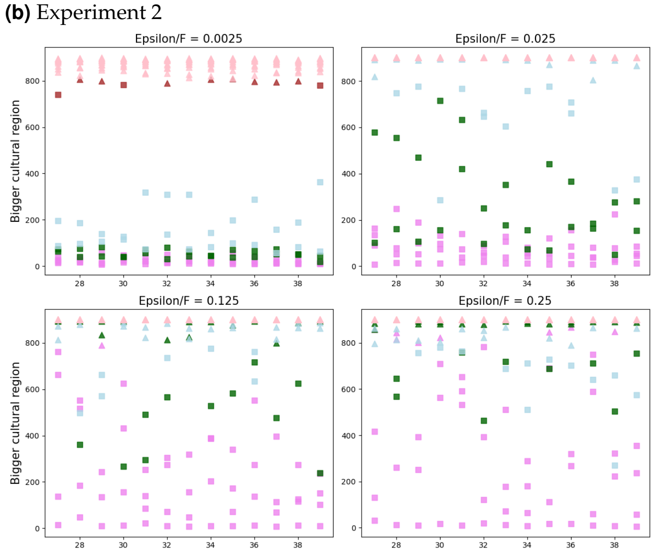
Experiment 2 ($14 \leq q \leq 26$) from (Figure 3b) shows a balance between monocultural and n-cultural states, with both appearing in similar proportions within the range of parameters analyzed. n-cultural states are characterized by low levels of information provided by the VF, ranging between 0% and 20% (represented by pink, green, light blue and yellow colors). Conversely, monocultural states are associated with high levels of information, reaching between 90% and 100% of VF. Notably, some monocultural states with low levels of VF information (0% to 20%) are distributed in green, light blue, yellow, and fuchsia colors, depending on the specific conditions of the parameters F and q.



(a) Experiment 1



(b) Experiment 2



(c) Experiment 3

Figure 3 Bigger cultural region $S_{\max}$ as a function of q. Each panel corresponds to a specific ϵ/F scenario (0.0025, 0.025, 0.125, and 0.25), illustrating the variation of $S_{\max}$ for different values of q in the experiments.

- **Green and light blue states** are primarily found with $\epsilon/F = 0.025, 0.125, 0.250$ and $(14 \leq q \leq 26)$. Their lower frequency suggests that these specific conditions do not favor the formation of monoculture as much as states with high information levels.
- **Fuchsia states** are mostly located in $\epsilon/F = [0.125, 0.25]$ and $(14 \leq q \leq 26)$. This indicates that even with slightly higher levels of ϵ/F, monocultures can occur, albeit in smaller proportions.
- **Yellow states** are even rarer and are observed when $\epsilon/F = 0.25$ and $(14 \leq q \leq 20)$. This restricted range implies that monocultures with these characteristics are unusual and limited to specific combinations of parameteras.

The results of Experiment 3 $(27 \leq q \leq 39)$ from (Figure 3c) show that n-cultural states are predominant, accounting for approximately 70% of the total compared to 30% for monocultural states. A significant proportion of n-cultural states occurs in the range $(18 \leq q \leq 26)$, indicating a strong relationship between high q values and the prevalence of n-culturalism. n-cultural states appear with high VF information (90% to 100%, pastel pink) and low VF information (0% to 20%, green and light blue). This demonstrates that the stability of these states does not solely depend on the level of VF information in the system, but also on other parameters such as q and ϵ/F.

It is observed that when $\epsilon/F = 0.0025$, the system generates two types of cultural regions: one with a maximum region size of up to 0.2 and another with a size exceeding 0.8. As ϵ/F increases, the proportions of the largest cultural region sizes become more evenly distributed across the spectrum of values between 0 and 1. In all cases, monoculture always occurs when $C > 0.35$, some with high VF information (90% or more, pastel pink) and others with almost no VF information (up to 20%, green and light blue). The number of cultures complements the analysis of S_{max} to characterize the transition between monocultural and n-cultural states.

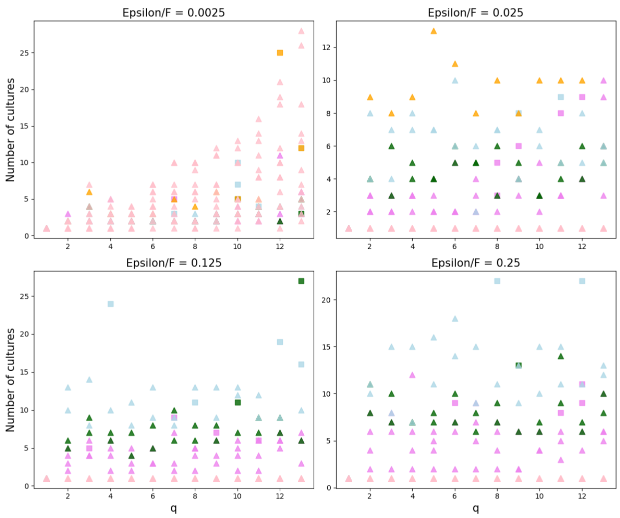
Cultural regions numerical results

Different types of cultural regions have been found and we show them for each experiment, we report two different views of each one to analyze accurately its relations with the parameters and the information measure of VF. For Experiment 1, Figure 4a reveals a pattern characterized by a low number of cultural regions in most of the configurations analyzed. For nearly all ϵ/F values, the number of cultural regions does not exceed 10. These regions can have low or high levels of VF information. In cultural regions with low VF information (0%–20%), states associated with low confidence levels $(C \leq 0.25)$ dominate, identified by pink, green, light blue and yellow colors. These configurations tend to generate multiple cultural regions, although in a limited number.

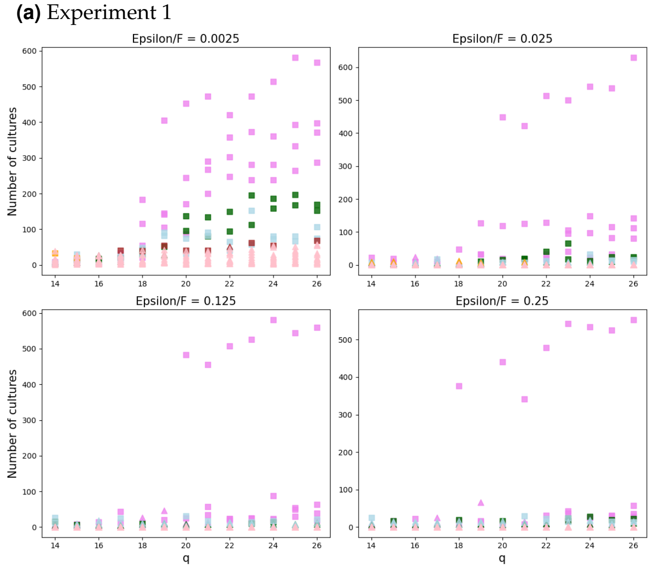
On the other hand, in cases with high VF information (above 90%), monocultural states $(C > 0.35)$ predominate, represented by triangles and pink coloring, which denote a high level of confidence. A notable observation is that even when there are fewer than 10 cultural regions, this configuration could be interpreted as a trend toward monoculture due to the system's low diversity, regardless of the ϵ/F value.

Note: Since the number of cultural regions is less than 10, this could also be seen as a monoculture in a certain sense (as triangles appear in the classification code).

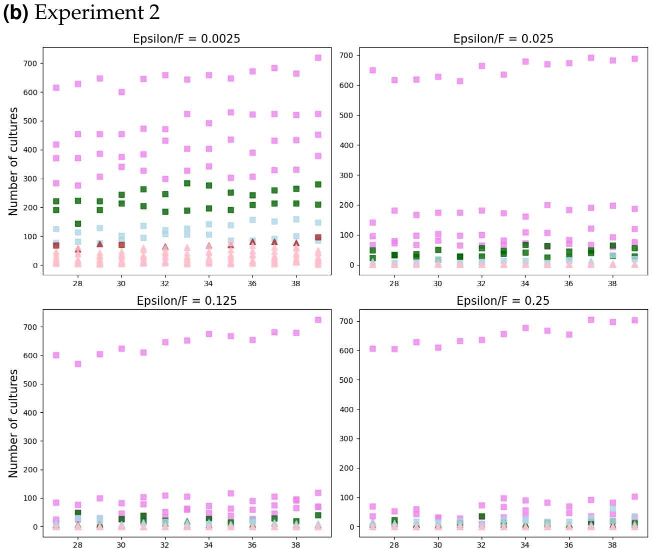
In Experiment 2, Figures 4b show a large number of cultural regions, with most configurations presenting around 50 cultural regions. However, extreme cases with significantly higher values, ranging between 200 and 500 cultural regions, are also observed.



(a) Experiment 1



(b) Experiment 2



(c) Experiment 3

Figure 4 Number of cultures as a function of q. Each panel corresponds to a specific ϵ/F scenario (0.0025, 0.025, 0.125, and 0.25), illustrating the variation in the number of cultures for different values of q in the experiments.

These cases of high cultural diversity occur predominantly for $q > 20$ and confidence levels $C \leq 0.15$ (pink color), highlighting strong cultural diversification under these conditions.

As the confidence level (C) increases, the number of cultural regions decreases significantly, stabilizing around 50. In these configurations, regions tend to have higher levels of VF information (above 90%, indicating greater integration and cultural homogeneity in systems with high levels of confidence. In general, for confidence values $C > 0.35$, the number of cultural regions is even smaller and the configurations are mostly limited to $14 \leq q \leq 20$ and $\epsilon/F = [0.0025, 0.025, 0.125, 0.25]$.

Finally, in Experiment 3, Figures 4c reveal remarkable cultural diversity, with configurations extending up to 700 cultural regions. This extreme level of cultural fragmentation occurs predominantly at low confidence levels ($C \leq 0.15$, pink color) and is observable across all q values within this range. This indicates that low confidence levels facilitate the existence of multiple cultures, particularly in systems with high q values. On the other hand, as the confidence level increases ($C > 0.15$), a progressive reduction is observed in the number of cultural regions, dropping to fewer than 100 in most configurations. A notable case arises in configurations with $C > 0.35$ and $\epsilon/F = 0.0025$ (red color), where the number of cultural regions is very limited, not exceeding 50.

Last square relation between bigger cultural region and number of cultures

The following graph (Figure 5), based on simulation data, illustrates the geometric relationship between S_{max} and number of cultural regions. Clearly, this relationship is not linear. However, what is the nature of the relationship between these two variables?

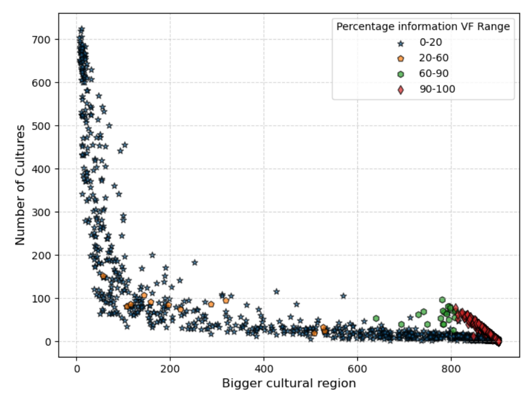


Figure 5 Bigger-region vs. number of cultures: This graph illustrates the relationship between the largest regions on the X-axis and number of cultures on the Y-axis, using different markers to represent varying percentages of VF information: blue stars (0–20%), orange pentagons (20–60%), green hexagons (60–90%), and red diamonds (90–100%). Star points and pentagons, which indicate low information levels, tend to be more dispersed, whereas hexagons and diamonds correspond to moderate and high information levels, respectively.

Figure 5 has a rectangular hyperbolic shape, with asymptotes parallel to Cartesian axes, so it has the form $y = \frac{k}{1+rx} + l$. A relationship between the model dependent variables was determined using the Levenberg-Marquardt method (Marquardt 1963), an iterative algorithm for solving nonlinear least squares problems to approximate their rate change. The resulting equation is as follows:

$$y = \frac{1029.91}{1 + 0.05935 \cdot x} \quad (8)$$

where:

- y: Number of cultures
- x: Bigger cultural region

For this hyperbola, the vertex is located at $(114.85, 131.7)$ and its eccentricity is $\sqrt{2}$.

DISCUSSION

The figures analysis and experimental results reveals that emergence of coexistence in the system does not significantly depend on individual parameters such as q or ϵ/F but rather on their joint interaction. N-cultural states span a wide range of conditions and are less sensitive to these variables, whereas monocultural states tend to arise in scenarios characterized by high information and confidence ($C > 0.35$). Categorization indicates that monocultural states are associated with high confidence and high information levels, while n-cultural states exhibit greater diversity, with confidence ranging from low ($C \leq 0.15$) to high and information levels varying widely. Regarding the parameter q, an evolution in cultural dynamics is observed. In Experiment 1 ($1 \leq q \leq 13$), monoculture predominates due to low number of cultural regions and high VF information, which limits diversity. In Experiment 2 ($14 \leq q \leq 26$), a transition occurs between highly fragmented and more integrated configurations, with n-cultural states prevailing under conditions of low confidence and VF information, while monocultural states emerge with high confidence. Finally, in Experiment 3 ($27 \leq q \leq 39$), high cultural diversity appears under low confidence conditions, whereas monoculture predominates with high confidence.

Overall, the model exhibits strong coexistence of n-cultural and monocultural states. Parameters q, ϵ/F, and C critically influence cultural transitions, offering information relevant to influence on social media, political polarization, and marketing strategies. Mathematically, the number of cultural regions (n_R) and maximum stability ($S_{\max}$) depend on F, q, ϵ/F, and C. Moreover, we hypothesize that $S_{\max} \propto (C + 0.15)$ and $n_R \propto (C + 0.15)^{-1}$, suggesting that small changes in these parameters can induce patterns of self-organization patterns and abrupt transitions between cultural states. The vertex $(114.85, 131.7)$ in equation 8 can be interpreted as a point where the relationship between number of cultural regions and the bigger cultural region changes most significantly. The eccentricity is $\sqrt{2}$, indicating a non-simple relationship between the two variables; small changes in one can result in extremely high or low values in the other. We analyze the relationship between the largest region and number of cultures using a logarithmic scale, applying a log-log transformation on both variables to identify patterns and model them with a power law.

$$num\_cultures = A \cdot (bigger\_region)^{\alpha}$$

where A is a proportionality constant and α is the exponent of scaling law. In log-log calculation (or plot), the fitted line's slope resulted in $\alpha = -1.53$ and $R^2 = 0.596$.

1. The power law exponent, α,indicates an inverse relationship between region size and number of cultures. As region size increases, the number of cultures decreases according to a power law
2. The proportionality constant, $A \approx 84950.89$, represents initial scale of the model and depends on unit of measurement used.
3. The coefficient of determination, R^2, indicates that the model explains about 59.6% of data variability, reflecting a moderately strong relationship.

The result showed a negative slope, suggesting that larger regions tend to support fewer cultures. This may be explained by cultural homogenization processes in expansive areas or the challenge of sustaining multiple cultures in large spaces without natural barriers. In contrast, smaller regions are more likely to maintain greater cultural diversity due to the reduced influence of dominant cultures.

CONCLUSION

This research on cultural dissemination in Axelrod's Model highlights the relevance of monoculture and n-culturalism based on the number of cultural regions. Cultural diversity was characterized and analyzed, showing that while it exists, full monoculturality is not achieved in this model. Two types of monoculture are identified: one influenced by mass media (VF) and one without. VF can generate unexpected results, since varying the value of q to a higher value makes the formation of a monoculture with VF more difficult; however, monoculture can emerge without VF influence if C is close to 1, although n-culturalism remains predominant.

Simulations were performed using NetLogo, optimizing computational performance, and dividing the experimental design into three parts to enhance computing efficiency. Using Python, data science methods, and scientific visualization, up to five variables were graphically represented, achieving a more comprehensive and cross-sectional understanding of the phenomenon. This research extends the model by González-Avella *et al.* (2005) and Rodríguez *et al.* (2009), developing new three-dimensional data visualizations on cultural dissemination and offering a novel analysis focused on the emergence of different cultural regions. In Gracia-Lázaro *et al.* (2021), the focus was on agents' internal mechanisms, emphasizing the adaptive advantage developed in agents, whereas our research introduces mass media to explore cultural stability conditions. Crokidakis (2012) investigates opinion spreading with an emphasis on critical phenomena and mathematical formalization, while our model incorporates cultural diversity within a sociological framework, both being analyzed from different perspectives.

Our study focused on a single population and the impact of strategic advertising on cultural diversity, whereas González-Avella *et al.* (2012) analyzes intergroup interactions through mass media in a model where two populations of social agents, each with its own internal dynamics. In González-Avella *et al.* (2014) explores interactions between two populations and phenomena such as localized coherence and symmetry breaking, while our research examines transitions between absorption states in a single population. Stivala *et al.* (2014) analyzes how hierarchical properties (ultrametricity) and cultural variations in agents structure cultural diversity, whereas our study focuses on how mass media induces monocultural states. In Tilles and Fontanari (2015), the focus was on the internal introduction of innovations, their dependence on network topology, and initial diversity. In contrast, our research focused on the impact of mass media and cultural stability. The work of Stivala *et al.* (2016) highlighted the relationship between culture and cooperation in the context of public goods games, while our study focused on the influence of external forces on cultural diversity. Hernández *et al.* (2018) incorporated a continuous updating dynamic of cultural traits, whereas our study explored the emergence of different states under external influences.

Raducha and Gubiec (2017) focused on the coevolution of cultural dynamics and complex network topology. In contrast, our research examined the stability of cultural states and explored cultural transitions. The work of Cosenza *et al.* (2020) centered on the impact of mass media represented as a global field reflecting predominant cultural traits, whereas our study modeled media as an external force directly influencing state transitions. In Gracia-Lázaro *et al.* (2021), a binary trait representing polarization was introduced, showing how it eliminates classical phase transitions. In contrast, our research demonstrated that n-cultural states can stabilize depending on external parameters, such as media intensity. Finally, Alvarez-Llamoza *et al.* (2024) introduced competition between two mass media sources, identifying conditions where a weaker medium can dominate, producing ordered patterns. In contrast, our study shows how mass media can drive transitions depending on their intensity, generating monocultural and n-cultural states.

Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

We appreciate the reviewers for their valuable suggestions and comments, which have significantly contributed to improving the quality and clarity of our work. We deeply appreciate the time and effort dedicated to reviewing our manuscript.

LITERATURE CITED

Alvarez-Llamoza, O., M. Cosenza, J. Gonzalez-Avella, M. Suarez, K. Tucci, *et al.*, 2024 Mass media competition and alternative ordering in social dynamics. Physical Review E **110**: 024311.

Axelrod, R., 1997 The dissemination of culture: A model with local convergence and global polarization. Journal of conflict resolution **41**: 203–226.

Campos Cantón, E., 2025 Orbits of families of discrete dynamical systems evolving in the natural numbers. Chaos: An Interdisciplinary Journal of Nonlinear Science **35**.

Carletti, T., D. Fanelli, S. Grolli, and A. Guarino, 2006 How to make an efficient propaganda. Europhysics Letters **74**: 222.

Centola, D., J. C. Gonzalez-Avella, V. M. Eguiluz, and M. San Miguel, 2007 Homophily, cultural drift, and the co-evolution of cultural groups. Journal of Conflict Resolution **51**: 905–929.

Cosenza, M. G., M. Gavidia, and J. C. González-Avella, 2020 Against mass media trends: Minority growth in cultural globalization. Plos one **15**: e0230923.

Crokidakis, N., 2012 Effects of mass media on opinion spreading in the sznajd sociophysics model. Physica A: Statistical Mechanics and its Applications **391**: 1729–1734.

Flache, A. and M. W. Macy, 2006 What sustains cultural diversity and what undermines it? axelrod and beyond. arXiv preprint physics/0604201 .

González-Avella, J. C., M. G. Cosenza, and M. San Miguel, 2012 A model for cross-cultural reciprocal interactions through mass media. PloS one **7**: e51035.

González-Avella, J. C., M. G. Cosenza, and M. San Miguel, 2014 Localized coherence in two interacting populations of social agents. Physica A: Statistical Mechanics and its Applications **399**: 24–30.

González-Avella, J. C., M. G. Cosenza, and K. Tucci, 2005 Nonequilibrium transition induced by mass media in a model for social influence. Physical Review E—Statistical, Nonlinear, and Soft Matter Physics **72**: 065102.

Gracia-Lázaro, C., E. Brigatti, A. R. Hernández, and Y. Moreno, 2021 Polarization inhibits the phase transition of axelrod's model. Physical Review E **103**: 062306.

Gracia-Lázaro, C., L. Floria, and Y. Moreno, 2011 Selective advantage of tolerant cultural traits in the axelrod-schelling model. Physical Review E—Statistical, Nonlinear, and Soft Matter Physics **83**: 056103.

Hernández, A. R., C. Gracia-Lázaro, E. Brigatti, and Y. Moreno, 2018 Robustness of cultural communities in an open-ended axelrod's model. Physica A: Statistical Mechanics and Its Applications **509**: 492–500.

Jacobmeier, D., 2005 Multidimensional consensus model on a barabási–albert network. International Journal of Modern Physics C **16**: 633–646.

Klemm, K., V. M. Eguíluz, R. Toral, and M. San Miguel, 2003a Global culture: A noise-induced transition in finite systems. Physical Review E **67**: 045101.

Klemm, K., V. M. Eguíluz, R. Toral, and M. San Miguel, 2003b Nonequilibrium transitions in complex networks: A model of social interaction. Physical review E **67**: 026120.

Marquardt, D. W., 1963 An algorithm for least-squares estimation of nonlinear parameters. Journal of the society for Industrial and Applied Mathematics **11**: 431–441.

Raducha, T. and T. Gubiec, 2017 Coevolving complex networks in the model of social interactions. Physica A: Statistical Mechanics and its Applications **471**: 427–435.

Railsback, S. F. and V. Grimm, 2019 *Agent-based and individual-based modeling: a practical introduction*. Princeton university press.

Rodríguez, A. H., M. del Castillo-Mussot, and G. Vázquez, 2009 Induced monoculture in axelrod model with clever mass media. International Journal of Modern Physics C **20**: 1233–1245.

Rodríguez, A. H. and Y. Moreno, 2010 Effects of mass media action on the axelrod model with social influence. Physical Review E—Statistical, Nonlinear, and Soft Matter Physics **82**: 016111.

Shibanai, Y., S. Yasuno, and I. Ishiguro, 2001 Effects of global information feedback on diversity: extensions to axelrod's adaptive culture model. Journal of Conflict Resolution **45**: 80–96.

Stivala, A., Y. Kashima, and M. Kirley, 2016 Culture and cooperation in a spatial public goods game. Physical Review E **94**: 032303.

Stivala, A., G. Robins, Y. Kashima, and M. Kirley, 2014 Ultrametric distribution of culture vectors in an extended axelrod model of cultural dissemination. Scientific reports **4**: 4870.

Tilles, P. F. and J. F. Fontanari, 2015 Diffusion of innovations in axelrod's model. Journal of Statistical Mechanics: Theory and Experiment **2015**: P11026.

How to cite this article: López Morales, S. J., González Silva, R. A. and González Silva M. I. Emergence of Coexistence in the Cultural Dissemination of the Axelrod Model with Mass Media. *Chaos Theory and Applications*, 7(1), 78-86, 2025.