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**Abstract**

This study aims to develop the Interactive Mathematics Learning Model that integrates digital tools with Realistic Mathematics Education. The model is designed specifically for teaching secondary school mathematics and consists of three basic components to make the mathematics learning process more meaningful, interactive and permanent: contextual problem scenarios, mathematical modeling and interactive discovery, and a collaborative and adaptive learning environment. The main purpose of the model is to provide middle school students with mathematical thinking skills through real-life problems, to develop mathematical models and to allow them to test these models with digital manipulatives. The model is supported by digital tools such as virtual reality environments, game-based learning platforms and artificial intelligence-supported analysis systems. Students create mathematical models using digital simulations in their problem-solving processes and make abstract concepts concrete by testing these models. The integration of the model into secondary school mathematics teaching processes aims to develop mathematical thinking skills by providing students with meaningful learning experiences. In this context, it is expected that the model will be tested with pilot applications, its effectiveness will be evaluated and it will offer an innovative approach in mathematics teaching.



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*Review Article*

# Realistic Mathematics Education and Integration of Digital Tools: A Model Development Study for Secondary School Mathematics Education [*]

**Tamer KUTLUCA** [1] **Koray AKRAN** [2]

**Abstract**

This study aims to develop the Interactive Mathematics Learning Model that integrates digital tools with Realistic Mathematics Education. The model is designed specifically for teaching secondary school mathematics and consists of three basic components to make the mathematics learning process more meaningful, interactive and permanent: contextual problem scenarios, mathematical modeling and interactive discovery, and a collaborative and adaptive learning environment. The main purpose of the model is to provide middle school students with mathematical thinking skills through real-life problems, to develop mathematical models and to allow them to test these models with digital manipulatives. The model is supported by digital tools such as virtual reality environments, game-based learning platforms and artificial intelligence-supported analysis systems. Students create mathematical models using digital simulations in their problem-solving processes and make abstract concepts concrete by testing these models. The integration of the model into secondary school mathematics teaching processes aims to develop mathematical thinking skills by providing students with meaningful learning experiences. In this context, it is expected that the model will be tested with pilot applications, its effectiveness will be evaluated and it will offer an innovative approach in mathematics teaching.

**Keywords:** Realistic mathematics education, digital tools, interactive mathematics learning model, collaborative and adaptive learning, mathematical thinking skills

## 1. INTRODUCTION

Mathematics is one of the basic disciplines that has existed in every aspect of life since the beginning of human history and that develops thinking and problem-solving skills. Since the early ages, mathematics has been one of the most effective tools that people have used to solve daily life problems. Applications such as measuring agricultural areas, making commercial calculations, creating calendar systems and building structures show that mathematics arises from practical needs (Freudenthal, 1991). Mathematics has become a building block that enables not only individuals but also societies to advance in the fields of science, technology and economics (Baykul, 2020). Especially in the age of information and technology, mathematics has become even more important as an element that develops critical thinking, analytical thinking and problem-solving skills (Niss, 2007). As John Dewey (1986) emphasized, the main purpose of education is to develop the individual's critical thinking, problem-solving and lifelong learning skills. In this context, mathematics education aims to develop individuals' logical thinking, problem solving, modeling and analysis skills (Altun, 2015). Thanks to mathematics, individuals develop their abstract thinking skills and can produce

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creative and analytical solutions to the problems they encounter in their daily lives (The National Council of Teachers of Mathematics [NCTM], 2000). In addition, mathematics enables individuals to become competent in fields such as information technologies, engineering, economics and science (Cai, 2014). It is necessary to ask the question, "What should we take into consideration in the process of teaching mathematics in order to develop the desired skills in individuals?" Because the teaching approaches adopted and the materials used in mathematics education directly affect the learning process of students. Traditional teaching methods have a teacher-centered structure and transfer information directly to the student (Güzel, 2009). In these methods, students focus on memorizing information and have difficulty relating mathematics to daily life (Coştu, 2020). This situation causes students to perceive mathematics as an abstract, incomprehensible and boring lesson and develop negative attitudes towards mathematics (Taş et al., 2016). Student-centered teaching approaches are being developed to overcome these problems. The constructivist approach to education advocates that student actively constructs knowledge and ensures that students take an active role in the learning process (Fosnot, 2013; Vygotsky, 1978). In teaching processes where methods that can be used within the scope of the constructivist approach are adopted, students' prior knowledge is taken into account and the discovery of new information is encouraged (Altun, 2006). Methods such as problem-based learning (PBL), inquiry-based learning, and project-based learning enable students to understand mathematical concepts in depth and use these concepts in different contexts (Hmelo-Silver, 2004). In particular, the use of mathematical modeling and multiple representations supports students' conceptual understanding and concretizes abstract concepts (Blum & Borromeo Ferri, 2009; Lesh & Doerr, 2003). Developing students' problem-solving skills and associating mathematics with daily life is one of the main goals of mathematics education (Niss, 2015). In this context, approaches such as Realistic Mathematics Education (RME) emphasize relating mathematics education to real-life problems (Freudenthal, 1991; Van den Heuvel-Panhuizen, 2003). RME allows students to explore mathematical concepts and learn them meaningfully (Doğan & Kurt, 2019).



Technological developments have also created a significant transformation in mathematics education. The use of digital tools makes mathematics teaching more interactive and student-centered (Kaput, 1992). Computer-based teaching software, dynamic geometry programs, virtual simulations and interactive learning platforms help students concretize and visualize mathematical concepts (Hegedus & Moreno-Armella, 2009; Sinclair & Bruce, 2015). Especially tools such as GeoGebra, Desmos and Wolfram Alpha support students' learning by using multiple representations in algebra, geometry and analysis (Selçik & Bilgici, 2014). Thanks to these tools, students can understand abstract mathematical concepts more easily and relate these concepts to daily life problems (Drijvers et al., 2010). Digital tools also offer various advantages for teachers. Through digital platforms, teachers can instantly monitor students' learning processes, provide individual feedback, and make teaching processes more flexible (Koehler & Mishra, 2009; Niess, 2005).

In order to use digital tools effectively, teachers must have sufficient knowledge and skills about these tools. Therefore, it is of great importance to support teachers in technology integration through in-service training programs (Department of Education Research and Development [EARGED], 2005; Mishra & Koehler, 2006). As a result, the effectiveness of mathematics teaching can be increased by adopting student-centered teaching approaches and integrating digital tools. Mathematics education that is associated with real life and supported by technological tools contributes to students' development of mathematical thinking skills and meaningful learning of mathematics (Turkish Ministry of Education [MoNE], 2018). With the cooperation of all stakeholders of the education system, mathematics education can be made more effective, efficient and student-centered. Thus, individuals not only have mathematical knowledge, but can also use this knowledge effectively in their daily and professional lives (NCTM, 2000).

### 1.1. Mathematics Education and Teaching

Mathematics education is a multidimensional teaching process that develops individuals' analytical thinking, problem solving, modeling, reasoning and logical inference skills (Baykul, 2020; Freudenthal, 1991). Mathematics not only forms the basis of science and technology, but is also important in terms of developing individuals' critical thinking skills and producing solutions to problems they encounter in daily life (Altun, 2015). Therefore, the main purpose of mathematics education is to increase individuals' mathematical literacy levels and to ensure that they integrate these skills into their lives (MoNE, 2018; NCTM, 2000).

However, throughout history, mathematics teaching has generally been limited to a rote-learning approach. Traditional teaching methods transferred information directly to the student and presented mathematics as a mass of abstract information (Güzel, 2009). This has limited students' opportunities to associate mathematical concepts with daily life, leading to mathematics being perceived as a "difficult" and "hard to understand" discipline (Taş et al., 2016).

Modern education approaches aim to change this negative perception. The purpose of mathematics education is not only for students to memorize formulas and rules, but also to use this information in their daily lives by making sense of it (Baykul, 2020). For example, algebraic equations should be taught not only as a process to be solved, but also with their real-life counterparts (NCTM, 2000). In this context, student-centered approaches and innovative teaching methods play an important role in increasing the effectiveness of mathematics education (Altun, 2006).

The constructivist education approach emphasizes the active role of the individual in teaching mathematics (Fosnot, 2013; Vygotsky, 1978). This approach is based on the student constructing knowledge in line with his/her own experiences and prior knowledge. During the learning process, the student ceases to be a recipient of information and becomes an individual who discovers knowledge (Altun, 2006). In this process, the teacher takes on a guiding role and supports the student in thinking, questioning and discovering new information. This understanding ensures that mathematics education is more permanent and meaningful (Baki, 2020).



Another important difficulty encountered in teaching mathematics is the disregard of individual differences. Each student's learning style, readiness and learning speed are different (Dağdelen & Ünal, 2017). For this reason, mathematics teaching should have a flexible and student-centered structure that takes individual differences into account. In this regard, innovative approaches such as RME provide a model that supports students in understanding mathematical concepts and relating them to daily life (Freudenthal, 1991; Van den Heuvel-Panhuizen, 2003). RME allows students to explore mathematical concepts through real-life problems, thus making mathematics learning more meaningful (Doğan & Kurt, 2019).

At the international level, the standards determined by NCTM constitute the basic elements of mathematics education (NCTM, 2000). According to NCTM, the five basic components of mathematics education are as follows:

*Problem Solving:* It is aimed for students to solve various problems and develop new strategies using their mathematical knowledge.

*Reasoning and Proof:* It is aimed for students to prove their results by developing their logical thinking and inference skills.

*Communication:* It is encouraged to express mathematical ideas clearly in written and oral form.

*Association:* It is ensured that mathematical concepts are associated with different disciplines and daily life.

*Representation:* It is aimed for mathematical ideas to be expressed with concrete and abstract representations (NCTM, 2000).

In Turkey, MoNE has developed curriculums compatible with these standards. With the 2005 and 2018 reforms, a constructivist approach was adopted, and skills such as problem solving, reasoning and association were emphasized (MoNE, 2005; MoNE, 2018, MoNE, 2024). The following goals are highlighted in the mathematics curriculum in Turkey:

*Mathematical Literacy:* It is aimed for students to use mathematical knowledge effectively and think critically (MoNE, 2018).

*Higher-Level Thinking Skills:* It is aimed to develop problem solving, reasoning, analysis and evaluation skills (Altun, 2015).

*Connection with Daily Life:* It is important to associate mathematical concepts with situations encountered in daily life (Baykul, 2020).

*Use of Technology:* Students are supported to learn interactively by using technology-supported tools such as GeoGebra while learning mathematics (MoNE, 2024).

### 1.2. Realistic Mathematics Education

RME is a teaching approach developed in the Netherlands in the 1970s and shaped under the leadership of Hans Freudenthal (Freudenthal, 1973). This approach sees mathematics not as a body of knowledge, but as a tool that individuals can relate to daily life and use to solve real problems (Van den Heuvel-Panhuizen, 2003). According to Freudenthal, mathematics should gain meaning through the environment and real-life contexts in which students are located (Freudenthal, 1991). In this direction, Realistic Mathematics Education offers a model in which students actively participate, problem-solving skills come to the fore, and they acquire meaningful information through rediscovery in the learning process (Gravemeijer & Doorman, 1999).

Traditional mathematics teaching mostly involves a structure based on abstract concepts and memorization (Skemp, 1976). However, RME aims to ground mathematics in concrete contexts and help students make sense of mathematical concepts (Treffers, 1987). In this approach, the teaching process is shaped by a process called "mathematization" (Freudenthal, 1991). Mathematization refers to students solving problems in real-life contexts through mathematical thinking (Gravemeijer, 1994). This process allows students to see mathematics not only as a course content but also as a system of thought that they can use throughout their lives (Van den Heuvel-Panhuizen, 2001).



Another important feature of RME is that it promotes a student-centered learning environment (Cobb et al., 1992). Students cease to be passive recipients in the learning process and actively explore and reconstruct mathematical concepts (Fosnot & Perry, 2005). In this context, the basic principles of RME, "guided reinvention", "didactic phenomenology" and "self-developed models" are important elements that support this process (Treffers, 1987). These principles help students develop mathematical thinking and gain in-depth understanding during the learning process (Van den Heuvel-Panhuizen & Drijvers, 2014).

RME encourages not only individual learning but also collaborative learning environments (Bakker & Van Eerde, 2015). Students develop their mathematical thinking and explore different perspectives through group work and discussions (Sfard, 2008). This process provides a deeper understanding of mathematical concepts and strengthens communication skills among students (Lave & Wenger, 1991). At this point, teacher guidance plays an important role (Wood et al., 1991). Teachers guide students in the processes of solving and making sense of mathematical problems, but do not directly present solutions (Boaler, 2016). In this way, students' problem-solving and critical thinking skills develop (Cai et al., 2017). While RME helps students develop a permanent understanding of mathematics, the use of digital tools further enriches this process (Zbiek et al., 2007). In particular, digital simulations and interactive learning platforms facilitate the concretization of abstract concepts and make the learning process more dynamic (Hoyles & Lagrange, 2010). Thus, when the opportunities offered by Realistic Mathematics Education are supported by technology, students' mathematical thinking skills and learning motivation increase significantly (Pierce & Stacey, 2010).

### 1.2.1. Basic principles of realistic mathematics education

In order for RME to be implemented effectively, certain basic principles have been adopted (Freudenthal, 1991). These principles differentiate mathematics teaching from the traditional understanding and ensure that students learn mathematics in a meaningful way (Van den Heuvel-Panhuizen, 2003).

*Guided Reinvention:* This principle refers to students discovering mathematical concepts and methods through their own experiences (Gravemeijer, 1994). The teacher provides guidance in this process but does not directly present the information (Treffers, 1987). This approach allows students to learn mathematical knowledge more permanently and develop critical thinking skills (Cobb et al., 1992). In this process, students create their own mathematical structures and deepen their level of understanding while discovering problems (Fosnot & Perry, 2005).

*Didactic Phenomenology:* Didactic phenomenology refers to students making sense of events in real-life contexts from a mathematical perspective (Freudenthal, 1973). This principle argues that mathematical concepts should be associated with daily life (Van den Heuvel-Panhuizen & Drijvers, 2014). For example, the concept of ratio can be taught through concrete contexts, such as adjusting the amounts of ingredients in a recipe (Gravemeijer & Doorman, 1999). Students' making sense of such contexts increases their mathematical understanding and makes learning permanent (Bakker & Van Eerde, 2015).

*Self-Developed Models:* In RME, students gradually discover mathematical concepts from concrete contexts to abstract concepts (Treffers, 1987). In this process, students make sense of mathematical concepts through models they develop (Gravemeijer, 1994). This approach helps students structure their mathematical thinking processes and makes their learning more flexible (Van den Heuvel-Panhuizen, 2003). Self-developing models allow students to discover abstract mathematical relationships by deriving them from concrete experiences (Cai et al., 2017). These basic principles are important elements that differentiate Realistic Mathematics Education from other teaching approaches (Boaler, 2016). Teaching processes shaped in line with these principles allow students to learn mathematics meaningfully (Pierce & Stacey, 2010).



### 1.2.2. Mathematization

Mathematization is a concept at the center of RME and refers to students solving problems in real-life contexts through mathematical thinking (Freudenthal, 1991). Mathematization consists of two basic components: horizontal mathematization and vertical mathematization (Treffers, 1987).

According to De Lange (2006), the mathematization process carried out by the student actually has a cyclical and dynamic structure. This process begins with the student encountering a meaningful problem and progresses step by step, building a bridge between the real world and mathematics. How the student moves in this cycle is given in Figure 1.

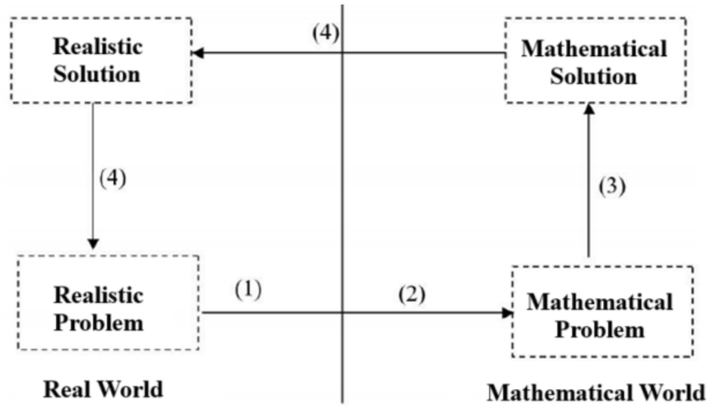


**Figure 1.** The mathematization cycle (De Lange, 2006)

In the first step, the student addresses a meaningful problem he encounters. For example, let's consider a situation such as placing trees to be planted in a garden at equal intervals. Here, the student tries to grasp the problem and determines the mathematical concepts hidden in it. At this stage, the student notices elements such as distances, numbers or symmetry. Understanding the problem allows the student to think actively and connect with real life.

In the second step, the student begins to transform the problem into a mathematical model. During this transformation, the student's attention is focused on the mathematical elements in the essence of the problem, disabling irrelevant details. If we return to the example of tree placement in the garden, the student can create a model using concepts such as distance measurement and regular arrangement. The mathematical model is shaped in front of the student as an abstract representation of the problem.

In the third step, the created mathematical model is studied. The student solves the problem using this model. For example, it calculates to find the optimal distance between trees. This stage is a process in which the student applies mathematical thinking and problem-solving skills. Working on the model allows the student to reflect the solution process and structure their thoughts.



In the last step, the student carries the mathematical solution he/she has obtained back to the real-world context. Let's assume that he plans the settlement of the trees in the garden; the student evaluates whether his calculations are really feasible. At this point, it reaches the result by interpreting the mathematical solution in a meaningful and concrete context. Returning to the real world allows the student to see mathematics not just as a theory, but as a part of life.

De Lange's (2006) cyclical mathematization process is based on the student actively understanding the problem, creating a mathematical model, producing a solution using the model, and interpreting the solution. This process allows the student to have a constant interaction between mathematics and the real world. In this way, the student experiences mathematics not only as a lesson, but as a tool that makes sense of life and provides solutions to problems.

Horizontal Mathematization: Horizontal mathematization refers to students mathematically formulating real-life problems (Gravemeijer & Doorman, 1999). For example, the budget calculation process in a market can be shown as an example of horizontal mathematization (Van den Heuvel-Panhuizen & Drijvers, 2014). This process allows mathematical thinking to be associated with daily life and makes it easier for the student to understand mathematics in concrete contexts (Bakker & Van Eerde, 2015).

Vertical Mathematization: Vertical mathematization refers to the use of abstract concepts and methods in the process of solving mathematical problems (Treffers, 1987). In this process, students make sense of abstract mathematical concepts based on the concrete problems at the beginning (Gravemeijer, 1994). For example, an equation-solving process can be given as an example of vertical mathematization (Freudenthal, 1991). Vertical mathematization deepens students' mathematical thinking skills and improves analytical thinking abilities (Hoyles & Lagrange, 2010).

The process of mathematization is an important process that develops students' both relation and abstract thinking skills with real life (Van den Heuvel-Panhuizen, 2003). This process emphasizes that mathematics is not only a course content, but also a system of thought (Pierce & Stacey, 2010).

These basic principles and mathematization processes of RME enable students to learn mathematical concepts in a more meaningful way (Boaler, 2016). RME teaching approach draws attention as a basic education model that allows students to learn mathematics in a meaningful and permanent way. Unlike the traditional understanding of education, this approach does not treat mathematics as a stack of abstract concepts, but as a viable tool in the context of the real world. Moreover, RME designs mathematics learning as a journey of discovery, and in this journey, students become active participants through guided rediscovery (Üzel, 2007).

The way to access information in RME is not similar to Bloom Taxonomy (Üzel, 2007). Bloom's Taxonomy usually starts from the information step and progresses towards high-level steps such as application, analysis, synthesis. RME reverses this order, starting from the implementation step and focusing on a real-life problem. The student tries to solve this problem first and experiences the process of horizontal math. Trying to understand real-world problems allows the student to act at the application level at the beginning of the process. Then, the student transforms these real problems into abstract mathematical models and moves on to the vertical mathematization stage. This loop is symbolized in Figure 2 (Üzel, 2007).

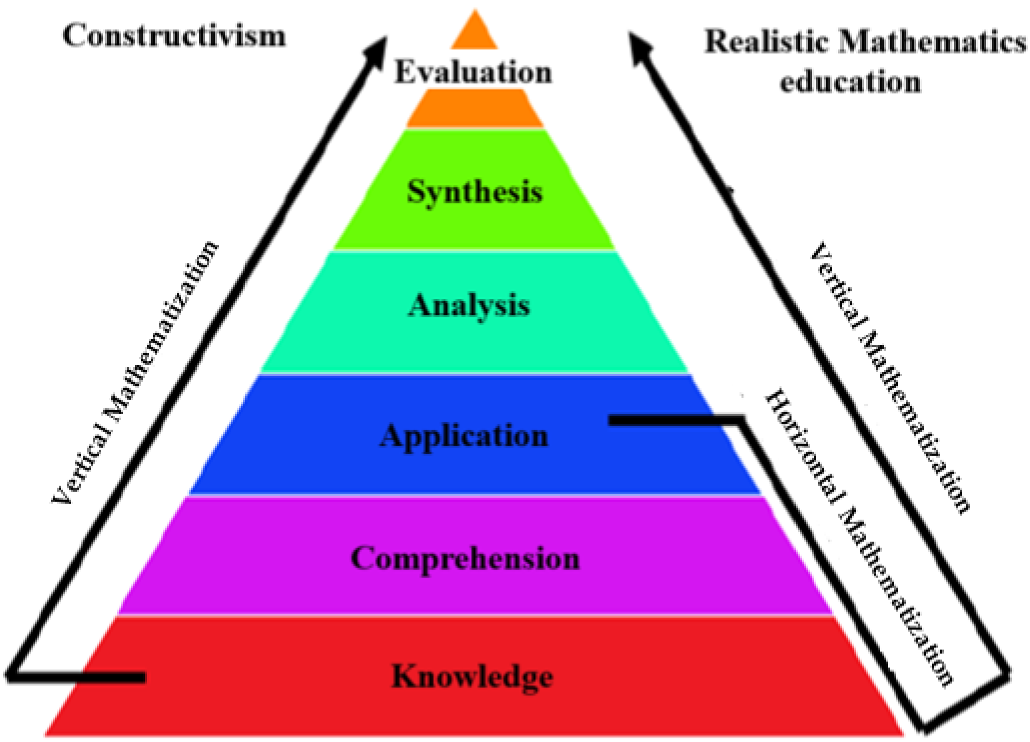


**Figure 2**. Representation of the stages in Bloom taxonomy in RME (Akran & Obay, 2022).



According to the classification made by Adri Treffers (1987), mathematics education is considered with four different approaches according to the processes of horizontal and vertical mathematization (Freudenthal, 1991). These approaches are; mechanical approach, empiristic approach, constructivism and realistic approach. These classifications are detailed in Table 1 by Freudenthal (1991).

**Table 1. Mathematical Approaches According to Horizontal and Vertical Mathematization**

| Approach | Horizontal Mathematization | Vertical Mathemathization |
|---|---|---|
| Mechanical | ✕ | ✕ |
| Empiristic Approach | ✓ | ✕ |
| Constructivism | ✕ | ✓ |
| Realistic | ✓ | ✓ |

*Mechanical Approach:* In the mechanical approach, students memorize arithmetic, algebraic or geometric operations and use this information in repetitive applications. Students memorize algorithms and perform operations, almost like a computer. However, horizontal and vertical mathematization processes do not come into play in this approach. The student cannot adapt the knowledge he/she learns to new and different situations, so he makes mistakes in the face of new problems he encounters. This type of mechanical learning provides superficial and non-permanent learning disconnected from mathematics.

*Empiristic Approach:* In the empiristic approach, students learn from real-world experiences. Here horizontal mathematization comes to the fore (Freudenthal, 1991). The student analyzes real-world problems with concrete experiences and develops practical solutions. However, in this approach, the process of vertical mathematization, that is, abstract thinking and mathematical model creation, does not come into play. Therefore, students find it difficult to reach deeper and more abstract concepts of mathematics. Although learning by experience is beneficial, learning is limited when abstract mathematical thinking is not developed.

*Constructivist Approach:* The constructivist approach advocates that student build knowledge with their own experiences and thinking processes. This approach allows horizontal mathematization using tools such as set theory and flowcharts (Freudenthal, 2002). However, constructivism creates temporary worlds that cannot be associated with the daily life of the student. The student works on abstract and artificial models. Tools such as Venn diagrams or diagrams lose their meaning when they are disconnected from real-world context. It was understood that this approach, which spread in the 1960s and 1970s under the name of "New Mathematics", was insufficient to improve students' mathematization skills (Freudenthal, 2002).



*Realistic Approach:* Realistic approach, on the other hand, stands out as the most effective and balanced method in mathematics learning. Real world problems are the starting point of learning in this approach. The student analyzes these problems with horizontal mathematization, explores regularities and relationships. Later, with the process of vertical mathematization, he transforms these concrete problems into abstract mathematical models (Zulkardi, 1999). For example, starting with a concrete problem such as budget calculation in a market, it progresses to abstract processes such as equation formation and algebraic thinking. This approach both connects with daily life and develops abstract thinking skills.

Realistic Mathematics Education is a model that supports the active learning of students and provides meaningful and lasting learning. It allows students to relate mathematics to real life and thus develop problem-solving skills. Thus, students see mathematics as a tool that they can use in all areas of life and enjoy the mathematics learning process (Freudenthal, 1991; Üzel, 2007).

*1.2.3. Basic characteristics of realistic mathematics education*

RME treats mathematical learning as both an individual and social process. The main features of this approach enable students to make connections between mathematics and real life, take an active role in their own learning processes, and develop mathematical thinking skills (Freudenthal, 1973, 1983; Gravemeijer, 1994; Treffers, 1987). In particular, Freudenthal's definition of mathematics as “a human activity” forms the theoretical basis of RME (Freudenthal, 1973).

*Connecting with Real Life*

The most prominent feature of RME is its emphasis on placing mathematical concepts in real-life contexts (Freudenthal, 1983; Gravemeijer, 1994). Students use mathematical thinking processes to solve the problems they encounter in daily life. For example, the ingredients in a recipe are proportioned to the correct measurement, or calculate the budget in a shopping list, reveal concrete uses of mathematics (Gravemeijer, 1994). Such realistic contexts support students to learn mathematics in a more meaningful and permanent way (Van den Heuvel-Panhuizen & Drijvers, 2014).

*Student Centered Approach*

RME refuses students to become passive recipients of knowledge; it encourages students to be active participants in the processes of exploring mathematical concepts, developing problem-solving strategies, and restructuring these strategies (Freudenthal, 1973; Treffers, 1987). This approach is sensitive to individual learning speed and student differences. The teacher allows students to build their own ways of thinking by remaining in a guide position (Gravemeijer, 1994).

*Supporting Mathematical Concepts with Models*

Models are an important component of RME. Students initially transform the models they create based on concrete experiences into more abstract mathematical structures over time. This process helps students to gradually generalize concepts and develop higher-level thinking skills (Gravemeijer, 1994; Treffers, 1987).

*Interaction and Collaboration*

RME emphasizes the social aspect of mathematical learning. Students share, compare, and restructure their thoughts through group work and in-class discussions (Van den Heuvel-Panhuizen & Drijvers, 2014). This interaction paves the way for students to deepen their mathematical thinking processes by developing different perspectives.

*Connected Structure of Mathematical Knowledge*

Mathematical concepts are often interconnected and intertwined with various fields. By making these relationships visible, RME allows students to understand mathematical knowledge in a systematic and holistic framework (Freudenthal, 1983; Treffers, 1987). For example, the concept of proportion is adapted to different problem situations in both algebra and geometry, making learning more consistent.

These key features distinguish RME from traditional mathematical teaching methods. Through RME, students have the opportunity to see mathematics not just as a pile of knowledge, but as a means of thinking and problem solving that they can use for life (Freudenthal, 1973; Gravemeijer, 1994; Van den Heuvel-Panhuizen & Drijvers, 2014).

#### *1.2.4. Teaching principles of realistic mathematics education*

The teaching principles of RME determine the methods used to achieve the goals of this approach. These principles guide students to learn mathematical concepts in a meaningful way (Freudenthal, 1973, 1983; Gravemeijer, 1994; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2014).

*Activity Principle:* Freudenthal's definition of mathematics as a "human activity" forms the basis of the principle of activity (Freudenthal, 1983). According to this principle, students should take an active role in the learning processes, develop and apply their own mathematical methods in problem solving processes. The teacher acts only as a guide in this process (Treffers, 1987).

*Reality Principle:* The principle of reality emphasizes the basing of mathematical learning on real-life contexts (Freudenthal, 1983; Gravemeijer, 1994). Students explore mathematical concepts by solving problems taken from daily life. For example, the task of calculating the optimal space for a parking lot arrangement in a parking lot allows students to use mathematical concepts in the real world (Van den Heuvel-Panhuizen & Drijvers, 2014).

*Level Principle:* Students make a gradual transition from informal methods to formal (official) methods (Gravemeijer, 1994; Treffers, 1987). Students who initially solve concrete problems with their own methods transform the understanding they develop in this process into abstract mathematical concepts. This approach supports a meaningful internalization of mathematical concepts.

*Interrelationship Principle:* The interconnection of mathematical concepts allows students to deepen their knowledge by establishing connections between different concepts (Freudenthal, 1983; Treffers, 1987). For example, a student who learns the concept of proportion understands the concept in a broader context by relating it to the problems of geometry, algebra and daily life.

*Interaction (Collaboration) Principle:* Mathematical learning is considered as a social process. Students have the opportunity to understand, question and develop each other's thoughts through group work and discussions (Van den Heuvel-Panhuizen & Drijvers, 2014). This process allows students to gain different perspectives and deepen their mathematical thinking skills.

*Guidance Principle*: At RME, the teacher is the person who guides and directs students' learning processes (Gravemeijer, 1994; Treffers, 1987). Teachers allow students to explore mathematical concepts by creating appropriate learning environments instead of presenting solutions directly. They provide the support students need on time by observing their requirements.

These principles, working in harmony with the core features of RME, enable students to see mathematics as a meaningful, interactive and life-related process (Freudenthal, 1973; Gravemeijer, 1994; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2014).

#### *1.2.5. Differences and similarities between constructivist education approach and realistic mathematics education*



*Similarities*

*Student-Centered Learning:* Both constructivist understanding and RME argue that the learner actively constructs knowledge and that in this process students are at the center of the learning environment (Freudenthal, 1983).

*Effective Participation and Discovery:* Both approaches foresee students to structure knowledge through problem solving, discussion, discovery and experience. In this process, students are not only passive recipients, but also active producers of knowledge (Piaget, 1952; Vygotsky, 1978).

*Meaningful Learning:* Constructivism is the student's construction of new knowledge on their pre-information; RME emphasizes the development of concepts in a meaningful context through realistic problems (Gravemeijer, 1994).

*Teacher's Guiding Role:* In both approaches, the teacher is a guide that facilitates, guides and supports the student's learning process. Instead of transmitting information directly, the teacher helps the student develop his own mental models (Treffers, 1987).

*Differences*

*Theoretical Focus and Application Area:* Constructivism is a general learning theory and can be applied to all disciplines (Piaget, 1952). RME, on the other hand, is a unique approach to teaching mathematics and integrates constructivist thinking with principles for teaching mathematical concepts (Freudenthal, 1983).

*The Emphasis of Realistic Contexts:* Constructivism emphasizes the meaningful construction of knowledge, but the use of realistic contexts is not mandatory (Vygotsky, 1978). RME, on the other hand, adopts the development of mathematical concepts based on realistic problem situations as a fundamental principle (Van den Heuvel-Panhuizen & Drijvers, 2014).

*Clearness of Mathematization Processes:* While the process of building knowledge in general is important in constructivism, in RME, students first transfer everyday problems to the language of mathematics (horizontal mathematicalization), then transform these representations into abstract and formal structures (vertical mathematicization) is specially defined (Treffers, 1987).

*Exploration Parallel to Historical Development:* While RME envisions students to "reinvent" mathematical concepts in a similar way to the process of historical development (Freudenthal, 1983), constructivism does not offer such a specific guideline. Constructivism evaluates knowledge construction on the basis of the individual's mental models (von Glasersfeld, 1989).

*Evaluation Practices:* While having a flexible approach to constructivism evaluation, RME evaluates the student's mathematical thinking levels, model-using skills and abstraction processes achieved through realistic problems in concrete contexts (Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2014).

Constructivistic Understanding of Education and Realistic Mathematics Education intersect by highlighting student-centered, meaningful and effective learning. However, RME uniques the constructivist framework in the field of mathematics education, offering unique components such as realistic contexts, gradual mathematization, and guided rediscovery (Ertem Akbaş & Yıldırım, 2024). In this way, RME transforms the abstract principles of constructivist theory into concrete, discipline-specific teaching strategies.

### 1.3. Use of Digital Tools in Education

The integration of technology into educational processes offers important opportunities, especially in the field of mathematics education (Drijvers, 2013). Digital tools are very effective in terms of visualizing mathematical concepts, diversifying learning materials and enriching students' learning experiences (Drijvers & Trouche, 2008). The use of digital tools in the context of Realistic Mathematics Education (RME) makes it easier for students to understand mathematical concepts and makes the learning process more interactive (Treffers, 1987). 

*1.3.1. The role of digital tools in mathematics education*

Digital tools are important tools that facilitate the understanding of abstract concepts in mathematics education and create interactive and collaborative learning environments (Drijvers, 2013). Visualization and Meaning: Digital tools allow the visualization of abstract mathematical concepts, helping students to interpret these concepts in concrete contexts (Gravemeijer, 1994). While dynamic geometry software allows interactive examination of function graphs, statistical software allows visualization of data in different dimensions (Drijvers, 2013).

*Interactive Learning Environments:* Digital platforms where students can progress according to their own learning speeds and strategies support individualized learning (Drijvers & Trouche, 2008). Virtual manipulatives create a learning process that is compatible with the RME's understanding of guided rediscovery by allowing the experience of mathematical relationships (Freudenthal, 1983).

*Problem Solving and Mathematization:* Digital tools make it easy for students to transform real-life problems into mathematical contexts (Van den Heuvel-Panhuizen & Drijvers, 2014). For example, a simulation of elevator systems to explore the relationship between speed, time and distance helps students make sense of concepts through a concrete scenario rather than abstract formulas (Drijvers, 2013).

*Collaboration and Interaction:* Online platforms allow students to collaborate not only with their own classmates, but also with their peers in different geographies (Drijvers & Trouche, 2008). This global interaction allows students to develop multifaceted perspectives by addressing mathematical discussions in a broad perspective (Freudenthal, 1983; Gravemeijer, 1994).

### 1.4. Use of Digital Tools in Realistic Mathematics Education

Realistic Mathematics Education (RME) addresses mathematical learning in context and encourages students' active participation. Digital instruments play an important role in the creation of these contexts (Van den Heuvel-Panhuizen & Drijvers, 2014). For example, a financial management simulation allows students to build mathematical relationships with everyday life problems such as budget planning and to make sense of these relationships (Freudenthal, 1983; Treffers, 1987).

Digital tools are also compatible with the basic principles of RME:

*Activity Principle:* Students develop solutions to mathematical problems through digital tools (Gravemeijer, 1994).

*Principle of Reality:* Simulations and virtual environments offer the opportunity to experience the repercussions of mathematical concepts in real life (Freudenthal, 1983).

*Principle of Interaction:* Online platforms support students to collaborate and exchange ideas (Drijvers & Trouche, 2008).

*1.4.1. Advantages of digital tools*

*Ease of Access:* Students can access learning materials anytime and from anywhere thanks to digital tools (Drijvers, 2013).

**Inclusive Learning:** Students with different learning styles can explore their own learning paths through digital tools (Gravemeijer, 1994).

*Assessment and Feedback:* Digital tools allow students to receive instant feedback. This allows quick and effective adjustments to be made in the learning process (Drijvers & Trouche, 2008).

*1.4.2. Difficulties encountered*

Access to Technology Issues: Not all students having equal access to digital tools can increase the risk of digital cliff (Van den Heuvel-Panhuizen & Drijvers, 2014).



*Quality of Educational Materials:* Failure to design digital tools correctly from a pedagogical point of view may cause students to develop misunderstandings (Freudenthal, 1983).

*Technology Literacy of Teachers:* For the effective use of digital tools, teachers must have sufficient knowledge of how to use these tools (Drijvers, 2013). Mathematics is not only a theoretical discipline, but also a tool used in all areas of daily life. Digital technologies make mathematics more easily correlated to its real-world applications (Gravemeijer, 1994).

*Real World Problems:* With digital tools, students can interpret probability and statistics in the context of everyday life by working with real-world problems such as weather simulations (Drijvers & Trouche, 2008).

### 1.5. Interactive Mathematics Learning Model (IMLM)

This study proposes the Interactive Mathematics Learning Model (IMLM), which integrates RME with digital tools. The model aims to enhance students' mathematical understanding by combining real-life problem-solving with interactive digital resources.

*1.5.1. Structure of the IMLM*

Mathematical concepts are introduced through authentic real-world problem scenarios, ensuring that students engage with mathematical ideas in meaningful contexts. According to Freudenthal (1991), mathematics should be experienced as a human activity where students construct knowledge through real-life applications, fostering deeper conceptual understanding. Recent studies support this notion, demonstrating that context-based learning enhances students' ability to connect mathematical concepts to real-world problems and improves problem-solving skills (Boaler, 2016; Van den Heuvel-Panhuizen & Drijvers, 2014). Furthermore, research highlights that incorporating digital tools in contextual learning environments significantly boosts engagement and retention (Pierce & Stacey, 2010).

*Digital Tools Used:*

Virtual Reality (VR) environments (e.g., mathematical simulations, virtual labs)

Game-based learning platforms (e.g., Prodigy, Minecraft Education Edition, Khan Academy)

Real-world datasets (e.g., Google Earth, Wolfram Alpha, Desmos)

*Example Application:*

*Problem:* Optimizing traffic flow in a city by determining which roads should be expanded.

*Tools:* Simulated traffic data and maps.

*Goal:* Applying ratio, proportion, and functions to model real-world situations (Gravemeijer, 1999).

*1.5.2. Mathematical modeling and interactive discovery*

In this phase, students create mathematical models based on the problem context and use digital manipulatives to explore and refine their understanding. According to Lesh and Doerr (2003), mathematical modeling provides a bridge between informal real-life understanding and formal mathematical reasoning, making abstract concepts more accessible to students.

*Digital Tools Used:*

Dynamic geometry software (e.g., GeoGebra, Desmos, Cabri 3D)

Probability and statistics simulators (e.g., TinkerPlots, CODAP)

AI-powered data analysis platforms

*Example Application:*

*Problem:* Developing an optimal pricing strategy for a business to maximize profit.

*Tools:* GeoGebra for analyzing price-profit functions.

*Goal:* Using linear functions and derivatives to make informed decisions (Blum & Ferri, 2009).

*1.5.3. Collaborative and adaptive learning environment*

Mathematical discussions and collaborative problem-solving help students develop diverse problem-solving strategies. Adaptive learning systems provide real-time feedback based on student progress (Akkuş & Gök, 2024). Sfard (2008) suggests that mathematical learning should be viewed as a social process where students co-construct knowledge through discussion and exploration.



*Digital Tools Used:*

Online collaboration platforms (e.g., Google Classroom, Padlet, Edmodo)

AI-driven adaptive learning systems (e.g., ALEKS, Knewton, DreamBox)

Team-based games (e.g., digital escape room puzzles, cooperative math games)

*Example Application:*

*Problem:* Designing an optimal training program for an athlete based on performance data.

*Tools:* AI-assisted data visualization tools.

*Goal:* Applying linear regression and data analysis to make evidence-based decisions (Pierce & Stacey, 2010).

*1.5.4. Implementation phases of interactive mathematics learning model (IMLM)*

The implementation of the IMLM involves a structured approach to integrate digital tools into secondary school mathematics teaching. This process ensures that students actively engage in mathematical problem-solving through real-life scenarios, digital simulations, and collaborative learning environments. The following table provides a detailed breakdown of the implementation phases:

| Phase | Description |
|---|---|
| 1. Preparation and Planning | Identify key mathematical topics and real-world problems suitable for IMLM integration. Teachers receive training on digital tools and pedagogical strategies. |
| 2. Introduction of Contextual Problems | Students are presented with a real-life problem scenario that is relevant to their mathematical learning objectives. |
| 3. Investigation with | Students explore the problem using digital tools such as simulations, VR |

| | |
|---|---|
| Digital Tools | environments, or data analysis platforms to gather insights and formulate questions. |
| 4. Mathematical Modeling | Students construct mathematical models to represent the problem, using digital manipulatives or dynamic geometry software. |
| 5. Testing and Refining Models | The models are tested through digital simulations and modified based on feedback, ensuring deeper understanding and accuracy. |
| 6. Collaborative Discussion and Presentation | Students present their findings, compare solutions with peers, and refine their approaches through teacher-guided discussions. |
| 7. Assessment and Reflection | Students complete assessments measuring conceptual understanding and problem-solving ability. Teachers collect feedback for iterative model improvement. |

A pilot study is necessary to assess the effectiveness of the IMLM before full-scale integration into the curriculum. This study will evaluate the model’s impact on students’ engagement, problem-solving skills, and overall mathematical proficiency. The following table outlines the key stages of the pilot implementation:

| Pilot Phase | Description |
|---|---|
| 1. Selection of Participants | Secondary school students (Grades 6-8) from various learning backgrounds participate in the study. |
| 2. Pre-Test Assessment | Students' mathematical knowledge and problem-solving skills are assessed before the implementation. |
| 3. Implementation of IMLM Lessons | The model is introduced in selected classrooms, where students engage with digital tools and real-world problem-solving. |
| 4. Data Collection | Student engagement levels, problem-solving strategies, and conceptual understanding are recorded using qualitative and quantitative methods. |
| 5. Post-Test Evaluation | A post-test is conducted to measure learning progress and improvements in mathematical thinking skills. |
| 6. Teacher and Student Feedback | Surveys and focus groups are conducted to gather insights on the usability and effectiveness of the model. |
| 7. Comparative Analysis | The pre-test and post-test results are analyzed to assess the impact of IMLM on student learning. |
| 8. Final Reporting and Recommendations | The findings are compiled into a report, highlighting key takeaways and potential refinements for broader implementation. |



This structured implementation and pilot study ensure that the IMLM is rigorously tested and refined to maximize its impact on mathematics education. The IMLM presents a technology-integrated, real-life-oriented framework for mathematics education. By incorporating authentic problems, interactive digital tools, and collaborative learning, the model aims to make mathematics more engaging, applicable, and conceptually rich. Research indicates that digital tools and collaborative learning strategies significantly enhance students' conceptual understanding and engagement in mathematics (Drijvers, 2013; Hoyles & Lagrange, 2010). The integration of RME into mathematics teaching has been shown to significantly improve students’ academic performance with a very large effect size (g = 1.107), while also yielding a moderate improvement in their attitudes toward mathematics (g = 0.694) according to recent meta-analytic evidence (Kutluca & Gündüz, 2022). These findings provide strong justification for embedding the RME principles into the IMLM. The effectiveness of the model will be tested through pilot studies, ensuring its potential to enhance mathematical thinking skills and improve learning outcomes in secondary school mathematics education.

*Ethics Committee Decision*
*Due to the scope and method of the study, ethics committee permission was not required.*

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