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Research Article

Investigating the Change in Mathematical Modeling Competencies of Middle School Students During the Educational Process Designed with Mathematical Modeling Activities*

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Abstract – This study aims to determine how the teaching process designed with mathematical modeling activities affects the mathematical modeling competencies of seventh grade middle school students. The embedded design, one of the mixed methods research designs, was adopted. The embedded design was adopted, one of the mixed methods research designs. The research participants consisted of 27 students studying at the seventh grade level of a middle school. During the 10-week implementation process, an opinion form consisting of open-ended questions and nine mathematical modeling activities developed by the researcher were used as data collection tools. Three of these activities were used for the pretest and posttest, and the remaining six were used in the implementation process. When analyzing quantitative data, the Wilcoxon signed-rank test was used. In the analysis of qualitative data, descriptive analysis was used. According to the results, during the implementation process, students showed the most improvement in understanding the problem and studying mathematically. In contrast, they showed the slightest improvement in the interpretation and verification stages. It is thought that it is essential to constitute environments where students can establish relationships with mathematics and to design mathematical modeling problems in these environments to attract students' attention as much as possible.

Keywords: Modeling competencies, mathematical modeling, middle school students

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Introduction

Mathematical modeling is a dynamic method that makes it easier to see the relationships inherent in problems in all areas of life, discover them and express the relationships between them in mathematical terms, classify, generalize, and draw conclusions (Fox, 2006). Mathematical modeling is the process of expressing a real-life situation mathematically (Kertil, 2008). Borromeo Ferri (2006) defined mathematical modeling as a complex and cyclical process that involves transformations between the mathematical world and real life. According to Kaiser and Maaß (2007), mathematical modeling competence means completing the mathematical modeling process willingly and purposefully. As can be understood from the definitions, the main emphasis in modeling is on the process. Many researchers have studied the cyclicity of the modeling process, including Berry and Davies (1996), Doerr (1997), Mason (1988), Voskoglou (2006), and Borromeo Ferri (2006).

These definitions and studies reveal that mathematical modeling is a structured process that involves multiple stages and competencies. When the stages of the modeling process described in the literature are examined, it is observed that although there are some differences in terms of competencies, most studies commonly emphasize understanding, simplifying, mathematizing, working mathematically, interpreting, and verifying (e.g., Blum & Borromeo Ferri, 2009; Galbraith & Stillman, 2006). Borromeo Ferri (2006) defined mathematical modeling by including this six-stage process. The ability to create a model by performing these six stages correctly and to analyze or compare given models is defined as "Modeling Competency" (Blum & Borromeo Ferri, 2009). Many studies have shown that students are relatively competent in the early stages of the modeling process understanding the problem, simplifying it by identifying relevant variables, expressing the situation symbolically, and performing mathematical operations. However, it has been emphasized that students often have difficulties in stages such as interpreting mathematical results in real-life contexts and verifying solutions (Blum & Borromeo Ferri, 2009; Galbraith & Stillman, 2006; Kaiser, 2020). In the context of this research, it was of particular importance to examine how students performed in these stages and to reveal both their strengths and the challenges they faced.

In parallel with the increasing interest in mathematical modeling worldwide**,** its importance has also grown in the context of education in Türkiye. When the middle school (5th, 6th, 7th, and 8th grade) mathematics curriculum is examined, it seems that it aims to raise individuals who can easily express their thoughts and reasoning in the problem-solving process, understand mathematical concepts, and associate and use these concepts with daily life (Ministry of National Education [MoNE], 2018a). Doruk (2010) stated that teachers must find practical methods to make students feel that mathematics is a part of their lives, enable them to enjoy mathematics and engage them in more meaningful mathematical learning. Çora (2018) states that mathematical modeling problems related to many fields, well-defined, covering rich information and cognitive processes can be included in the classroom environment instead of traditional problem activities for this purpose. Deniz (2014) says modeling is essential in associating mathematical concepts with daily life, considering the difficulties individuals experience in establishing the relationship between the natural and mathematical worlds. If mathematical modeling is used in mathematics education, students will better understand real-world situations and learn the subject in lessons, and various mathematical skills can be developed (Blum, 2002). In this context, the importance of mathematical modeling applications for solving daily life problems emerges.

In this context, an important distinction emerges between traditional problem types and modeling activities. Traditional problems solved in the classroom environment help students improve their computing skills. However, it does not contain a structure that will reveal the mathematical structures in students' minds. Mathematical modeling activities stand out as powerful tools that enable students to create their essential mathematical ideas and processes rather than directly applying the knowledge they have previously learned within the scope of the curriculum (English, 2006). According to Lesh and Doerr (2003), mathematical modeling activities are problems that allow students to work on a problem taken from real life, create their mathematical thoughts, and revise their thoughts. Therefore, confronting students with activities in which they will use their ideas rather than traditional problem-solving activities is a more effective teaching method (Blum, 2002). This study examined how students developed modeling competencies using their ideas and abilities in the teaching process designed with modeling activities.

There are also several studies in the literature that have examined students' competencies in modeling processes through similar learning environments. In their study with 3rd grade primary school students, English and Watters (2004) stated that students' ability to make sense of meaning, problematize, create hypotheses, and mathematize were seen with the help of mathematical modeling activities. Maaß (2006), in his study to determine students' mathematical modeling skills, stated that students showed improvement in mathematical modeling activities and that students with low achievement levels also participated in the process. In their study, Hıdıroğlu et al. (2014) examined students' solution approaches to the comet problem within the framework of the mathematical modeling process. As a result of the study, they stated that their students' performance decreased as the stages of the modeling process progressed. In their study, Tekin Dede and Yılmaz (2015) sought an answer to how the cognitive modeling competencies of 6th grade students could be improved. In the study, where Borromeo Ferri's (2006) cognitive modeling perspective was used as a theoretical framework, the development of students' cognitive modeling competencies was examined. As a result of the study, they stated that the students quickly achieved the desired development in terms of understanding the problem, simplifying, mathematizing, and studying mathematically. On the other hand, they stated that the students did not develop sufficiently in interpretation and verification competence. Özgen and Şeker (2020) examined the modeling competencies of 6th grade students in the context of Borromeo Ferri's (2006) cognitive modeling perspective. As a result of this study, they emphasized that the students showed improvement in the context of all competencies. In their study, where they examined the modeling competencies of 7th grade students in the context of Borromeo Ferri's (2006) cognitive modeling perspective, Alkan and Aydın (2021) stated that the students showed improvement at all stages. Kılıç (2020) used the same cognitive perspective in his study with middle school students. He stated that students improved in all competencies except verification competency in the study. These studies are related to the current study in terms of their focus on mathematical modeling competencies, but they differ in terms of the grade levels studied, the specific modeling tasks used, and the emphasis placed on different stages of the modeling process.

Despite these contributions, mathematical modeling research is still limited in terms of scope and depth. Albayrak and Çiltaş (2017) state that there are few experimental studies on mathematical models and modeling in Türkiye. Most of them were conducted with university students, and how teaching with mathematical modeling is carried out in these studies is not explained in detail. Didiş Kabar and İnan (2018) stated that studies with middle school students studying under different conditions in different socio-economic regions of the country should be increased and that the applicability of mathematical modeling activities in middle school classes studying under different conditions and the possible benefits of these applications for students and teachers should be evaluated. All this shows that mathematical modeling problems are not widely applied in classrooms in our country, and therefore, their use by teachers in the classroom environment should be expanded, and research on mathematical modeling should be diversified. Although there are studies examining mathematical modeling competencies in our country, the number of studies that evaluate the teaching process with more than one activity, as in this study, is low. Considering the methods, samples, and approaches used in previous modeling studies, the number of experimental studies that include qualitative data is also low. This study aims to address this gap by combining multiple modelling activities with a detailed examination of the instructional process through both quantitative and qualitative methods**.** Since this study is an experimental study that includes qualitative data and provides a comprehensive description of the implementation, it is thought that it can guide teachers who want to implement modeling activities in their classrooms on how the process will work. In this respect, it differs from previous studies that often focus on single activities or lack in-depth pedagogical analysis.

Purpose of the Research

The research aims to examine the development of the modeling competencies of seventh grade students in the teaching process designed with mathematical modeling activities. Specifically, answers were sought to the following problems:

1. How are the mathematical modeling competencies of 7th grade middle school students before and after the implementation?

2. How is the change in the mathematical modeling competencies of 7th grade middle school students during the implementation process?

Method

Research Design

In this research, nested design was used, one of the mixed method designs. In the mixed method design organized according to the nested (embedded) design, qualitative and quantitative data can be collected simultaneously, and a data set can support the study's secondary function (Creswell & Plano Clark, 2018).

In the current study, the scores obtained from students' pretest-posttest mathematical modeling activities constitute the quantitative data of the research. The students' solutions to modeling problems (activities) during the implementation and their answers to the opinion form constitute the qualitative data of the research. The research examined student solution sheets of three mathematical modeling problems used for the pretest and posttest to support the quantitative data obtained from the pretest and posttest. At the end of the implementation process, student opinions were also taken through an opinion form to support the quantitative data obtained from the pretest and posttest. Qualitative data obtained by observing the students' situations during the process, examining their activity sheets, and gathering their opinions were used to portray the development of the students' mathematical modeling competencies, especially throughout the teaching process.

Before the implementation, students' solution papers were examined to support the quantitative data obtained from the pretest. Similarly, after the implementation, students' solution sheets and answers to the opinion form were examined to explain the data obtained from the posttest and quantitatively describe the students' situations. In the quantitative dimension of the research, a single-group pretest-posttest design was used.

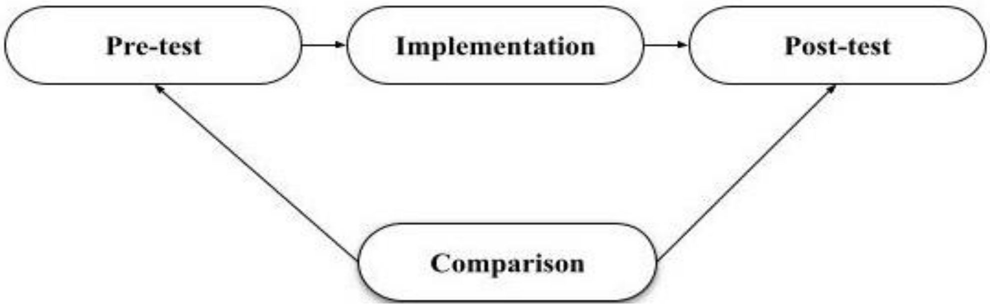


Figure 1 Single Group Pretest-Posttest Design (Johnson & Christensen, 2014)

Study Group

The study group consists of 27 seventh grade students attending a public middle school located in a district where one of the researchers was employed during the 2018–2019 academic year. Among the participants, there are 16 female and 11 male students. The study group was selected using the convenience sampling method, which is one of the purposeful sampling techniques. In this method, the researcher selects participants that are close and easy to reach, which brings speed and practicality to the research process (Yıldırım & Şimşek, 2016). The selected class was one of three sections at the school where the research was conducted. Although the sampling method was based on accessibility, this particular class was preferred because the students generally demonstrated characteristics such as being able to understand what they read, attending school regularly, and showing interest in mathematics. During the implementation process, students were divided into heterogeneous groups of four according to their mathematics course grades. Each designed activity was implemented during two class hours as part of the mathematics applications course.

Data Collection

As a data collection tool in the research,

- Personal information form,
- Three "mathematical modeling activities" to be used in the pretest and posttest,
- Six "mathematical modeling activities" to be used in the implementation process and
- Student opinion form was used.

Personal Information Form

A personal information form was used to obtain personal information about the students. With this form, information about the gender of the students, the general average of the last semester's mathematics course, and the educational status of the parents were obtained.

Mathematical Modeling Activities and Development Process

Nine mathematical modeling activities were used in the study, three of which were used in the pretest and posttest. The researcher developed mathematical modeling activities by scanning the relevant literature, Programme for International Student Assessment (PISA), Trends in International Mathematics and Science Study (TIMSS), seventh grade mathematics textbooks, and seventh grade elective mathematics textbooks published in previous years. Since the developed mathematical modeling activities were implemented for seventh grade students, care was taken to ensure that the activities were appropriate to the real-life context and the 7th grade objectives in the secondary school mathematics curriculum (MoNE, 2018a).

Table 1 Contents of Mathematical Modeling Activities

Weeks	Name of activity	Related objectives	Daily life context
Pretest-Posttest	Perimeter of Lake Hazar	Recognizes rectangle, parallelogram, trapezoid and rhombus.	Field, Hedge
Pretest-Posttest	Population of Küçükkadı	Solves problems that require operations with integers.	Population
Pretest-Posttest	Tomato Garden	Solves problems that require operations with integers.	Daily shopping
1st week	Teacher's time	Solves problems that require operations with integers. Calculates one quantity as a percentage of another quantity.	Time
2nd week	Let's Build Shelters for Sheep	Recognizes rectangle, parallelogram, trapezoid, and rhombus. Solves problems related to the area.	Shelter, warehouse
3rd week	Which Seed Should We Plant?	Finds and interprets a data group's mean, median, and mode values.	Field, average expense
4th week	Let's Repair the Ten-Eyed Bridge	Recognizes rectangle, parallelogram, trapezoid, and rhombus. Solves problems related to space.	Historical place
5th week	Footprint	Given one of two quantities whose ratio is known, finds the other.	Weight, depth
6th week	Let's Find the Suitable Fuel	Solves problems that require operations with integers. Solves problems that require operations with rational numbers.	Vehicle fuel

Modeling activities were created to include an introductory essay, preparation questions, problem situations, and presentation of solutions, which are the principles of modeling activities specified by Tekin Dede and Bukova Güzel (2014). Tekin Dede and Bukova Güzel (2014) described the purpose of the introductory essay and preparation questions as warming up and preparing students for the problem situation. They stated the problem situation as the primary component of modeling activities. They stated that the presentation of solutions involves students presenting their presentations to their friends and reviewing their solutions when necessary. Borromeo Ferri's (2006) modeling cycle under the cognitive perspective was chosen as the theoretical framework. Under this cognitive perspective, cognitive modeling competencies are understanding the problem, simplifying, mathematizing, studying mathematically, interpreting, and verifying. In this modeling process, at the stage of understanding the problem, the student makes sense of it and re-represents it. In the simplification stage, the student extracts the information necessary to solve the problem and makes assumptions. In the mathematization stage, the student creates the mathematical model necessary to solve the problem with the help of existing mathematical knowledge. In the mathematical studying stage, the students make mathematical solutions in the context of their obtained model. The mathematical solution formed in the interpretation stage is interpreted in the context of real life. In the verification stage, the entire process is checked, corrections are made at the necessary stages, and the process is terminated. To determine these mathematical modeling competency levels, students were required to follow the following instructions along with the given activity:

1. Express the problem in your own words.
2. Explain what information you need to solve the problem.
3. Explain mathematically what method you would follow to solve the problem.
4. Complete the necessary operations to solve the problem.
5. Do you think the solution you found is suitable? Write your comments about the result of your solution.
6. How can you be sure that your result is correct? Explain.

Each of these guiding questions corresponds to the mathematical modeling competencies put forward by Borromeo Feri (2006). The "Perimeter of the Lake Hazar Lake" modeling activity used in the pretest and posttest is given in Figure 2 below.

Lake Hazar is a tectonic lake located in the southeast of Elazığ province. Lake Hazar has an essential position in terms of the climate and geographical features of the region. Lake Hazar hides significant riches under its blue cover. Recent studies have led to the discovery of traces of the settlement, which was submerged due to an earthquake in the 13th century and dates back to the 11th century. Since Lake Hazar is located close to Diyarbakır, it is a place where people frequently go daily to cool off during the summer months.

Above is a satellite image of Lake Hazar. Accordingly, estimate the perimeter of Lake Hazar.

1. Express the problem in your own words.
2. Explain what information you need to solve the problem.
3. Explain mathematically what method you would follow to solve the problem.
4. Write and solve the appropriate stage to solve the problem.
5. Do you think the solution you found is suitable? Write your comments about the result of your solution.
6. How can you be sure that your result is correct? Explain.

Figure 2 How Many Kilometers is the Perimeter of the Lake Hazar? (Modeling Activities)

While developing the activities, ten mathematical modeling activities were designed considering mathematical situations that students may encounter in daily life. In addition, the cultural and socioeconomic levels of the students were taken into account in the activities, and care was taken to ensure that the activities were related to the student's immediate environment. The developed activities were examined by two faculty members who are experts in mathematics education and three mathematics teachers with master's degrees in this field. While examining the activities, experts expressed their opinions by considering the introductory essay, preparation questions, problem situation, and presentation of solutions, which are the principles of model-building activities stated by Tekin Dede and Bukova Güzel (2014). As a result of the expert reviews, it was stated that one of the developed activities did not comply with the principles of model-building activities, because the problem situation was too routine and did not provide sufficient opportunity for model development. Therefore, this activity was not evaluated. Experts stated that there were semantic deficiencies in some activities, especially in the preparation questions and the wording of the problem situations. Based on the expert feedback, the semantic deficiencies were revised by clarifying the preparation questions and rewording the problem situations to better guide the modeling process. Thereupon, all activities were examined by a Turkish language teacher. The Turkish language teacher examined the activities regarding meaning, spelling, and spelling-punctuation marks. Within the framework of the suggestions, semantic errors in the questions were eliminated, spelling and punctuation errors in the activities were corrected, and after all revisions were completed, a pilot implementation was conducted with 25 seventh grade students.

Opinion Form

The opinion form, which was developed by the researcher and consisted of open-ended questions, was developed after the posttest to obtain the opinions of the study group about the process carried out with mathematical modeling activities (the difficulties they encountered while solving the activities and their reasons, their differences from routine problems, etc.). The questions in the opinion form were prepared by examining the relevant literature and in line with the sub-problems of the research. The questions were presented to the opinion of two experts in the field, and the experts suggested clarifying the wording of certain questions, especially regarding the distinction between modeling problems and routine problems and recommended making the language more accessible for students. In line with these suggestions, the expressions were revised for clarity, ambiguous terms were replaced with simpler alternatives and the questions were given their final form.

Implementation Process of the Research

Pilot Implementation Process

A pilot study was conducted in the second semester of the 2017-2018 academic year with 25 seventh grade students at the school where the researcher works. The study carried out each activity in the mathematics applications course and two weekly lesson hours for ten weeks. The personal information form was administered in the first lesson hour of the first week of implementation. In the 2nd lesson hour, the first activity of the pretest was administered to the students individually. The second pretest activity was administered to the students in the 1st lesson hour of the second week. The third pretest activity was applied to the students in the second lesson of the second week and the pre-implementation procedures were completed. Then, a 6-week implementation period was started, and six mathematical modeling activities were administered. In the first week after the end of the implementation process, the first activity of the posttest was administered to the students in the first lesson of the mathematics applications course, and the second activity of the posttest was administered to the students in the second lesson. The following week, the third activity of the posttest was administered in the first lesson hour, and the opinion form was administered in the second. With the pilot implementation, necessary corrections were made in the parts that were not understood, such as the implementation time of the activities, the suitability and adequacy of the materials used, the difficulties experienced in the implementation, and the language and expression of the problems. Student feedback was collected orally at the end of the pilot implementation, and based on this feedback, the questions' ambiguities were eliminated, and the activities were finalized. Students were asked which parts they found unclear, difficult to understand, or hard to implement, and their suggestions were taken into account during the revision process. In addition, the pilot implementation process gave the researcher experience for the main implementation.

Actual Implementation Process

The study was conducted with seventh grade students who chose the mathematics applications course, which is two hours a week. When the explanations regarding the implementation of the 2018 mathematics applications course curriculum are examined, it is stated that the modeling method is taken as the basis in the mathematics applications course. It was emphasized that activities to solve and establish problems should be included in developing mathematical models. When developing mathematical models, it is recommended to encourage student discussions within and between groups based on realistic and daily life situations and to allow students to develop their models (MoNE, 2018b). For these reasons, the implementation process with mathematical modeling activities in the mathematics applications course was deemed more appropriate.

The primary implementation process was carried out in the first semester of the 2018-2019 academic year. No information was given about modeling before the implementation, but necessary information was given about how the process would work. The personal information form was administered in the first lesson hour of the first week of the implementation. In the 2nd lesson hour, the first activity of the pretest was implemented to the students individually. The second pretest activity was implemented for the students individually in the 1st lesson hour of the second week. The third pretest activity was implemented to the students individually in the 2nd lesson hour, and the pre-implementation procedures were completed.

Before starting the 6-week implementation process, the students were divided into seven heterogeneous groups of four according to their mathematics course grades. It was stated that the implementation process with mathematical modeling activities would be held in the block course for 80 minutes every week, that the groups would be given 40 minutes for the modeling activities, that the groups would present their solutions after the solutions were completed, and that the students will be given detailed information about the process, such as what is expected from the students after the process is completed.

In the mathematics applications course, an 80-minute block every week, students tried to be motivated by asking interesting questions about the activity to be distributed before the activity sheet. For example, before distributing the "teacher's time" activity, students were asked interesting questions such as: "Have you ever thought about how you spent your time in a day?" or "As you enter a new year, have you ever thought about where and how you spent most of your time in the previous year?". Then, an activity sheet was distributed to each group, and an attempt was made to understand what all groups understood in the question by reading the activity through in-class discussions. After understanding the problem, solutions to other sub-questions were started respectively. It was impossible to move on to the next sub-problem until the solution of each sub-problem was completed. All sub-problems were tried to be solved by discussing them within the groups. The ideas and problems that emerged were guided by the researcher in a way that did not directly provide the answer. After completing the solutions to all questions, two groups presented their solutions on the board each week. The process was completed with six different modeling activities over six weeks.

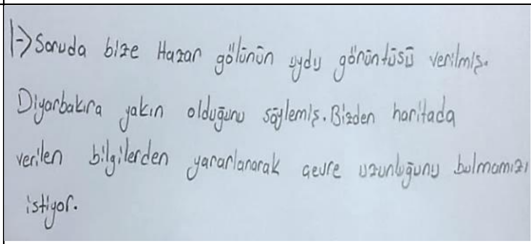
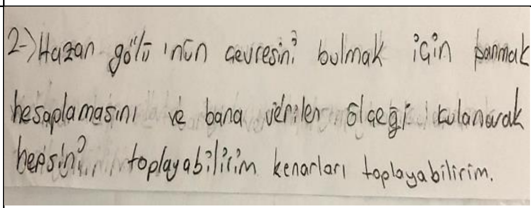
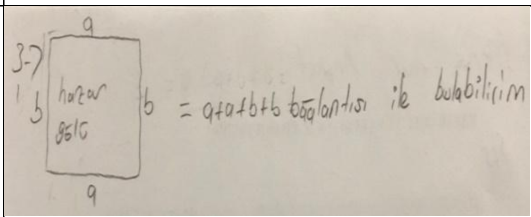
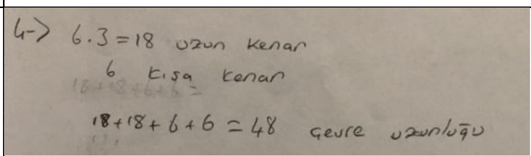
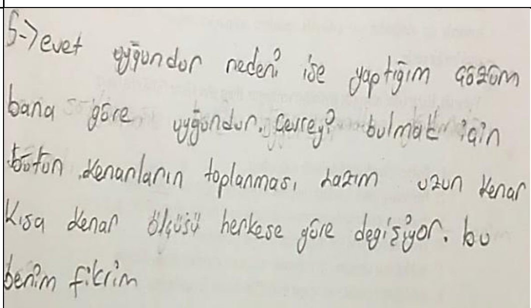
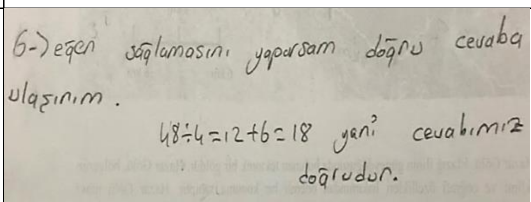
In the first week after the end of the implementation process, the first activity of the posttest was implemented to the students in the first lesson of the mathematics applications course, and the second activity of the posttest was implemented to the students in the second lesson. The following week, the third activity of the posttest was implemented in the first lesson hour, and the opinion form was administered in the second. Thus, the data collection process of the research was completed.

Data Analysis

Quantitative and qualitative analysis methods were used when analyzing the research data.

In the first sub-problem of the research, the Modeling Competencies Evaluation Rubric (Tekin Dede & Bukova Güzel, 2014) was used to determine the scores students received from the pretest and posttest mathematical modeling activities. It is possible to evaluate cognitive modeling competencies quantitatively with an analytical scoring key by examining students' solution papers with the modeling competencies evaluation rubric (Tekin Dede & Bukova Güzel, 2014). The mathematical modeling competencies evaluation rubric consists of 6 dimensions: understanding the problem, simplifying, mathematizing, studying mathematically, interpreting, and verifying. The highest score that can be obtained from the competence of understanding the problem, mathematizing, studying mathematically, and interpreting is four, the highest score that can be obtained from the competence of simplification is three, and the highest score that can be obtained from the competence of verification is six. In this study, the modeling competencies evaluation rubric was rearranged to compare competencies and determine the competencies in which students are successful so that the highest score obtained in all dimensions is 12, and the lowest score is 0. For example, in the simplification stage evaluated at four levels in which Level 1 corresponds to "0 points"; Level 2 corresponds to "4 points"; Level 3 corresponds to "8 points"; Level 4 corresponds to "12 points". Below is the posttest "Lake Hazar" activity sheet of student S24 and the relevant scoring.

Table 2 Quotations and Modeling Competency Levels of S24's Posttest Lake Hazar Activity

Modeling competence	Quotation	Reason	Score/ Competency level
Understanding the problem		It can be said that while scoring the ability to understand the problem, the student used the expression "we will find it by using the information given on the map" and included expressions indicating that the problem was understood and determined what was desired with the given information.	12 points/ Level 5
Simplification		The student stated that he benefited from the scale but did not explain how to use the rule of thumb. Therefore, it can be said that he determined the necessary variables and made an acceptable assumption to some extent.	8 points/ Level 3
Mathematizing		It can be said that the student compared the shape given in the problem to a rectangle, presented an accurate model suitable for the solution, and explained the model correctly.	12 points/ Level 5
Studying mathematically		The student did not specify precisely how he solved the answer he found while solving the model and did not find the answer exactly. It was observed that the mathematical model he created accurately contained deficiencies in its solution.	9 points/ Level 3
Interpretation		The student tried to emphasize mathematically by saying that the edges should be added together and that the length of the short and long sides will vary from person to person. Therefore, it can be said that he interpreted the mathematical solution incompletely in the context of real life.	9 points/ Level 4
Verification		He tried to perform mathematical calculations and verify them. While taking the verification approach, he tried to verify only the long side but made mistakes. It has been observed that he does not correct errors in the verification approach	8 points/ Level 5

Before analyzing the quantitative data for the first sub-problem of the research, the Shapiro-Wilks normality test was performed to understand whether the scores were normally distributed. The Shapiro-Wilks test is a test used to find out whether the scores are normally distributed if the group size is less than 50 (Büyüköztürk, 2016). Mathematical modeling pretest-posttest total scores and scores of each sub-stage of mathematical modeling did not show a normal distribution. Therefore, the “Wilcoxon Signed Rank Test” was used in this sub-problem to analyze the relationship between pretest and posttest scores.

In the second sub-problem of the research, students’ activity sheets were analyzed with a qualitative approach (descriptive analysis) to describe the students’ situations during the implementation process with modeling activities. In descriptive analysis, data are described clearly and systematically and presented in an interpreted manner with cause-effect relationships (Çepni, 2010). In this study, students’ written responses were reviewed in relation to each modeling activity and interpreted descriptively without detailed coding or categorization. The aim was to present a holistic picture of students’ approaches, difficulties, and strategies during the modeling process. In descriptive analysis, direct quotations are frequently used in order to strikingly reflect the views of the individuals interviewed or observed (Yıldırım & Şimşek, 2016). This type of analysis aims to convey the findings to the reader in an organized and interpreted way (Yıldırım & Şimşek, 2016).

In addition, the “Modeling Competencies Evaluation Rubric” was used, as in the pretest and posttest, to describe quantitatively the possible development or change in students’ mathematical modeling competencies throughout the implementation process. The quantitative data obtained through the evaluation rubric were interpreted in connection with the students’ written solution papers for the modeling tasks. These qualitative and quantitative data sets were integrated to provide a more comprehensive understanding of students’ performances, and the results were presented descriptively throughout the implementation process.

To ensure the reliability of the scoring process, each activity was evaluated twice by the first author at different times, both before and after the implementation. The comparison of the two scoring rounds revealed an 85% consistency rate. This level of agreement exceeds the 70% threshold recommended by Miles and Huberman (1994) for acceptable reliability. In cases where discrepancies arose, the first and second authors reviewed the scores together and reached a consensus.

Findings

In this section, the findings obtained throughout the implementation process and comments on these findings are given.

Findings Regarding the First Sub-Problem

This section includes findings regarding the first sub-problem of the research, "How are the mathematical modeling competencies of 7th grade middle school students before and after the implementation?". The students' pretest and posttest Wilcoxon signed-rank test analysis results are given in Table 3.

Table 3 Wilcoxon Signed Rank Test Results of Pretest and Posttest Mathematical Modeling Scores of the Implementation Process with Mathematical Modeling Activities

Posttest-pretest	n	Mean rank	Sum of ranks	z	p
Negative rank	0	.00	.00	-4.541^{a}	.000
Positive rank	27	28.00	378.00		
Equal	0				

The analysis results show that there is a statistically significant difference between the pretest and posttest scores of the students participating in the research from modeling competencies, $z=-4.541$, $p<.05$. Considering the rank average and total of the difference scores, it is seen that this observed difference is in favor of positive ranks, in other words, the posttest score. This finding shows that the implementation process with mathematical modeling activities significantly affects the development of students' mathematical modeling competencies.

Pretest and posttest analysis results regarding students' mathematical modeling sub-competencies are given in Table 4.

Table 4 Wilcoxon Signed-Rank Test Results of Pretest and Posttest Scores Regarding Modeling Competencies

Modelling competencies	Posttest-pretest	n	Mean rank	Sum of ranks	z	p
Understanding the problem	Negative Rank	1	1.50	1.50	-4.430[a]	.000
	Positive Rank	25	13.98	349.50		
	Equal	1				
Simplification	Negative Rank	0	.00	.00	-4.388[a]	.000
	Positive Rank	25	13.00	325.00		
	Equal	2				
Mathematizing	Negative Rank	0	.00	.00	-4.471[a]	.000
	Positive Rank	26	13.50	351.00		
	Equal	1				
Studying mathematically	Negative Rank	0	.00	.00	-4.564[a]	.000
	Positive Rank	26	14.00	378.00		
	Equal	1				
Interpretation	Negative Rank	0	.00	.00	-3.537[a]	.000
	Positive Rank	16	8.50	136.00		
	Equal	11				
Verification	Negative Rank	1	2.50	2.50	-4.314[a]	.000
	Positive Rank	24	13.44	322.50		
	Equal	2				

It is seen that there is a statistically significant difference between the pretest and posttest scores of the students' modeling competencies: Understanding the problem (z=-4.430, p<.05), simplifying (z=-4.338, p<.05), Mathematizing (z=-4.471, p<.05), Studying Mathematically (z=-4.564, p <.05), Interpretation (z=-3.537, p<.05), Verification (z=-4.314, p<.05). Considering the rank average and total of the difference scores, it is seen that this observed difference is in favor of positive ranks, in other words, posttest scores. This finding shows that the implementation process significantly affects the development of all sub-competencies of mathematical modeling in students. The average scores students received from each competency of mathematical modeling are given in the chart below

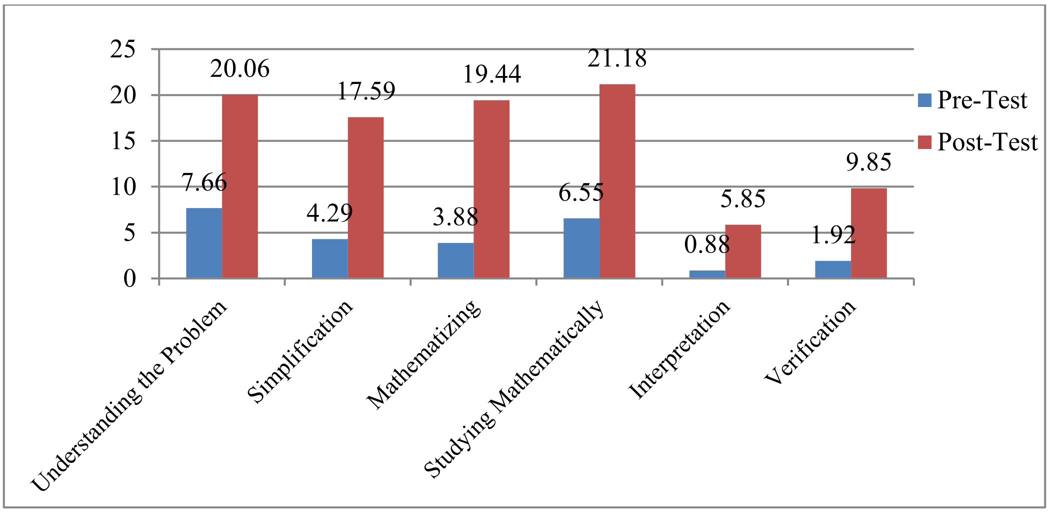


Figure 3 Students' Mathematical Modeling Competencies Pretest and Posttest Average Scores

According to the figure, there was an increase in students' posttest scores in all modeling competencies. In the posttest, the competency in which the students had the highest mean score was "Understanding the Problem", while the competency in which they had the lowest mean score was "Interpretation". The increase in students' scores in all modeling competencies can be interpreted as the implementation process with mathematical modeling activities positively affecting their mathematical modeling competencies.

The students' expressions and related solutions in the activity sheets supported the above quantitative data. Some of the expressions and solutions of the students in the activity sheets are exemplified below in the context of modeling competencies and the relevant order. A section from student S24's solution sheet for the "Population of the Village" activity used in the pretest and posttest regarding competency in understanding the problem is given in Figure 4 as an example.

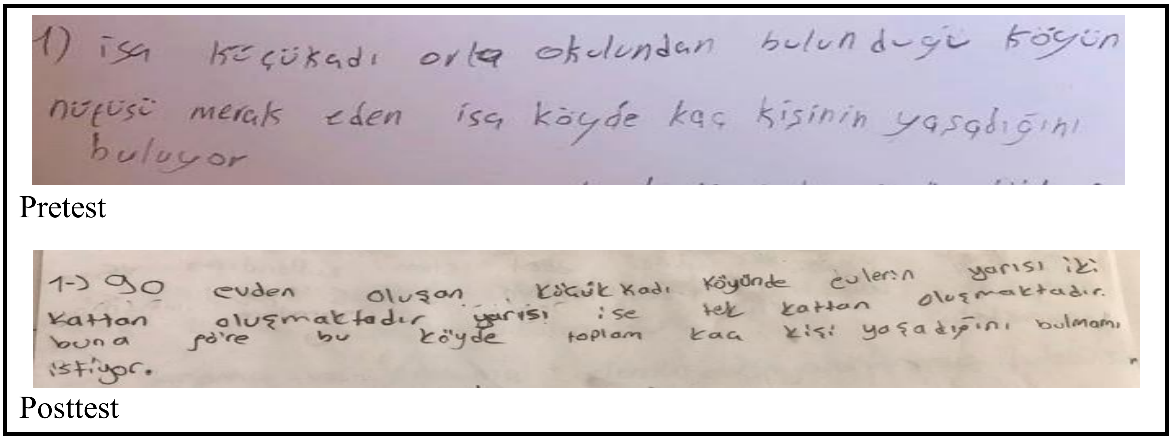
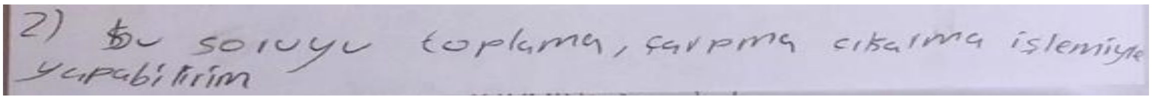
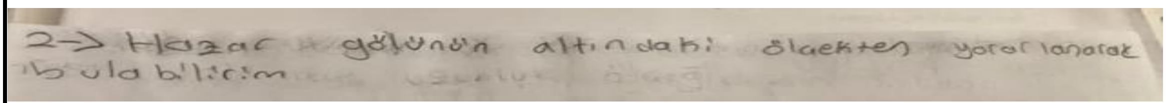


Figure 4 A Section from the Pretest-Posttest Village Population Activity on S24's Proficiency in Understanding the Problem

While scoring for the ability to understand the problem, it was determined that S24 was at level 2 in the pretest and was given 6 points. In the posttest, since the expression "half of the houses are two-floor and half are single-floor" was used in the student's answer, it was determined that he was at level 5, and 12 points were given because he understood the problem and determined what was wanted with the given information. Below is a section from the pretest and posttest solution paper of the S19 student in the Surrounding of the Lake Hazar activity regarding simplification proficiency.



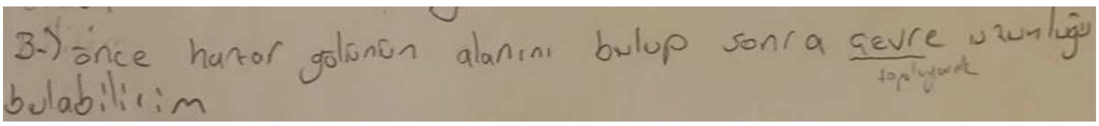
Pretest



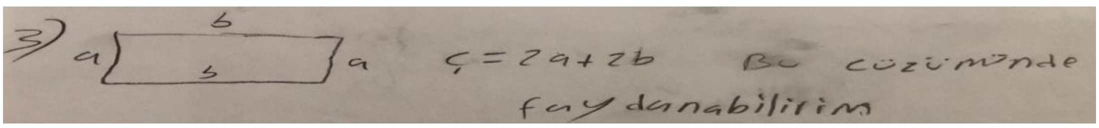
Posttest

Figure 5 A Section from the Lake Hazar Activity Pretest-Posttest Regarding S19 Student's Simplification Proficiency

While scoring for simplification skills, student S19 was seen to be at level 2 because she simplified the problem to some extent with the statement "I will use four operations" in the pretest and was given a score of 4. In the posttest, S19 stated that she would use the scale with the statement "I can use the scale below", but did not fully explain how she would do this. For this reason, since it determined the necessary variables and made acceptable assumptions to some extent, it was determined to be at level 3 in the evaluation rubric and was given 8 points. Below is a section from the pretest and posttest solution paper of S2 in the Surrounding of the Lake Hazar activity regarding mathematization proficiency.



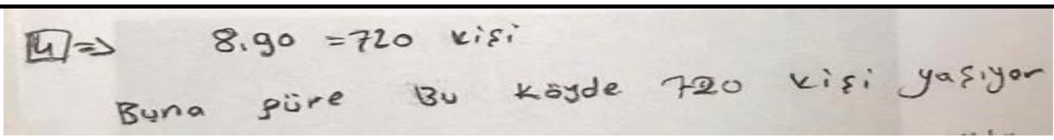
Pretest



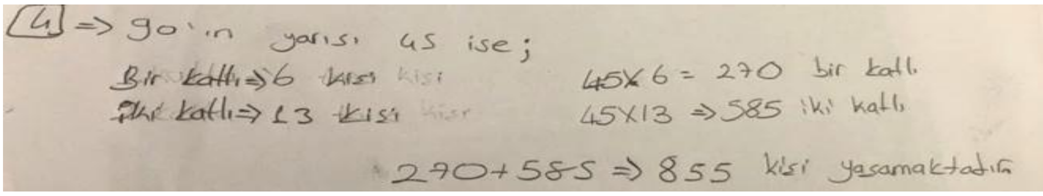
Posttest

Figure 6 A Section from the Pretest-Posttest Hazar Lake Activity on S2 Student's Mathematization Proficiency

Student S2 was determined to be at level 1 because she did not create a mathematical model in the pretest with the statement “I will find the area of the Hazar Lake and add the perimeter” and was given 0 points. The posttest determined that he was at level 5 in the evaluation rubric and was given 12 total points because he compared the shape given in the problem to a rectangle, presented an accurate model suitable for the solution, and explained the model correctly. A section from the solution sheet of the S6 student in the “Population of the Village” activity regarding Competency in Studying Mathematics and the solution scoring are given below.



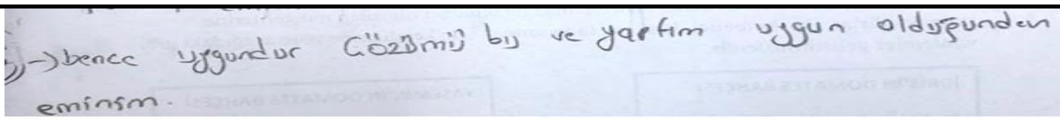
Pretest



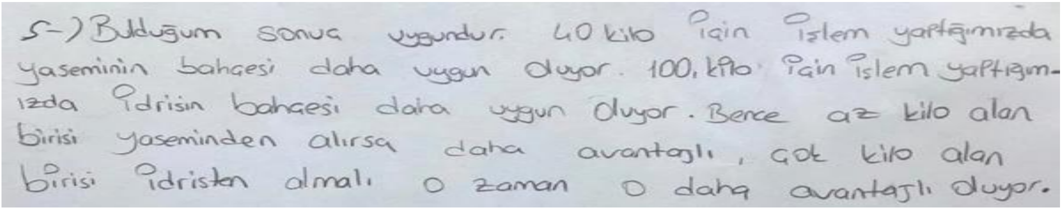
Posttest

Figure 7 A Section from the Pretest-Posttest Village Population Activity on S6 Student’s Proficiency in Studying Mathematically

In the pretest, the student S6, while solving the mathematical model, did a multiplication operation and stated that there were 720 people. Since he solved it incorrectly and incompletely, he was determined to be at level 2 and was given 3 points. In the posttest, he did a mathematical operation while solving the model and found the answer as 855, but he did not state exactly how he solved the answer he found and did not find the answer exactly. It was observed that the mathematical model he created accurately contained deficiencies in its solution. For this reason, the evaluation rubric was determined to be at level 4, and 9 points were given. A section from student S1’s solution paper in the Tomato Garden activity regarding interpretation proficiency is below.


5-) bence uygundur çözümü bu ve yaptım uygun olduğundan eminim.

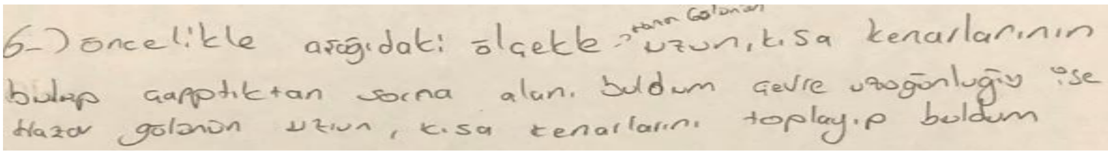
Pretest


5-) Bulduğum sonuç uygundur. 40 kilo için işlem yaptığımızda yaseminin bahçesi daha uygun oluyor. 100 kilo için işlem yaptığımızda idrisin bahçesi daha uygun oluyor. Bence az kilo alan birisi yaseminden alırsa daha avantajlı, çok kilo alan birisi idristen almalı o zaman o daha avantajlı oluyor.

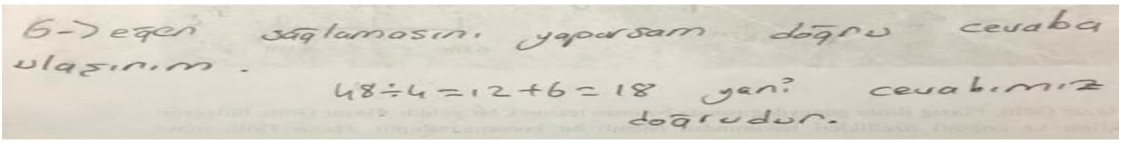
Posttest

Figure 8 A Section from the Tomato Garden Activity Pretest-Posttest Regarding S1's Interpretation Competence

For the interpretation stage, it was determined that student S1 was at level 2 because he partially made the mathematical interpretation in the pretest. In the posttest, she tried to emphasize mathematics by saying that "the sides should be added together, the length of the short and long sides will vary from person to person". Therefore, it was accepted that it was at level 4 because he interpreted the mathematical solution incompletely in the context of real life. A section from student S25's solution paper regarding verification proficiency in the Surrounding of the Lake Hazar activity is given.


6-) Öncelikle aşağıdaki ölçekle hazar gölünün uzun, kısa kenarlarının bulup çarptıktan sonra alanı buldum çevre uzunluğu ise Hazar gölünün uzun, kısa kenarlarını toplayıp buldum

Pretest


6-) eğer sağlamasını yaparsam doğru cevaba ulaşırım. 48÷4=12+6=18 yani cevabımız doğrudur.

Posttest

Figure 9 Pretest-Posttest Regarding S25's Verification Competency A Section from the Hazar Lake Activity

In the verification step, it was determined that student S25 tried to make a verbal verification with the statement "I found the long and short sides of the lake by adding them together" in the pretest, but it was at Level 2 because it contained errors. In the posttest, he said that he would verify it and tried to make a mathematical calculation and to do so. While taking the verification approach, he tried to verify only the long edge. He was determined at level 5 because he did not correct errors in the verification approach.

Findings Regarding the Second Sub-Problem

In this sub-problem, it was tried to reveal how the groups developed during the process. Figure 10 shows the groups' average scores regarding their ability to understand the problem.

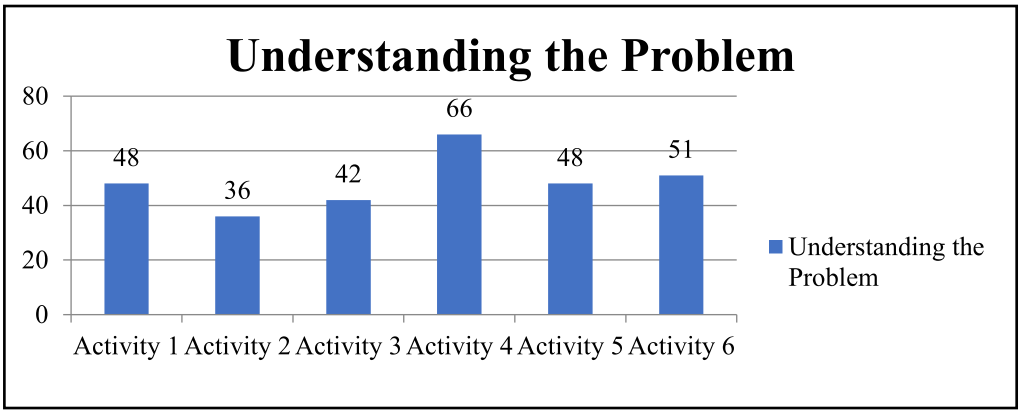


Figure 10 Average Scores of the Groups Regarding the Adequacy of Understanding the Problem

The figure shows that the average problem-understanding scores of the activities are high. It can be said that the average scores in this competency are close to each other, except for the fourth activity. This situation can be interpreted as students not having difficulty in understanding the problem. Below is the solution sheet for the 7th group's ability to understand the problem in the Which Seed Should We Plant activity.

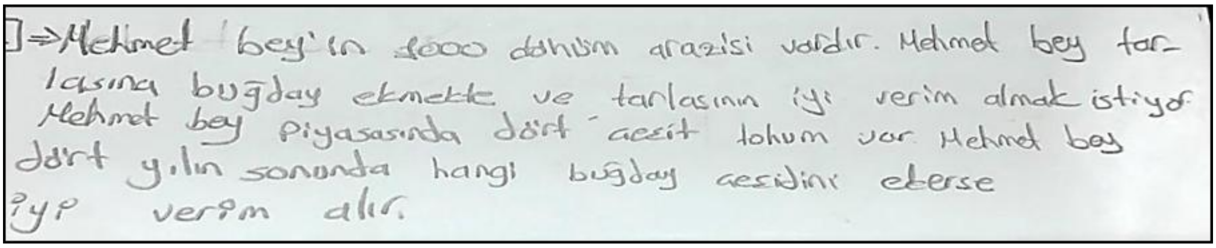


Figure 11 An Excerpt from the Seventh Group's Opinions on the Adequacy of Which Seed Should We Plant Activity in Understanding the Problem

The students in this group expressed the problem in their own words by saying, "There are four types of seeds and whichever seed is planted will yield the best yield at the end of four years". It was determined that they were at level 4 in their ability to understand the problem because they determined what was given and wanted in the problem. This finding is essential for developing students' ability to understand the problem. Figure 12 shows the average scores of the groups regarding simplification competence.

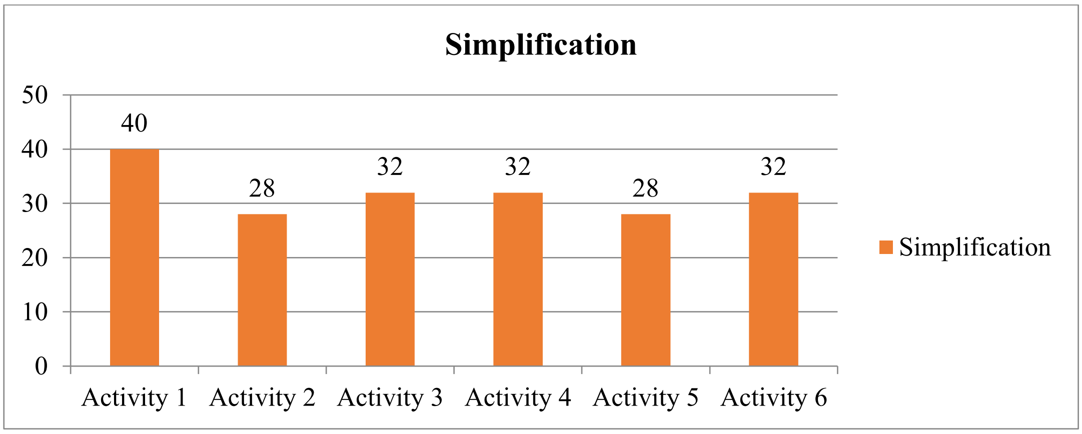


Figure 12 Average Scores of the Groups Regarding Simplification Competency

The students' average scores regarding simplification competence are given in the figure above. It is seen that students' scores in this competency are lower than the problem understanding competency. It can be seen that the highest score in this competency belongs to activity 1. The difference in the scores in the activities can be explained by the difficulty levels of the activities and the student's psychological state while performing the activity. Below is the solution sheet for the simplification competence of group 7.

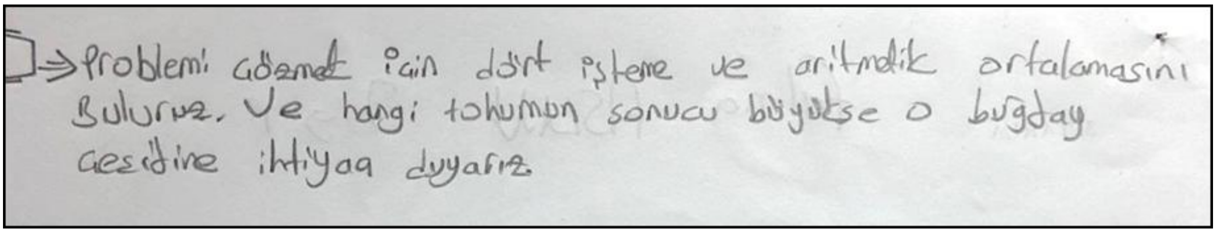


Figure 13 An Excerpt from the opinions of the Seventh Group on the Adequacy of Simplification of Which Seed Should We Plant Activity

It is seen that the students in this group are at level 4 because they determined the necessary and unnecessary variables and made realistic assumptions with the statement "we need four operations and the arithmetic mean" in the Simplification step. In this competence, students must emphasize the arithmetic mean and the four operations. This shows that the students have a high level of simplification competence. Figure 14 shows the average scores of the groups regarding Mathematization competence.

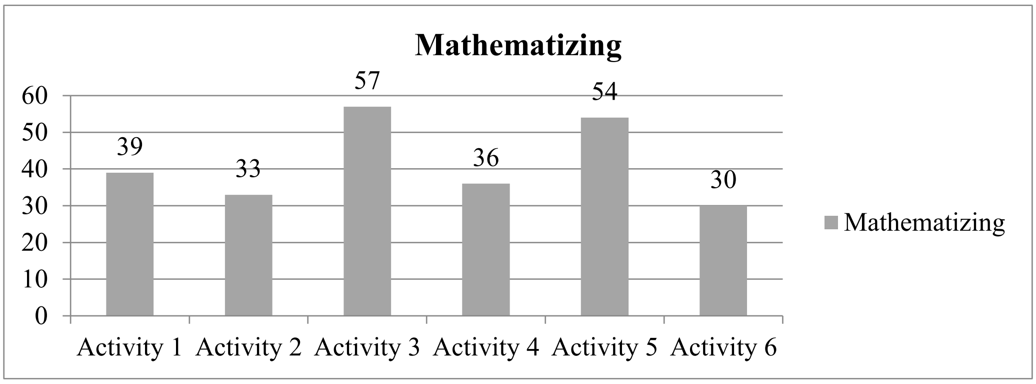


Figure 14 Mean Scores of the Groups on Mathematization Competence

As seen in the figure, it can be seen that the average scores of the activities are generally high. It is seen that the third and fifth activity scores are high in this competence. The reason why other activities are low can be explained by the fact that students have difficulty in these activities. Below is the solution sheet for the Mathematization competency in the Which Seed Should We Plant activity of the seventh group.

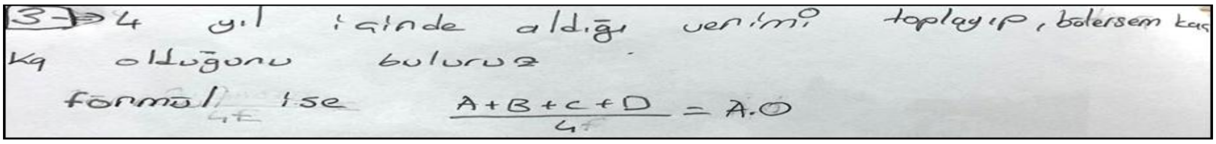


Figure 15 An Excerpt from the views of the Seventh Group on the Mathematization Competence of the Which Seed Should We Plant Activity

The students in this group correctly created and explained the average formula, which is the necessary mathematical model, in the mathematization step with the statement "If we add the yield in 4 years and divide by 4". It was determined that they were at level 5 in the evaluation rubric because they correctly constituted the necessary mathematical model. This situation reveals that the students' Mathematization competencies are high. Figure 16 shows the groups' average scores regarding competency in Studying Mathematically.

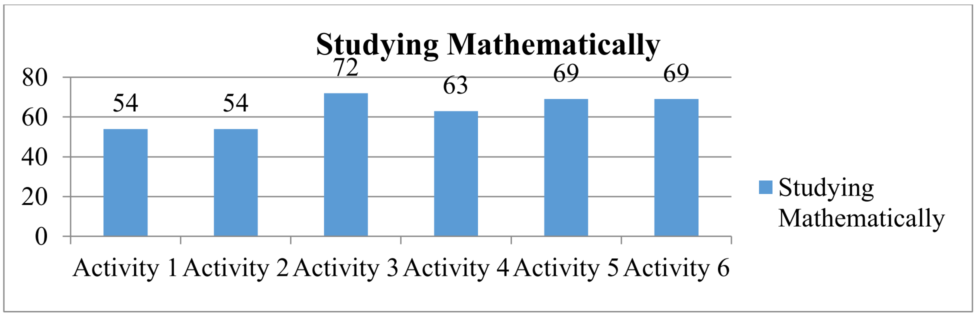


Figure 16 Average Scores of the Groups Regarding Mathematical Studying Proficiency

Considering Figure 15, it can be seen that the average scores in the activities are high. It can be said that the average scores in the activities are close to each other. Students focus on solutions to routine problems. The groups' high scores in this competence can be interpreted as their familiarity with this situation.

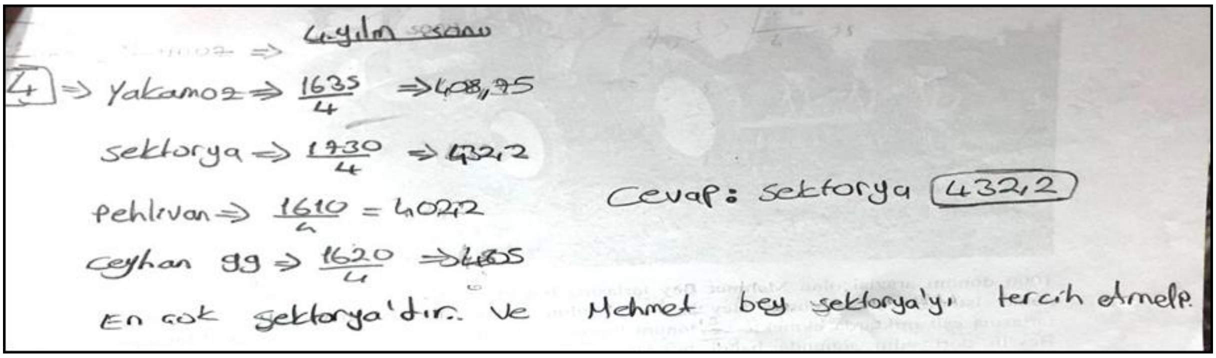


Figure 17 An Excerpt from the views of the Seventh Group on the Competence of Studying Mathematically in the Which Seed Should We Plant Activity

It is seen that the students in the group made solutions using the average relation according to the mathematical model they constituted correctly. According to the evaluation rubric, the students in this group are at level 5. This situation is essential regarding the students' status in this proficiency. Figure 18 shows the average scores of the groups regarding Interpretation competence.

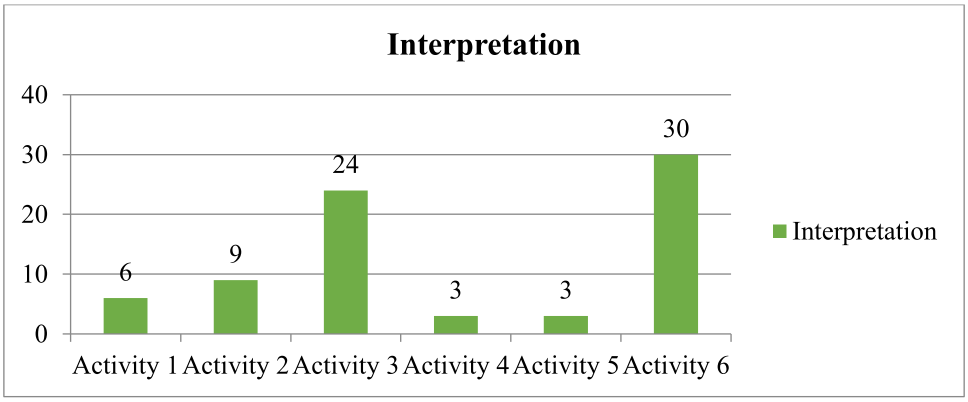


Figure 18 Average Scores of the Groups Regarding Interpretation Competence

Considering the activity scores in Figure 18, it can be seen that the scores are low. It can be seen that the activities with the highest scores in this competence are the third and sixth activities. This situation shows that students have difficulty in this competence. Students' difficulty in this competence can be interpreted as not being used to this situation. In addition, the scores in the two activities are higher than the other activities, which can be explained by the fact that the other two activities are related to their close environment.

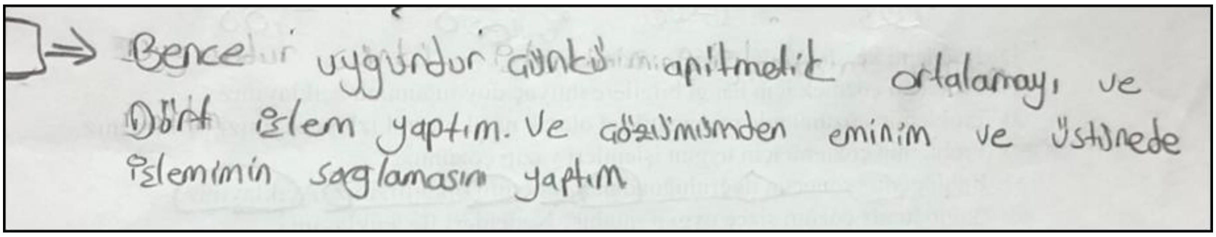


Figure 19 An Excerpt from the Joint Views of the Seventh Group on the Adequacy of Interpretation of Which Seed Should We Plant Activity

When the solution paper of the seventh group regarding interpretation competence was examined, it was seen that the students could not interpret in the context of daily life with the statement "We are sure that we did the average and the four operations". In this case, it was determined that the students were at level 1 in interpretation competence. This situation shows that students have difficulty in this competence. Figure 20 shows the average scores of the groups regarding Verification competence.

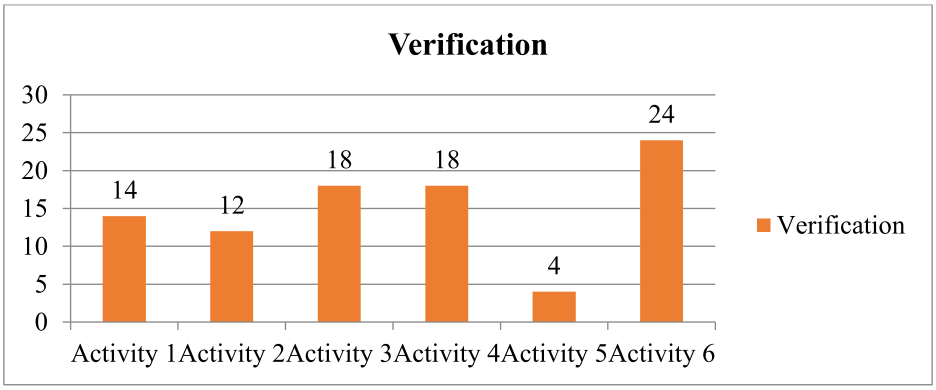


Figure 20 Average Scores of the Groups Regarding Verification Adequacy

Considering the verification competence scores, it was seen that the activity scores are at a low level. The score in the fifth activity was determined to be lower than the other activities. This situation can be explained by students having difficulty in this activity. Students' low scores can also be interpreted as unfamiliarity with this competency.

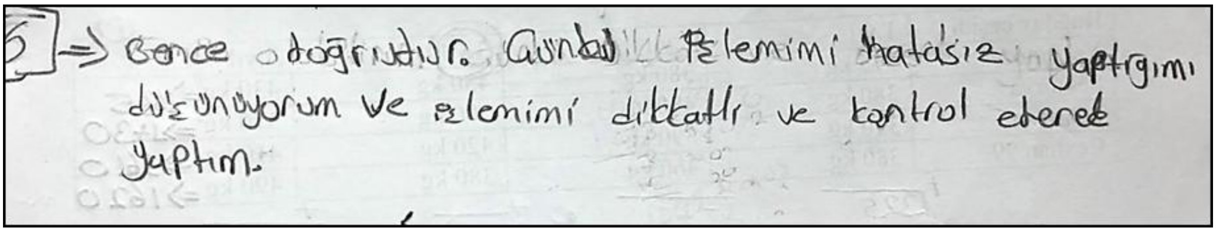


Figure 21 An Excerpt from the joint views of the Seventh Group on Which Seed Should We Plant Activity Verification Adequacy

When we look at the solution paper for the verification step of the seventh group, we see that they did not take a verification approach with the return of "We did the operation without any errors". This shows that they are at level 1 in the evaluation rubric. The data of this group in the solution sheet and the data in the graph are parallel.

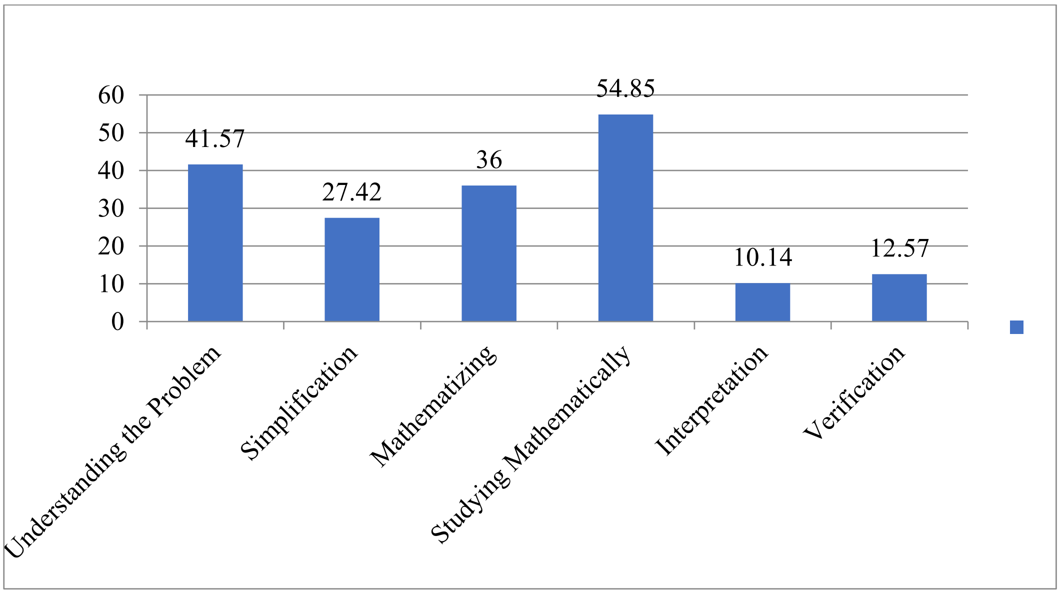


Figure 22 Average scores of the groups for all competencies

While the stage in which the students had the highest mean score was "Studying Mathematically", the stage in which they had the lowest mean score was "Interpretation". The average scores obtained by the students during the process and those received by the students in the posttest are similar. Students' opinions about their difficulties while solving the activities are summarized in Table 5 below.

Table 5 Student Opinions on the Difficulties Encountered While Solving the Activities

Category	Opinions	Students	f
Difficulty situations	I had a hard time in general	S2, S3, S5, S9, S11, S13, S19, S21, S23, S26	11
	I had a hard time understanding the problem	S6, S16, S27	3
	I had a hard time finding a solution	S4, S15, S16,	3
	I had difficulty interpreting	S1, S12, S14, S18, S20, S24, S25	6
	I was not forced	S7, S8, S10, S22	4
	Total		27

As seen in Table 5, after the implementation, it was observed that the student's opinions on the question "What difficulties did you encounter while solving the activities" were mostly "I Had Difficulty in General" and secondly "I Had Difficulty in Interpreting". Some student opinions according to the category in the table are given below.

S5: I had a hard time and put in a lot of effort while solving these activities.

S9: I already have difficulty in mathematics and also had difficulty while solving these activities.

S6: The difficulty I encounter while solving these activities is that I do not understand the problem well.

S27: Sometimes, I did not understand the question when I read it.

S4: I had difficulty analyzing these activities.

S15: I had the most difficulty solving these activities' operations.

S12: While solving these activities, I had the most difficulty in the comments section because I could not express my thoughts much.

S14: I had difficulty making interpretations and could not add any comments.

After the implementation, opinions were received from the students about the implementation process. Most students stated that they had difficulty in all steps of the activities, some had difficulty understanding the problems, and some stated that they had difficulty in the interpretation and verification stages. It can be said that the difficulties students face in activities have changed. After the implementation process, the results of the content analysis of the opinion form regarding the difference between mathematical modeling activities and mathematical problems encountered in the lessons are given in Table 6.

Table 6 Student Opinions on the Difference Between Modeling Activities and Traditional Mathematics Problems

Opinions	Students	f
There is a difference in revealing our thoughts	S1, S2, S5, S6, S8, S9, S12, S13, S14, S16, S18, S20, S24, S25, S27	15
No difference	S3, S4, S7, S10, S11, S15, S17, S19, S21, S22, S23, S26	12
Total		27

As seen in Table 6, after the implementation, it was observed that the students' opinions regarding whether there was a difference between these activities and the mathematical problems they encountered in the lessons were mostly "There is a difference". Some student opinions according to the category in the table are given below.

S5: The difference between these activities is that they want to know how much we use our minds and how much we think.

S6: I think everyone's answers differ in the implementations we make differently. Everyone has their thoughts and hypotheses.

S12: I think there is a difference. In these activities, we put our thoughts on paper because they relate to daily life, whereas in regular lessons, we only need information for the questions.

S11: I think there is no difference because we answer mathematics questions and do the same in the activities.

S23: I think there is no difference because whatever we study in mathematics class, we encounter in these activities.

After the implementation, students' opinions were received regarding the difference between the activities and routine mathematics problems. While some students stated that they revealed their thoughts that there was a difference between them, others stated that there was no difference. It can be said that these activities impact revealing some students' thoughts.

Discussion and Conclusion

In this part of the research, the findings were discussed in the light of the relevant literature considering the sub-problems of the research.

According to the findings obtained from the pretest in the research, the students had low levels of all modeling competencies (understanding the problem, simplifying, mathematizing, working mathematically, interpreting, and verifying). In his study conducted with secondary school students, Kılıç (2020) reported that students demonstrated limited competencies in mathematical modeling prior to the implementation. He attributed this to the fact that mathematical modeling is not sufficiently embedded in the structure and practice of the mathematics curriculum. Similarly, Tekin Dede and Yılmaz (2015) examined the cognitive modeling competencies of sixth grade students in the study and concluded that modeling competencies were mainly at a low level. Students' modeling skills were low before the implementation could be interpreted as the students encountering modeling activities for the first time. They were unfamiliar with these activities and did not have sufficient experience with them.

As a result of the analysis, it was concluded that the teaching process with mathematical modeling activities provided a significant increase in students' mathematical modeling competency scores. In a similar study, Tekin Dede and Yılmaz (2015) applied twelve action plans in their study on the development of modeling skills of 6th grade students. As a result of the study, it was stated that the student's cognitive modeling skills showed a statistically significant difference, and the students had the most difficulty in the interpretation stage. Similar to this study, Özgen and Şeker (2020) determined that at the end of the implementation process with modeling activities in the experimental and control group study, they constituted with the participation of 6th grade students, the modeling competencies of the students in the experimental group were better than the modeling competencies of the students in the control group. In his study, Maaß (2006) concluded that students showed positive development in mathematical modeling activities, low-level students also participated in the process, and students could enter the modeling process individually even if they did not demonstrate all of the sub-modeling competencies. As a result of another study conducted by Maaß (2005) stated that 8th grade students could improve their modeling competencies through modeling applications, and modeling could be taught at the middle school level. In their study examining the relationship between 7th grade students' modeling and reading comprehension skills, Alkan and Aydın (2021) stated that their modeling competencies improved after the 8-week implementation period. When the studies are examined, it is seen that teaching processes with mathematical modeling activities improve modeling competencies. This can be interpreted as an appropriate teaching process improving modeling competencies.

Although students showed improvement in all mathematical modeling competencies in this study, when the literature was examined, it was stated that students had difficulty improving particularly in interpreting and verifying competencies in some studies (Didiş Kabar & İnan, 2018; Kılıç, 2020; Şahin & Eraslan, 2016; Tekin Dede & Yılmaz, 2015). In this study, opinions were taken from the students after the implementation. Most students stated that mathematical modeling activities differed from routine problems and that mathematical modeling activities revealed their thoughts differently from routine problems. In addition, most students stated that they had difficulty in all stages, while most stated difficulty interpreting competence. Considering that the most difficult competencies in the modeling process were interpretation and verification competencies (Blum & Borromeo Ferri, 2009; Tekin Dede & Yılmaz, 2013), results consistent with this finding were obtained in the study. Most of the students could not analyze the mathematical results they obtained and interpret them completely in a real-life situation. This result of the study is similar to many studies in the literature that concluded that students had difficulties in interpreting and verifying the mathematical operations and results they performed in the modeling process (Alkan & Aydın, 2021; Blum & Borromeo Ferri, 2009; Kaya & Keşan, 2022; Sarı Uzun et al., 2023; Şahin & Eraslan, 2016). The opinions expressed by the students support the statistical data. Şahin and Eraslan (2016), in their study where they tried to reveal the modeling processes of 4th grade primary school students, stated that the students had difficulty in competencies for understanding and interpreting the problem. He stated that this difficulty stems from students' limited experience in such activities inside and outside the school. Likewise, Hıdıroğlu et al. (2014), in their study examining secondary school students' solution approaches to the comet problem, stated that as the modeling process progressed, the performance of the students decreased and that the students could not find any approach in the verification stage. This situation is attributed to students who focus on the result tend to stop interpreting and verifying the problem and their actions as soon as they find the mathematical result, so they do not show much progress in the last two stages. In their study with middle school students, Şahin and Eraslan (2017) stated that students generally accepted the accuracy of the solutions of modeling activities without commenting or verifying them. Sarı Uzun et al. (2023) stated in their study with 5th grade middle school students that almost all of the students could not approach the verification and interpretation step. They emphasized that this caused the students to have difficulty in verification and interpretation competence by generally sticking to a single result and performing a procedural problem solution. Considering the conducted studies, it is seen that students generally have difficulties in interpretation and verification competencies. It can be said that the students' difficulties in some of the modeling competencies are due to reasons such as not having knowledge about modeling activities before, being accustomed to the multiple-choice exam system, and teachers not attaching importance to the verification and interpretation stages when solving routine problems in the classroom environment.

In the second sub-problem of the research, the students' situations were examined in modeling activities throughout the implementation process. As a result of the analysis, it was seen that the students' modeling competencies were generally in good condition throughout the process. It was observed that the students were better at understanding the problem, simplifying, mathematizing, and studying mathematically than interpreting and verifying competencies. When the literature was examined, Kılıç (2020) found in his study with middle school students that there was a significant difference in favor of the posttest between the pretest and posttest comprehension, simplification, mathematization, mathematical working, and interpretation competencies of the experimental group However, he stated that there was no significant difference between verification competencies. It was emphasized that this may be because students are not asked to interpret the problem solutions in the context of daily life and are not asked to verify as in the verification stage, which is one of the modeling competencies. In their study with middle school students, Alkan and Aydın (2021) stated that the stage where the students' development was the least was the interpretation stage. Tekin Dede (2017), in his study with middle school students at different grade levels, found that all competencies except verification and modeling competencies increased as the grade level increased. In another study, İnan Tutkun and Didiş Kabar (2018) emphasized in their case study with middle school students that after reaching the desired solution during the modeling process, the students did not interpret the mathematical results in the context of real life and did not check the accuracy of the results. They stated that this might be due to their students' habits of result-oriented problem-solving in the solution process of traditional verbal problems in mathematics classes and that the students participating in the study had their first experience with a modeling problem. When the process is examined, in parallel with the studies conducted, it is seen that in this study, students' modeling competencies decreased at the verification and interpretation stage, as in the last test. It was determined that the modeling competencies were not regular from the first activity to the last activity. In addition, it was observed that students' modeling competencies were better in some activities, and in others, their modeling competencies were not as good as in others. It can be said that the reason for this is that the difficulty levels of the activities are different based on the informal observations of the researcher. In addition, based on observations, it can be interpreted that activities related to students' immediate environment attract their attention more. Therefore, they have less difficulty in the activities. In this context, it can be said that it is essential to constitute environments where students can establish relationships with mathematics and daily life and to design mathematical modeling problems to attract students' attention as much as possible.

Suggestions

The following implications were made in the context of the results of the research:

This study showed that teaching with mathematical modeling activities generally improved the mathematical modeling competencies of 7th grade students. Therefore, mathematical modeling activities can be developed. The development of modeling competencies of students at different grade levels can be examined, and a contribution to the literature can be made.

As a result of this study, when examining modeling competencies, it was determined that students showed low levels of development in the "interpretation" and "verification" stages. When the literature is examined, it is possible to encounter similar results. In this context, qualitative studies can be conducted to investigate the reasons for this in depth.

The findings of this study were conducted with students studying in a class at a public middle school in a rural area. Future studies can examine the modeling processes of students studying in various regions of Türkiye with different socio-cultural characteristics.

It is thought that it is essential to create environments where students can establish relationships with mathematics and to design mathematical modeling problems in these environments to attract students' attention as much as possible.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

The authors declare that they have no competing interests

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CRediT author statement

First author: research design, analysis, methodology, data collection, resources, discussion, conclusion, writing-original draft, writing - review & editing. The second author: methodology, writing -original draft, writing - review & editing, resources, discussion, conclusion, supervision.

Research involving Human Participants and/or Animals

The study involves human participants. Ethics committee permission was obtained from Dicle University, Social and Human Sciences Ethics Committee.

Matematiksel Modelleme Etkinlikleri ile Tasarlanan Öğretim Sürecinde Ortaokul Öğrencilerinin Matematiksel Modelleme Yeterliklerindeki Değişimin İncelenmesi

Özet:

Bu çalışmanın amacı, matematiksel modelleme etkinlikleri ile tasarlanan öğretim sürecinin ortaokul yedinci sınıf öğrencilerinin matematiksel modelleme yeterliklerini nasıl etkilediğini belirlemektir. Araştırmada karma yöntem araştırma desenlerinden iç içe (gömülü) desen benimsenmiştir. Araştırmanın çalışma grubu bir devlet okulunun ortaokul yedinci sınıf düzeyinde öğrenim gören 27 öğrenciden oluşmaktadır. Katılımcılar, amaçlı örnekleme yöntemlerinden kolay ulaşılabilir durum örnekleme yöntemiyle oluşturulmuştur. 10 hafta süren uygulama sürecinde veri toplama aracı olarak araştırmacı tarafından geliştirilen ve açık uçlu sorulardan oluşan görüş formu ile dokuz matematiksel modelleme etkinliği kullanılmıştır. Bu etkinliklerden üçü ön test ve son testte kullanılmış, geri kalan altı etkinlik ise, uygulama sürecinde kullanılmıştır. Nicel veriler analiz edilirken, Wilcoxon işaretli sıralar testi kullanılmıştır. Nitel verilerin analizinde ise betimsel analiz kullanılmıştır. Elde edilen sonuçlara göre, uygulama süreci boyunca öğrenciler en çok problemi anlama ve matematiksel olarak çalışma basamağında gelişim gösterirken, en az gelişimi yorumlama ve doğrulama basamağında göstermişlerdir. Öğrencilerin matematik ile ilişki kurabileceği ortamların yaratılması ve bu ortamlarda matematiksel modelleme problemlerinin mümkün olduğunca öğrencilerin dikkatini çekecek şekilde tasarlanmasının önemli olduğu düşünülmektedir.

Anahtar kelimeler: Modelleme yeterlikleri, matematiksel modelleme, ortaokul öğrencileri.

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