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On Nano $\wedge_{g^\star}$-Closed Sets

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Abstaract − In this paper, we introduce and study nano topological properties of nano $\wedge_{g^\star}$-closed sets and nano $\wedge_{g^\star}$-open sets and its relationships with other nano generalized closed sets are investigated.

Keywords − *Nano $\wedge_{g^\star}$-open sets, nano λ-closed set, nano $\wedge$-set*

1 Introduction

Thivagar et al. [4] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space.

In 2017, Rajasekaran et al. [7, 8] introduced nano $\wedge$-sets and nano $\wedge_g$-sets in nano topological spaces and we introduced the notion of nano λ-closed set and nano λ-open sets. In this paper, we introduce and study nano topological properties of nano $\wedge_{g^\star}$-closed sets and nano $\wedge_{g^\star}$-open sets and its relationships with other nano generalized closed sets are investigated.

2 Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a

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space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. *[4] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. *[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*
2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. *[4] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[4] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.*

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. *[4] A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

1. *nano pre-open set if $H \subseteq Nint(Ncl(H))$.*

2. *nano semi-open set if $H \subseteq Ncl(Nint(H))$.*

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.7. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

1. *nano g-closed [1] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*

2. *nano gs-closed set [2] if $Nscl(H) \subseteq G$ whenever $H \subseteq G$, G is nano open.*

3. *nano gp-closed set [3] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*

Definition 2.8. *[5] Let $(U, \tau_R(X))$ be a nano topological spaces and $H \subseteq U$. The nano $Ker(H) = \bigcap\{U : H \subseteq U, U \in \tau_R(X)\}$ is called the nano kernal of H and is denoted by $\mathcal{N}Ker(H)$.*

Definition 2.9. *[7] A subset H of a space $(U, \tau_R(X))$ is called;*

1. *nano $\wedge$-set if $H = \mathcal{N}Ker(H)$.*

2. *nano λ-closed if $H = L \cap F$ where L is a nano $\wedge$-set and F is nano closed.*

Definition 2.10. *[8] A subset H of a space $(U, \tau_R(X))$ is called;*

1. *nano $\wedge_g$-closed set if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano λ-open.*

2. *a nano ${}_g\wedge$-closed set if $N\lambda cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*

3. *a nano $\wedge$-g-closed set if $N\lambda cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano λ-open.*

The complement of the above mentioned sets are called their respective open sets.

Lemma 2.11. *[7] For a subset H of a space $(U, \tau_R(X))$, the following conditions are equivalent.*

1. *H is nano λ-closed.*
2. *$H = L \cap Ncl(H)$ where L is a nano $\wedge$-set.*
3. *$H = \mathcal{N}Ker(H) \cap Ncl(H)$.*

Lemma 2.12. *[7]*

1. *Every nano $\wedge$-set is nano λ-closed.*
2. *Every nano open set is nano λ-closed.*
3. *Every nano closed set is nano λ-closed.*

3 Nano $\wedge_{g^\star}$-Closed Sets

Definition 3.1. *A subset H of a space $(U, \tau_R(X))$ is called a nano $\wedge_{g^\star}$-closed if $N\lambda cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano g-open.*

The complement of nano $\wedge_{g^\star}$-open if $H^c = U - H$ is nano $\wedge_{g^\star}$-closed.

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$.*

1. *Then $\{a, b\}$ is nano $\wedge_{g^\star}$-closed.*
2. *Then $\{a, d\}$ is not nano $\wedge_{g^\star}$-closed.*

Theorem 3.3. *In a space $(U, \tau_R(X))$, every nano λ-closed is nano $\wedge_{g^\star}$-closed.*

Proof. Let $H \subseteq G$, where G is nano g-open. Since H is nano λ-closed, we have $\lambda cl(H) = H \subseteq G$. Hence H is nano $\wedge_{g^\star}$-closed.

Remark 3.4. *The converse of statements in Theorem 3.3 are not necessarily true as seen from the following Example.*

Example 3.5. *In Example 3.2, then $\{a, c, d\}$ is nano $\wedge_{g^\star}$-closed but not nano λ-closed.*

Theorem 3.6. *In a space $(U, \tau_R(X))$, every nano closed is nano $\wedge_{g^\star}$-closed.*

Proof. Proof follows from Lemma 2.12 and Theorem 3.3.

Remark 3.7. *The converse of statements in Theorem 3.6 are not necessarily true as seen from the following Example.*

Example 3.8. *In Example 3.2, then $\{d\}$ is nano $\wedge_{g^\star}$-closed but not nano closed.*

Theorem 3.9. *In a space $(U, \tau_R(X))$, every nano open is nano $\wedge_{g^\star}$-closed.*

Proof. Obvious by the Definitions.

Remark 3.10. *The converse of statements in Theorem 3.9 are not necessarily true as seen from the following Example.*

Example 3.11. *In Example 3.2, then* $\{c\}$ *is nano* $\wedge_{g^\star}$*-closed but not nano open.*

Theorem 3.12. *Let* H *be a nano* g*-open. Then* H *is nano* λ*-closed if* H *is nano* $\wedge_{g^\star}$*-closed.*

Proof. Let H is nano $\wedge_{g^\star}$-closed and nano g-open. Since as $H \subseteq H$, $N\lambda cl(H) \subseteq H$. Hence H is nano λ-closed.

Theorem 3.13. *In a space* $(U, \tau_R(X))$*, every nano* $\wedge_{g^\star}$*-closed is nano* ${}_g\wedge$*-closed.*

Proof. Let H is nano $\wedge_{g^\star}$-closed and $H \subseteq G$, with G is nano open. Since every nano open is nano g-open and H is nano $\wedge_{g^\star}$-closed, we have $\lambda cl(H) \subseteq G$. Hence H is nano ${}_g\wedge$-closed.

Remark 3.14. *The converse of statements in Theorem 3.13 are not necessarily true as seen from the following Example.*

Example 3.15. *In Example 3.2, then* $\{a\}$ *is nano* ${}_g\wedge$*-closed but not nano* $\wedge_{g^\star}$*-closed.*

Theorem 3.16. *In a space* $(U, \tau_R(X))$*, every nano* $\wedge_{g^\star}$*-closed is nano* $\wedge$*-*g*-closed.*

Proof. Obvious.

Remark 3.17. *The converse of statements in Theorem 3.16 are not necessarily true as seen from the following Example.*

Example 3.18. *In Example 3.2, then* $\{a\}$ *is nano* $\wedge$*-*g*-closed but not nano* $\wedge_{g^\star}$*-closed.*

Theorem 3.19. *In a space* $(U, \tau_R(X))$*, every nano* g*-closed is nano* $\wedge_{g^\star}$*-closed.*

Proof. Obvious.

Remark 3.20. *The converse of statements in Theorem 3.19 are not necessarily true as seen from the following Example.*

Example 3.21. *In Example 3.2, then* $\{a, b, d\}$ *is nano* $\wedge_{g^\star}$*-closed but not nano* g*-closed.*

Remark 3.22. *In a space* $(U, \tau_R(X))$*, every nano* $\wedge_g$*-closed is nano* $\wedge_{g^\star}$*-closed.*

Example 3.23. *In Example 3.2, then* $\{a, b, d\}$ *is nano* $\wedge_{g^\star}$*-closed but not nano* $\wedge_g$*-closed.*

Remark 3.24. *The concepts of nano* $\wedge_{g^\star}$*-closed and being nano gs-closed, nano gp-closed are independent.*

Example 3.25.

1. *Let* $U = \{a, b, c\}$ *with* $U/R = \{\{b\}, \{a, c\}\}$ *and* $X = \{c\}$*. Then the nano topology* $\tau_R(X) = \{\phi, \{a, c\}, U\}$*. Then* $\{a, c\}$ *is nano* $\wedge_{g^\star}$*-closed but not nano gp-closed.*
2. *In Example 3.2, then* $\{a\}$ *is nano gp-closed but not nano* $\wedge_{g^\star}$*-closed.*

Example 3.26. *In Example 3.2,*

1. *then $\{a, b, d\}$ is nano $\wedge_{g^\star}$-closed but not nano gs-closed.*
2. *then $\{a\}$ is nano gs-closed but not nano $\wedge_{g^\star}$-closed.*

Remark 3.27. *In a space $(U, \tau_R(X))$,*

1. *the intersection of two nano $\wedge_{g^\star}$-open sets but not nano $\wedge_{g^\star}$-open.*
2. *the union of two nano $\wedge_{g^\star}$-closed sets but not nano $\wedge_{g^\star}$-closed.*

Example 3.28. *In Example 3.2,*

1. *then $P = \{b\}$ and $Q = \{c\}$ is nano $\wedge_{g^\star}$-open sets. Hence $P \cup Q = \{b, c\}$ is not nano $\wedge_{g^\star}$-open.*
2. *then $P = \{a, b\}$ and $Q = \{a, c\}$ is nano $\wedge_{g^\star}$-closed sets. Hence $P \cap Q = \{a\}$ is not nano $\wedge_{g^\star}$-closed.*

4 Properties of $\wedge_{g^\star}$-Closed Sets

Theorem 4.1. *If a subset H is nano $\wedge_{g^\star}$- closed set, then nano $N\lambda cl(H) - H$ does not contain any non empty nano closed in U.*

Proof. Let H be nano $\wedge_{g^\star}$-closed, suppose K is a non empty nano closed contained in $N\lambda cl(H) - H$, which clearly implies $H \subseteq K^c$, where K^c is nano open. Since H is nano $\wedge_{g^\star}$-closed and as every nano open is nano g-open, we have $N\lambda cl(H) \subseteq K^c$. Hence $K \subseteq U - N\lambda cl(H)$. Also we have $K \subseteq N\lambda cl(H)$. Therefore $K \subseteq (U - N\lambda cl(H)) \cap N\lambda cl(H) = \phi$. Hence $N\lambda cl(H) - H$ does not contain any non empty nano closed.

Theorem 4.2. *If a subset H is nano $\wedge_{g^\star}$- closed, then $N\lambda cl(H) - H$ does not contain any non empty nano g-closed.*

Proof. Let H be nano $\wedge_{g^\star}$-closed. Suppose K is a nano g-closed contained in $N\lambda cl(H) - H$, which clearly implies $H \subseteq K^c$, where K^c is nano g-open. Since H is nano $\wedge_{g^\star}$-closed and $N\lambda cl(H) \subseteq K^c$. Hence $K \subseteq U - N\lambda cl(H)$. Also we have $K \subseteq N\lambda cl(H)$. Therefore $K \subseteq (U - N\lambda cl(H)) \cap N\lambda cl(H) = \phi$. Hence $N\lambda cl(H) - H$ does not contain a non empty nano g-closed.

Theorem 4.3. *In a space $(U, \tau_R(X))$, for each $x \in U$, $\{x\}$ is nano g-closed or nano $\wedge_{g^\star}$-open.*

Proof. Suppose $\{x\}$ is not nano g-closed then $U - \{x\}$ is not nano g-open, then the only nano g-open containing $U - \{x\}$ is U. That is $U - \{x\} \subseteq U$. So $N\lambda cl(U - \{x\}) \subseteq U$. Hence $U - \{x\}$ is nano $\wedge_{g^\star}$-closed set. Hence $\{x\}$ is nano $\wedge_{g^\star}$-open.

Theorem 4.4. *Let H be nano $\wedge_{g^\star}$-closed. Then H is nano λ-closed $\iff$ $N\lambda cl(H) - H$ is nano closed.*

Proof. Necessity : Suppose H be nano $\wedge_{g^\star}$-closed and nano λ-closed. H is nano λ-closed implies $N\lambda cl(H) = H$. Hence $N\lambda cl(H) - H = \phi$ is nano closed.

Sufficiency : Suppose H is nano $\wedge_{g^\star}$-closed and $N\lambda cl(H) - H$ is nano closed. Then by Theorem 4.1. $N\lambda cl(H) - H$ contains no non empty nano closed. Hence we should have $N\lambda cl(H) - H = \phi$, which in turn implies $N\lambda cl(H) = H$. Therefore H is nano λ-closed.

Theorem 4.5. *If every nano $\wedge_{g^\star}$-closed is nano λ-closed then $\{x\}$ is nano g-closed or nano λ-open.*

Proof. Suppose $\{x\}$ is not a nano g-closed, then $U - \{x\}$ is not a nano g-open. Hence we have U is the only nano g-open containing $U - \{x\}$. Obviously $N\lambda cl(U - \{x\}) \subseteq U$. Therefore $U - \{x\}$ is nano $\wedge_{g^\star}$-closed. By hypothesis $U - \{x\}$ is nano λ-closed set. Hence $\{x\}$ is nano λ-open.

Theorem 4.6. *Let H is nano g-open set and nano $\wedge_{g^\star}$-closed. If K is nano λ-closed then $H \cap K$ is nano $\wedge_{g^\star}$-closed.*

Proof. By Theorem 3.12 if a set H is both nano g-open and nano $\wedge_{g^\star}$-closed then H is nano λ-closed. Hence if K is nano λ-closed then $H \cap K$ is nano λ-closed as the intersection of nano λ-closed sets is a nano λ-closed. Hence by Theorem 3.3 $H \cap K$ is nano $\wedge_{g^\star}$-closed set.

Theorem 4.7. *For a subset H of a space $(U, \tau_R(X))$, the following are equivalent:*

1. *every nano g-open set is nano λ-closed.*
2. *every subset is a nano $\wedge_{g^\star}$- closed.*

Proof. (1) $\Rightarrow$ (2). Let H be any subset of $(U, \tau_R(X))$ such that $H \subseteq G$ where G is nano g-open. Hence we get $N\lambda cl(H) \subseteq N\lambda cl(G)$. By hypothesis G is nano λ-closed set. Then we get $N\lambda cl(H) \subseteq N\lambda cl(G) = G$. Hence H is nano $\wedge_{g^\star}$-closed.

(1) $\Rightarrow$ (2). Let H be a nano g-open. By hypothesis H is nano $\wedge_{g^\star}$-closed. Then we have $N\lambda cl(H) \subseteq H$. Therefore H is nano λ-closed. Hence every nano g-open is nano λ-closed.

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