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## Soft $\beta$ -Separation Axioms in Soft Quad Topological Spaces

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**Abstract** - The major purpose of this article is to publicize soft  $\beta$ -separations axioms in soft quad topological spaces. We talk over and focus our attention on soft  $\beta$ -separation axioms in soft quad topological spaces with respect to ordinary points and soft points. Moreover study the inherited characteristics at different respects with respect to ordinary points and soft points. Some of their principal properties in soft quad topological spaces are also brought under consideration.

**Key words** - Soft sets, soft topology, soft  $\beta$  open set, soft  $\beta$  closed set, soft quad topological space, soft  $\beta qT_0$  structure, soft  $\beta qT_1$  structure, soft  $\beta qT_2$  structure, soft  $\beta qT_3$  structure and soft  $\beta qT_4$  structure.

### 1 Introduction

In actual life situation the complexity in economics, engineering, social sciences, medical science etc. we cannot attractively use the outdated classical methods because of different kinds of uncertainties existing in these problems. To finish out these complexity, some types of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, inwhich we can safely use mathematical methods for dealing with uncertainties. But, all these theories have their inherent worries. To bury these difficulties in the year 1999, Russian scholar Molodtsov [4] introduced the idea of soft set as a new mathematical methods to deal with uncertainties. Which is free from the above difficulties? Kelly [5] studied bi-topological spaces and discussed different results. Tapi et al. [53] beautifully discussed separation axioms in quad topological spaces. Hameed and Abid [54] discussed separation axioms in Tri-topological spaces.

Recently, in 2011, Shabir and Naz [7] opened the idea of soft topological space and discussed different results with respect to ordinary points. They beautifully defined soft topology as a collection of  $\tau$  of soft sets over  $X$ . they also defined the basic idea of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axioms, soft regular and soft normal spaces and published their several performances. Soft separation axioms are also discussed at detail. Aktas and Cagman [9] discussed Soft sets and soft groups. Chen [10] discovered the parameterization reduction of soft sets and its applications. Feng et al. [11]. Studied Soft semi rings and its applications. In the recent years, many interesting applications of soft sets theory and soft topology have been discussed at great depth [12,13,14,,15,16,17,18,19,20,21,22]. Kandil at al. [25] explained Soft connectedness via soft ideal developed soft set theory. Kandil et al. [27] launched Soft regularity and normality based on semi open soft sets and soft ideals.

In [28,29,30,31,32,33,34,35,36] discussion is launched soft semi hausdorff spaces via soft ideals, semi open and semi closed sets, separation axioms ,decomposition of some type supra soft sets and soft continuity are discussed. Hussain and Ahmad [51] defined soft points, soft separation axioms in soft topological spaces with respect to soft points and used it in different results, Kandil et al. [52] studied Soft semi separation axioms and some types of soft functions and their characteristics.

In this present paper, concept of soft  $\beta$ -separation axioms in Soft quad topological spaces is introduced with respect to ordinary and soft points.

Many mathematicians threw light on soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open, soft  $\alpha$ -open set and soft  $\beta$ -open set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present article bridge is built over the gap that exists in soft quad-topology related to soft  $\beta qT_0$ , soft  $\beta qT_1$ , soft  $\beta qT_2$ , soft  $\beta qT_3$  and soft  $\beta qT_4$  structures. Some propositions in soft quid topological spaces are discussed with respect to ordinary points and soft points. When we talk about distances between the points in soft topology then the concept of soft separation axioms is automatically taking birth. That is why these structures are catching our attentions. It is hoped that these results will be the driving force for the future study on soft quad topological spaces to achieve general framework for the practical applications and to solve the most complicated problems containing doubts in economics, engineering, medical, environment and in general mechanic systems of various varieties. In future these beautiful soft topological structures may be extended to soft n-topological spaces provided n is even.

## 2. Preliminaries

The following Definition s which are pre-requisites for present study

**Definition 1** [4]. Let  $X$  be an initial universe of discourse and  $E$  be a set of parameters. Let  $P(X)$  denotes the power set of  $X$  and  $A$  be a non-empty sub-set of  $E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(X)$

In other words, a set over  $X$  is a parameterized family of sub set of universe of discourse  $X$ . For  $e \in A, F(e)$  may be considered as the set of e-approximate elements of the soft set  $(F, A)$

and if  $e \notin A$  then  $F(e) = \phi$  that is  $F_A = \{F(e) : e \in A \subseteq E, F : A \rightarrow P(X)\}$  the family of all these soft sets over  $X$  denoted by  $SS(X)_A$

**Definition 2** [4]. Let  $F_A, G_B \in SS(X)_E$  then  $F_A$  is a soft subset of  $G_B$  denoted by  $F_A \bar{\subseteq} G_B$ , if

1.  $A \subseteq B$  and
2.  $F(e) \subseteq G(e), \forall e \in A$

In this case  $F_A$  is said to be a soft subset of  $G_B$  and  $G_B$  is said to be a soft super set  $F_A, G_B \bar{\supseteq} F_A$

**Definition 3** [6]. Two soft subsets  $F_A$  and  $G_B$  over a common universe of discourse set  $X$  are said to be equal if  $F_A$  is a soft subset of  $G_B$  and  $G_B$  is a soft subset of  $F_A$

**Definition 4** [6]. The complement of soft subset  $(F, A)$  denoted by  $(F, A)^c$  is defined by  $(F, A)^c = (F^c, A)$   $F^c \rightarrow P(X)$  is a mapping given by  $F^c(e) = U - F(e) \forall e \in A$  and  $F^c$  is called the soft complement function of  $F$ . Clearly  $(F^c)^c$  is the same as  $F$  and  $((F, A)^c)^c = (F, A)$

**Definition 5** [7]. The difference between two soft subset  $(G, E)$  and  $(F, E)$  over common of universe discourse  $X$  denoted by  $(F, E) \setminus (G, E)$  is defined as  $F(e) \setminus G(e)$  for all  $e \in E$

**Definition 6** [7]. Let  $(G, E)$  be a soft set over  $X$  and  $x \in X$  We say that  $x \in (F, E)$  and read as  $x$  belong to the soft set  $(F, E)$  whenever  $x \in F(e) \forall e \in E$  The soft set  $(F, E)$  over  $X$  such that  $F(e) = \{x\}, \forall e \in E$  is called singleton soft point and denoted by  $x$ , or  $(x, E)$

**Definition 7** [6]. A soft set  $(F, A)$  over  $X$  is said to be Null soft set denoted by  $\bar{\emptyset}$  or  $\emptyset_A$  if  $\forall e \in A, F(e) = \emptyset$

**Definition 8** [6]. A soft set  $(F, A)$  over  $X$  is said to be an absolute soft denoted by  $\bar{A}$  or  $X_A$  if  $\forall e \in A, F(e) = X$

Clearly, we have,  $X_A^c = \emptyset_A$  and  $\emptyset_A^c = X_A$

**Definition 9** [51]. The soft point  $e_F$  is said to be in the soft set  $(G, A)$ , denoted by  $e_F \in (G, A)$  if for the element  $e \in A, F(e) \subseteq G(e)$ .

**Definition 10** [44]. Two soft sets  $(G, A), (H, A)$  in  $SS(X)_A$  are said to be soft disjoint, written  $(G, A) \cap (H, A) = \emptyset_A$  If  $G(e) \cap H(e) = \emptyset$  for all  $e \in A$ .

**Definition 11** [51]. The soft point  $e_G, e_H$  in  $X_A$  are disjoint, written  $e_G \neq e_H$  if their corresponding soft sets  $(G, A)$  and  $(H, A)$  are disjoint.

**Definition 12** [6]. The union of two soft sets  $(F, A)$  and  $(G, B)$  over the common universe of discourse  $X$  is the soft set  $(H, C)$ , where,  $C = A \cup B, \forall e \in C$

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

Written as  $(F, A) \cup (G, B) = (H, C)$

**Definition 13** [6]. The intersection  $(H, C)$  of two soft sets  $(F, A)$  and  $(G, B)$  over common universe  $X$ , denoted  $(F, A) \bar{\cap} (G, B)$  is defined as

$$C = A \cap B \text{ and } H(e) = F(e) \cap G(e), \forall e \in C$$

**Definition 14** [2]. Let  $(F, E)$  be a soft set over  $X$  and  $Y$  be a non-empty sub set of  $X$ . Then the sub soft set of  $(F, E)$  over  $Y$  denoted by  $(Y_F, E)$ , is defined as follow  $Y_{F(\alpha)} = Y \cap F(\alpha), \forall \alpha \in E$  in other words

$$(Y_F, E) = Y \cap (F, E).$$

**Definition 16** [3]. Let  $\tau$  be the collection of soft sets over  $\tilde{X}$  then  $\tau$  is said to be a soft topology on  $\tilde{X}$  if

1.  $\emptyset, \tilde{X} \in \tau$
  2. The union of any number of soft sets in  $\tau$  belongs to  $\tau$
  3. The intersection of any two soft sets in  $\tau$  belong to  $\tau$
- The triplet  $(\tilde{X}, \tau, E)$  is called a soft topological space.

**Definition 17** [1]. Let  $(\tilde{X}, F, E)$  be a soft topological space over  $X$ , then the member of  $\mathcal{I}$  are said to be soft open sets in  $X$ .

**Definition 18** [1]. Let  $(X, \tau, E)$  be a soft topological space over  $X$ . A soft set  $(F, A)$  over  $\tilde{X}$  is said to be a soft closed set in  $X$  if its relative complement  $(F, E)^c$  belong to  $\mathcal{I}$ .

**Definition 20** [51]. Let  $(\tilde{X}, \tau, E)$  be a soft topological space and  $(F, E) \subseteq SS(\tilde{X})_A$  then  $(F, E)$  is called  $\beta$ -open soft set if  $((F, E) \subseteq Cl(int(Cl(F, E)))$ ). The set of all  $\beta$ -open soft set is denoted by  $S\beta O(X, \tau, E)$  or  $S\beta O(\tilde{X})$  and the set of all  $\beta$ -closed soft set is denoted by  $S\beta C(\tilde{X}, \tau, E)$  or  $S\beta C(\tilde{X})$ .

**Proposition 1.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ . If  $(X, \tau, E)$  is soft  $\beta T_3$ -space, then for all  $x \in X, x_E = (x, E)$  is  $\beta$ -closed soft set.

**Proposition 2.** Let  $(Y, \tau_Y, E)$  be a soft sub space of a soft topological space  $(X, \tau, E)$  and  $(F, E) \in SS(X)$  then

1. If  $(F, E)$  is soft  $\beta$  open set in  $Y$  and  $Y \in \tau$ , then  $(F, E) \in \tau$
2.  $(F, E)$  is soft  $\beta$  open soft set in  $Y$  if and only if  $(F, E) = Y \cap (G, E)$  for some  $(G, E) \in \tau$ .
3.  $(F, E)$  is soft  $\beta$  closed soft set in  $Y$  if and only if  $(F, E) = Y \cap (H, E)$  for some  $(H, E)$  is  $\tau$  soft  $\beta$  closed.

#### 4. Soft $\beta$ -Separation Axioms in Soft Quad Topological Spaces

In this section we inaugurated soft  $\beta$  separation axioms in soft quad topological space with respect to ordinary points and discussed some results with respect to these points in detail.

**Definition 27.** Let  $(X, \tau_1, E), (X, \tau_2, E), (X, \tau_3, E)$  and  $(X, \tau_4, E)$  be four different soft topologies on  $X$ . Then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called a *soft quad topological space*. The soft four topologies  $(X, \tau_1, E), (X, \tau_2, E), (X, \tau_3, E)$  and  $(X, \tau_4, E)$  are independently satisfying the axioms of soft topology. The members of  $\tau_1$  are called  $\tau_1$  soft open set. And complement of  $\tau_1$  Soft open set is called  $\tau_1$  soft closed set. Similarly, the member of  $\tau_2$  are called  $\tau_2$  soft open sets and the complement of  $\tau_2$  soft open sets are called  $\tau_2$  soft closed set. The members of  $\tau_3$  are called  $\tau_3$  soft open set. And complement of  $\tau_3$  Soft open set is called  $\tau_3$  soft closed set and the members of  $\tau_4$  are called  $\tau_4$  soft open set. And complement of  $\tau_4$  Soft open set is called  $\tau_4$  soft closed set.

**Definition 28.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$  and  $Y$  be a non-empty subset of  $X$ . Then  $\tau_{1Y} = \{(Y_F, E): (F, E) \in \tau_1\}, \tau_{2Y} = \{(Y_G, E): (G, E) \in \tau_2\}, \tau_{3Y} = \{(Y_H, E): (H, E) \in \tau_3\}$  and  $\tau_{4Y} = \{(Y_I, E): (I, E) \in \tau_4\}$  are said to be the relative topological on  $Y$ . Then  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is called relative soft quad-topological space of  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ .

Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$ , where  $(X, \tau_1, E), (X, \tau_2, E), (X, \tau_3, E)$  and  $(X, \tau_4, E)$  be four different soft topologies on  $X$ . Then a sub set  $(F, E)$  is said to be quad-open (in short hand q-open) if  $(F, E) \subseteq \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$  and its complement is said to be soft q-closed.

#### 4.1 Soft $\beta$ -Separation Axioms of Soft Quad Topological Spaces with Respect to Ordinary Points.

In this section inauguration of soft  $\beta$  separation axioms in soft quad topological space with respect to ordinary points is launched and discussed some eye-catching results with respect to these points in detail.

**Definition 29.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$  and  $x, y \in X$  such that  $x \neq y$ . If we can find soft q-open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called soft  $qT_0$  space.

**Definition 30.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$  and  $x, y \in X$  such that  $x \neq y$ . If we can find two soft q-open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  and  $y \in (G, E)$  and  $x \notin (G, E)$  then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called soft  $qT_1$  space.

**Definition 31.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$  and  $x, y \in X$  such that  $x \neq y$ . If we can find two q- open soft sets such that  $x \in (F, E)$  and  $y \in (G, E)$  moreover  $(F, E) \cap (G, E) = \phi$ . Then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called a soft  $qT_2$  space.

**Definition 32.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft topological space  $(G, E)$  be q-closed soft set in  $X$  and  $x \in X_A$  such that  $x \notin (G, E)$ . If there occurs soft q-open sets  $(F_1, E)$  and  $(F_2, E)$  such that  $x \in (F_1, E), (G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called soft q-regular spaces. A soft q-regular  $qT_1$  Space is called soft  $qT_3$  space.

Then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called a soft  $q$ -regular spaces . A soft  $q$ -regular  $T_1$  Space is called soft  $qT_3$ space.

**Definition 33.**  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space  $(F_1, E), (G, E)$  be closed soft sets in  $X$  such that  $(F, E) \cap (G, E) = \varnothing$  If there exists  $q$ - open soft sets  $(F_1, E)$  and  $(F_2, E)$  such that  $(F, E) \subseteq (F_1, E), (G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \varnothing$ . Then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called a  $q$ -soft normal space. A soft  $q$ -normal  $qT_1$  Space is called soft  $qT_4$  Space.

**Definition 34.** Let  $(X, \tau, A)$  be a soft Topological space over  $X$  and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if there can happen at least one soft semi open set  $(F_1, A)$  or  $(F_2, A)$  such that  $e_G \in (F_1, A), e_H \notin (F_1, A)$  or  $e_H \in (F_2, A), e_G \notin ((F_2, A))$  then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called a soft  $qT_0$  space.

**Definition 35.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft Topological spaces over  $X$  and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if there can happen soft  $q$ -open sets  $(F_1, A)$  and  $(F_2, A)$  such that  $e_G \in (F_1, A), e_H \notin (F_1, A)$  and  $e_H \in (F_2, A), e_G \notin ((F_2, A))$  then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called soft  $qT_1$  space.

**Definition 36.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft Topological space over  $X$  and  $e_G, e_H \in X_A$  such that  $e_G \neq e_H$  if there can happen soft  $q$ -open sets  $(F_1, A)$  and  $(F_2, A)$  such that  $e_G \in (F_1, A)$ , and  $e_H \in (F_2, A), (F_1, A) \cap (F_2, A) = \varnothing_A$ . Then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called soft  $qT_2$  space

**Definition 37.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft topological space  $(G, E)$  be  $q$ -closed soft set in  $X$  and  $e_G \in X_A$  such that  $e_G \notin (G, E)$ . If there occurs soft  $q$ -open sets  $(F_1, E)$  and  $(F_2, E)$  such that  $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \varnothing$ . Then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is called soft  $q$ -regular spaces . A soft  $q$ -regular  $qT_1$  Space is called soft  $qT_3$ space.

**Definition 38.** In a soft quad topological space  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$

1)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_0$  space with respect to  $\tau_3 \cup \tau_4$  if for each pair of points  $x, y \in X$  such that  $x \neq y$  there exists  $\tau_1 \cup \tau_2$  soft  $\beta$ -open set  $(F, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$ -open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (G, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_0$  space with respect to  $\tau_1 \cup \tau_2$  if for each pair of points  $x, y \in X$  such that  $x \neq y$  there exists  $\tau_3 \cup \tau_4$  soft  $\beta$ open set  $(F, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$ . Soft quad topological spaces  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is said to be pair wise soft  $\beta T_0$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_0$  space with respect to  $\tau_3 \cup \tau_4$  and to  $\tau_3 \cup \tau_4$  and is soft  $\beta T_0$  space with respect to  $\tau_1 \cup \tau_2$ .

2)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$  if for each pair of points  $x, y \in X$  such that  $x \neq y$  there exists  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and to  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (G, E)$  and  $y \in (G, E)$  and  $x \notin (G, E)$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_1$  space with respect to  $\tau_1 \cup \tau_2$  if for each pair of distinct points  $x, y \in X$  such that  $x \neq y$  there exists  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and a to  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  and  $y \in (G, E)$  and  $x \notin (G, E)$  . Soft quad topological spaces  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is said to be pair wise soft

$\beta T_1$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$  and to  $\tau_3 \cup \tau_4$  is soft  $\beta T_1$  space with respect to  $\tau_1 \cup \tau_2$ .

3)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_2$  space with respect to  $\tau_1 \cup \tau_2$  if for each pair of points  $x, y \in X$  such that  $x \neq y$  there exists a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \in (G, E)$ ,  $(F, E) \cap (G, E) = \emptyset$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_2$  space with respect to  $\tau_1 \cup \tau_2$  if for each pair of points  $x, y \in X$  such that  $x \neq y$  there exists a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$ ,  $y \in (G, E)$  and  $(F, E) \cap (G, E) = \emptyset$ . The soft quad topological space  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is said to be pair wise soft  $\beta T_2$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_2$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_2$  space with respect to  $\tau_1 \cup \tau_2$ .

**Definition 39.** In a soft quad topological space  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$

1)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_3$  space with respect to a  $\tau_3 \cup \tau_4$  if  $\tau_1 \cup \tau_2$  is soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$  and for each pair of points  $x, y \in X$  such that  $x \neq y$  there exists  $\tau_1 \cup \tau_2$  Soft  $\beta$  closed set  $(G, E)$  such that  $x \notin (G, E)$ ,  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \emptyset$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_3$  space with respect to  $\tau_1 \cup \tau_2$  if  $\tau_3 \cup \tau_4$  is soft  $\beta T_1$  space with respect to  $\tau_1 \cup \tau_2$  and for each pair of points  $x, y \in X$  such that  $x \neq y$  there exists a  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(G, E)$  such that  $x \notin (G, E)$ ,  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \emptyset$ .  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is said to be pair wise soft  $\beta T_3$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_3$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_3$  space with respect to  $\tau_1 \cup \tau_2$ .

2)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_4$  space with respect to  $\tau_3 \cup \tau_4$  if  $\tau_1 \cup \tau_2$  is soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$ , there exists a  $\tau_1 \cup \tau_2$  soft  $\beta$  closed set  $(F_1, E)$  and  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \emptyset$ . Also there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_1 \cup \tau_2$   $\beta$  open set,  $(G_1, E)$  is soft  $\tau_3 \cup \tau_4$   $\beta$  open set such that  $(F_1, E) \subseteq (F_3, E)$ ,  $(F_2, E) \subseteq (G_1, E)$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_4$  space with respect to  $\tau_1 \cup \tau_2$  if  $\tau_3 \cup \tau_4$  is soft  $\beta T_1$  space with respect to  $\tau_1 \cup \tau_2$ , there exists  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(F_1, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \emptyset$ . Also there exist  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_3 \cup \tau_4$   $\beta$  open set,  $(G_1, E)$  is soft  $\tau_1 \cup \tau_2$   $\beta$  open set such that  $(F_1, E) \subseteq (F_3, E)$ ,  $(F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \emptyset$ . Thus,  $(X, \tau_1, \tau_2, E)$  is said to be pair wise soft  $\beta T_4$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_4$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_4$  space with respect to  $\tau_1 \cup \tau_2$ .

**Proposition 3.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$ . Then, if  $(X, \tau_1, \tau_2, E)$  and  $(X, \tau_3, \tau_4, E)$  are soft  $\beta T_3$  space then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is a pair wise soft  $\beta T_2$  space.

**Proof:** Suppose  $(X, \tau_1, \tau_2, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_3, \tau_4, E)$  then according to definition for  $x, y \in X$ , which distinct, by using Proposition 1,  $(Y, E)$  is soft  $\beta$  closed set in  $\tau_3 \cup \tau_4$  and  $x \notin (Y, E)$  there exists a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$ ,  $y \in (Y, E) \subseteq (G, E)$  and  $(F, E) \cap (G, E) = \emptyset$ . Hence  $\tau_1 \cup \tau_2$  is soft  $\beta T_2$  space with respect to  $\tau_3 \cup \tau_4$ . Similarly, if  $(X, \tau_3, \tau_4, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_1, \tau_2, E)$  then according to definition for  $x, y \in X, x \neq y$ , by using Theorem 2,  $(x, E)$  is  $\beta$  closed soft set in  $\tau_1 \cup \tau_2$  and  $y \notin (x, E)$  there exists a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $y \in (F, E)$ ,  $x \in (x, E) \subseteq$

$(G, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Hence  $\tau_3 \cup \tau_4$  is soft  $\beta T_2$  space. This implies that  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is a pair wise soft  $\beta T_2$  space.

**Proposition 4.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over X. if  $(X, \tau_1, \tau_2, E)$  and  $(X, \tau_3, \tau_4, E)$  are soft  $\beta T_3$  space then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is a pair wise soft  $\beta T_3$  space.

**Proof:** Suppose  $(X, \tau_1, \tau_2, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_3, \tau_4, E)$  then according to definition for  $x, y \in X, x \neq y$  there exists a  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(F, E)$  and a  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  and for each point  $x \in X$  and each  $(X, \tau_1, \tau_2, E)$   $\beta$  closed soft set  $(G_1, E)$  such that  $x \notin (G_1, E)$  there exists a  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(F_1, E)$  and  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(F_2, E)$  such that  $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Similarly, to  $(X, \tau_3, \tau_4, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_1, \tau_2, E)$ . So according to definition for  $x, y \in X, x \neq y$  there exists a  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(F, E)$  and a  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  and for each point  $x \in X$  and each  $(X, \tau_3, \tau_4, E)$   $\beta$  closed soft set  $(G_1, E)$  such that  $x \notin (G_1, E)$  there exists  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(F_1, E)$  and  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(F_2, E)$  such that  $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Hence  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_3$  space.

**Proposition 5.** If  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over X. if  $(X, \tau_1, \tau_2, E)$  and  $(X, \tau_3, \tau_4, E)$  are soft  $\beta T_4$  space then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_4$  space.

**Proof:** Suppose  $(X, \tau_1, \tau_2, E)$  is soft  $\beta T_4$  space with respect to  $(X, \tau_3, \tau_4, E)$ . So according to definition for  $x, y \in X, x \neq y$  there exist a  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(F, E)$  and a  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  each  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  closed set  $(F_1, E)$  and a  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . There exist  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $(X, \tau_3, \tau_4, E)$  and soft  $\beta$  open set  $(G_1, E)$  is soft  $(X, \tau_1, \tau_2, E)$   $\beta$  open set  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$ . Similarly,  $(X, \tau_3, \tau_4, E)$  is soft  $\beta T_4$  space with respect to  $(X, \tau_1, \tau_2, E)$  is so according to definition for  $x, y \in X, x \neq y$  there exists a  $(X, \tau_3, \tau_4, E)$  soft semi open set  $(F, E)$  and a  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  and for each  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  closed set  $(F_1, E)$  and  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . there exists soft  $\beta$  open sets  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $(X, \tau_3, \tau_4, E)$   $\beta$  open set  $(G_1, E)$  is soft  $(X, \tau_1, \tau_2, E)$   $\beta$  open set such that  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$ . Hence  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_4$  space.

**Proposition 6.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over X and Y be a non-empty subset of X. if  $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_3$  space. Then  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_3$  space.

**Proof:** First we prove that  $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_1$  space. Let  $x, y \in X, x \neq y$  if  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise space then this implies that  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft space. So there exists  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open  $(F, E)$  and  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$  now



$x \in Y$  and  $x \notin (G, E)$ . Hence  $x \in Y \cap (F, E) = (Y_F, E)$  then  $y \notin Y \cap (\alpha)$  for some  $\alpha \in E$ . this means that  $\alpha \in E$  then  $y \notin Y \cap F(\alpha)$  for some  $\alpha \in E$ .

Therefore,  $y \notin Y \cap (F, E) = (Y_F, E)$ . Now  $y \in Y$  and  $y \in (G, E)$ . Hence  $y \in Y \cap (G, E) = (G_Y, E)$  where  $(G, E) \in (X, \tau_3, \tau_4, E)$ . Consider  $x \notin (G, E)$  this means that  $\alpha \in E$  then  $x \notin Y \cap G(\alpha)$  for some  $\alpha \in E$ . There fore  $x \notin Y \cap (G, E) = (G_Y, E)$  thus  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_1$  space.

Now we prove that  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_3$  space then  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta$  regular space.

Let  $y \in Y$  and  $(G, E)$  be a soft  $\beta$  closed set in  $Y$  such that  $y \notin (G, E)$  where  $(G, E) \in (X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  then  $(G, E) = (Y, E) \cap (F, E)$  for some soft  $\beta$  closed set in  $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ . Hence  $y \notin (Y, E) \cap (F, E)$  but  $y \in (Y, E)$ , so  $y \notin (F, E)$  since  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is soft  $\beta T_3$  space  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is soft  $\beta$  regular space so there exists  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(F_1, E)$  and  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(F_2, E)$  such that  $y \in (F_1, E), (G, E) \subseteq (F_2, E), (F_1, E)(F_2, E) = \phi$

Take  $(G_1, E) = (Y, E) \cap (F_2, E)$  then  $(G_1, E), (G_2, E)$  are soft  $\beta$  open set in  $Y$  such that  
 $y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$   
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$   
 $(G_1, E) \cap (G_2, E) = \phi$

There fore  $\tau_{1Y} \cup \tau_{2Y}$  is soft  $\beta$  regular space with respect to  $\tau_{3Y} \cup \tau_{4Y}$ . Similarly, Let  $y \in Y$  and  $(G, E)$  be a soft  $\beta$  closed sub set in  $Y$  such that  $y \notin (G, E)$ , where  $(G, E) \in (X, \tau_3, \tau_4, E)$  then  $(G, E) = (Y, E) \cap (F, E)$  where  $(F, E)$  is some soft  $\beta$  closed set in  $(X, \tau_3, \tau_4, E)$ .  $y \notin (Y, E) \cap (F, E)$  But  $y \in (Y, E)$  so  $y \notin (F, E)$  since  $(X, \tau_1, \tau_2, E)$  is soft  $\beta$  regular space so there exists  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(F_1, E)$  and  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(F_2, E)$ . Such that

$y \in (F_1, E), (G, E) \subseteq (F_2, E)$   
 $(F_1, E) \cap (F_2, E) = \phi$

Take  $(G_1, E) = (Y, E) \cap (F_1, E)$   
 $(G_2, E) = (Y, E) \cap (F_2, E)$

Then  $(G_1, E)$  and  $(G_2, E)$  are soft  $\beta$  open set in  $Y$  such that

$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$   
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$

There for  $\tau_{3Y} \cup \tau_{4Y}$  is soft  $\beta$  regular space with respect  $\tau_{1Y} \cup \tau_{2Y} \Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_3$  space.

**Proposition 7.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$  and  $Y$  be a soft  $\beta$  closed sub space of  $X$ . if  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_4$  space then  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_4$  space.

**Proof:** Since  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_4$  space so this implies that  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_1$  space as proved above.

We prove  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta$  normal space.

Let  $(G_1, E), (G_2, E)$  be soft  $\beta$  closed sets in  $Y$  such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Then  $(G_1, E) = (Y, E) \cap (F_1, E)$

And  $(G_2, E) = (Y, E) \cap (F_2, E)$

For some soft  $\beta$  closed sets such that  $(F_1, E)$  is soft  $\beta$  closed set in  $\tau_1 \cup \tau_2$  soft  $\beta$  closed set  $(F_2, E)$  in  $\tau_3 \cup \tau_4$ .

And  $(F_1, E) \cap (F_2, E) = \phi$

From Proposition 2. Since,  $Y$  is soft  $\beta$  closed sub set of  $X$  then  $(G_1, E), (G_2, E)$  are soft  $\beta$  closed sets in  $X$  such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta$  normal space. So there exists soft  $\beta$  open sets  $(H_1, E)$  and  $(H_2, E)$  such that

$(H_1, E)$  is soft  $\beta$  open set in  $\tau_1 \cup \tau_2$  and  $(H_2, E)$  is soft  $\beta$  open set in  $\tau_3 \cup \tau_4$  such that

$$(G_1, E) \subseteq (H_1, E)$$

$$(G_2, E) \subseteq (H_2, E)$$

$$(H_1, E) \cap (H_2, E) = \phi$$

Since  $(G_1, E), (G_2, E) \subseteq (Y, E)$

Then  $(G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And  $[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where  $(Y, E) \cap (H_1, E)$  and  $(Y, E) \cap (H_2, E)$  are soft  $\beta$  open sets in  $Y$  there fore  $\tau_{1Y} \cup \tau_{2Y}$  is soft  $\beta$  normal space with respect to  $\tau_{3Y} \cup \tau_{4Y}$ . Similarly, let  $(G_1, E), (G_2, E)$  be soft  $\beta$  closed sub set in  $Y$  such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Then  $(G_1, E) = (Y, E) \cap (F_1, E)$

And  $(G_2, E) = (Y, E) \cap (F_2, E)$

For some soft  $\beta$  closed sets such that  $(F_1, E)$  is soft  $\beta$  closed set in  $\tau_3 \cup \tau_4$  and  $(F_2, E)$  soft  $\beta$  closed set in  $\tau_1 \cup \tau_2$  and

$$(F_1, E) \cap (F_2, E) = \phi$$

From Proposition 2. Since,  $Y$  is soft  $\beta$  closed sub set in  $X$  then  $(G_1, E), (G_2, E)$  are soft  $\beta$  closed sets in  $X$  such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta$  normal space so there exists soft  $\beta$  open sets  $(H_1, E)$  and  $(H_2, E)$

Such that  $(H_1, E)$  is soft  $\beta$  open set in  $\tau_3 \cup \tau_4$  and  $(H_2, E)$  is soft  $\beta$  open set in  $\tau_1 \cup \tau_2$  such that

$$(G_1, E) \subseteq (H_1, E)$$

$$(G_2, E) \subseteq (H_2, E)$$

$$(H_1, E) \cap (H_2, E) = \phi$$

Since  $(G_1, E), (G_2, E) \subseteq (Y, E)$

Then  $(G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And  $[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where  $(Y, E) \cap (H_1, E)$  and  $(Y, E) \cap (H_2, E)$  are soft  $\beta$  open sets in  $Y$  there fore  $\tau_{3Y} \cup \tau_{4Y}$  is soft  $\beta$  normal space with respect to  $\tau_{1Y} \cup \tau_{2Y}$

$\Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_4$  space.

## 4.2 Soft $\beta$ -Separation Axioms in Soft Quad Topological Spaces with Respect to Soft Points.

In this section, we brought out soft topological structures known as  $\beta$  separation axioms in soft quad topology with respect to soft points. With the applications of this soft  $\beta$  separation axioms different result are brought under examination.

**Definition 40.** In a soft quad topological space  $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$

1)  $\tau_1 \cup \tau_2$  said to be soft  $\beta T_0$  space with respect to  $\tau_3 \cup \tau_4$  if for each pair of distinct points  $e_G, e_H \in X_A$  there happens  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$ , Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_0$  space with respect to  $\tau_1 \cup \tau_2$  if for each pair of distinct points  $e_G, e_H \in X_A$  there happens  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$ . Soft quad topological spaces  $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$  is said to be pair wise soft  $\beta T_0$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_0$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_0$  spaces with respect to  $\tau_1 \cup \tau_2$ .

2)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$  if for each pair of distinct points  $e_G, e_H \in X_A$  there happens a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$  and  $e_H \in (G, E)$  and  $e_G \notin (G, E)$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_1$  space with respect to  $\tau_1 \cup \tau_2$  if for each pair of distinct points  $e_G, e_H \in X_A$  there exist a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$  and  $e_H \in (G, E)$  and  $e_G \notin (G, E)$ . Soft quad topological space  $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$  is said to be pair wise soft  $\beta T_1$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_1$  spaces with respect to  $\tau_1 \cup \tau_2$ .

3)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_2$  space with respect to  $\tau_3 \cup \tau_4$ , if for each pair of distinct points  $e_G, e_H \in X_A$  there happens a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (G, E)$  and  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_2$  space with respect to  $\tau_1 \cup \tau_2$  if for each pair of distinct points  $e_G, e_H \in X_A$  there happens a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_G \in (G, E)$  and  $(F, E) \cap (G, E) = \phi$ . The soft quad topological space  $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$  is said to be pair wise soft  $\beta T_2$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_2$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_2$  space with respect to  $\tau_1 \cup \tau_2$ .

**Definition 41.** In a soft quad topological space  $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$

1)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_3$  space with respect to  $\tau_3 \cup \tau_4$  if  $\tau_1 \cup \tau_2$  is soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$  and for each pair of distinct points  $e_G, e_H \in X_A$ , there exists a  $\tau_1 \cup \tau_2$   $\beta$  closed soft set  $(G, E)$  such that  $e_G \notin (G, E)$ ,  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_2, E)$  such that  $e_G \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_3$  space with respect to  $\tau_1 \cup \tau_2$  if  $\tau_3 \cup \tau_4$  is soft  $\beta T_1$  space with respect to  $\tau_1 \cup \tau_2$  and for each pair of distinct points  $e_G, e_H \in X_A$  there exists a  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(G, E)$  such that  $e_G \notin (G, E)$ ,  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_2, E)$  such that  $e_G \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ .  $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$  is said to be pair wise soft  $\beta T_3$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_3$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_3$  space with respect to  $\tau_1 \cup \tau_2$ .

2)  $\tau_1 \cup \tau_2$  is said to be soft  $\beta T_4$  space with respect to  $\tau_3 \cup \tau_4$  if  $\tau_1 \cup \tau_2$  is soft  $\beta T_1$  space with respect to  $\tau_3 \cup \tau_4$ , there exists a  $\tau_1 \cup \tau_2$  soft  $\beta$  closed set  $(F_1, E)$  and  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \emptyset$ , also, there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_1 \cup \tau_2$   $\beta$  open set,  $(G_1, E)$  is soft  $\tau_3 \cup \tau_4$   $\beta$  open set such that  $(F_1, E) \subseteq (F_3, E)$ ,  $(F_2, E) \subseteq (G_1, E)$ . Similarly,  $\tau_3 \cup \tau_4$  is said to be soft  $\beta T_4$  space with respect to  $\tau_1 \cup \tau_2$  if  $\tau_3 \cup \tau_4$  is soft  $\beta T_1$  space with respect to  $\tau_1 \cup \tau_2$  there exists  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(F_1, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \emptyset$ . Also there exists  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_3 \cup \tau_4$   $\beta$  open set,  $(G_1, E)$  is soft  $\tau_1 \cup \tau_2$   $\beta$  soft set such that  $(F_1, E) \subseteq (F_3, E)$ ,  $(F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \emptyset$ . Thus,  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is said to be pair wise soft  $\beta T_4$  space if  $\tau_1 \cup \tau_2$  is soft  $\beta T_4$  space with respect to  $\tau_3 \cup \tau_4$  and  $\tau_3 \cup \tau_4$  is soft  $\beta T_4$  space with respect to  $\tau_1 \cup \tau_2$ .

**Proposition 8.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft topological space over  $X$ .  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is soft  $\beta T_3$  space, then for all  $e_G \in X_E$   $e_G = (e_G, E)$  is soft  $\beta$ -closed set.

**Proof:** We want to prove that  $e_G$  is  $\beta$  closed soft set, which is sufficient to prove that  $e_G^c$  is  $\beta$  open soft set for all  $e_H \in \{e_G\}^c$ . Since  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is soft  $\beta T_3$  space, then there exists soft  $\beta$  sets  $(F, E)_{e_H}$  and  $(G, E)$  such that  $e_{H,E} \subseteq (F, E)_{e_H}$  and  $e_{G,E} \cap (F, E)_{e_H} = \emptyset$  and  $e_{G,E} \subseteq (G, E)$  and  $e_{H,E} \cap (G, E) = \emptyset$ . It follows that,  $\cup_{e_H \in (e_G)^c} (F, E)_{e_H} \subseteq e_{G,E}^c$ . Now, we want to prove that  $e_{G,E}^c \subseteq \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$ . Let  $\cup_{e_H \in (e_G)^c} (F, E)_{e_H} = (H, E)$ . Where  $H(e) = \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$  for all  $e \in E$ . Since  $e_{G,E}^c(e) = (e_G)^c$  for all  $e \in E$  from Definition 9, so, for all  $e_H \in \{e_G\}^c$  and  $e \in E$   $e_{G,E}^c(e) = \{e_G\}^c = \cup_{e_H \in (e_G)^c} \{e_H\} = \cup_{e_H \in (e_G)^c} (F, E)_{e_H} = H(e)$ . Thus,  $e_{G,E}^c \subseteq \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$  from Definition 2, and so,  $e_{G,E}^c = \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$ .

This means that,  $e_{G,E}^c$  is soft  $\beta$  open set for all  $e_H \in \{e_G\}^c$ . Therefore,  $e_{G,E}$  is  $\beta$  closed soft set.

**Proposition 9.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$ . Then, if  $(X, \tau_1, \tau_2, E)$  and  $(X, \tau_3, \tau_4, E)$  are soft  $\beta T_3$  space, then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is a pair wise soft  $\beta T_2$  space.

**Proof:** Suppose if  $(X, \tau_1, \tau_2, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_3, \tau_4, E)$ , then according to definition for,  $e_G \neq e_H, e_G, e_H \in X_A$ , by using Theorem 8,  $(e_H, E)$  is soft  $\beta$  closed set in  $(X, \tau_3, \tau_4, E)$  and  $e_G \notin (e_H, E)$  there exist a  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(F, E)$  and a  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$ ,  $e_H \in (y, E) \subseteq (G, E)$  and  $(F_1, E) \cap (F_2, E) = \emptyset$ . Hence,  $(X, \tau_1, \tau_2, E)$  is soft  $\beta T_2$  space with respect to  $(X, \tau_3, \tau_4, E)$ . Similarly, if  $(X, \tau_3, \tau_4, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_1, \tau_2, E)$ , then according to definition for,  $e_G \neq e_H, e_G, e_H \in X_A$ , by using Theorem 8,  $(e_G, E)$  is  $\beta$  closed soft set in  $(X, \tau_1, \tau_2, E)$  is and  $y \notin (x, E)$  there exists a  $(X, \tau_3, \tau_4, E)$  soft  $\beta$  open set  $(F, E)$  and a  $(X, \tau_1, \tau_2, E)$  soft  $\beta$  open set  $(G, E)$  such that  $e_H \in (F, E)$ ,  $e_G \in (x, E) \subseteq (G, E)$  and  $(F_1, E) \cap (F_2, E) = \emptyset$ . Hence,  $(X, \tau_3, \tau_4, E)$  is a soft  $\beta T_2$  space. Thus  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is a pair wise soft  $\beta T_2$  space.

**Proposition 10.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over  $X$ . If  $(X, \tau_1, \tau_2, E)$  and  $(X, \tau_3, \tau_4, E)$  are soft  $\beta T_3$  space then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is a pair wise soft  $\beta T_3$  space.

**Proof:** Suppose  $(X, \tau_1, \tau_2, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_3, \tau_4, E)$  then according to definition for  $e_G, e_H \in X_A, e_G \neq e_H$  there happens  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and for each point  $e_G \in X_A$  and each  $\tau_1 \cup \tau_2$   $\beta$  closed soft set  $(G_1, E)$  such that  $e_G \notin (G_1, E)$  there happens a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_2, E)$  such that  $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Similarly  $(X, \tau_3, \tau_4, E)$  is a soft  $\beta T_3$  space with respect to  $(X, \tau_1, E)$ . So according to definition for  $e_G, e_H \in X_A, e_G \neq e_H$  there exists a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $e_H \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and for each point  $e_G \in X_A$  and each  $\tau_3 \cup \tau_4$   $\beta$  closed soft set  $(G_1, E)$  such that  $e_G \notin (G_1, E)$  there exists  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_2, E)$  such that  $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ . Hence  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_3$  space.

**Proposition 11.** If  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over X.  $(X, \tau_1, \tau_2, E)$  and  $(X, \tau_3, \tau_4, E)$  are soft  $\beta T_4$  space then  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_4$  space.

**Proof:** Suppose  $((X, \tau_1, \tau_2, E))$  is soft  $\beta T_4$  space with respect to  $(X, \tau_3, \tau_4, E)$ . So according to definition for  $e_G, e_H \in X_A, e_G \neq e_H$  there happens a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  each  $\tau_1 \cup \tau_2$  soft  $\beta$  closed set  $(F_1, E)$  and a  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . There occurs  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_3 \cup \tau_4$   $\beta$  open set  $(G_1, E)$  is soft a  $\tau_1 \cup \tau_2$   $\beta$  open set  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$ . Similarly,  $\tau_3 \cup \tau_4$  is soft  $\beta T_4$  space with respect to  $\tau_1 \cup \tau_2$  so according to definition for  $e_G, e_H \in X_A, e_G \neq e_H$  there happens a  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F, E)$  and a  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  and for each  $\tau_3 \cup \tau_4$  soft  $\beta$  closed set  $(F_1, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  closed set  $(F_2, E)$  such that  $(F_1, E) \cap (F_2, E) = \phi$ . there occurs  $(F_3, E)$  and  $(G_1, E)$  such that  $(F_3, E)$  is soft  $\tau_3 \cup \tau_4$   $\beta$  open set  $(G_1, E)$  is soft  $\tau_1 \cup \tau_2$   $\beta$  open set such that  $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$  and  $(F_3, E) \cap (G_1, E) = \phi$  hence  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\beta T_4$  space.

**Proposition 12.** Let  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  be a soft quad topological space over X and Y be a non-empty subset of X. if  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_3$  space. Then  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_3$  space.

**Proof:** First we prove that  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_1$  space.

Let  $e_G, e_H \in X_A, e_G \neq e_H$  if  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft space then this implies that  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is pair wise soft  $\tau_1 \cup \tau_2$  space. So there exists  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(G, E)$  such that  $e_G \in (F, E)$  and  $e_H \notin (F, E)$  or  $e_H \in (G, E)$  and  $e_G \notin (G, E)$  now  $e_G \in Y$  and  $e_G \notin (G, E)$ . Hence  $e_G \in Y \cap (F, E) = (Y_F, E)$  then  $e_H \notin Y \cap F(\alpha)$  for some  $\alpha \in E$ . this means that  $\alpha \in E$  then  $e_H \notin Y \cap F(\alpha)$  for some  $\alpha \in E$ .

There fore,  $e_H \notin Y \cap (F, E) = (Y_F, E)$ . Now  $e_H \in Y$  and  $e_H \in (G, E)$ . Hence,  $e_H \in Y \cap (G, E) = (G_Y, E)$  where  $(G, E) \in \tau_3 \cup \tau_4$ . Consider  $x \notin (G, E)$ . this means that  $\alpha \in E$  then  $x \notin Y \cap G(\alpha)$  for some  $\alpha \in E$ . There fore  $e_G \notin Y \cap (G, E) = (G_Y, E)$  thus  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_1$  space.

Now, we prove that  $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$  is pair wise soft  $\beta T_3$  space.

Let  $e_H \in Y$  and  $(G, E)$  be soft  $\beta$  closed set in  $Y$  such that  $e_H \notin (G, E)$  where  $(G, E) \in \tau_1 \cup \tau_2$  then  $(G, E) = (Y, E) \cap (F, E)$  for some soft  $\beta$  closed set in  $\tau_1 \cup \tau_2$  hence  $e_H \notin (Y, E) \cap (F, E)$  but  $e_H \in (Y, E)$ , so  $e_H \notin (F, E)$  since  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is soft  $\beta T_3$  space  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is soft  $\beta$  regular space so there happens  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_2, E)$  such that  $e_H \in (F_1, E), (G, E) \subseteq (F_2, E), (F_1, E)(F_2, E) = \phi$

Take  $(G_1, E) = (Y, E) \cap (F_2, E)$  then  $(G_1, E), (G_2, E)$  are soft  $\beta$  open sets in  $Y$  such that  
 $e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$   
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$   
 $(G_1, E) \cap (G_2, E) = \phi$

Therefore,  $(\tau_{1Y}, \tau_{2Y})$  is soft  $\beta$  regular space with respect to  $(\tau_{3Y}, \tau_{4Y})$ . Similarly, Let  $e_H \in Y$  and  $(G, E)$  be a soft  $\beta$  closed sub set in  $Y$  such that  $e_H \notin (G, E)$ , Where  $(G, E) \in \tau_3 \cup \tau_4$  then  $(G, E) = (Y, E) \cap (F, E)$  where  $(F, E)$  is some soft  $\beta$  closed set in  $\tau_3 \cup \tau_4$ .  $e_H \notin (Y, E) \cap (F, E)$  but  $e_H \in (Y, E)$  so  $e_H \notin (F, E)$  since  $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$  is soft  $\beta$  regular space so there happens  $\tau_3 \cup \tau_4$  soft  $\beta$  open set  $(F_1, E)$  and  $\tau_1 \cup \tau_2$  soft  $\beta$  open set  $(F_2, E)$ . Such that

$e_H \in (F_1, E), (G, E) \subseteq (F_2, E)$   
 $(F_1, E) \cap (F_2, E) = \phi$

Take  $(G_1, E) = (Y, E) \cap (F_1, E)$   
 $(G_2, E) = (Y, E) \cap (F_2, E)$

Then  $(G_1, E)$  and  $(G_2, E)$  are soft  $\beta$  open set in  $Y$  such that  
 $e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$   
 $(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$ .

Therefore  $(\tau_{3Y}, \tau_{4Y})$  is soft  $\beta$  regular space.

#### 4. Conclusion

A soft set with single specific topological structure is unable to shoulder up the responsibility to construct the whole theory. So to make the theory healthy, some additional structures on soft set has to be introduced. It makes, it more springy to develop the soft topological spaces with its infinite applications. In this regard we introduce strong topological structure known as soft quad topological structure in this paper.

Topology is the supreme branch of pure mathematics which deals with mathematical structures. Freshly, many scholars have studied the soft set theory which is coined by Molodtsov [4] and carefully applied to many difficulties which contain uncertainties in our social life. Shabir and Naz [7] familiarized and deeply studied the origin of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we constantly study the behavior of soft semi separation axioms in soft quad topological spaces with respect to soft points as well as ordinary points of a soft topological space. We introduce soft  $\beta qT_0$  structure, soft  $\beta qT_1$  structure, soft  $\beta qT_2$  structure, Soft  $\beta qT_3$  and soft  $\beta qT_4$  structure with respect to soft and ordinary points. In future we will plant these structures in different results. More over defined soft  $\beta T_0$  structure

w.r.t. soft  $\beta T_1$  structure and vice versa, soft  $\beta T_1$  structure w.r.t soft  $\beta T_2$  structure and vice versa and soft  $\beta T_3$  space w.r.t soft  $\beta T_4$  and vice versa with respect to ordinary and soft points in soft quad topological spaces and studied their activities in different results with respect to ordinary and soft points . We also planted these axioms to different results. These soft semi separation axioms in quad structure would be valuable for the development of the theory of soft topology to solve complicated problems, comprising doubts in economics, engineering, medical etc. We also attractively discussed some soft transmissible properties with respect to ordinary as well as soft points. We expect that these results in this article will do help the researchers for strengthening the toolbox of soft topological structures. In the forthcoming, we spread the idea of soft  $\alpha$ - open, and soft  $b^{**}$  open sets in soft quad topological structure with respect to ordinary and soft points.

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