



## Efficient Reliability Analysis and Reliability-Based Design Optimization of Mechanical Systems by Using Latin Hypercube Sampling

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### Abstract

Considering uncertainty in engineering design is computationally more expensive than solving traditional deterministic problems. This challenge force researches to search for more efficient methods. For that purpose, in this work, the superiority of the LHS over MCS in the process of reliability analysis and RBDO of a mechanical system is investigated. Accordingly, the reliability analysis and RBDO process with both LHS and MCS is implemented separately on the tension-compression spring design problem. According to these results, in both reliability analysis and RBDO process, LHS was more stable in convergence compared to MCS. Moreover, LHS can be considered to be more efficient than MCS in RBDO of the spring problem.

## 1. INTRODUCTION

Most of engineered structures or systems include uncertainties stemming from human factors, accuracies of instruments, working environments, manufacturing and production. In case of low level of uncertainty, the deterministic design approaches are more reasonable for an efficient analysis and design optimization process [1,2]. However, the stochastic approaches, such as reliability analysis, are required to deal with the high level of uncertainty in a system [3-5]. Reliability analysis relies on examining the probability that failures occur in a state where a system is unable to perform its function as required. For that purpose, many reliability analysis methods have been developed, including sampling methods (Monte Carlo Simulation (MCS), Importance Sampling, Latin Hypercube Sampling (LHS), etc.) and structural methods (First-order reliability methods, Second-order reliability methods, etc.). Among these methods, LHS and MCS has been used for both reliability analysis and reliability-based design optimization (RBDO). Furthermore, LHS is known to be much less computationally expensive, compared with MCS [4]. In this work, it is aimed to investigate the superiority of the LHS over MCS in the process of reliability analysis and RBDO of mechanical systems. For that purpose, first, the reliability analysis with both LHS and MCS is implemented on a well-known benchmark design problem, which is the tension-compression spring design problem. Second, the RBDO of the spring problem is carried out by using both LHS and MCS. The comparison results obtained in terms of probability of failure, elapsed time and the objective minimum value are discussed for each process.

The rest of the work is organized as follow: in Section 2, the background information of the MCS and LHS are briefly introduced. In Section 3, the formulation of the design optimization problems is expressed under deterministic and stochastic case. In Section 4, how to implement the reliability analysis and the RBDO of the spring problem along with both LHS and MCS is illustrated, and also the comparison results and their discussion are given. In Section 5, a conclusion about all of these analysis and optimization efforts is drawn.

## 2. MONTE CARLO SIMULATION AND LATIN HYPERCUBE SAMPLING

MCS is a method to generate randomly a large number of values from uncertain variables with known distributions in order to reach the most probable point to be searched. MCS herein is used to find the probability of failure of a mechanical structure in respect to specified limit-state functions that determine the failure margin of the structure. The probability of failure is found approximately by using the following formula:

$$P_f = \frac{N_f}{N_s} \quad (1)$$

where,  $N_f$  denotes the number of failures of a design case in which the limit-state violations are observed.  $N_s$  represents the total number of samples being generated for the design case.

Similar to MCS, LHS is a sampling method that guarantees non-overlapping designs, and also has been effectively utilized to generate multivariate samples for given distribution types. LHS has four basic steps to simply be applied as [4];

- 1) Divide the relevant distribution for each variable into  $n$  non-overlapping intervals (usually five intervals) having equal probability.
- 2) Select randomly one value from each interval with respect to its probability density.
- 3) The desired number of random values are generated for each variable.
- 4) Associate the values of the variables with each other according to a specified criterion such as concentrating midpoints of the intervals or reducing the correlation between these variables.

## 3. DEFINITION OF DESIGN OPTIMIZATION

A generic deterministic design problem, which has been commonly used in the literature, can be defined as

$$\left. \begin{array}{l} \text{Min } f(X) \\ \text{subject to: } h_j(X) \leq 0, \quad j = 1, \dots, r \\ \quad \quad \quad g_k(X) = 0, \quad k = 1, \dots, n \\ \quad \quad \quad X_i^L \leq X \leq X_i^U, \quad i = 1, \dots, s \end{array} \right\} \quad (2)$$

where,  $f$  represents an objective function.  $X$  is the vector of deterministic variables.  $h$  and  $g$  are constraint functions.  $X_i^L$  and  $X_i^U$  are the lower and upper limits of  $i$ th design variable, respectively.

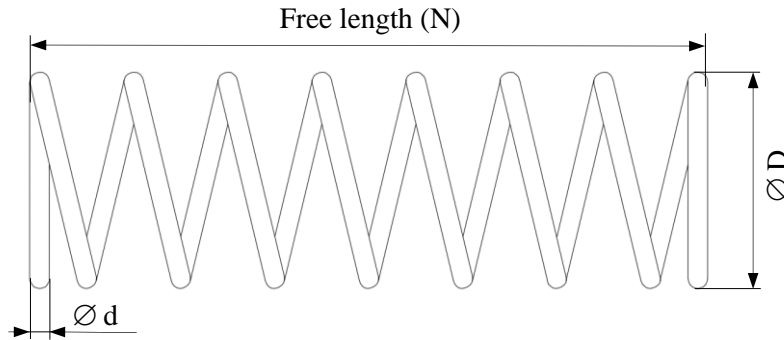
A generic definition of the RBDO can be expressed as follows [4]:

$$\left. \begin{array}{l} \text{Min } f(X, \mu_B) \\ \text{subject to: } \\ \quad \quad \quad P(L_m(X, B)) \geq R_m, \quad m = 1, \dots, t \\ \quad \quad \quad \text{or } P(L_m(X, B)) \leq P_f \\ \quad \quad \quad X_i^L \leq X \leq X_i^U, \quad i = 1, \dots, s \end{array} \right\} \quad (3)$$

where  $f$  stands for the objective function.  $B$  is the vector of random design variables or parameters.  $\mu_B$  represents the mean value of the random variable.  $L_m$  represents a limit-state function or a specified probabilistic constraint.  $R_m$  and  $P_f$  are the levels of target reliability and the allowed probability of failure, respectively.

#### 4. RELIABILITY-BASED DESIGN OPTIMIZATION OF THE SPRING PROBLEM

There are a lot of benchmark test optimization problems to assess the performance of an optimization method. In this work, the design optimization problem of a tension/compression spring, which has often been used, is considered as a benchmark design problem. The spring design problem consists of minimizing the weight of the spring depending on the constraints on minimum deflection, shear stress, surge frequency, limits on outside diameter and design variables [6]. The design variables which are the mean coil diameter ( $D$ ), the wire diameter ( $d$ ) and the number of active coils ( $N$ ), are assumed to be continuous, and the design variable vector is given as  $X = (D, d, N)$  (Figure 1).



**Figure 1.** The design variables of the spring design problem

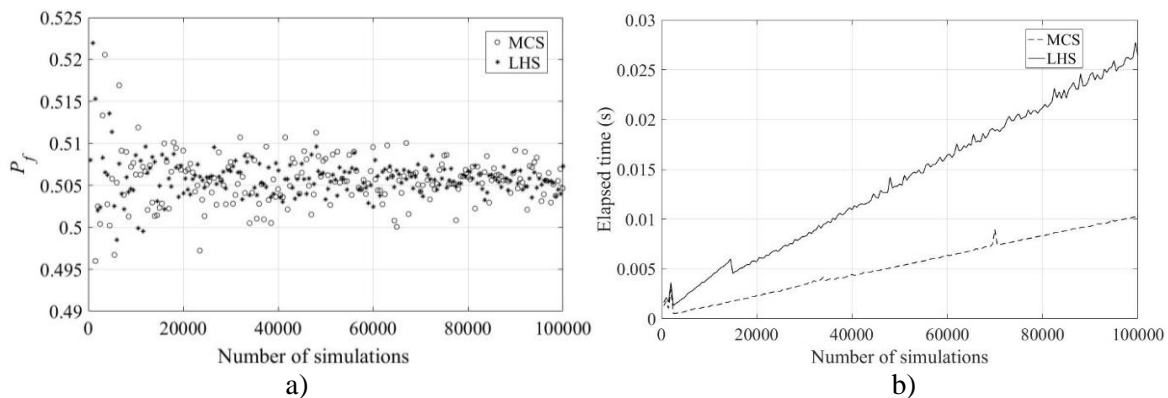
The deterministic optimization formulation for the spring problem is defined as follows [7, 8]:

$$\left. \begin{aligned}
 & \text{Minimize :} \\
 & f(X) = (N + 2)Dd^2 \\
 & \text{Subject to :} \\
 & L_1(X) = 1 - \frac{D^3 N}{(71875d^4)} \leq 0 \\
 & L_2(X) = \frac{D(4D - d)}{12566d^3(D - d)} + \frac{2.46}{12566d^2} - 1 \leq 0 \\
 & L_3(X) = 1 - \frac{140.54d}{D^2 N} \leq 0 \\
 & L_4(X) = \frac{D + d}{1.5} - 1 \leq 0 \\
 & 0.05 \leq d \leq 2, 0.25 \leq D \leq 1.3, 2 \leq N \leq 15
 \end{aligned} \right\} \quad (4)$$

There are a lot of optimum design results for that problem in the literature. One of the best optimum deterministic results for this problem are  $d=0.0517$  mm;  $D=0.357$  mm and  $N=11.287$  given in the reference [6], and this deterministic point is accepted as a starting point in searching for the best reliable design under uncertainty. Although this design points are known to be the best safe points for the spring design, because the optimum deterministic design does not account for the uncertainty in the design variables, it may not possible to conclude that this is absolutely a reliable and robust design under real conditions. To compensate for the realistic safety problem, the design variables are considered having a coefficient of variance (COV) of 0.01, and a probability of failure of 0.01 (it can be also referred to a desired reliability level of % 99) in the reliability analysis and design optimization under uncertainty. The formulation of the RBDO problem is presented by

$$\left. \begin{array}{l}
 \text{Min } f(d, D, N) \\
 \text{Subject to :} \\
 P(L_m(d, D, N)) \leq 0.01, \quad m = 1, \dots, 4 \\
 0.05 \leq d \leq 2 \\
 0.25 \leq D \leq 1.3 \\
 2 \leq N \leq 15
 \end{array} \right\} \quad (5)$$

When incorporating these uncertain variables into the process of reliability analysis, the  $P_f$  and corresponding elapsed time (s) were obtained, as shown in Figure 2a and 2b respectively. From these graphs, it can be pointed out that as the number of simulations increases, both MCS and LHS converges more to the true  $P_f$  value. More specifically, the number of simulations in LHS and MCS that they started to converge around the true or acceptable  $P_f$  value, were nearly 60000 and 100000 simulations, respectively. Accordingly, LHS was observed to be more stable in convergence compared to MCS. In other words, LHS can converge an optimum or a true point even with small number of simulations. On the other hand, the elapsed time in LHS is much more than that in MCS at the same number of simulations because LHS needs additional efforts for generation of samples subjected to a specified criterion. Despite the disadvantage of time-consuming of LHS, this method can be considered to be more efficient because the elapsed time in LHS with 60000 simulations and in MCS with 100000 simulations (nearly 0.016 s and 0.0105 s) are compared, the difference time of 0.0055 s can be trivial for today's computers.



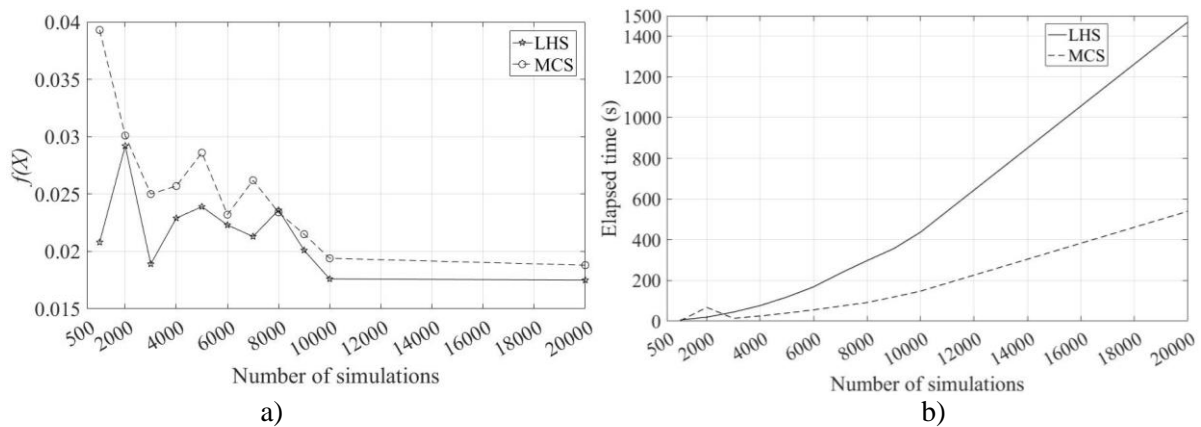
**Figure 2.** The  $P_f$  (a) and corresponding elapsed time (b) in LHS and MCS for the reliability analysis

After the reliability analysis, to clearly illustrate the performance of LHS over MCS for the RBDO process, both two methods were applied to the spring design problem under the same objective and constraint conditions given before in Eq. 5. The obtained objective values ( $f(X)$ ) and corresponding elapsed time in both two methods are presented in Table 1.

**Table 1.** The obtained objective values and corresponding elapsed time in LHS and MCS

Number of simulations	LHS		MCS	
	$f(X)$	Elapsed time (s)	$f(X)$	Elapsed time (s)
1000	0.0208	6	0.0393	2
2000	0.0292	20	0.0301	68
3000	0.0189	45	0.0250	14
4000	0.0229	77	0.0257	26
5000	0.0239	119	0.0286	40
6000	0.0223	169	0.0232	56
7000	0.0213	236	0.0262	74
8000	0.0236	298	0.0234	92
9000	0.0201	357	0.0215	119
10000	0.0176	438	0.0194	148
20000	0.0175	1470	0.0188	540

According to the RBDO process implemented, the best optimum design point, including  $D=0.7490$ ,  $d=0.0650$  and  $N=3.5381$  at  $f(X)=0.0175$ , was achieved by LHS with 20000 simulations. Similar to the performance history in the reliability analysis, LHS (converged at 10000 simulations) was more stable in convergence compared to MCS (converged over 20000 simulations) in the RBDO process, as seen in Figure 3a and 3b. In terms of the elapsed time, LHS again can be considered to be more efficient than MCS in RBDO of the spring problem because the elapsed time in LHS with 10000 simulations and in MCS with 20000 simulations are around 450 s, and at least 600 s (not converged even at 20000 simulations), respectively.

**Figure 3.** The  $P_f$  (a) and corresponding elapsed time (b) in LHS and MCS for the RBDO process

## 5. CONCLUSION AND FUTURE WORKS

In this work, the superiority of the LHS over MCS in the process of reliability analysis and RBDO of a mechanical system was investigated. Accordingly, the reliability analysis and RBDO process with both LHS and MCS is implemented separately on the tension-compression spring design problem. In both reliability analysis and RBDO process, LHS was more stable in convergence compared to MCS. Moreover, LHS can be considered to be more efficient than MCS in RBDO of the spring problem because the elapsed time in LHS with 10000 simulations and in MCS with 20000 simulations are around 450 s, and at least 600 s (not converged even at 20000 simulations), respectively. For the future work, the comparison of LHS and first-order/second-order reliability methods can be investigated towards computational efficiency and effectiveness.

## CONFLICT OF INTEREST

The author declares that there is no conflict of interest regarding the publication of this paper.

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