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Research Article

Some Results on Eigenvalue Intervals for Positive Solutions of the 3^{rd}-order Impulsive Boundary Value Problem's Iterative System with *p*-Laplacian Operator

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Abstract: The objective of this paper is to determine the eigenvalue intervals for which positive solutions are guaranteed for the iterative system of the 3^{rd}-order impulsive boundary value problem. The existence of solutions is established by applying the well-known Guo-Krasnosel'skii fixed point theorem. An illustrative example is provided to demonstrate the applicability of the theoretical results.

Keywords: Fixed point, impulsive BVP, iterative systems, eigenvalue interval.

1. Introduction

Boundary value problems (BVPs) serve as fundamental tools for modeling complex phenomena in physics, biology, and engineering. A specific subclass, impulsive boundary value problems (IBVPs), offers a robust framework for analyzing systems subject to sudden, discontinuous changes. Foundational contributions by Lakshmikantham [12], Bainov [5] and Simeonov [4] established the core theory of impulsive differential equations, extending to higher-order systems. Subsequently, advanced mathematical techniques-such as fixed point theorems and variational methods-have been employed to address more complex formulations [3, 12]. In addition to their theoretical strength, IBVPs are widely applicable across various scientific and engineering domains. For instance, in mechanical engineering, they are used to analyze structural vibrations under sudden loads, such as during seismic events affecting bridges [17]. In biomedical modeling, they support the optimization of oscillatory behavior in drug delivery systems [30]. In control theory, they aid in investigating the controllability of impulsive dynamic systems [1]. Furthermore, they find meaningful applications in population dynamics [26] and financial market modeling [27]. Moreover, the study by Zhang et al. [33] in the references demonstrates significant potential for applications in autonomous robot

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swarms, drone fleets, distributed sensor networks, and intelligent transportation systems.

Determining eigenvalue intervals is crucial for analyzing the existence and uniqueness of solutions to boundary value problems, as eigenvalues characterize the spectral properties of differential operators and thus dictate the system's stability and behavior [16]. These intervals reveal the parameter values (typically denoted by λ) at which nontrivial solutions arise, which is vital for various applications-including vibration analysis in mechanical systems to determine resonance frequencies in beams [34], estimation of energy levels in quantum mechanics via Schrödinger equations [8]. Recent studies have also emphasized their significance in fractional and nonlinear BVPs, thereby enhancing system design and optimization across disciplines [9].

Although extensive research has been conducted on third-order impulsive boundary value problems (BVPs) [6, 10, 11] and iterative systems [13, 18, 19, 22, 24], the literature still lacks focused investigations on eigenvalue intervals for third-order impulsive systems with iterative structures. Addressing this gap, the present study is, to the best of our knowledge, the first to explore eigenvalue intervals in this specific context. By employing the Guo–Krasnosel'skii fixed point theorem [12], we establish the existence of positive solutions and identify the corresponding eigenvalue intervals.

Compared to previous studies, this work significantly advances the field by unifying third-order dynamics, impulsive effects, iterative structures, and eigenvalue intervals into a single framework. In contrast to earlier studies-for instance, Zhang and Yao [31], who investigated solution multiplicity for second-order p-Laplacian impulsive equations using variational methods, or Oz and Karaca [19], who examined eigenvalue intervals for second-order m-point impulsive BVPs via fixed-point theory-our study focuses on third-order systems. Likewise, although Zhang and Ao [32] studied some third-order BVPs with eigenparameter-dependent boundary conditions on specific time scales, they did not consider iterative systems. Other works, such as those by Bi and Liu [6], Feliz and Rui [10], primarily addressed the existence of solutions, without investigating the role of eigenvalue intervals. In 2022, Bouabdallah et al. [7] studied eigenvalue boundary value problem with impulsive conditions, but the problem they considered is neither of third order nor does it involve an iterative system. Therefore, our study not only fills a significant gap in the existing literature but also provides a novel and comprehensive perspective for future research on complex impulsive systems. Based on the above-mentioned results and the importance of theoretical solutions to contribute to the application areas, in this work, we handle the following nonlinear

3^{rd}-order with p-Laplacian impulsive boundary value problem (IBVP)'s iterative system:

$$\begin{cases} (\phi_p(\kappa_i''(t)))' + \mu_i q_i(t) h_i(\kappa_{i+1}(t)) = 0, \ t \in I = [0,1], \ t \neq t_m, i \in \{1,2,3,\cdots,n\} \\ \kappa_{n+1}(t) = \kappa_1(t) \\ \triangle \kappa_i|_{t=t_m} = \mu_i I_{im}(\kappa_{i+1}(t_m)), \ m \in \{1,2,\cdots...,k\} \\ \triangle \kappa_i'|_{t=t_m} = -\mu_i J_{im}(\kappa_{i+1}(t_m)), \ m \in \{1,2,\cdots...,k\} \\ a_1\kappa_i(0) - a_2\kappa_i'(0) = 0 \\ a_3\kappa_i(1) + a_4\kappa_i'(1) = 0 \\ \kappa_i''(0) = 0, \end{cases} \tag{1}$$

where $t \neq t_m$, $m \in \{1,2,3,\cdots\ldots,k\}$ such that $0 < t_1 < t_2 < \cdots\ldots < t_k < 1$. Furthermore, for $i \in \{1,2,3,\cdots\ldots,n\}$, the functions $\triangle\kappa_i$ and $\triangle\kappa_i'$ at the point $t = t_m$ stand for the jump of $\kappa_i(t)$ and $\kappa_i'(t)$ at the point $t = t_m$, i.e.,

$$\triangle\kappa_i|_{t=t_m} = \kappa_i(t_m^+) - \kappa_i(t_m^-), \quad \triangle\kappa_i'|_{t=t_m} = \kappa_i'(t_m^+) - \kappa_i'(t_m^-),$$

where the values $\kappa_i(t_m^+)$, $\kappa_i'(t_m^+)$ state the right-hand limit of $\kappa_i(t)$ and $\kappa_i'(t)$ at the point $t = t_m$, $m \in \{1,2,3,\cdots,k\}$, and similarly $\kappa_i(t_m^-)$, $\kappa_i'(t_m^-)$ state left-hand limit of $\kappa_i(t)$ and $\kappa_i'(t)$ at the point $t = t_m$, $m \in \{1,2,3,\cdots,k\}$. In addition, the function $\phi_p(s)$ is a p-Laplacian operator, i.e., $\phi_p(s) = |s|^{p-2}s$ for $p > 1$.

In this paper, we assume that the following conditions are given:

$(C1)$ a_1, a_2, a_3, a_4 are positive real constants.

$(C2)$ For $i = 1,\cdots,n$, h_i is a continuous function from the set $\mathbb{R}^+$ to $\mathbb{R}^+$.

$(C3)$ For $i \in \{1,2,3,\cdots,n\}$, $q_i \in C(I,\mathbb{R}^+)$ and on any closed subinterval of I, q_i does not vanish identically.

$(C4)$ For $i \in \{1,2,3,\cdots,n\}$, $I_{im} \in C(\mathbb{R},\mathbb{R}^+)$ and $J_{im} \in C(\mathbb{R},\mathbb{R}^+)$ are bounded functions and the inequality $[a_4 + a_3(1-t_m)]J_{im}(\eta) > a_3 I_{im}(\eta)$, $t < t_m$, $m \in \{1,2,3,\cdots,k\}$ is satisfied, where η be any nonnegative number.

$(C5)$ Each of the following expressions is a positive real number:

$$h_i^0 = \lim_{\kappa\to 0^+} \frac{h_i(\kappa)}{\kappa^{p-1}}, \quad I_{im}^0 = \lim_{\kappa\to 0^+} \frac{I_{im}(\kappa)}{\kappa}.$$

$$J_{im}^0 = \lim_{\kappa\to 0^+} \frac{J_{im}(\kappa)}{\kappa}, \text{ and } h_i^\infty = \lim_{\kappa\to\infty} \frac{h_i(\kappa)}{\kappa^{p-1}}, \ i \in \{1,2,3,...,n\},$$

where positive solutions of the nonlinear 3^{rd}-order IBVP (1)'s iterative system with p-Laplacian exist for μ_i, $i \in \{1,2,3,\cdots,n\}$.

The primary structure of this manuscript unfolds as follows. Section 2 introduces several definitions and fundamental lemmas, which serve as key tools for establishing our main result. Section 3 determines the eigenvalue intervals that ensure the existence of positive solutions in the 3^{rd}-order IBVP (1)'s iterative system with the p-Laplacian operator. Section 4 provides an illustrative example to demonstrate the applicability of the main results.

2. Preliminaries

In this section, we introduce fundamental definitions in Banach spaces and supply several supplementary lemmas that will be utilized later.

Define $I' = I \backslash \{t_1, t_2, \cdots, t_k\}$. The space $C(I)$ denotes the Banach space of all continuous mappings $\kappa : I \to \mathbb{R}$ equipped with the norm $\|\kappa\| = \sup_{t\in I} |\kappa(t)|$. The space $PC(I)$ consists of functions $\kappa : I \to \mathbb{R}$ such that $\kappa \in C(I')$, $\kappa(t_m^+)$ and $\kappa(t_m^-)$ exist and $\kappa(t_m^-) = \kappa(t_m)$ for $m \in \{1,2,\cdots,k\}$. $PC(I)$ is also a Banach space with the norm $\|\kappa\|_{PC} = \sup_{t\in I} |\kappa(t)|$. Additionally, The space $C^2(I')$ consists of all twice continuously differentiable functions defined on an interval I' to $\mathbb{R}$.

Let $\mathbb{B} = PC(I) \cap C^2(I')$. A function $(\kappa_1, \cdots, \kappa_n) \in \mathbb{B}^n$ is considered a solution of the 3^{rd}-order IBVP (1)'s iterative system if it satisfies the conditions of the 3^{rd}-order IBVP (1)'s iterative system.

We first consider the case $i = 1$ in the 3^{rd}-order IBVP (1). Accordingly, the solution κ_1 of the 3^{rd}-order IBVP (2) is obtained. Once κ_1 is determined, we proceed to compute κ_n. Continuing in this manner, we successively determine $\kappa_{n-1}, \kappa_{n-2}, \ldots,$ until we reach κ_2. In this way, the complete solution $(\kappa_1, \ldots, \kappa_n)$ of the iterative system associated with the 3^{rd}-order IBVP (1) is constructed.

Assume that $x(t) \in C(I)$, then we deal with the following 3^{rd}-order IBVP:

$$\begin{cases} (\phi_p(\kappa_1''(t)))' + x(t) = 0, \quad t \in I = [0,1], \ t \neq t_m \\ \triangle \kappa_1|_{t=t_m} = \mu_1 I_{1m}(\kappa_2(t_m)), \ m \in \{1,2,\cdots,k\} \\ \triangle \kappa_1'|_{t=t_m} = -\mu_1 J_{1m}(\kappa_2(t_m)), \ m \in \{1,2,\cdots,k\} \\ a_1\kappa_1(0) - a_2\kappa_1'(0) = 0 \\ a_3\kappa_1(1) + a_4\kappa_1'(1) = 0 \\ \kappa_1''(0) = 0. \end{cases} \tag{2}$$

The following homogeneous equation's solutions are specified via τ and η.

$$\phi_p(\kappa_i''(t))' = 0,\ t \in I \tag{3}$$

under the initial conditions

$$\begin{cases} \tau(0) = a_2, & \tau'(0) = a_1 \\ \eta(1) = a_4, & \eta'(1) = -a_3. \end{cases} \tag{4}$$

Using the initial conditions (4), we can deduce from (3) for τ and η the following equations:

$$\tau(t) = a_2 + a_1 t,\ \textit{and}\ \eta(t) = a_4 + a_3(1-t). \tag{5}$$

Set

$$\delta := a_1a_4 + a_1a_3 + a_2a_3. \tag{6}$$

Lemma 2.1 *Assume that the conditions (C1)-(C5) are satisfied. If κ_1, which is belonging to set $\mathbb{B}$, is a solution of the following equation*

$$\kappa_1(t) = \int_0^1 \mathcal{G}(t,s)\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds + \sum_{m=1}^{k} H_{1m}(t,t_m), \tag{7}$$

where

$$\mathcal{G}(t,s) = \frac{1}{\delta}\begin{cases} (a_2 + a_1 s)[a_4 + a_3(1-t)], & s \le t \\ (a_2 + a_1 t)[a_4 + a_3(1-s)], & t \le s \end{cases} \tag{8}$$

and

$$H_{1m}(t,t_m) = \frac{1}{\delta}\begin{cases} (a_2 + a_1 t)[-a_3\mu_1 I_{1m}(\kappa_2(t_m)) + (a_4 + a_3(1-t_m))\mu_1 J_{1m}(\kappa_2(t_m))], & t < t_m \\ (a_4 + a_3(1-t))[a_1\mu_1 I_{1m}(\kappa_2(t_m)) + (a_2 + a_1 t_m)\mu_1 J_{1m}(\kappa_2(t_m))], & t_m \le t, \end{cases} \tag{9}$$

then κ_1 is a solution of the 3^{rd}-order IBVP (2).

Proof Let κ_1 satisfy (7), then we will show that y is a solution of the IBVP (2). Because κ_1 satisfies (7), then we obtain

$$\kappa_1(t) = \int_0^1 \mathcal{G}(t,s)\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds + \sum_{m=1}^{k} H_{1m}(t,t_m),$$

i.e.,

$$\begin{aligned}\kappa_1(t) \quad = \quad & \frac{1}{\delta}\int_0^t (a_2 + a_1 s)[a_4 + a_3(1-t)]\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds \\ & +\frac{1}{\delta}\int_t^1 (a_2 + a_1 t)[a_4 + a_3(1-s)]\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds \\ & +\frac{1}{\delta}\sum_{0<t_m<t}(a_4 + a_3(1-t))[a_1\mu_1 I_{1m}(\kappa_2(t_m)) + (a_2 + a_1 t_m)\mu_1 J_{1m}(\kappa_2(t_m))] \\ & +\frac{1}{\delta}\sum_{t<t_m<1}(a_2 + a_1 t)[-a_3\mu_1 I_{1m}(\kappa_2(t_m)) + (a_4 + a_3(1-t_m))\mu_1 J_{1m}(\kappa_2(t_m))],\end{aligned}$$

$$\begin{aligned}\kappa_1'(t) \quad = \quad & \frac{1}{\delta}\int_0^t (-a_3)(a_2 + a_1 s)\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds \\ & +\frac{1}{\delta}\int_t^1 (a_1)[a_4 + a_3(1-s)]\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds \\ & +\frac{1}{\delta}\sum_{0<t_m<t}(-a_3)[a_1\mu_1 I_{1m}(\kappa_2(t_m)) + (a_2 + a_1 t_m)\mu_1 J_{1m}(\kappa_2(t_m))] \\ & +\frac{1}{\delta}\sum_{t<t_m<1}(a_1)[-a_3\mu_1 I_{1m}(\kappa_2(t_m)) + (a_4 + a_3(1-t_m))\mu_1 J_{1m}(\kappa_2(t_m))].\end{aligned}$$

Thus,

$$\begin{aligned}\kappa_1''(t) \quad & = \frac{1}{\delta}\left[-a_3(a_2 + a_1 t) - a_1(a_4 + a_3(1-t))\right]\phi_p^{-1}\left(\int_0^t x(\omega)d\omega\right) \\ & = -\phi_p^{-1}\left(\int_0^t x(\omega)d\omega\right)\end{aligned}$$

and

$$\kappa_1''(0) = 0.$$

So that

$$(\phi_p(\kappa_1''(t)))' = -x(t),$$

i.e.,

$$(\phi_p(\kappa_1''(t)))' + x(t) = 0.$$

Since

$$\kappa_1(0) = \frac{1}{\delta}\int_0^1 a_2[a_4 + a_3(1-s)]\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds$$

$$+\frac{1}{\delta}\sum_{m=1}^{k} a_2[-a_3\mu_1 I_{1m}(\kappa_2(t_m)) + (a_4 + a_3(1-t_m))\mu_1 J_{1m}(\kappa_2(t_m))]$$

and

$$\kappa_1'(0) = \frac{1}{\delta}\int_0^1 (a_1)[a_4 + a_3(1-s)]\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds$$

$$+\frac{1}{\delta}\sum_{m=1}^{k} a_1[-a_3\mu_1 I_{1m}(\kappa_2(t_m)) + (a_4 + a_3(1-t_m))\mu_1 J_{1m}(\kappa_2(t_m))],$$

we get

$$a_1\kappa_1(0) - a_2\kappa_1'(0) = 0.$$

Since

$$\kappa_1(1) = \frac{1}{\delta}\int_0^1 (a_2 + a_1 s)(a_3 + a_4)\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds$$

$$+\frac{1}{\delta}\sum_{m=1}^{k} (a_3 + a_4)[a_1\mu_1 I_{1m}(\kappa_2(t_m)) + (a_2 + a_1 t_m)\mu_1 J_{1m}(\kappa_2(t_m))]$$

and

$$\kappa_1'(1) = \frac{1}{\delta}\int_0^1 (-a_3)(a_2 + a_1 s)\phi_p^{-1}\left(\int_0^s x(\omega)d\omega\right)ds$$

$$+\frac{1}{\delta}\sum_{m=1}^{k} (-a_3)[a_1\mu_1 I_{1m}(\kappa_2(t_m)) + (a_2 + a_1 t_m)\mu_1 J_{1m}(\kappa_2(t_m))],$$

we have

$$a_3\kappa_1(1) + a_4\kappa'(1) = 0.$$

□

Lemma 2.2 *Let (C1)-(C5) hold. For* $\kappa_1 \in \mathbb{B}$ *with* $x(t) \geq 0$ *for* $t \in I$*, the solution* κ_1 *of the* 3^{rd}*-order IBVP (2) satisfies, for* $t \in I$*,* $\kappa_1(t) \geq 0$.

Proof Initially, for $t, s \in I \times I$, it is apparent from the description that Green's function $\mathcal{G}$ is positive. In addition, since the functions I_{1m} and J_{1m} are positive, we have the positivity of H_{1m}. Consequently, for $t \in I$, $\kappa_1(t)$ is positive. □

Lemma 2.3 *[13] Assume that (C1)-(C5) are satisfied. For $t \in I$, the 3^{rd}-order IBVP (2)'s solution, i.e., $\kappa_1 \in \mathbb{B}$ satisfy the inequality $\kappa_1'(t) \geq 0$.*

Lemma 2.4 *Suppose that the conditions (C1)-(C5) are satisfied. Therefore, for any $t, s \in I$, we get the following inequality*

$$\mathcal{G}(s,s) \geq \mathcal{G}(t,s) \geq 0, \tag{10}$$

where the function $\mathcal{G}(t,s)$ defined as in (8).

Proof The claimed inequality can be easily obtained from (8). □

Lemma 2.5 *[13] Assume that the conditions (C1)-(C5) are fulfilled. Let $\sigma \in (0, \frac{1}{2})$. Therefore, for any $t, s \in I$, we get*

$$\mathcal{G}(s,s) \leq \frac{1}{\gamma}\mathcal{G}(t,s), \tag{11}$$

where $\gamma := \min\left\{\frac{a_2 + a_1\sigma}{a_2 + a_1}, \frac{a_4 + a_3\sigma}{a_4 + a_3}\right\}$.

The set $\mathcal{P}$ defined as $\mathcal{P} = \{\kappa_1 \in PC(I) : \kappa_1(t)$ is nonnegative, nondecreasing and concave on $I\}$ is a cone of the set $PC(I)$.

Lemma 2.6 *Assume that the conditions (C1)-(C5) are satisfied and $\kappa_1(t) \in \mathcal{P}$. Then, the following inequality is satisfied,*

$$\min_{t \in [\sigma, 1-\sigma]} \kappa_1(t) \geq \sigma \|\kappa_1\|_{PC}, \tag{12}$$

where $\sigma \in (0, \frac{1}{2})$ and $\|\kappa_1\|_{PC} = \sup_{t \in I} |\kappa_1(t)|$.

Proof Since κ_1 is an element of $\mathcal{P}$, we can say that $\kappa_1(t)$ is concave on I. As a consequence of this, $\|\kappa_1\|_{PC} = \sup_{t \in I} |\kappa_1(t)| = \kappa_1(1)$ and $\min_{t \in [\sigma, 1-\sigma]} \kappa_1(t) = \kappa_1(\sigma)$. As κ_1's graph is concave downward on the interval I, we achieve

$$\frac{\kappa_1(1) - \kappa_1(0)}{1 - 0} \leq \frac{\kappa_1(\sigma) - \kappa_1(0)}{\sigma - 0},$$

i.e., $\kappa_1(\sigma) \geq \sigma\kappa_1(1) + (1-\sigma)\kappa_1(0)$. Thus, $\kappa_1(\sigma) \geq \sigma\kappa_1(1)$. □

If and only if

$$\kappa_1(t) = \int_0^1 \mathcal{G}(t,s_1)\,\phi_p^{-1}\Bigg(\mu_1 \int_0^{s_1} q_1(\omega_1)\,h_1\Bigg(\cdots h_{n-1}\Bigg(\int_0^{s_n} \mathcal{G}(\omega_{n-1},s_n)\,\phi_p^{-1}\Bigg(\mu_n \int_0^{s_n} q_n(\omega_n)\,h_n\big(\kappa_1(\omega_n)\big)\,d\omega_n\Bigg)ds_n$$

$$+\sum_{m=1}^{k} H_{nm}(\omega_{n-1},t_m)\Bigg)\cdots\Bigg)d\omega_1\Bigg)ds_1 + \sum_{m=1}^{k} H_{1m}(t,t_m)$$

where for $i = 1,2,\cdots,n$

$$\kappa_i(t) = \int_0^1 \mathcal{G}(t,s)\phi_p^{-1}\left(\mu_i \int_0^s q_i(\omega)h_i(\kappa_{i+1}(\omega))d\omega\right)ds + \sum_{m=1}^{k} H_{im}(t,t_m),\ \ t \in I,$$

$$\kappa_{n+1}(t) = \kappa_1(t),$$

$$H_{im}(t,t_m) = \frac{1}{\delta}\begin{cases}(a_2+a_1t)[-a_3\mu_i I_{im}(\kappa_{i+1}(t_m)) + (a_4+a_3(1-t_m))\mu_i J_{im}(\kappa_{i+1}(t_m))], & t < t_m\\ (a_4+a_3(1-t))[a_1\mu_i I_{im}(\kappa_{i+1}(t_m)) + (a_2+a_1t_m)\mu_1 J_{im}(\kappa_{i+1}(t_m))], & t_m \leq t.\end{cases}$$

We state that an n-tuple $(\kappa_1(t),\kappa_2(t),\cdots,\kappa_n(t))$ is a solution of the 3^{rd}-order IBVP (1)'s iterative system. We will employ a fixed point theorem called Guo-Krasnosel'skii [12] to determine the eigenvalue intervals wherein the 3^{rd}-order IBVP (1)'s iterative system possesses at least one positive solution within a cone.

Theorem 2.7 *[12] Let X denote a Banach space and $P \subset X$ be a cone within X. Suppose Ω_1 and Ω_2 are two bounded open subsets of X such that $0 \in \Omega_1$ and $\bar{\Omega}_1 \subset \Omega_2$. Consider $A : P \cap (\bar{\Omega}_2 \setminus \Omega_1) \to P$ as a completely continuous operator, satisfying either of the following conditions:*

i. For all $x \in P \cap \partial\Omega_1$, $\|Ax\| \leq \|x\|$, and for all $x \in P \cap \partial\Omega_2$, $\|Ax\| \geq \|x\|$,

ii. For all $x \in P \cap \partial\Omega_1$, $\|Ax\| \geq \|x\|$, and for all $x \in P \cap \partial\Omega_2$, $\|Ax\| \leq \|x\|$.

Under these conditions, the operator A possesses at least one fixed point in $P \cap (\bar{\Omega}_2 \setminus \Omega_1)$.

3. Main Result

In this section, we establish the conditions necessary to identify the eigenvalues for which the iterative system associated with the third-order impulsive boundary value problem (2) has at least one positive solution in a cone. Then, we define an integral operator $T : \mathcal{P} \to \mathbb{B}$ for $\kappa_1 \in \mathcal{P}$, where

$$T\kappa_1(t)=\int_0^1 \mathcal{G}(t,s_1)\,\phi_p^{-1}\Bigg(\mu_1\int_0^{s_1} q_1(\omega_1)\,h_1\Bigg(\cdots h_{n-1}\Bigg(\int_0^{s_n}\mathcal{G}(\omega_{n-1},s_n)\,\phi_p^{-1}\Bigg(\mu_n\int_0^{s_n} q_n(\omega_n)\,h_n(\kappa_1(\omega_n))\,d\omega_n\Bigg)ds_n$$
$$+\sum_{m=1}^{k} H_{nm}(\omega_{n-1},t_m)\Bigg)\cdots\Bigg)d\omega_1\Bigg)ds_1+\sum_{m=1}^{k} H_{1m}(t,t_m), \tag{13}$$

thereby setting the foundation for analyzing the behavior of the solutions within this framework.

From conditions (C1)–(C5), Lemmas 2.2 and 2.3, and the definition of T, it follows that for $\kappa_1\in\mathcal{P}$, the following hold: $T\kappa_1(t)\geq 0$, $(T\kappa_1)'(t)\geq 0$, and $(T\kappa_1)'(t)$ is concave on I. Therefore, $T(\mathcal{P})\subset\mathcal{P}$. Moreover, one can show that the operator T is completely continuous by applying the Arzelà–Ascoli Theorem.

We now explore the relevant fixed points of T within the cone $\mathcal{P}$. For convenience, we introduce the following notation. Let

$$N_1 := \max_{1\leq i\leq n}\left\{\left[\phi_p\left(\gamma\sigma\int_\sigma^{1-\sigma}\mathcal{G}(s,s)\left(\int_0^s q_i(\omega)d\omega\right)ds\right)h_i^\infty\right]^{-1}\right\}$$

and

$$N_2=\min_{1\leq i\leq n}\left\{\left[\mu_i^{\frac{2-p}{p-1}}\left(\int_0^1\mathcal{G}(s,s)\left(\int_0^s q_i(\omega)d\omega\right)ds+\frac{k}{\delta}(2a_1+a_2)(a_3+a_4)\right)\cdot\left(\max\{\phi_p^{-1}(h_i^0),I_{im}^0,J_{im}^0\}\right)\right]^{-1}\right\}.$$

Theorem 3.1 *Suppose that the conditions (C1)-(C5) are met. Therefore, for each $\mu_1,\mu_2,\cdots,\mu_n$ satisfying*

$$N_1<\mu_i<N_2,\ \ i=1,2,\cdots,n \tag{14}$$

an n-tuple $(\kappa_1,\kappa_2,\cdots,\kappa_n)$ exists, satisfying (1), with each $\kappa_i(t)>0$ for $i\in\{1,2,3,...,n\}$ on I.

Proof Assume μ_r, for $1\leq r\leq n$, be as defined in (14). Choose $\varepsilon>0$ such that

$$\max_{1\leq i\leq n}\left\{\left[\phi_p\left(\gamma\sigma\int_\sigma^{1-\sigma}\mathcal{G}(s,s)\phi_p^{-1}\left(\int_0^s q_i(\omega)d\omega\right)ds\right)(h_i^\infty-\varepsilon)\right]^{-1}\right\}\leq\min_{1\leq r\leq n}\mu_r$$

and

$$\max_{1\leq r\leq n}\mu_r\leq\min_{1\leq i\leq n}\left\{\left[\left(\mu_i^{\frac{2-p}{p-1}}\int_0^1\mathcal{G}(s,s)\phi_p^{-1}\left(\int_0^s q_i(\omega)d\omega\right)ds+\frac{k}{\delta}(2a_1+a_2)(a_3+a_4)\right)\right.\right.$$
$$\left.\left.\cdot\left(\max\{\phi_p^{-1}(h_i^0+\varepsilon),I_{im}^0+\varepsilon,J_{im}^0+\varepsilon\}\right)\right]^{-1}\right\}.$$

We investigate the fixed points of the completely continuous operator $T:\mathcal{P}\to\mathcal{P}$, as defined in (13). Utilizing the definitions of $h_i^0, I_{im}^0, J_{im}^0$, there exists a constant $K_1>0$ such that, for each $i\in\{1,2,3,\cdots,n\}$ and $1\le m\le k$,

$$h_i(\kappa)\le (h_i^0+\varepsilon)\kappa^{p-1},\quad I_{im}(\kappa)\le (I_{im}^0+\varepsilon)\kappa,\quad J_{im}(\kappa)\le (J_{im}^0+\varepsilon)\kappa,\quad 0<\kappa<K_1.$$

Suppose that $\kappa_1\in\mathcal{P}$ with $\|\kappa_1\|=K_1$. We begin by verifying that $\kappa_n\le K_1$ holds in the case when $i=n$. For $0\le s_{n-1}\le 1$, by applying Lemma 2.4 and the choice of ε, we obtain

$$\begin{aligned}
&\int_0^1 \mathcal{G}(s_{n-1},s_n)\phi_p^{-1}\left(\mu_n\int_0^{s_n} q_n(\omega_n)h_n(\kappa_1(\omega_n))d\omega_n\right)ds_n+\sum_{m=1}^{k}H_{nm}(s_{n-1},t_m)\\
&\le \mu_n\Bigg[\left(\mu_n^{\frac{2-p}{p-1}}\int_0^1 \mathcal{G}(s_n,s_n)\phi_p^{-1}\left(\int_0^{s_n} q_n(\omega_n)d\omega_n\right)ds_n+\frac{k}{\delta}(2a+b)(c+d)\right)\\
&\qquad\cdot\left(\max\{\phi_p^{-1}(h_n^0+\varepsilon),I_{nm}^0+\varepsilon,J_{nm}^0+\varepsilon\}\right)\Bigg]\|\kappa_1\|\\
&\le K_1.
\end{aligned}$$

Proceeding with the case $i=n-1$, we now demonstrate that κ_{n-1} is also less than K_1. This pattern persists with Lemma 2.4, where, for $0\le s_{n-2}\le 1$, it holds that

$$\begin{aligned}
&\int_0^1 \mathcal{G}(s_{n-2},s_{n-1})\,\phi_p^{-1}\Bigg(\mu_{n-1}\int_0^{s_{n-1}} q_{n-1}(\omega_{n-1})\,h_{n-1}\Bigg(\int_0^1 \mathcal{G}(\omega_{n-1},s_n)\,\phi_p^{-1}\left(\mu_n\int_0^{s_n} q_n(\omega_n)\,h_n(\kappa_1(\omega_n))\,d\omega_n\right)ds_n\\
&\quad+\sum_{m=1}^{k}H_{nm}(\omega_{n-1},t_m)\Bigg)d\omega_{n-1}\Bigg)ds_{n-1}+\sum_{m=1}^{k}H_{n-1,m}(s_{n-2},t_m)\\
&\le \mu_{n-1}\Bigg[\left(\mu_{n-1}^{\frac{2-p}{p-1}}\int_0^1 \mathcal{G}(s_{n-1},s_{n-1})\,\phi_p^{-1}\left(\int_0^{s_{n-1}} q_{n-1}(\omega_{n-1})\,d\omega_{n-1}\right)ds_{n-1}+\frac{k}{\delta}(2a_1+a_2)(a_3+a_4)\right)\\
&\qquad\cdot\left(\max\{\phi_p^{-1}(h_{n-1}^0+\varepsilon),\,I_{n-1,m}^0+\varepsilon,\,J_{n-1,m}^0+\varepsilon\}\right)\Bigg]\|\kappa_1\|\\
&\le \|\kappa_1\|=K_1.
\end{aligned}$$

Proceeding with this argument, we obtain

$$\begin{aligned}
&\int_0^1 \mathcal{G}(t,s_1)\phi_p^{-1}\left(\mu_1\int_0^{s_1} q_1(\omega_1)h_1(\mu_2\cdots)d\omega_1\right)ds_1+\sum_{m=1}^{k}H_{1m}(t,t_m)\\
&\le \mu_1\Bigg[\left(\mu_1^{\frac{2-p}{p-1}}\int_0^1 \mathcal{G}(s_1,s_1)\phi_p^{-1}\left(\int_0^{s_1} q_1(\omega_1)d\omega_1\right)ds_1+\frac{k}{\delta}(2a_1+a_2)(a_3+a_4)\right)\\
&\qquad\cdot\left(\max\{\phi_p^{-1}(h_1^0+\varepsilon),I_{1m}^0+\varepsilon,J_{1m}^0+\varepsilon\}\right)\Bigg]K_1\\
&\le K_1=\|\kappa_1\|.
\end{aligned}$$

Thereby, $\|T\kappa_1\| \leq K_1 = \|\kappa_1\|$. If we define $\Omega_1 = \{\kappa \in \mathbb{B} : \|\kappa\| < K_1\}$, then the inequality

$$\|T\kappa_1\| \leq \|\kappa_1\| \text{ holds for } \kappa_1 \in \mathcal{P} \cap \partial\Omega_1. \tag{15}$$

From the definitions of h_i^∞, $i = 1, 2, \cdots, n$, there is a $\bar{K}_2 > 0$ such that, for each $1 \leq i \leq n$, $h_i(\kappa) \geq (h_i^\infty - \varepsilon)\kappa^{p-1}$, $\kappa \geq \bar{K}_2$. Let $K_2 = \max\{2K_1, \frac{\bar{K}_2}{\sigma}\}$. Let $\kappa_1 \in \mathcal{P}$ and $\|\kappa_1\| = K_2$. Therefore, $\min_{t\in[\sigma,1-\sigma]} \kappa_1(t) \geq \sigma\|\kappa_1\| \geq \bar{K}_2$ is gained with the help of the Lemmas 2.5 and 2.6. We begin by verifying that $\kappa_n \geq K_2$ holds in the case when $i = n$.

Consequently, utilizing Lemmas 2.5 and 2.6, and given the selection of ε, we obtain

$$\begin{aligned}
&\int_0^1 \mathcal{G}(s_{n-1}, s_n)\phi_p^{-1}\left(\mu_n \int_0^{s_n} q_n(\omega_n)h_n(\kappa_1(\omega_n))d\omega_n\right) ds_n + \sum_{m=1}^{k} H_{nm}(s_{n-1}, t_m) \\
&\geq \gamma \int_\sigma^{1-\sigma} \mathcal{G}(s_n, s_n)\phi_p^{-1}\left(\mu_n \int_0^{s_n} q_n(\omega_n)h_n(\kappa_1(\omega_n))d\omega_n\right) ds_n \\
&\geq \phi_p^{-1}(\mu_n)\phi_p^{-1}(h_n^\infty - \varepsilon)\gamma \int_\sigma^{1-\sigma} \mathcal{G}(s_n, s_n)\phi_p^{-1}\left(\int_0^{s_n} q_n(\omega_n)d\omega_n\right)\kappa_1(s_n)ds_n \\
&\geq \phi_p^{-1}(\mu_n)\phi_p^{-1}(h_n^\infty - \varepsilon)\gamma\sigma \int_\sigma^{1-\sigma} \mathcal{G}(s_n, s_n)\phi_p^{-1}\left(\int_0^{s_n} q_n(\omega_n)d\omega_n\right) ds_n\|\kappa_1\| \\
&\geq \|\kappa_1\| = K_2 \ for \ 0 \leq s_{n-1} \leq 1.
\end{aligned}$$

We now consider the case $i = n - 1$ and show that $\kappa_{n-1} > K_2$. Following the approach used in Lemmas 2.5 and 2.6, and using the selected ε, we obtain

$$\begin{aligned}
&\int_0^1 \mathcal{G}(s_{n-2}, s_{n-1})\phi_p^{-1}\Bigg(\mu_{n-1} \int_0^{s_{n-1}} q_{n-1}(\omega_{n-1})h_{n-1}\Bigg(\int_0^1 \mathcal{G}(\omega_{n-1}, s_n)\phi_p^{-1}\Bigg(\mu_n \int_0^{s_n} q_n(\omega_n)h_n(\kappa_1(\omega_n))d\omega_n\Bigg)ds_n \\
&\quad + \sum_{m=1}^{k} H_{nm}(\omega_{n-1}, t_m)\Bigg)d\omega_{n-1}\Bigg)ds_{n-1} + \sum_{m=1}^{k} H_{n-1,m}(s_{n-2}, t_m) \\
&\geq \phi_p^{-1}(h_{n-1}^\infty - \varepsilon)\gamma \int_\sigma^{1-\sigma} \mathcal{G}(s_{n-2}, s_{n-1})\phi_p^{-1}\left(\mu_{n-1} \int_0^{s_{n-1}} q_{n-1}(\omega_{n-1})d\omega_{n-1}\right) ds_{n-1}K_2 \\
&\geq \phi_p^{-1}(\mu_{n-1})\phi_p^{-1}(h_{n-1}^\infty - \varepsilon)\gamma\sigma \int_\sigma^{1-\sigma} \mathcal{G}(s_{n-1}, s_{n-1})\phi_p^{-1}\left(\int_0^{s_{n-1}} q_{n-1}(\omega_{n-1})d\omega_{n-1}\right) ds_{n-1}K_2 \\
&\geq K_2 \quad for \ 0 \leq s_{n-2} \leq 1.
\end{aligned}$$

Once more, employing a bootstrapping argument leads us to conclude that

$$\int_0^1 \mathcal{G}(t, s_1)\phi_p^{-1}\left(\mu_1 \int_0^{s_1} q_1(\omega_1)h_1\left(\int_0^1 \cdots\right) d\omega_1\right) ds_1 + \sum_{m=1}^{k} H_{1m}(t, t_m) \geq K_2.$$

Thus, $T\kappa_1(t) \geq K_2 = \|\kappa_1\|$.

Therefore, $\|T\kappa_1\| \geq \|\kappa_1\|$. Putting $\Omega_2 = \{\kappa \in \mathbb{B} : \|\kappa\| < K_2\}$, then

$$\|T\kappa_1\| \geq \|\kappa_1\|, \qquad \kappa_1 \in \mathcal{P} \cap \partial\Omega_2. \tag{16}$$

Applying Lemma 2.1 to (15) and (16), we can conclude that T has a fixed point $\kappa_1 \in \mathcal{P} \cap (\bar{\Omega}_2 \backslash \Omega_1)$. In conclusion, setting $\kappa_{n+1} = \kappa_1$ yields a positive solution $(\kappa_1, \kappa_2, \cdots, \kappa_n)$ for the 3^{rd}-order IBVP (1)'s iterative system, where iteratively,

$$\kappa_r(t) = \int_0^1 \mathcal{G}(t,s)\phi_p^{-1}\left(\mu_r \int_0^s q_r(\omega)h_r(\kappa_{r+1}(\omega))d\omega\right)ds + \sum_{m=1}^{k} H_{rm}(t,t_m), \quad r \in \{n, n-1, \cdots, 1\}.$$

□

Example 3.2 *Assume that* $k = 4$, $n = 4$ *and* $p = 2$, $q_i(t) = 1$ *for* $1 \leq i \leq 4$, $a_1 = a_3 = 4, a_2 = a_4 = 2, \sigma = \frac{1}{4}$ *in the IBVP* (1)*'s iterative system, i.e.,*

$$\begin{cases} (\phi_1(\kappa_i''(t)))' + \mu_i h_i(\kappa_{i+1}(t)) = 0, \ t \neq t_m, \ t \in I = [0,1], \ t \neq t_m, \ i \in \{1,2,3,4\} \\ \kappa_{n+1}(t) = \kappa_1(t) \\ \triangle\kappa_i|_{t=t_m} = \mu_i I_{im}(\kappa_{i+1}(t_m)), \ m = 1,2 \\ \triangle\kappa_i'|_{t=t_m} = -\mu_i J_{im}(\kappa_{i+1}(t_m)), \ m = 1,2 \\ 3\kappa_i(0) - 2\kappa_i'(0) = 0 \\ 3\kappa_i(1) + 2\kappa_i'(1) = 0 \\ \kappa_i''(0) = 0, \end{cases} \tag{17}$$

where

$$h_1(\kappa_2) = \kappa_2\left(3\cdot 10^4 - \frac{29999}{\ln(e+\kappa_2)}\right), \ h_2(\kappa_3) = 2\kappa_3(10^4 - 9999e^{-5\kappa_3}),$$

$$h_3(\kappa_4) = \kappa_4\left(4\cdot 10^4 - 39999\frac{e^{-4\kappa_4}}{\ln(e+\kappa_4)}\right), \ h_4(\kappa_1) = \frac{\kappa_1}{5}\kappa_1(10^5 - (99995)e^{-\kappa_1}),$$

$$I_{1m}(\kappa_2) = \frac{6\kappa_2^2 + 4\kappa_2}{3+\kappa_2}, \ I_{2m}(\kappa_3) = \frac{2\kappa_3^3 + 4\kappa_3}{8+\kappa_3^2}, \ I_{3m}(\kappa_4) = \frac{8\kappa_4^3 + 4\kappa_4}{7+4\kappa_4^2}, \ I_{4m}(\kappa_1) = \frac{10\kappa_1^2 + 2\kappa_1}{11+\kappa_1},$$

$$J_{1m}(\kappa_2) = \frac{9\kappa_2^2 + 6\kappa_2}{2+\kappa_2}, \ J_{2m}(\kappa_3) = \frac{3\kappa_3^3 + 6\kappa_3}{5+\kappa_3^2}, \ J_{3m}(\kappa_4) = \frac{12\kappa_4^3 + 6\kappa_4}{5+4\kappa_4^2}, \ J_{4m}(\kappa_1) = \frac{15\kappa_1^2 + 3\kappa_1}{8+\kappa_1}.$$

Using the definitions of the functions h_i, I_{im} *and* J_{im} *for* $i \in \{1,2,3,4\}$, *we achieve the following numbers:*

$h_1^0 = 1$, $h_2^0 = 2$, $h_3^0 = 1$ *and* $h_4^0 = 1$, $h_1^\infty = 3\cdot 10^4$, $h_2^\infty = 2\cdot 10^4$, $h_3^\infty = 4\cdot 10^4$ *and* $h_4^\infty = 2\cdot 10^4$,

$I_{1m}^0 = \frac{4}{3}$, $I_{2m}^0 = \frac{1}{2}$, $I_{3m}^0 = \frac{4}{7}$ *and* $I_{4m}^0 = \frac{2}{11}$, $J_{1m}^0 = 3$, $J_{2m}^0 = \frac{6}{5}$, $J_{3m}^0 = \frac{6}{5}$ *and* $J_{4m}^0 = \frac{3}{8}$.

It is easy to see that conditions (C1)-(C5) are satisfied. With the help of some basic *computations, for* $1 \le i \le 4$*, we obtain* $\rho = 21, \gamma = \dfrac{11}{20}$ *and*

$$\mathcal{G}(t,s) = \frac{1}{21}\begin{cases}(2+3s)(5-3t), & s \le t\\(2+3t)(5-3s), & t \le s.\end{cases}$$

Additionally, if we use descriptions, we get $N_1 = 0,025322$ *and* $N_2 = 0,081632$*. With the help of the Theorem 3.1, we determine that the optimal eigenvalue interval is*

$$0,025322 < \mu_i < 0,081632 \ for \ i = 1,2,3,4$$

ensuring a positive solution of the 3^{rd}*-order IBVP (17)'s iterative system.*

4. Conclusion

This study explores eigenvalue intervals for third-order impulsive boundary value problems (IBVPs) with p-Laplacian and iterative structures, addressing a previously underexplored area. By applying the Guo–Krasnosel'skii fixed point theorem [12], we establish the existence of positive solutions for the iterative system (1) and determine the eigenvalue intervals of parameters $\mu_1, \mu_2, \cdots, \mu_n$. Beginning with the initial solution $\kappa_1(t)$ of the third-order IBVP (2), the iterative construction of the solution set $(\kappa_1(t), \cdots, \kappa_n(t))$ provides a robust analytical framework for understanding such systems' dynamics.

Beyond theoretical contributions, this work has significant practical implications. In mechanical engineering, it aids in analyzing vibrational modes of structures subjected to impulsive forces (e.g., seismic events or explosions), contributing to safer designs [17, 32]. In biological modeling, these intervals reveal oscillatory patterns in drug delivery systems, optimizing dosing strategies [30]. For control theory, they enhance stability algorithms in robotics and signal processing where abrupt changes occur [1, 15].

Our study advances the field by unifying third-order impulsive systems with iterative structures-a gap in existing literature. Unlike prior work on second-order impulsive BVPs [19, 31] or non-iterative third-order systems [6, 32], we incorporate eigenvalue intervals and higher-order dynamics, offering novel perspectives for complex impulsive systems.

Future research could extend this framework to higher-order systems or complex boundary conditions – for instance, the boundary parameters $a_i's$ could be generalized from positive constants to functions. Furthermore, combining numerical solution methods may enhance computational efficiency, stability analyses under parameter variations [15, 16, 23] will provide critical insights for engineering applications.

In summary, this study comprehensively advances the theory and applications of third-order iterative IBVPs. By elucidating eigenvalue intervals and their cross-disciplinary relevance, we pave the way for mathematical and practical breakthroughs.

Declaration of Ethical Standards

The author declares that the materials and methods used in her study do not require ethical committee and/or legal special permission.

Conflicts of Interest

The author declares no conflict of interest.

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