Journal for the Mathematics Education and Teaching Practices, 6(1), 1-19, June 2025
e-ISSN: 2717-8587
dergipark.org.tr/jmetp

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Math Teaching Practice

Explorations with patty paper focusing on polygons: properties and area measurements

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Article Info

Received: 7 January 2025
Accepted: 8 April 2025
Available online: 30 June 2025

Keywords:
Field measurements
Learning by doing
Math teaching practise
Patty paper geometry
Polygons
Teaching polygons

Abstract

Doing mathematics is not an activity such as applying the rules or methods. Doing mathematics is developing methods to solve problems testing whether the answers given are meaningful or not and being able to model the work of doing mathematics in the real world as accurately as possible (Van de Walle, Karp, & Bay-Williams, 2013). Doing mathematics is a prerequisite for students to recognize meaningful mathematics. Meaningful mathematics can be discovered by students in many different ways and methods. One of these ways paper folding activities. Paper folding is an engaging and educational way to "do math." Therefore, an instructional activity was developed in order to let students and teachers see and appreciate the educational function of these activities. Patty paper folding activities were carried out to explore the properties and areas of polygons. For this purpose, "Patty Paper Folding Activity in the Focus of Polygons" (PFA) and "Patty Paper Folding Activity Opinion Form" (PFO) were administrated. The study was held with six graduate students who were also mathematics teachers studying at the Department of Mathematics Education of a state university. PFA was held for 4 hours. The results of the study showed that the patty paper folding activity is a useful and functional tool that can be used in teaching polygons. After the implementation of the activity, it was seen that teachers generally did not have difficulties with the patty paper folding activities; they only needed the researcher's guidance in using the initial assumptions of patty paper and emphasized that the planning of folding activities should be detailed; the order of instruction is important and more time should be given for activity tasks. All of the teachers stated that the patty paper folding activities were beneficial and they willingly made the applications in discovery process. They stated that they intend to use these activities in their classrooms.

To cite this article

Çontay, E.G. (2024). Explorations with patty paper focusing on polygons: properties and area measurements. *Journal for the Mathematics Education and Teaching Practices, 6*(1), 1-19. DOI: https://doi.org/10.5281/zenodo.1

Introduction

"What does it mean to know a topic in mathematics?" The answer lies beyond applying the rule/formula or method related to that topic. Most mathematics courses are presented to students with a limited understanding that will not allow them to know what they learn. In order for students to know in mathematics, they must be given the opportunity to do mathematics. Doing mathematics means developing methods to solve problems beyond solving a bunch of examples by repeating the methods explained by the teacher, applying them, testing whether the answers given are meaningful by trying to see if these methods lead to a result, and being able to model the work of doing mathematics in the real world as accurately as possible. This begins with providing students with activities to think about and creating

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environments where they can share their thoughts (Van de Walle et al., 2003). When we think of doing mathematics, we think of learning through discovery. Learning through discovery was introduced in the 1960's by Bruner, but Piaget's (Cognitive Development Theory), Vygotsky's (Social Development Theory) is a closely related learning model. According to him, learning by discovery is new knowledge from previous knowledge and active experiences knowledge-building, is a teaching approach and reflects a constructivist perspective. Bruner (1961) focused on the intellectual intelligence of the individual and built four foundations from the experience of learning through discoveries. Accordingly, the first was the "Intellectual potency". After a series of studies, he concluded that discovery in learning makes the learner to be constructive, to organize what he encounters not only to discover order and relationality, but also to organize the knowledge in a good way and is a necessary condition for learning a variety of problem-solving techniques. Secondly, "Intrinsic and extrinsic motives" take place. To free the child from immediate control of environmental rewards and punishments is important. The child must be put in a position to experience success and failure not as a reward and punishment. Thirdly, "Learning the heuristics of discovery" is important. This is the process of trying to find out something in a process of problem solving that converts a puzzle form into a problem that can be solved in a way that gets the child where she wants to be. It is a way that recasts the difficulty into a form that one knows how to work with. And the fourth is "Conservation of memory", that is not storage, but retrieval. "*In sum, the very attitudes and activities that characteize "figuring out" or "discovering" things for oneself also seem to have the effect of making material more accesible in memory*" (Bruner, 1961, p. 9). This constructivist perspective, combined with learning by discovery, paved the way for the development of the concept of meaningful learning in mathematics education. In fact, the "Meaning Theory" introduced by Brownell led the way to the full recognition of the value of the childrens' experiences and studied on how to make mathematics meaningfull (Viewer & Suydam, 1972). Brownell (1954) in his investigations about the revolution searched for a better organization of content for better methods of teaching also emphasized learning through discovery. According to Brownell (1954), to be intelligent children had to see sense in mathematics and for this, instruction had to be meaningful organized around the ideas and relations inherent in arithmetic as mathematics. Besides, he added: "*They must also have experiences in using the arithmetic they learn in ways that are significant to them at the time of learning, and this requirement makes it necessary to build arithmetic into the structure of living itself* (Brownell, 1954, p. 5). The modern mathematics movement at that time sharpened and extended a focus on mathematical meaning and meaning has began to be recognized as a vital component in teaching and learning process (Viewer & Suydam, 1972). The emphasis on experiences and heuristic learning in teaching led to the inclusion of the use of manipulatives in teaching. Kennedy (1986) in his study mentioned that many teachers began to believe strongly in the benefits of manipulative materials in building a foundation for students' mathematical concepts. He said that in the constructivist point of view of Piaget and Skemp; mental images and abstract ideas of students are based on their experiences and manipulative materials are learnings aids in all four stages of constructivist theory. So, students learning with manipulatives have clearer mental images and represent their abstract ideas better than those who don't. Dienes also holds the idea that students should use manipulatives in learning. Learning theories advocate that student whose mathematical experiences grounded in manipulative experiences are likely to bridge the gap between the world in which they live and the abstract world of mathematics (Kennedy, 1986). From this perspective, paper folding is one of the most suitable manipulative materials for discovery learning and provides a suitable learning environment for doing mathematics which is embedded in the persoective of constructivist theory. Paper folding is not a new method or way of thinking and many researchers studied to make links and relationships between thking geometry and folding papers. T. Sundara Row in his writings about the geometric exercise in paper folding in 1893; mentioned that paper folding was a way to afford mathematical recreation to all people in an attractive and economical form. This geometry was not rigidly bounded to folding as the Euclidean Geometry was to compass and ruler study, his work showed that many relationships could be illustrated without ruler and compass by using papers (Row, 1917). At the same time, Wiener's folded paper models were exhibited in his manuscripts and formed an operative geometry of folding (Friedman, 2016). Serra (1994) after his geometry studies (in his book of discovering geometry); introduced Patty Paper Geometry with open investigations to encourage students to explore their own methods of discoveries by using another form of paper folding. Haga (2008) later in his book: "Origamics" showed several ways of folding with explorations. He exhibited the art of origami (paper folding) and its manipulative and experimental nature with relating it to Euclidean Geometry via axioms and theorems. So, linking geometry and folding has a history and has no doubt that focuses on exploration and learning by discovery.

Paper folding activities provide students with both enjoyment and the opportunity to make discoveries, justifications, and hypotheses by thinking about them. It is an engaging way to add realism and interest to mathematics teaching and to add active experiences is through paper folding. Creating lines by folding on a sheet of paper is a simple way to demonstrate and explore the relationships between lines and angles. Once a relationship is found by folding paper, formal explanations about the subject do not seem as difficult to students. Therefore, paper folding not only makes learning mathematics easier, but it also develops understanding and appreciation of mathematics and provides the experiential foundation necessary for further learning. (Johnson, 1957, Olson, 1975). Paper folding is an engaging and educational way to "do math". Paper folding is frequently used in geometry instruction to provide students with a visual representation of geometric concepts such as shapes, properties of shapes, congruence, similarity, and symmetry, to establish cross-cultural connections to mathematical ideas, and to support the development of spatial perception, and the activities discussed align with the National Council of Teachers of Mathematics (NCTM, 2000) in geometry standards (Robichaux & Rodrigue, 2003). Geometry standards emphasize that students from preschool through high school analyze the properties of two- and three-dimensional shapes, develop arguments about the relationships between these properties, and use geometric modeling, spatial reasoning, and visualization to solve problems (NCTM, 2000, p. 41).

Paper folding activities are also an important method for students to comprehend spatial relationships and develop spatial skills (Boakes, 2008). The connection between paper folding and geometry is easy for students or teachers to discover. Because folds and edges represent lines; angles are formed by their intersections. The intersections of folds with themselves form points. Therefore, due to its manipulative and experiential nature, paper folding always has the potential to be an effective content for learning and teaching geometry (Haga, 2008). Patty paper folding is a type of paper folding and, in addition to paper folding, it brings many advantages in teaching geometry

Literature Review

Patty papers used for geometric explorations are light, waxy and transparent square papers. They are used in uncooked hamburger patties, which are called "meatball papers" in restaurants. Serra (1994) states in his book "Patty Paper Geometry" that many of the geometric rules and formulas and the properties of geometric shapes can be discovered by folding patty paper. In this study, baking papers sold in markets were used as "patty papers" and it is recommended to be used in classroom applications because they are more easily accessible and economical. In addition, it should be cut into squares. Any property discovered using compass and ruler can be discovered by folding patty papers and useful for discovering most of the properties in school geometry (Serra, 1994). Thanks to its transparency, patty papers can be used to measure and compare lengths and angles. A triangle equivalent to a triangle can be drawn by placing one patty paper on top of another; the interior angles of a triangle (obtuse, acute, etc.) can be examined by taking advantage of the property that one corner of patty paper is 90 degrees (initial assumpsions); or many different examinations and research activities can be carried out such as creating angle bisectors and side bisectors by making different folds. Many rules and formulas can be proven with patty paper folding activities.

Many studies (Adom &Adu, 2020; Aksoy& Işıksal Bostan, 2024; Bornasal, Sulatra, Gasapo, & Gasapo, 2021; Febriani, Susanti, & Hapizah, 2023; Patkin & Canner, 2010; Subaar, Asechoma, Asigri, Alebna, & Adams, 2010) revealed the positive effects of paper folding activities based instruction on students improvement in an experimental manner.

Adom & Adu (2020) investigated the use of paper folding on the performance of 9th graders on the focus of learning fractions. Quasi-experimental design was used, and they found that there was a significant difference between pretest and post test scores. In other words, instruction with paper folding as a manipulative material had a positive impact on learners' academic performance in fractions. Similarly, Febriani et al. (2023) conducted their studies on fractions at different grade level. They aimed to evaluate a learning pathway's effect with paper folding activities on fourth graders' comprehension of fractions, with a focus on their comparison and sequencing. According to the findings of the study, it was revealed that student learning trajectory encompassed three principal activities. Initially, students used folding and gluing of paper to discern fraction values. Subsequently, they engaged in coloring and illustrating folds for fraction comparison. The final activity involved drawing, coloring boxes, and fraction comparison and sequencing. Students showed proficiency in understanding and determining fraction values and comparing them yet struggled with ordering certain fractions. The structured learning path facilitated students' understanding of basic fraction concepts, especially in comparing them. Aksoy & Işıksal Bostan (2024) investigated the effect of a paper folding activity on sixth-grade students' concept definitions and concept images of parallelism and perpendicularity. The study investigated how the concept definition and concept images changed after the paper folding activity. The findings revealed that the paper folding activity had a significant positive effect on students' concept definitions and concept images. In addition, the interviews after pre and post tests indicated that the students' personal concept definitions of parallelism and perpendicularity of two lines/line segments began to match the formal concept definitions of these concepts after the paper folding activity. Bornasal et al. (2021) investigated the effect of paper folding (origami) instruction in teaching 8th graders geometry. They used a quasi experimental model and found that both groups achieved better performance through paper folding and non-paper folding instruction. However, the experimental group recorded higher mathematics performance compared to the control group. They concluded that paper folding instruction promoted more effective learning in geometry. Patkin and Canner (2010) explored the extent of influence of learning abstract terms, such as "special segments of triangles", using illustrations of paper folding (of 8th grades). They found that students improved their definition capability and their comprehension of terms learnt in this way, demonstrating a change for the better in their attitude towards the geometry studies. Subaar et al. (2010) in their experimental study aimed to reveal if there was a difference in the performance of students who were taught using sets of objects (sets model) with paper folding activities and withour using them; to solve word problems involving addition and subtraction of proper fractions. The students' level of performance had improved drastically with the help of paper folding method. In conclusion, paper folding activities helped students to appreciate word problems involving addition and subtraction of proper fractions.

There were also a considerable number of studies which have also described the positive effects of folding activities on learning in differents design and methods manner. Robichaux & Rodrigue (2003) in their study reported that teachers who have used origami in their lessons said that students' understandings of the concepts incresed and they were well motivated. The level of understandings of the concepts were also evidents in students' writings. At the end of the foldings, students were able to name all the shapes and able to understand the properties. Gürbüz, Ağsu & Güler (2018) investigted habits of mind of 11th grade students while they were using paper folding activities. Ther searched for the potential of paper folding to improve students' geometric thinking skills and to enhance their achievement in national exams. They concluded that the students were able to reach solutions more easily by concretizing the intangible questions through paper folding. The students were able to comprehend the fact that the main components of triangles didn't change; that was, they were preserved and the students' thinking processes have improved in the study. They have permanently maintained the indicators of geometric habits of mind they have gained. Demirci & Çontay (2023) designed a paper folding activity task, which involved reaching the Pythagorean Theorem with a series of steps. The task was conducted in order to reach deductive reasoning and logical inference. Besides, it was aimed to examine the effectiveness of the task and to share the patty paper folding task with the teachers. The patty paper folding activity task was carried out for a total of 6 lesson hours for three weeks. According to the results of the study, it was revealed that the students did not have difficulty while folding, while they had difficulties in performing algebraic operations and expressing the concepts mathematically. According to the findings, the patty paper folding task helped students understand why the theorem was true and contributed to meaningful learning in terms of being a explanatory proof. Empson & Turner (2006) investigated students' solutions to folding tasks, which involved predicting the number of equal parts created by a succession of given folds and determining a sequence of folds to create a given number of equal parts. Analyzing a combination of cross-sectional data and case studies from standardized clinical interviews, they found first, third and fiftht grade students were most successful at coordinating folding sequences with multiplicative thinking when they used a conceptualization of doubling based upon recursion. This conceptualization tended to generate more sophisticated solutions. Friedman and Rittberg (2021) searched for the ways in which paper folding constituted a mathematical practice and prompted common mathematical activities. They presented the paper folding as a material reasoning practice. They concluded that mathematical paper folding is a reasoning practice which helps understanding of the of mathematical proof. Morye (2025) investigated the symbiotic relationship between mathematics and origami. He explored the utility of origami in education while finding out how origami could become an effective way of teaching methods of geometry because of its experiential nature. Galicha & Lazaro (2022) aimed to studied the level of acceptability of a research-based supplementary learning material in geometry for six grade students. The mentioned paper folding based material was found a useful learning aid. It was found that the paper folding material possessed adequacy, clarity, suitability and usefulness. Van Wijk, Bos, Shvarts and Doorman (2023) investigated what reasons did teachers from France and Germany report for implementing mathematical folding activities in authentic classroom situations. Study showed that the reasons were "to activate students by letting them manipulate paper" and "to visualise mathematics". Teachers stated that folding allowed for dynamic representations that supported the transition from informal to formal mathematics and the practice of skills. They memorized these findings n two classifications: "to activate students by providing folding tasks" and "to grow mathematical understanding by folding". Boakes (2008) discussed how the use of Origami-mathematics lessons implemented into a geometry unit impacted students' math and spatial skills. She intended to provide with a perspective on using Origami in the mathematics classroom and concluded that the act of folding an Origami model holded great potential in the classroom in many ways like growing spatial abilites and knowing concepts, or engaging students in mathematics in a more exiting way. She reported that origami-mathematics lessons blended a variety of approaches to instruction in a way that students benefited mathematically.

Puspose of the Instructional Activity

The purpose of this study is to present an instructional activity that will demonstrate the use of patty paper activities that can be used for students to "do mathematics" for meaningful learning" in classroom applications. For this purpose, it was deemed appropriate to test the application that will be designed for students on teachers. For this purpose, it was planned to reveal whether this activity was useful under the guidance of teachers and through their eyes and to provide inferences about the education they will provide to their students in the future. In short, this study aims to test the teaching practice designed with patty paper activities on teachers and to provide inferences about patty paper activities for both teachers and researchers.

Focus of the Instructional Activity

The most effective method of classroom teaching in patty paper geometry is cooperative learning, and the best group structure is recommended as pair-sharing. Here, students are divided into groups of four and divided into two pairs. While one student from each pair does the joint folding, the other student reads the instructions. Then the pairs compare their results with the other pair in the group of four. Each pair creates and shares their assumptions. For the next research, the students who fold and the students who read the rules switch places. This cooperative group structure helps all students share the excitement of learning by discovering. Therefore, it helps students reduce their mathematical anxiety and provides permanent learning through discovery (Serra, 1994).

This study was conducted with teachers who were graduate students at the same time; it was thought that working with teachers in pairs would support them to teach in a double-shared structure in their classes. Patty paper explorations in this study were carried out with the focus on polygons. Focus on polygons has existed since the Babylonian and Egyptian mathematical writings, which are considered the first mathematics. So much so that; geometric shapes have been used since ancient times. The need to clarify the land boundaries with the overflow of the Nile River in ancient Egypt led to the necessity of calculating the areas of polygons with simple measurements. It later became systematic with Greek mathematics and has become a subject that we still encounter today in architecture, geography, art, nature and many other fields (Ulusoy, 2022). When the middle school mathematics curriculum of the Republican period is examined (MEB, 2024), it is seen that the concept of polygon is one of the most intensively included concepts in the curriculum. Polygons have been heavily included in all programs since 1938 (Yavuzsoy Köse & Özen Ünal, 2020). Therefore, it was deemed important to examine the subject of polygons, which is frequently included in middle school curriculum, with the focus of using patty paper activities.

Teachers' Readiness for Folding Patty Paper

Teachers who participated in the study voluntarily were students studying in master's program of the mathematics education department of the university where the researcher studied. The participants had previously participated in the TUBITAK 4004 project in which the author of the study was an educator and, in a workshop, organized by the author at an international congress on mathematics education, and they were familiar with the activities of folding patty papers on different subjects. Therefore, it was thought that they had relevant prior learning.

Information About the Implementation

This study was carried out in 4 lesson hours with 5 female and 1 male mathematics teachers who were master's students in the Department of Mathematics Education of a university in the Aegean Region. In the selection of the participants in the study, it was desired to study with mathematics teachers who were familiar with folding patty papers. As mentioned above, they were familiar to the paper folding activities. In addition, the fact that the teachers were also postgraduate students was effective in their appreciation of the effectiveness and their willingness. From this point of view, although it seems that the purposive sampling method was adopted in the sample selection, the fact that the teachers were graduate students in the author's department played a role in this selection. From this point of view, it can also be considered that it is close to the convenience sampling method. However, from both perspectives, this study sought to meet a variety of interests, multiple purposes and needs; therefore, hybrid sampling, one of the qualitative research sampling methods, was used (Baltacı, 2018). The data collection process was carried out in the mathematics classroom in a suitable environment in the main mathematics department. Since the participants were familiar with folding patty papers, they were not given information about how to use it. The purpose of the study was stated, and the study was carried out smoothly. The activities were carried out in pairs for the discovery of patty paper geometry. Patty Paper Folding Activity Focusing on Polygons (PFA) was conducted. Teachers were given a task with written instructions on what to do and they were also given additional explanations via powerpoint when needed in order to guide the teachers while exploring the properties. Teachers were in contact at every stage of the activity. The teachers first discussed the folds they made within the framework of learning by discovery, shared their ideas among themselves (in groups of two; in pairs), then each group individually explained to the other groups how they made the folds at each step. How these folds and discoveries could be made from the students' point of view was discussed and different ideas were shared with other groups. They were enabled to take part in an independent discovery process but were also guided. In this way, they were able to gain different experiences in the discovery processes they would carry out with their students and discuss how to build new knowledge on their students' old knowledge. They were encouraged to discuss to what extent and from which knowledge to start, thus enabling them to develop intuition about how to scaffold their stundets' knowledge. During the implementation, the teachers and their actions were audio and video recorded, and the folds they constructed were photographed. At the same time, observations were made, and notes were taken. In the light of all these data, the application was analyzed with a descriptive approach. Observation notes were analyzed in detail; POF was analyzed by considering common statements. This study focuses on the extent to which teachers perform multiplication rather than their performance and whether the PFA works effectively.

During the implementation, Square-cut patty papers sold in markets were used and compasses and rulers were not used for any measurement purposes (Any property that can be discovered using compasses and rulers can be discovered by folding the patty papers (Serra, 1994). Compasses were used to draw circles, and rulers were used to mark the folds properly. In this study, "Patty Paper Folding Activity Focusing on Polygons" (PFA) (Figure 1) and "Patty Paper Folding Activity Opinion Form" (POF) were applied. PFA was applied to teachers in groups of two. PFA consisted of two parts. In the first part, discovering the properties of certain polygons were focused, and in the second part, calculating the area measurements of the polygons whose properties were discovered took place. In the first part, folding activities related to triangles and their properties, auxiliary elements of triangles were carried out and Euler Line was formed. Then, the properties of parallelogram, rhombus, deltoid, rectangle and square were discovered. In the second part, the area formulas of triangle, rectangle, parallelogram and trapezoid were obtained through patty paper folding activities. (Figure 1)

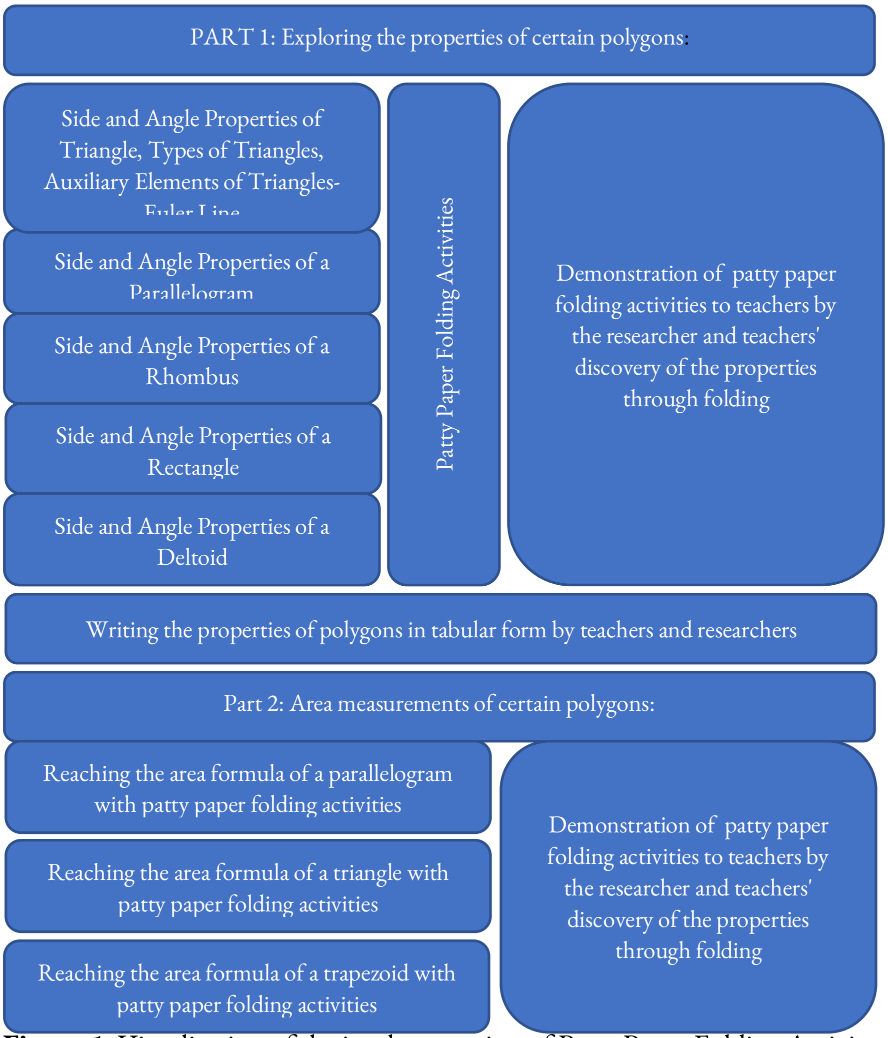

Figure 1. Visualization of the implementation of Patty Paper Folding Activity

After the implementation, POF was directed in a written form to determine the teachers' opinions about the activity. Questions in the POF were as follows:

- Do you find the patty paper folding activities useful? Why? Please explain in detail.
- Would you consider applying patty paper folding activities in your own classrooms? Please explain.
- Can patty paper folding activities be adapted to every math topic? Why?
- Would you recommend patty paper folding activities to your fellow math teachers?
- What areas do you think need improvement in the activities we conducted?
- What part/topic of the wax paper folding activities did you like most or find most useful?
- Is there anything else you would like to add? Please write and explain if applicable.

Limitations

Due to time constraints, there were limitations in the detail of information obtained from teachers. Not all teachers have the same knowledge and skill levels and pegadagogical competencies. While these differences were useful in terms of revealing the different implementations of PFA, on the other hand, imbalances in terms of time and implementation may have occurred between one group of teachers with different lower levels of pedagogical content knowledge and the other group. For this reason, each teacher's way of implementing this activity on their students may differ. Therefore, the learning outcomes for each teacher in this study may differ.

Implementation

In the first part of the PFA, teachers discussed how they could introduce the angle and side properties of triangles to students using patty paper. In this way, the initial assumptions of patty paper (patty paper being square, therefore a corner angle measure being 90º; all side lengths being equal and these side lengths being usable in measuring) were discussed with the teachers. Then, the question; "How do we fold triangles according to their angles and sides?" was asked to the teachers. All groups made triangle folds according to their angles and sides. One group first started folding with isosceles right triangles from special triangles; when asked why they started with a special triangle, they answered "we thought it would be easier to find". From here, it can be concluded that folding behaviors about reaching generalizations are related to starting from special situations. This group stated that they had difficulty in creating an equilateral triangle.

They used the expression "We though about creaing triangles by forming a hexagon and using a square." It was thought that the reason for this problem was that the teachers did not use the initial assumptions of the patty paper. So much so that; the other two groups did not use the initial assumptions in the same way. The second group stated that they started by folding a scalene triangle because they could compose the most irregular and random ones by acting with the behavior of reaching the correct generalization compared to the first group. They formed a scalene triangle with three different foldings, then they said that they composed the axis of symmetry by first determining a line and then lowering the isosceles triangle perpendicular to that line from a point outside it and obtained a triangle with a third folding (the exercise of lowering a perpendicular from a point outside a line to that line was experienced with the group before. For this, while folding a line; folding another line from a point outside it to that line, it is necessary to pay attention to the overlapping of the lines in hand. In this way, the right angle is divided into two to form a right angle and this right angle is formed by the line folded from that point to the line). For the relevant folding, see. (Çontay, 2018). Teachers stated that they used the height of the isosceles right triangle as the axis of symmetry. They stated that they constructed the equilateral triangle by forming three axes of symmetry in a similar manner after they started folding the isosceles triangle (Figure 2).

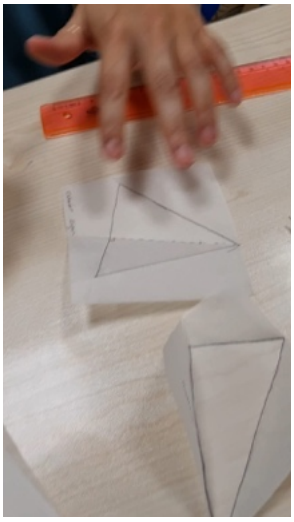

Figure 2. Folding an equilateral triangle

While these two groups started with triangle types according to their sides, the other group started with triangle foldings according to their angles. First, they formed the right triangle by using the folding of the perpendicular distance of a point to a line; then they formed the right triangle. They told how they formed the acute-angled triangle with the following expression:

> "*We drew a line segment with an angle smaller than the right angle, but we determined the right angle of this one in case it was obtuse or right angle, by folding it over and over again, to draw an angle smaller than this, we drew a narrower angle like this, all of them were acute, by testing the right angle, it turned out to be narrow when it was smaller than that, we did the same thing for the width, this time we chose an angle wider than the right angle, when*

we combined it, we didn't feel the need to test it because since it was an obtuse angle, it is certain that both of them would be acute."

The researcher stated that it would be easier for teachers to act with their initial assumptions after this stage. The researcher reminded them that they could use two sheets of patty papers with their transparency and benefit from their squareness. After this reminder, the teachers were able to easily identify right-angled, acute-angled and obtuse-angled triangles by placing one sheet of patty paper on top of the other (Figure 3).

Figure 3. Using the transparency Property of patty paper

All groups used the initial assumptions of the patty paper and folded the triangles by varying them according to both their sides and angles. Meanwhile, the second group developed a new strategy while finding the equilateral triangle according to its angles. They formed an angle of an equilateral triangle by folding one corner into three without measuring and using the transparency feature of patty paper, then they showed the equality of the other two sides by folding the angle measurements of 60° from the fold lines of the equality, using the axis of symmetry (Figure 4).

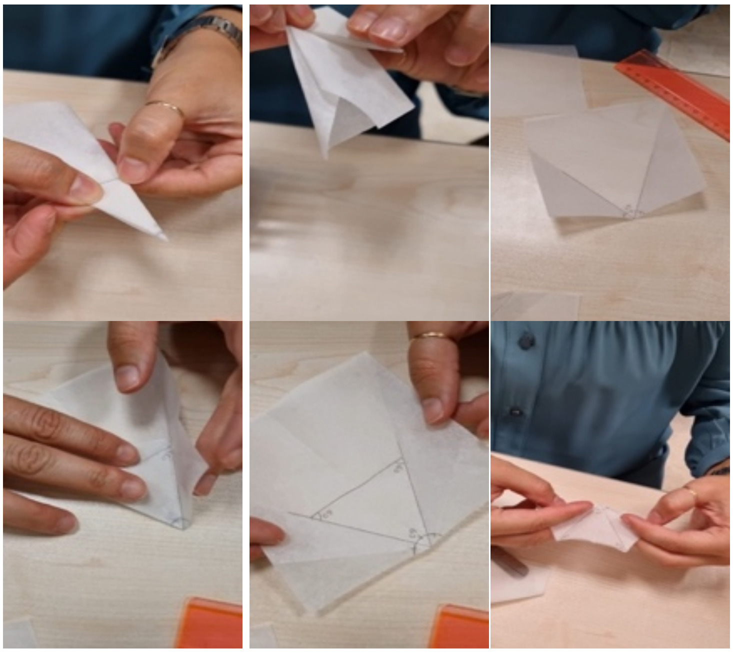

Figure 4. Folding an equilateral triangle by taking advantage of the transparency and squarenedd properties of patty paper

In the next folding activity of PFA, teachers were asked to find the auxiliary elements of the triangle (perpendicular bisectors, angle bisectors, medians and altitudes and their intersection points). For this, the same triangle would be used, so they were asked to fold and draw four identical, acute-angled wide scalene triangles. Teachers folded the perpendiculars of the triangle in groups of two. One group measured the distance from the intersection point to one corner of the triangle with a second piece of patty paper and placed this second piece of patty paper on top of the first one, measuring this length from the intersection point of the perpendiculars of the triangle to the other corners, and they saw that the lengths were the same (Figure 5).

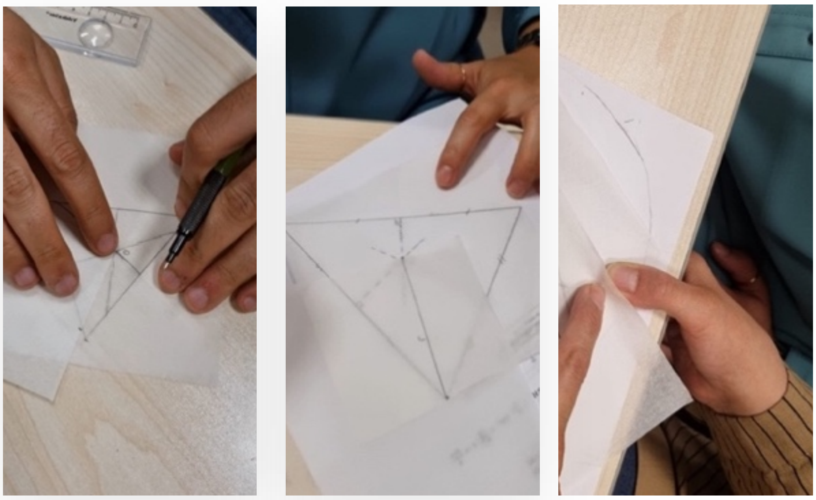

Figure 5. Finding the intersecion point of the perpendicular midpoints of a triangle

The second group also made the same measurement and stated that these were radius measurements because they were the same lengths. Thus, they concluded that the intersection point of the triangle's perpendiculars was the center of the circumscribed circle. The teachers were asked how they could teach this to students if they wanted to. (It was said that although the teachers could see that the measured length was the same as the radius, the students would not be able to see this, so the teachers were asked to assume that the students knew the definition of a circle). The first group performed measurements using a ruler; it was then reminded that there was no need to measure with a ruler when using patty paper. Teachers placed the pointed end of the compasses at the intersection of the perpendicular bisectors, continued by measuring the distances to the vertices of the triangle, and constructed the circle (Figure 6).

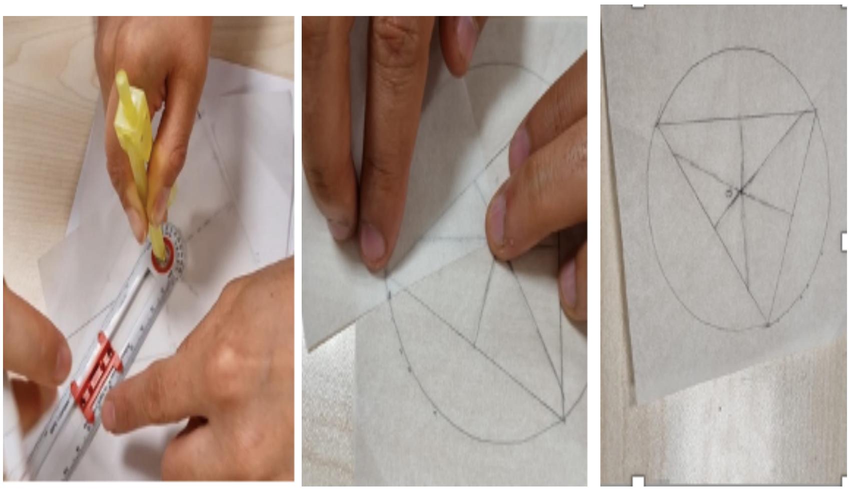

Figure 6. Finding the center of the circumcircle of a triangle

Teachers formed the points of intersection of the angle bisectors. One group folded a perpendicular from the point of intersection of the angle bisectors to one of the sides of the triangle without using a compass (Çontay, 2018); they marked the cases where this length was the same on the other sides (Figure 7).

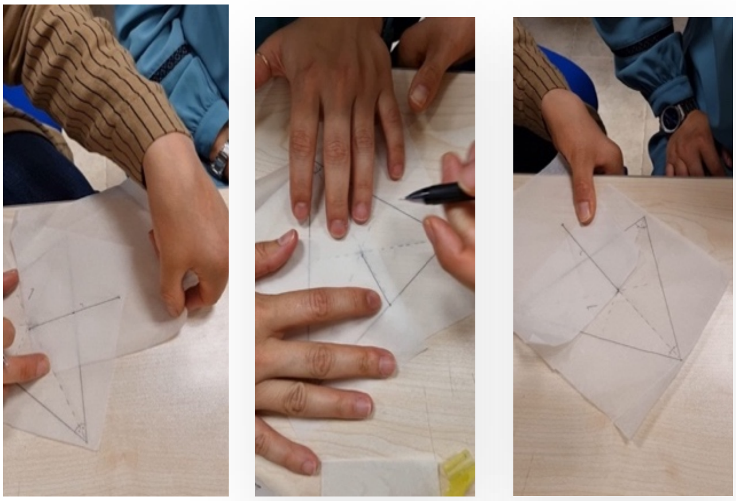

Figure 7. Finding the intersection point of the angle bisectors of a triangle

The other two groups reached the radius measurement by marking with compasses. They wrote their assumptions by saying that this circle is an inscribed circle (Figure 8).

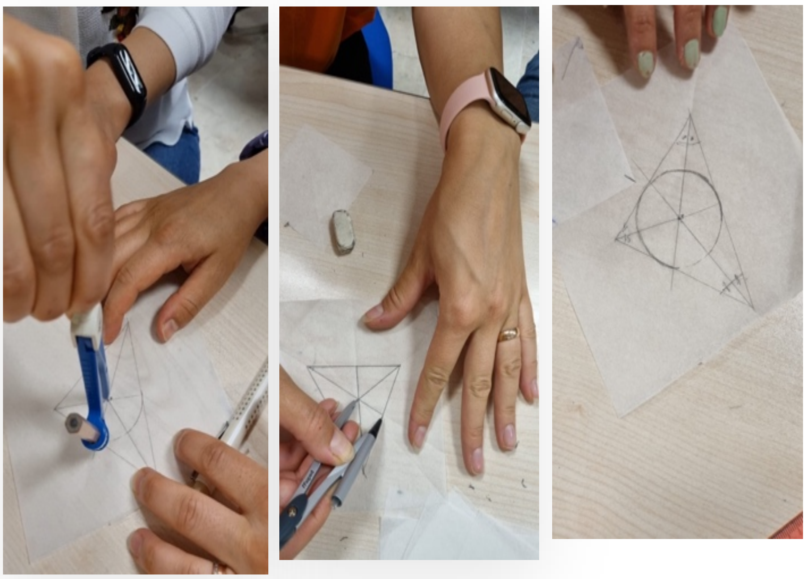

Figure 8. Finding the inscribed circle of a triangle

In the next stage, the teachers folded the medians of the triangles drawn identically to the other triangles on another sheet of patty paper and found their cutting points. After this, the teachers were given a thicker cardboard on which to copy the patty paper. They marked the same triangle and its medians on this cardboard and the teachers were asked how they could find the center of barycentre of the triangle (with the pencils in their hands). Thereupon, the teachers determined that the center of barycentre of the triangle they copied was balanced by placing the barycentre on the tip of the pencil. The teachers were asked how they would find the ratio in which the medians divided barycentre. All groups marked the length of the lower part of the barycentre on the patty paper with the help of another sheet of patty paper and wrote their assumptions by measuring that the upper part was twice the lower part. Here, the transparency property of the patty paper helped them make the measurement (Figure 9).

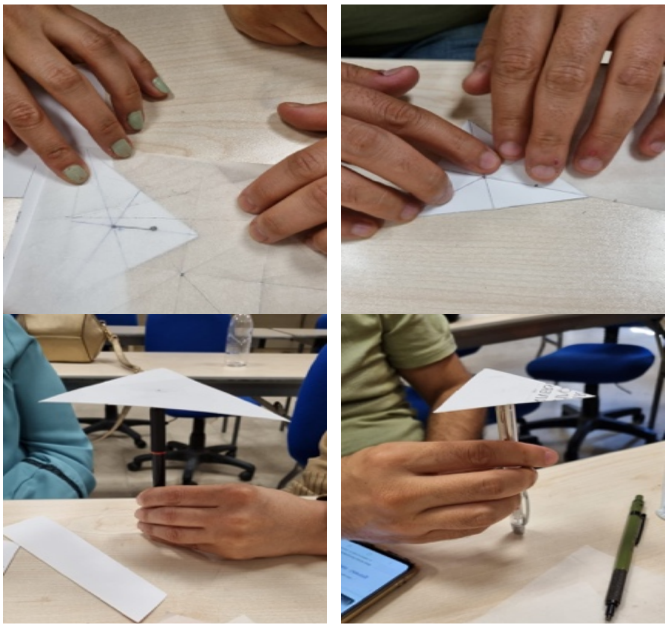

Figure 9. Discovery of properties related to the intersection point of the medians of a triangle

Then the teachers were asked to copy the same triangle onto another piece of patty paper and fold the heights of the same triangle to find the intersection point of the heights. The teachers reached the heights by finding the shortest distance from a point to a line. They folded the height from the point they took at one corner of the triangle to the opposite side of the triangle as the shortest distance; while doing this, they paid attention to the fact that the opposite side coincided while folding and formed a right angle (Figure 10).

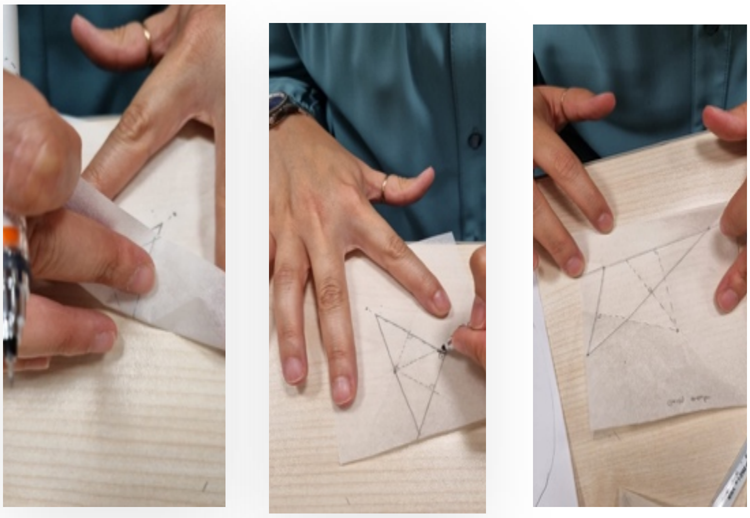

Figure 10. Finding the intersection point of the heights of a triangle

The teachers were given time to find the Euler line. When the teachers were asked what the Euler line was, they stated that they did not know. The teachers were asked to put the identical triangles on top of each other and it was stated that the points they found (three out of four points) had a property. Due to the transparency property of the patty paper, all groups concluded that the intersection points of the medians, medians and altitudes could be on a side-by-side line. All groups determined the Euler line (Figure 11).

Figure 11. Constructing the Euler Line

After the foldings and measurements related to triangles were completed, folding activities were carried out with the teachers to discover the properties of certain quadrilaterals. The groups were given two rulers, one thick and one thinner than the other, to fold the properties related to rhombuses and parallelograms. First, the teachers were asked to draw a rhombus and a parallelogram using these rulers (assuming that both sides of the rulers are parallel to each other). The groups drew the rhombus using a single ruler and the parallelogram using two rulers (Figure 12). Later, the teachers were asked to define the rhombus and parallelogram, and patty paper activities were conducted on their properties. The groups revealed the properties of the rhombus and parallelogram by folding patty paper along their diagonals, sides, and angles and performing measurements.

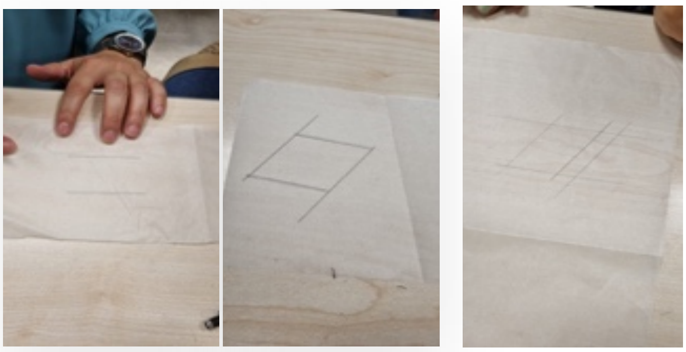

Figure 12. Constructing the rhombus and parralellogram

They folded the rhombus along its diagonals and then demonstrated in various ways that these diagonals bisect each other perpendicularly. For example, the second group folded the diagonals twice to form a right triangle, showing that the rhombus is composed of four right triangles. Another group divided the rhombus into two equal parts, dropped a perpendicular from the vertex to the diagonal, and showed that this line coincided with the diagonal.
In the case of the parallelogram, they showed that the diagonals do not intersect perpendicularly, are of different lengths, but still bisect each other. They folded the diagonals, then placed another piece of patty paper over the existing one to compare the lengths, demonstrating that the diagonals bisect each other (Figure 13).

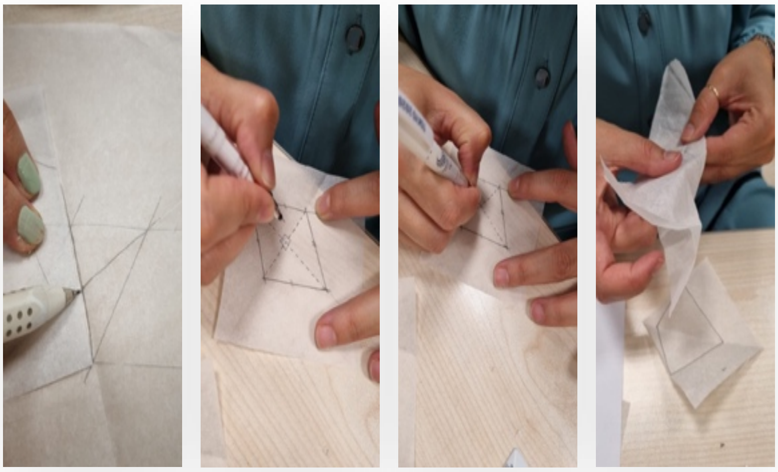

Figure 13. Diagonal properties of rhombus and parralellogram

Teachers have discovered properties such as the sum of adjacent angles of a rhombus and parallelogram being supplementary, and the opposite angles being equal, by measuring with another piece of patty paper (Figure 14).

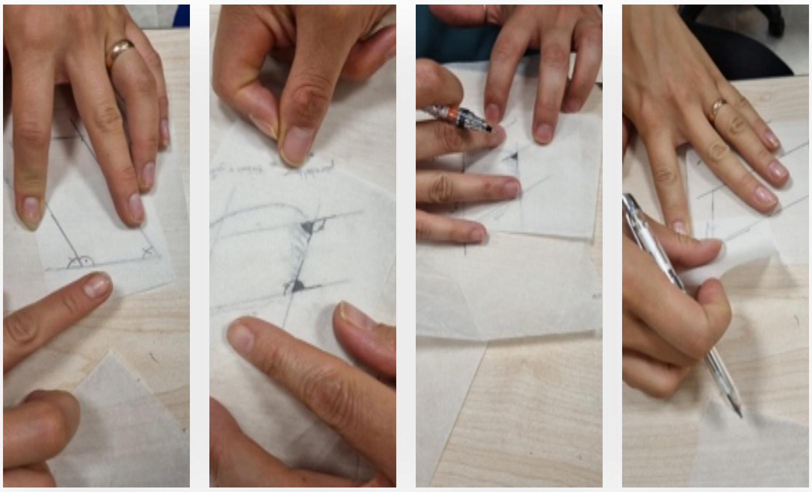

Figure 14. Angle properties of parralelogram and rhombus

Later, the teachers made folds to explore the properties of the rectangle. First, they used a ruler to form the parallel long sides; then, they folded perpendicular lines to the parallel sides of the long sides (when they folded the other long sides on top of each other, a right angle was formed, dividing the shape exactly in half (Figure 15).

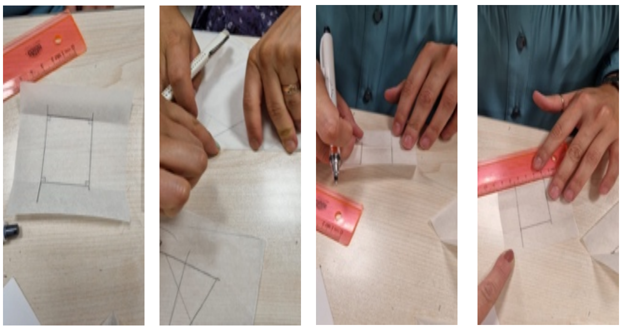

Figure 15. Discovery of the properties of the rectangle

In the next step, the teachers proceeded with the researcher's statement, "Let’s form a quadrilateral with two pairs of equal adjacent sides," to form the deltoid. The teachers, using an experimental approach, made observations about the angles, and with another piece of patty paper, they discovered the angle bisectors and perpendicularity conditions. They also discovered the equality of the angle measures using the patty paper again (Figure 16).

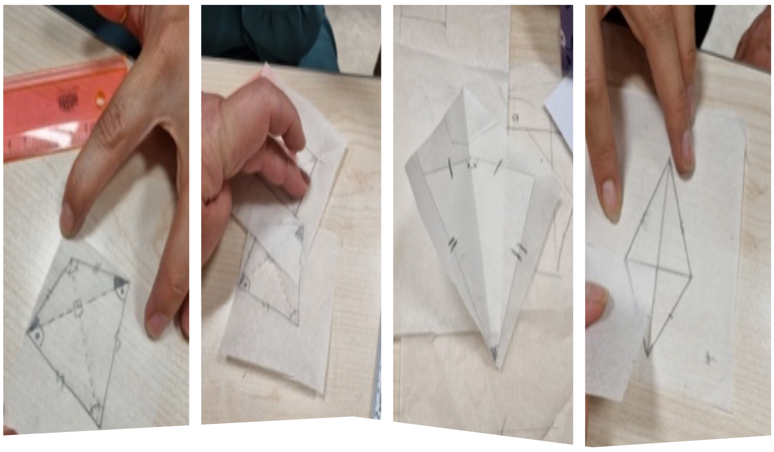

Figure 1. Discovery of deltoid and its properties

After this stage, the second part of the lesson plan shifted to area measurements. First, teachers were asked how they taught the area of a rectangle to their students. Teachers explained the reason why the formula for the area of a rectangle long side times short side with expressions is such as “*repeated sum of width and length*” and “*repeated sum of length and width*”; “*sum of unit squares*”, “*covering*”. Then, teachers were asked to reach the formula for the area of a parallelogram based on this. All groups drew the area of the triangle they created based on the short side inside the parallelogram on another piece of patty paper and moved it to the side where the other short side was and showed that the area did not change by creating a rectangle. Thus, they reached the conclusion that the area of a parallelogram with the same base length and height is the same as the area of a rectangle (Figure 17).

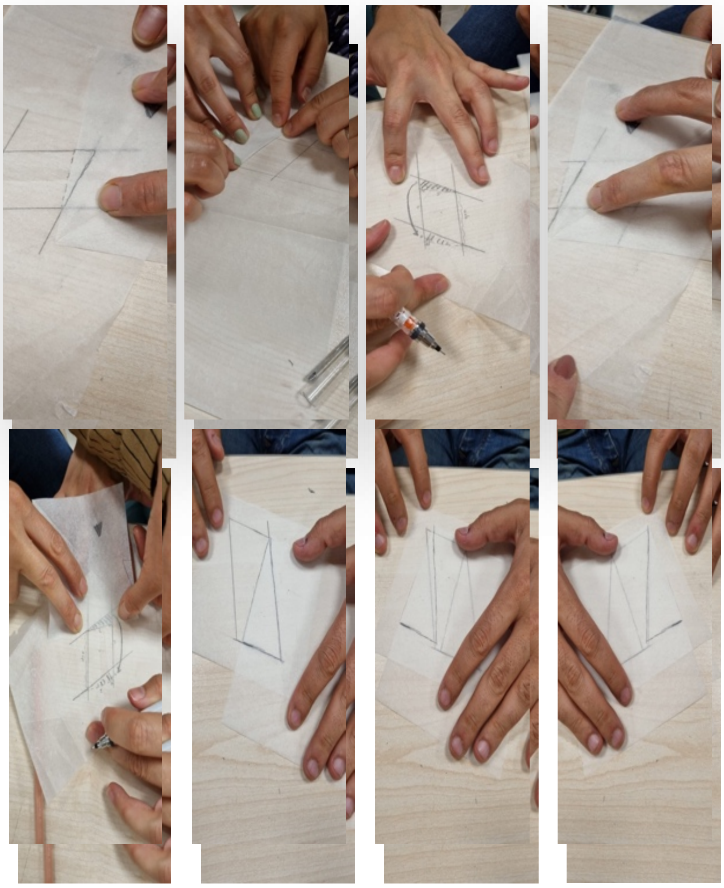

Figure 17. Finding the area formula of a parralellogram

In the next step, the teachers were asked to use the area of the parallelogram to derive the area of the triangle and to start by folding the triangle onto patty paper. Following this, the teachers copied an acute-angled triangle onto another piece of patty paper, flipped it, and joined the two halves to form a parallelogram. They easily demonstrated that the area of the parallelogram is twice the area of the triangle (Figure 18).

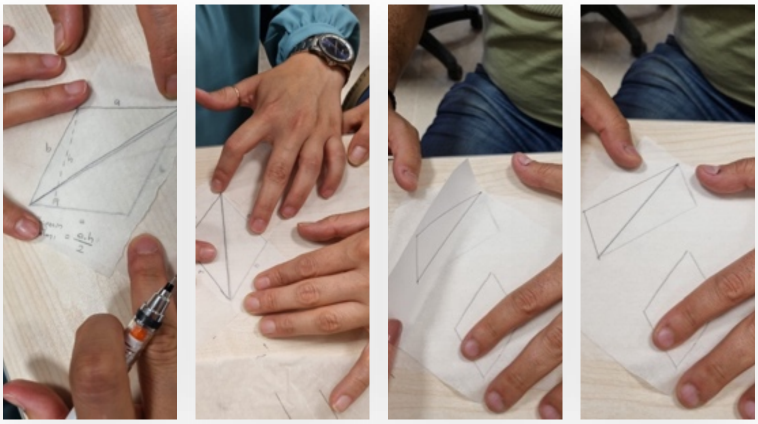

Figure 18. Finding the area formula of a triangle

In the final stage, the teachers were asked to use the formula for the area of the parallelogram to derive the formula for the area of the trapezoid. The teachers were asked to draw a trapezoid. All the groups, in a similar manner, copied the trapezoid onto another piece of patty paper and placed it by flipping it next to the original trapezoid. They easily observed that a parallelogram was formed and concluded that the area of the trapezoid is half the area of the parallelogram, thus deriving the formula for the area of the trapezoid.

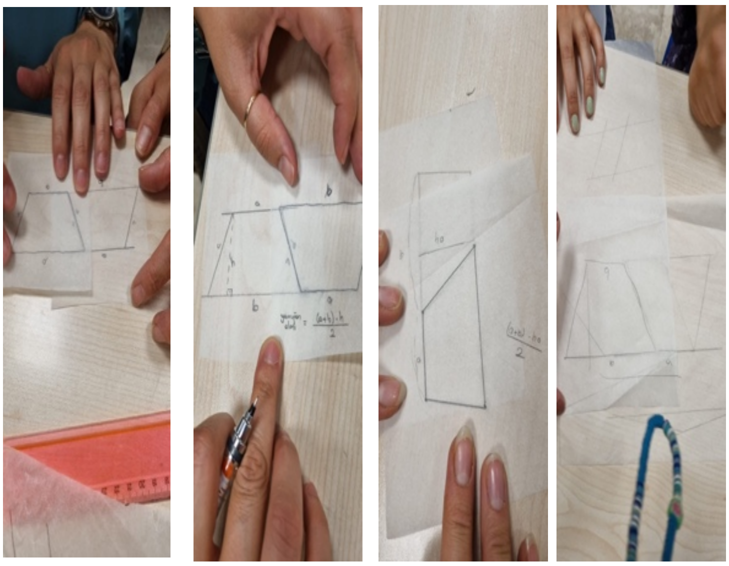

Figure 19. Finding the area formula of a trapezoid

Conclusion and Recommendations

In this study, patty paper folding activities (PFA) were conducted with mathematics teachers, focusing on the properties of specific polygons and area measurements. In general, it was observed that the teachers displayed a positive attitude during these activities, eagerly performed the folds, and enjoyed the discovery process. The fact that teachers find folding activities useful is an important factor in the implementation of these practices in their classrooms. Parallel to this assumption; Robichaux & Rodrigue (2003) concluded that teachers who have used origami in their lessons mentioned that students' understandings of the concepts incresed and they were well motivated. Van Wijk et al. (2023) in their study said that teachers who participated their study reported that folding allowed for dynamic representations that supported the transition from informal to formal mathematics and the practice of skills.

The activities were carried out in pairs for the discovery of patty paper geometry. It is suggested that these activities be conducted in pairs with students, as recommended by Serra (1994), through shared pairing. For future studies, it is recommended to conduct these activities with middle and high school students and, if possible, with larger samples. Since the activities were conducted with teachers, there was no need for shared pairing. It was determined that the teachers needed support to use the initial assumptions of the patty paper. For example, they were asked to start the folds by assuming the patty paper was a square, but from the first folds onwards, even though they knew that one of the corners of the patty paper formed a 90° angle or that the sides were equal, they preferred more complicated methods to discover these properties. For instance, instead of using a corner to form a right angle, they chose to fold perpendicularly by considering the distance of a point from a line. Later, with the researcher's guidance, they began using the properties of the patty paper more effectively. For example, after guidance, they made folds to form a right angle by placing one piece of patty paper over another, utilizing the transparency property of the patty paper. The reason the teachers did not immediately use these properties at the beginning is believed to stem from their limited experience with making assumptions in classroom teaching. Therefore, it is recommended that preparatory activities emphasizing the use of these properties be conducted at the beginning of patty paper folding activities.

It was observed that as the teachers gained experience in the PFA, they became more comfortable with discovering properties and measuring areas. Therefore, it is recommended that patty paper folding activities can be conducted frequently with students. As students gain experience, they will be more comfortable discovering properties and making assumptions. The teachers stated that they were not familiar with Euler's line. Since Euler's line is the intersection of the altitudes, medians, and heights of a triangle, it is considered beneficial to apply this activity at the 8th grade level. This way, students will have the opportunity to explore the auxiliary elements and properties of a triangle, leading to more lasting learning. In this study, the time allocated for PFA was limited for all polygon properties and area measurements. It is recommended that practitioners plan with wider intervals and provide longer time frames.

After the PFA was conducted with the teachers, the "Patty Paper Folding Activity Opinion Form" (PAF) was administered. The teachers responded in writing in the same classroom setting. All the teachers gave a positive response to the first question. Three of the teachers emphasized the transparency property. One teacher found it effective for seeing the reference measurements, while the other two teachers stated that the folds were drawable due to the transparency, allowing the activity to progress. They also mentioned that this helped make the concepts of perpendicularity, parallelism, and equivalence clearer, and the activity could continue by stacking the patty papers due to transparency. Two teachers mentioned that the idea of "*learning through discovery*" emerged as a result of the activities, and two other teachers stated that learning types such as "*permanent learning*," "*discovery learning*," and "*learning by doing*" were formed through these activities. Additionally, they described the activities as "*functional*," "*highly efficient*," and "*useful*". All the teachers gave a positive response to the second question about whether they would apply the patty paper folding activities in their own classrooms. One teacher mentioned that they already used patty paper folding activities in some classroom activities and would use them more frequently in the future. They explained this as "*the opportunity to achieve the highest outcomes with inexpensive materials*." Another teacher stated that they would use it as "*pre-discovery*" in some classes. When asked why, they responded with the phrase "*to encourage fruitful mathematical dialogues*."

The other four teachers indicated that they were considering using patty paper, explaining their reasons with phrases such as "*a material that provides learning by doing in a fun way*," "*stimulates prior knowledge*," "*accessible, economical*," "*promotes learning by doing*," "*increases interest in the lesson*," and "*enhances retention*." One teacher emphasized that it was important to check whether students folded correctly. Another teacher stated, "*I was only using it in the baking tray, but now it will be in the classroom too.*" In response to the third question about whether the activity could be adapted to any math topic, most teachers gave a positive answer. Four teachers specifically thought it could be more adaptable in the field of geometry, while one teacher pointed out, "*Although it can be adapted to any math topic, how functional it* *would be is confusing; the teacher must definitely do their own preparation before the lesson*," highlighting the need for the teacher to be prepared and patient. A teacher who said it could be adapted to geometry found it applicable in terms of providing visual representation, while another teacher stated that it could not be applied to topics like probability, but it could be applied to subjects such as fractions, with about 90 % adaptability. All the teachers gave a positive response to the fourth question about whether they would recommend patty paper folding activities to their fellow math teachers. When discussing areas for improvement, the teachers suggested that the process should be planned in detail, that instructions should be prepared, and that the sequence of topics covered was important. Similar to these expressions, Van Vijk et al. (2023) concluded that mathematics teachers find paper folding activities useful "to activate students by providing folding tasks' and 'to grow mathematical understanding by folding'.

In addition, the teachers agreed that students should be given enough time, that guidance should be done well, and that groups should consist of two-three people. One teacher also pointed out that there was an issue when copying some drawings and suggested that a material with less chance of slipping would be better. In response to the question, "*What part/topic of the patty paper folding activity did you find most useful or enjoyable*?" one teacher found Euler's line surprising, while two teachers found the topic of area in quadrilaterals very useful. They stated that it would increase retention and reduce misconceptions. Two teachers expressed that they found it useful for students to make assumptions. Two teachers mentioned that patty paper was a versatile material, as it replaced tools like rulers and protractors. One teacher talked about the benefits of transparency and usability with minimal materials. They said that the properties of geometric shapes became naturally visible with the patty paper. One teacher found the process of forming congruent shapes by stacking them to be useful. When asked if they had anything else to add, three responses were given. One teacher stated that the activity was enjoyable and instructive and that they were happy to participate. Another teacher reiterated that in-class time management and attention to groups should be handled carefully during the folding activity. One teacher emphasized the necessity of being well-prepared to manage the process. So, there were limitations in the details of the information received from the teachers due to time constraints. More efficiency can be obtained in more time. It is recommended to conduct proctracted studies.

Overall, it can be said that the patty paper folding activities conducted with the teachers were useful. It is recommended that teachers carry out patty paper folding activities in their classrooms. This activity, which was found useful by the teachers, is recommended to be practiced on students. Indeed, many studies (Adom &Adu, 2020; Aksoy& Işıksal Bostan, 2024; Bornasal et al., 2021; Boakes, 2008; Galicha & Lazaro, 2022; Demirci & Çontay, 2023; Empson & Turner, 2006; Friedman & Rittberg, 2021; Gürbüz et al., 2018; Febriani, et al., 2023; Morye, 2025; Patkin & Canner, 2010; Robichaux & Rodrigue, 2003; Subaar, et al., , 2010; Van Wijk et al., 2023) revealed the positive effects of paper folding activities based instruction on students improvement. In this sense, experimental studies are recommended. As a continuation of the explorations with patty paper in the focus of polygons, studies in which the hierarchical properties of polygons will be discussed in an experimental manner are recommended.

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