Modelling and Vibration Analysis of Powertrain System

Mehmet Onur Genç[1], Burak Budak[1], Necmettin Kaya[2]

[1]*Valeo Automotive Systems, Bursa, Turkey*

Abstract

   Developments in vehicle technology reveal that increased demands on vehicle comfort and fuel efficiency in the vehicles require to design more efficient powertrain systems. Vibrations originated in the engine need to be well optimized and controlled at desired limits. For this purpose, modal analysis for vehicle components is the main analysis method to evaluate the vibration behaviours. During vehicle design, important parameters for vehicle vibration such as inertia and stiffness are investigated and modified values to be replaced to get lower vibration values by means of the modal analysis. In this study, the truck powertrain system is modelled with real system values by the 1-D analysis software program and modal analysis is performed to find mode shapes and vibration magnitudes of the powertrain system components. Mode shapes of the clutch damper was investigated by changing damper stiffness values and the other components variables were taken constant to evaluate the stiffness effects on the system by modal analysis.

Keywords: Clutch damper spring stiffness; modal analysis; vehicle vibration; powertrain system modelling; powertrain system design; vehicle comfort; vibration simulation

1. Introduction

   Vibration is the mechanical oscillation movement for a mass or system around an equilibrium point. A statically balanced object oscillates at a certain frequency depending on its mass and its stiffness. It is called the natural frequency of that mass. Calculation of these frequencies and determination of their shape is important for engineers in terms of solving vibration-induced problems. Vibrations originating from engine are present their effects in almost all components of the vehicles. Vibration is an unwanted movement in vehicles and causes disturbing noises and energy loss at the same time. For this reason, it is aimed to bring the vibration to the minimum level while designing the systems in the vehicle. Damping is the reduction, limitation or prevention of vibration. For this purpose, vibration damping systems are designed to make the vehicle has worked more efficiently and more comfortable. In the vehicle design, the vibration performance of the systems is analyzed by simulations with finite element and tests, so their effects are evaluated. In this study, the system model and vibration analysis of the powertrain in the vehicle were performed and their effect on vehicle performance was examined. For this purpose, damper stiffness values of clutch were considered as variable and the vibration behavior of the powertrain system was evaluated in detail. The clutch disc showed at figure 1. is a high-strength structure that allows torque transmission between the flywheel and the clutch plate by friction, allowing the torque interruption at desired time to perform gear shifting.
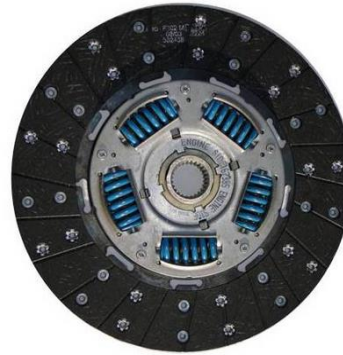

Figure 1. Clutch Disc (Valeo technical documentations)

   This construction is expected to damp engine vibrations during torque transmission, so the damper springs used in the disc are an important damping element. Helical compression springs with various stiffness values are used according to the vehicle torque levels. The most important point that when these springs are used in clutch discs, springs have to compansate the vehicle torque. Stiffness values and geometric properties are of great importance while the spring is selected according to the vehicle torque (figure 2).

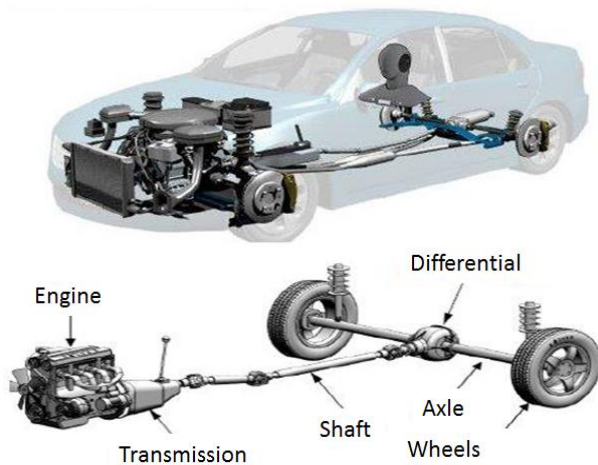Figure 2. Clutch Disc Damper Springs (Valeo technical documentations)

Figure 3. Vehicle powertrain system

Many studies have been carried out in the literature regarding the vibration of the vehicles. These studies have examined that the vibration effects of the clutch system as well as the other components such as engine, transmission.

Smith [1] examined the natural frequency of vehicle components and determined that the optimal weight increments to be made in the transmission system by dividing the vehicle's forced frequencies into modes increase the moment of inertia of the system reduced the natural frequency and improved the comfort. When the critical damping value is reached, it is observed that the increase in amplitude at the resonance frequency does not continue. It has been observed that this improves comfort by reducing vehicle vibrations. Acar et al. [2] proposed improvements to the clutch design by analysing frequency levels in a Matlab program and performing vibration level optimization studies for 3-cylinder engines with higher fuel economy which have more effective design and comfort requirements than the 4-cylinder engine for vibration damping. In their study, the design of the three-stage clutch spring, which meets the

vehicle vibrations at different stages has been studied. Sofian et al. [3] examined the frequency of the gearbox vibrations and compared their vibrations according to different gearbox types. They have observed that engine vibrations must be within the ranges specified by the gearbox manufacturers and vibration values not in this range causes comfort reduction in vehicle and reduced life of the gearbox mechanical components. Brandt et al. [4] investigated the time-dependent vibrations at the frequency level by FFT method and interpreted the dynamic optimization study of the power transfer system by harmonic analysis. They used harmonic values in vibration and noise analysis by grouping the components in the vehicle according to their natural frequencies. Keeney and Shih [5] conducted harmonic investigations in order to reduce the noise effect of the radial vibrations in the power transmission system and investigated the causes of vibration and noise originating from the engine by order track analysis. Jadhav [6] studied the design of transmission gears and the optimization of clutch-damping factors by applying harmonic analysis, interpreted the frequency ranges at which gearbox vibrations and noises occur. Hwang et al. [7] interpreted the dynamic responses of the system by examining natural frequencies and engine-forced vibrations in the frequency range by making a dynamic model of a rear-wheel drive vehicle in their work.

Miyasato et al. [8] studied the vibration frequency ranges generated by the forced frequencies in the automotive power transmission system in their work, and interpreted the improvements that occurred with the variation of the clutch spring coefficient in the transmission frequency and noise frequency range. Mazzei et al. [9] have determined the frequency ranges by performing modal analysis on the heavy vehicles in their work and interpreted the work to reach the result by making to the stiffness and mass changes at the places where improvement is required.

In this study, the dynamic behaviour of the system is examined by modelling the powertrain system of a heavy vehicle. The modal analysis of the power transmission system was used to investigate the response of the vibrations at the frequency levels.

2. Material and Method

2.1 Comfortable driving range, frequency analysis and vibration

The comfortable driving range represents the engine revolution per minute (RPM) most frequently used by drivers under normal driving conditions. It is requested vibrations caused by the engine in the comfortable driving RPM range should be low, therefore suitable comfort RPM range is one of the most important issues in vibration and acoustics during vehicle design. This range is more limited in heavy trucks compare to passenger cars. For instance, in a gasoline passenger car, the most commonly used RPM

range is between 1500 to 3500 RPM (rev./min.), whereas the average comfort RPM range of truck it corresponds to 1000-1800 RPM.

When the vibrations coming from the engine are high, the comfort of the vehicle will reduce and in the long term it leads to damage to the mechanical system components. The damped natural frequency in dynamical systems provides benefits in terms of lifetime and cost of mechanical systems in addition to improving vehicle use comfort. In order to rotate the crank while the vehicle engine is running, explosions occur in a certain row in the cylinders, causing sinusoidal vibrations in the vehicle. In case of the force of vibration frequency is equal to the natural frequency on vibrating structure, the vibration amplitude tends to increase. This increase in vibration levels prevents the system operating correctly and also causes damage to the mechanical system. For this purpose, damper spring stiffness is determined by frequency analysis in order to reduce vibrations in resonance regions.

The application of damped natural frequency in dynamical systems provides benefits in terms of lifetime and cost of mechanical systems, in addition to increasing vehicle comfort. The damper springs used on the clutch discs reduce the resonance frequency of the system and the vibrational amplitudes resulting from the force of the system, thereby providing more comfortable and safe working intervals in the vehicle. If a structure is given a motion input while in equilibrium point, the structure begins to vibrate at a certain frequency, called natural frequency which is dependent on its mass and its stiffness. In case of a structure that makes forced vibrations under dynamic force, the vibration amplitude tends to increase its value if the frequency of the force is equal to natural frequency of the structure.

The frequency is the number of oscillations per unit time, expressed in Hertz (Hz). When a system's natural frequency f (Hz) is calculated, equation 2 is used together with stiffness k (N / mm) and mass m (kg);

$$f = \frac{1}{2}\sqrt{\frac{k}{m}} \qquad (1)$$

For the vibration analysis of rotational components such as powertrain systems, we may pay attention to angular frequency. It can be called 'circular frequency' as well. Angular frequency is used to measure rotation rate. One complete revolution is equal to 2π radians, therefor angular frequency will be equal to $2\pi*$ Frequency (Eq. 3)

$$Wn = 2\pi \ f \ rad/sec \qquad (2)$$

$$Wn = \sqrt{\frac{k}{m}} \qquad (3)$$

If we consider the unit as period 'T' which represents the one complete revolution, the equation can be shown at Equation 4. Period is the duration of one cycle in repeating events.

$$Wn = 2\pi/T \qquad (4)$$

Critical damping ratio ζ is used to define the damping rate in the system. The critical damping ratio has an important role in determining the characteristics of the oscillation amplitudes occurring in the system. In addition, the actual damping c (Ns / m) in the system and the critical damping ζ can be correlated. Critical damping ratio can be classified under 3 different situations; overdamped: critical damping ζ >1, critically damping: critical damping ζ =1, underdamped: critical damping ζ < 1. If the system is overdamped, there would be no any oscillations or if the system has continuous vibrations, oscillations decrease within the time. For the critically damping situation, no any change will occur during the dynamic conditions. If the system is underdamped, continous oscillations are expected under dynamic conditions. Critical damping ratio is given in Eq.5

$$\zeta = \frac{c}{2\sqrt{k \times m}} \qquad (5)$$

2.2 Damped natural frequency and vibration

Damped natural frequency in dynamic systems is requested properly due to life, cost and comfort issues for the vehicles. When the clutch damper stiffness is lowered, the oscillation amplitudes occurred as a result of engine forcing vibrations is reduced. As a result RPM values, at which high vibration is occurred, will be lower and enable to wider comfort ranges. The system's damped natural frequency W_d, critical damping ratio ζ, and undamped natural frequency W_n, is shown in the Equation 6. Also same equation was illustrated by means of the different way in Equation 7;

$$W_d = Wn\sqrt{1 - \zeta^2} \qquad (6)$$

$$W_d = \sqrt{Wn^2 - \left(\frac{c}{2m}\right)^2} \qquad (7)$$

Angular acceleration (rad / s^2) is the change in angular velocity in unit time. The angular acceleration is defined as the first derivative of angular velocity with respect to time, and the second derivative with respect to time of angular displacement. In the power transmission system, the vibrations originating from the engines are represented by the angular acceleration (Figure 4). The amount of vibration can be expressed by amplitude. Amplitude is the amount at which the greatest amount of motion is separated from the average value or equilibrium point in harmonic vibration. The vibration amplitude can be examined in 3 groups as peak to peak (P-P), zero to peak (0-P) and RMS (root mean square) (square root of sum of squares). Peak to peak

(P-P) and RMS values were taken into account in this study. The RMS value is the value that represents the effective value of the vibration and the vibration which is felt in the vehicle by passengers. The RMS value is equal to 0.7 times the value of 0-P in simple harmonic motion (Figure 4).
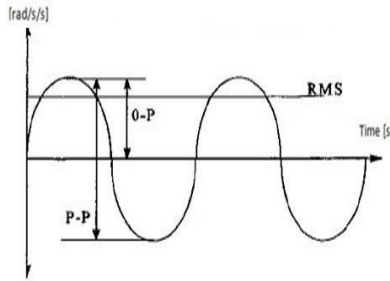

Figure 4. RMS (Root mean square) amplitude values

2.3 Modal analysis

Modal analysis is a method used to determine the dynamic characteristics of a system, including the natural frequency, damping values and mode format which is a value dependent on the structural phase. In the power transmission system, the frequency of the engine excitation frequency and the resonance frequencies of the system components are compared with each other to see the vibration characteristics. By collecting the same directional vibration coordinates at the frequency level, it finds the natural frequencies of the individually defined system components. In this way, system vibration levels and desired frequency levels can be controlled to determine the most comfortable driving needs.
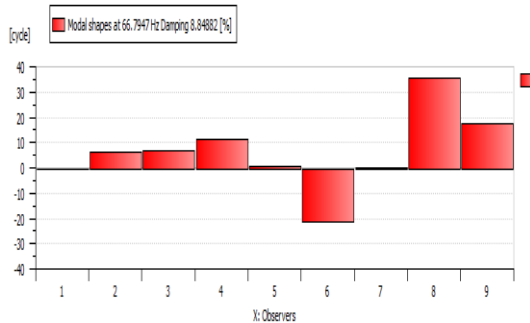

Figure 5. LMS Amesim modal analysis (Valeo technical do cumentations)

By analysing the modal analysis graphs above (figure.5), you will find out that which component has high amplitude in a total of 9 parts powertrain system and you have an idea about which improvements should be made in these components. First of all, it can be seen what kind of reactions the system gives in the forced vibrations coming from the engine by finding the frequencies. The limitations of the system have to be known before the improvements to be

made in vibration analysis. Before optimizing the values of stiffness and mass in the system components to change the vibration values, dynamic constraints must be considered. For example, the increase in mass should be controlled by calculations to match vehicle geometry, compatibility with dynamic forces, and the effect of fuel economy. With this method, the frequency of the vibration that forced the system was found. In this way, while examining the system vibrations and noises, engine-induced vibrations and non-engine-induced vibrations can be distinguished using the FFT method. Time-dependent vibrations are analysed at the frequency level to perform dynamic analysis of system vibrations. Time-dependent vibrations are transformed from the time level to the frequency level by the FFT method (Fast Fourier Transformation). In the FFT method, mixed signals are separated and harmonic ones are determined and separated into harmonic orders.
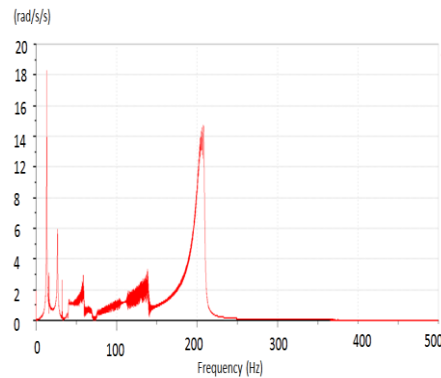

Figure 6. FFT Method for vibration analysis

As can be understood from the graph, when the harmonic frequencies 3N, 6N and 9N produced by the engine reach the frequency 205 Hz, the gearbox enters the resonance area and the highest vibrations are exposed to this frequency. To reduce these vibrations, we increase the amount of damping by lowering the clutch damper spring stiffness. In this way, at the transmission it can be obtained that lower-amplitude vibrations to prevent damage to the mechanical parts and provide a more comfortable driving option for the vehicle users.

First, the vibration analysis and interpretation of the vehicle by the 'eigenvalues' command mentioned in the previous section in which the resonance modes of the vehicle are located in the same direction coordinates can be examined. This command uses the FFT base to give parts and frequencies that are high in the vehicle's vibration values. The effects of changes in mass and stiffness values on vehicle vibration will be observed in these analyses.

Powertrain system in which power and torque flow occurs is the main part of the vehicle. Clutch provides torque transmission by friction between engine and gearbox, in addition high vibrations are expected to damped by clutch

to prevent gearbox from any mechanical damages. Gearbox producers share the vibration limits of their own gearbox with vehicle producers in order to be in desired vibration limits to prevent possible mechanical damages. Otherwise, gearbox is damaged within the time and results in noise, vibration and mechanical problems. Recent years, low-stiffness dampers have been demanded widely by many vehicle brands. As seen in Figure 7., owing to using of low-stiffness dampers, gearbox vibrations are decrease drastically and resonance frequency goes lower RPM. This graph shows that by means of the usage of low stiffness damper in clutch discs lower vibration values and wider RPM comfort ranges are acquired.
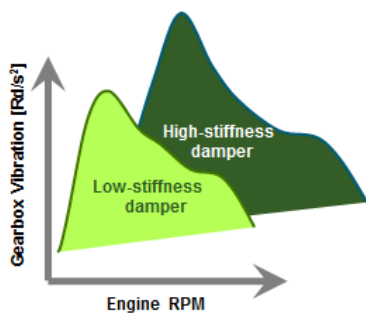
Figure 7. Low-High Stiffness Damper Comparison
(Valeo technical documentations)

3. Modelling and analysis

In this section, the vehicle power transmission system is modelled and the vibrations of the components in the power transmission system are examined. The basic parts of the power transmission system consist of the engine, clutch, gearbox, main shaft, fittings, wheel and equivalent vehicle mass.

LMS Imagine.Lab 1D physical model of powertrain system created in Amesim software is given in Figure 8. When the model is constructed, the stiffness, inertia, damping that represent the relevant parts of the vehicle were selected in powertrain library. System modelling has been done by assigning such properties. In the model the signal which enables the system initation is obtained from the library and installed to engine in modelling of vehicle. The vibration analysis of the installed model can be done by the modal analysis commands mentioned in the previous section. In this section, the values given in the table will be entered as input and comparative analysis will be done.

For 300 Nm / ° and 100 Nm / ° clutch disc damper stiffness values, which represent the situation in which two different clutch damper stiffnesses are used, the effect of the power transmission system on the vibrations will be investigated and compared.
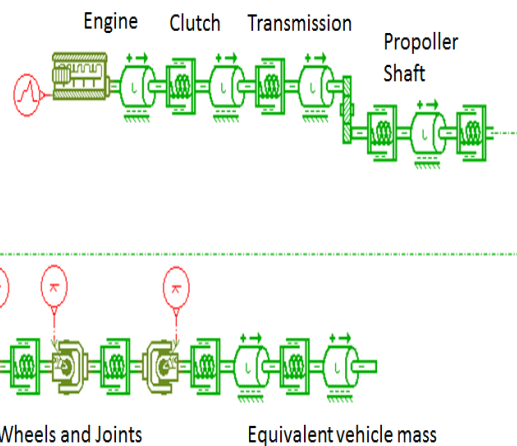

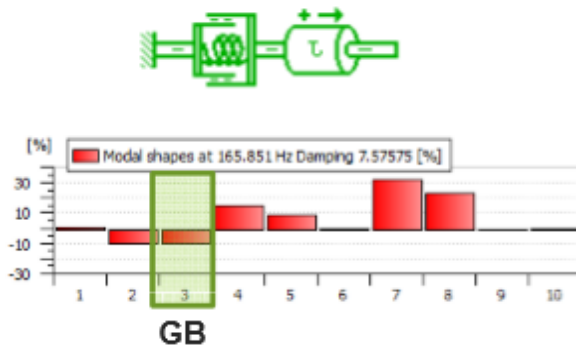Figure 8. Powertrain System Modeling

Figure 9. Modal Analysis Representing on Powertrain Sytem
(Valeo technical documentations)

Powertrain system components vibration behaviours can be illustrated on the modal analysis graphs to see its level and magnitude. Figure 9. tells us that Gearbox which is represented with number 3, covers the %10 of the total oscillations at 165.81 Hz. At this frequency it can be seen that column 7 and 8 have high vibration levels and we may say that this frequency level is the mod of the components which are represented with number 7 and number 8.

Table 1. Powertrain System Appointed Values

Component	Stiffness (NM / °)	Inertia (kg.m2)
1 - Engine	-	1
2 - Clutch	1. case 300	0.005
	2. case 100	
3 - Transmission	3000	0.1
4 - Shaft	1000	0.5
5 - Wheels and Joints - 1	1500	0.07
6 - Wheels and Joints - 2	1000	0.09
7 - Wheels and Joints - 3	500	0.12
8 - Wheels and Joints - 4	1000	0.12
9 - Equivalent vehicle mass	1000	250

4. Results and Discussions

4.1 Powertrain system analysis for 300 NM/° Clutch Damper Stiffness

The frequency values of the different modes of 9 different parts in our model were obtained. In the first case, the values obtained by taking the clutch stiffness 300 Nm / ° are as follows. The 7 eigenvalues are given by the analysis program shows us that 7 different frequency vibration modes are exist and its characteristics depend on the stiffness and inertia values given in table 1. In modal analysis components numbers were shown from 1 to 9 in table 1. The frequency values of the different modes of 9 different parts in our model were obtained. In the first case, the values obtained by taking the clutch stiffness 300 Nm / ° are as follows. The 7 eigenvalues are given by the analysis program shows us that 7 different frequency vibration modes are exist and its characteristics depend on the stiffness and inertia values given in table 1. In modal analysis components numbers were shown from 1 to 9 in table 1.
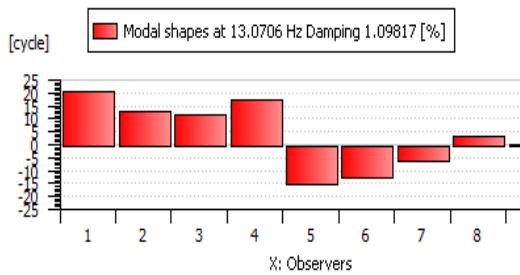
Figure 10. 300 Nm/° 13 Hz modal analysis

Figure 10. explains us that at 13.0706 Hz frequency level, when the vibrations coming from engine are equal to 13.0706 Hz, the biggest oscillations, which showed with number 1, occur in the engine. It gives us a idea that this frequency level is overlap with the natural frequency of engine. It covers the %20 of total vibration values. In contrast, at this frequency the low oscillation about %1 occurred in the equivalent mass vehicle. In addition, as we can observed that the total oscillations ratio will be %100 for 9 components in total. The vibrational directions in the modal graph show that the vibration modes vibrate in different directions. We can called it 'out of phase' situation. Therefor we are able to classified that the components number 1, 2, 3, 4, 8 and 9 are vibrate in the same direction and they are in the same phase. However the powertrain system components which showed with 5, 6, 7 are vibrate at reverse direction and can be called 'out of phase'.
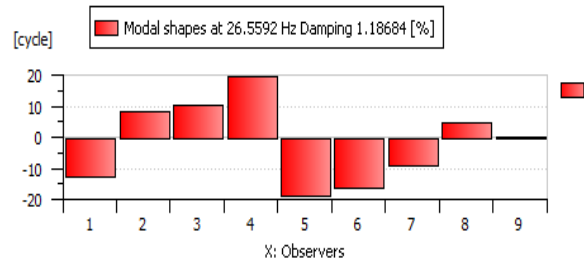
Figure 11. 300 Nm/° 26 Hz modal analysis

It is clearly seen that at 26.5592 Hz frequency level the biggest oscillations occur in the shaft (propeller shaft) which indicated with number 4 (Figure 11). In addition to propeller shaft, wheels and joints which showed with number 5 have big oscillations close to propeller shaft. The following modal analysis at different frequencies have been illustrated below;
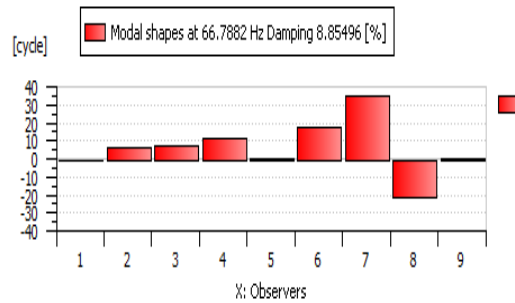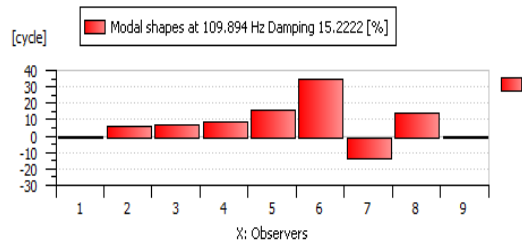
Figure 12. 300 Nm/° 66 Hz modal analysis

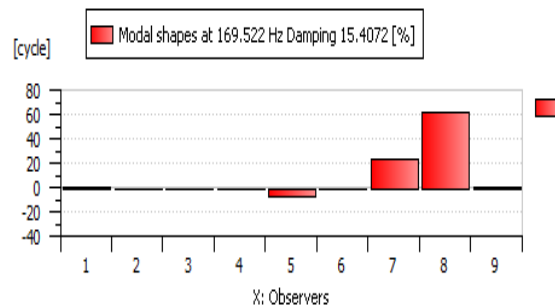Figure 13. 300 Nm/° 109 Hz modal analysis
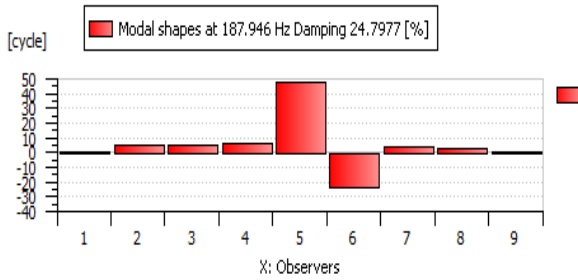
Figure 14. 300 Nm/° 169 Hz modal analysis

Figure 15. 300 Nm/° 187 Hz modal analysis

Figure 12, 13, 14 and 15 have shown similar vibration characteristics as a result of modal analysis and the maximum oscillations are clearly seen on the wheels and joints groups.
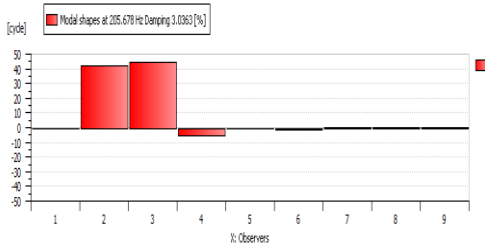

Figure 16. 300 Nm/° 205 Hz modal analysis

Figure 16. tells us that at 205.678 Hz frequency level the biggest oscillations occur in the clutch and transmission, in conversely the other components of the powertrain system have the low oscillations at this frequency level. It means that when the vehicle is excited at 205 Hz forced vibration by engine, forced vibration frequency will overlap with natural frequency mode of the clutch and transmission systems. This overlapping means that high vibration will be seen on these parts, then if the stiffness and inertia values of the components change, it leads to change on natural frequencies.

If this frequency level is considered inconvenient for the members of the powertrain system, in order to get desired design, inertia and stiffness values of the components would be changed to requested level. For this purpose, clutch damper stiffness value was changed and examination was done for the 100 Nm/° stiffness value at next section.

4.2 Powertrain system analysis for 100 NM/° clutch damper stiffness

At this stiffness value, 7 vibration modes were given by the analysis program. The maximum vibration frequency of a engine with a damper stiffness 300 Nm / ° was occured at 13 Hz frequency level, whereas in the usage of 100 Nm / ° clutch damper stiffness, it is seen that it drops to 10 Hz frequency level. This situation is directly proportional to

the 1 st equation given in chapter 2. When the powertrain system stiffness value k is reduced, the engine natural frequency dropped and it overlapped with the forced vibration at the level 10 Hz frequency (figure 17).
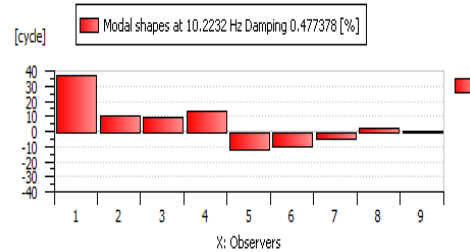

Figure 17. 100 Nm/° 10 Hz modal analysis

This comparison clearly shows us that changes on the power transmission system components effect the whole system and cause variations in the vibration modes of the powertrain by changing the equivalent stiffness and inertia values. As will be seen also in the following graphs, this change in clutch damper stiffness affects the equivalent stiffness of the entire powertrain system and reduces the vibration mode frequency of the other components which causes the vehicle to vibrate at lower rpm values. This result also shows us that in the design of the vehicle, changing the stiffness and inertia values of the components in terms of vibration optimization enables to change the natural frequency of the components and to vibrate at the desired frequency levels.
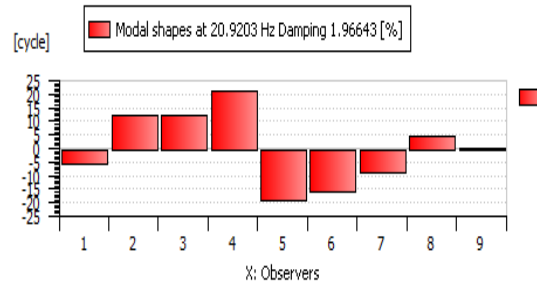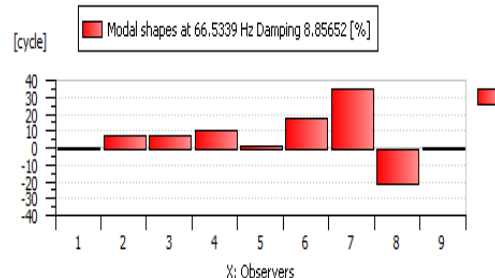

Figure 18. 100 Nm/° 20 Hz modal analysis

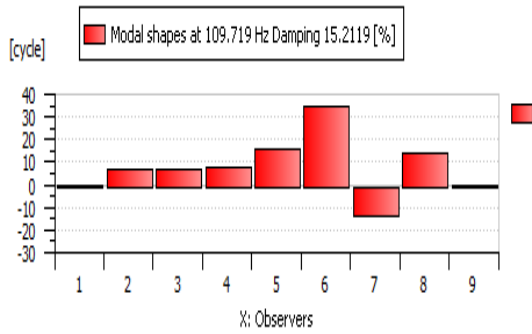Figure 19. 100 Nm/° 66 Hz modal analysis

Figure 20. 100 Nm/° 109 Hz modal analysis
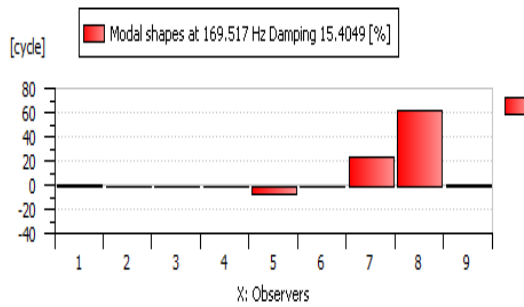

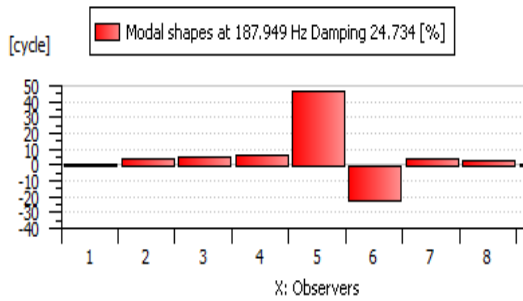Figure 21. 100 Nm/° 169 Hz modal analysis
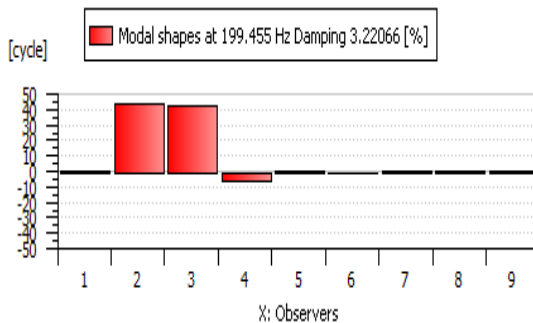

Figure 22. 100 Nm/° 187 Hz modal analysis

Figure 23. 100 Nm/° 199 Hz modal analysis

As expected, the equivalent rigidity of the system was reduced with lower clutch damper stiffness, then the maximum vibration mod levels of clutch and transmission shown by numbers 2 and 3 have been dropped from 205 Hz to 199 Hz. This graph also shows us that except the clutch and gearbox there is no other power transmission system member enters the vibration mode at the frequency of 199 Hz (Figure 23).

5. Conclusions

In this study, torsional vibration behaviours of the modelled truck powertrain system is investigated and analysed by modal analysed with 1-D modelling analysis. The truck powertrain system is modelled with LMS Amesim software and values are taken from the real truck systems. Powertrain system components vibration mode frequencies were found by modal analysis and their behaviours were analysed based on frequency levels. For this purpose, clutch damper spring stiffness was taken as variable and the other components were considered as constant. Clutch damper stiffness at different values were investigated and observed to evaluate vibration system behaviour of powertrain. According to analysis results, using of low stiffness dampers in powertrain system is lowers the vibration prominently and change the mode shape characterization. Results show that using of 100 Nm/ ° stiffness damper gives more damped vibration outputs and improves mode shapes instead of 300 Nm/ °. This approach has major importance during design process which provides cost and time saving. By taking countermeasures prior to system design such as modal analysis with AMESim, system vibration characterization can be constricted between required frequency levels. Consequently, this study shows us that vehicle vibrations should be investigated deeply with modal analysed and necessary countermeasures can be taken to prevent unwanted vibration values, for this reason clutch vibration levels at different stiffness values were compared with each other and mode shapes were found to illustrate the behavior change.

Nomenclature

c   damping coefficient (Ns /m)
f   frequency (Hz)
fd   damped natural frequency (Hz)
f_n undamped natural frequency (Hz)
fs   friction coefficient
k   stiffness   (NM/s)
N   number of friction surface
Rm   mean   radious of the torque transmission surface (mm)
RMS   vibration felt in vehicle (Root mean square)
RPM   rotation of engine per minute (Rev./Min.)
Sec   second
T   torque (NM)
ζ critical damping ratio

References

[1] Smith, R., 'Changing the Effective Mass to Control Resonance Problems.' Sound and Vibrations, Cilt no:242, 14-17, 2001.

[2] Acar, H. , Gul, C. ,Avcı, M., 'Clutch Disc Torsionel Characteristics Optimization to Reduce Idle and Gear Rattle on Passenger Car.' International Automotive Congress, Belgrade-Serbia, 22-28, 14-15 Nisan, 2015.

[3] Sofian, M., Hazry D, Ishak, I., 'A Study of Vibration Analysis for Gearbox Casing Using Finite Element Analysis.' Proceedings of International Conference on Applications and Design in Mechanical Engineering ,Malaysia, 10E1-10E7, 2009.

[4] Brandt, A. ,Lagö, T. ,Ahlin, K. ,Tuma, J., 'Main Principles and Limitations of Current Order Tracking Methods.' Sound and Vibrations, Cilt no:281, 19-22, 2005.

[5] Keeney, S. , Shih, S., 'Prediction and Control of Heavy Duty Powertrain Torsional Vibration.' Rockwell International, 1992-11-01, 922481, 1992.

[6] Jadhav, S. 2014. 'Powertrain NVH Analysis Including Clutch and Gear Dynamics.' SAE Technical Paper Series.

[7] Hwang, S. , Stout, J. , Ling, C., 'Modeling and Analysis of Powertrain Torsional Response.' SAE Technical Paper Series, 1998-02-23, 980276, 1998.

[8] Miyasato, H. , Siminatto, G. , Junior, M., 'Study of the Gear Rattle Phenomena in Automotive Powertrain Systems.' 21st International Congress of Mechanical Engineering, Natal-Brazil, 24-28 Ekim, 2011.

[9] Mazzei, A. , Alzahabi, B. , Natarajan, L.K. 2002. 'Analysis of Drivetrain Bending Response For a Heavy Truck Driveline.' *SAE Technical Paper Series*.

[10] Miyasato, H. , Siminatto, G. , Junior, M., 'Linear Powertrain Models For NVH Phenomena Evaluation.' International Symposium on Dynamic Problems of Mechanics', Buzios-Brazil, 143-152, 17-22 Şubat, 2013.

[11] Valeo Automotive , 'Valeo Technical Documentations', 2017.

[12] Skup, Z., 'Damping of Vibrations in a Power Transmission System Containing a Friction Clutch.' Journal of Theoretical and Applied Mechanics,Cilt 43(4), 875-892, 2005.

[13] Gambino, C., 'Vibration Damping Via Acoustic Treatment Attached to Vehicle Body Panels.' University of Windsor, Yüksek lisans tezi, Engineering department, Windsor-Canada, 2015.

[14] M. Y. Wang, W. Zhao, and R. Manoj, "Numerical modelling and analysis of automotive transmission rattle," Journal of Vibration and Control, Cilt no:8, 921–943, 2002.

[15] P. Fietkau and B. Bertsche, "Influence of tribological and geometrical parameters on lubrication conditions and noise of gear transmissions," Mechanism and Machine Theory, Cilt no: 69, 303–320, 2013.

[16] C. Canudas-de-Wit, P. Tsiotras, E. Velenis, M. Basset, and G. Gissinger, "Dynamic friction models for road/tire longitudinal interaction," Vehicle System Dynamics, Cilt no: 39, 189– 226, 2003.

[17] D. Centea, H. Rahnejat, and M. T. Menday, "Non-linear multibodydynamic analysis for the study of clutch torsional vibrations (judder)," Applied Mathematical Modelling, Cilt no:25, no. 3, 177–192, 2001.

[18] G. Zhao, Z. Jiang, and L.Chen, "Linear analysis for performance of dual mass flywheel with centrifugal pendulum vibration absorbers system," Telkomnika—Indonesian Journal of Electrical Engineering, Cilt no:11, 2371–2376, 2013.