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ELASTO-PLASTIC STRESS ANALYSIS IN A COMPOSITE PLATE WITH A SQUARE HOLE

Kare Delikli Kompozit Bir Plakta Elasto-Plastik Gerilme Analizi

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ÖZET

Bu çalışmada, çelik-alüminyum kompozitinden imal edilmiş kare delikli plağın üniform yayılı çekme yükleri altında elasto-plastik gerilme analizi yapılmıştır. Çözümde dokuz düğümlü izoparametrik dikdörtgen eleman kullanılmıştır. Problemin çözümünde sonlu eleman modeli olarak otomatik ağ dağılımı ve özel bilgisayar programı kullanılmıştır. Delik civarındaki plastik bölgelerin dağılımı ve artık gerilmeler değişik oryantasyon açılarında incelenmiştir.

ABSTRACT

In this study, the elasto-plastic stress analysis of the plates with a square hole manufactured steel-aluminum composite is made under the uniform tension loads. In the solution, two dimensional isoparametric rectangular element with nine nodes is used. The automatic mesh generation is used in finite element model and the special computer programmes are used to solve problem. Distributions of plastic regions near the hole and variations of residual stresses are investigated in the different oriantation angles.

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1. INTRODUCTION

With the increasing thecnology, it has been investigated to find new materials. Especially in space, aircraft and defence industries, light and high strength materials are required. So, the metal-matrix materials are used.

Karakuzu (1992), investigated increasing of strength of composite plates with semicircular holes under the elasto-plastic loads. Owen (1983), worked on anisotropic elasto-plastic finite element analysis of thick and thin plates and shells. Theo (1981), studied stress around rectangular holes in orthotropic plates. Yehia (1981), investigated finite element analysis of elasto-plastic fibrous composite structure. Zienkiewicz (1969), obtained solution of engineering problems " initial stress " finite element aproach.

In this study, the elasto-plastic stress analysis of the plates manufactured from steel- aluminum composite is made under the uniform tension loads. Distribution of plastic region near the hole is investigated in the different orientation angles.

2. STRESS-STRAIN RELATION IN AN ORTHOTROPIC MATERIALS

Two dimensional stress-strain relation in an orthotropic material can be written as Karakuzu (1993):

$$\begin{aligned} \varepsilon_1 &= C_{11}.\sigma_1 + C_{12}.\sigma_2 \\ \varepsilon_2 &= C_{12}.\sigma_1 + C_{22}.\sigma_2 \\ \gamma_3 &= C_{66}.\tau_{12} \end{aligned} \tag{1}$$

where, elastic constants, C_{11}, C_{12} and C_{22}, are the function of young modules, poisson ratio and shear modules.

Stress-strain relation:

$$\sigma_{1,2} = C.\varepsilon_{1,2} \tag{2}$$

where,

$$\begin{aligned} \sigma_{1,2} &= \{\sigma_1, \sigma_2, \sigma_3\}^T \\ \varepsilon_{1,2} &= \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}^T \end{aligned} \tag{3}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{4}$$

where,

$$\begin{aligned} C_{11} &= E_1/(1-\nu_{12}.\nu_{21}) \\ C_{22} &= E_2/(1-\nu_{12}.\nu_{21}) \\ C_{12} &= E_2.\nu_{12}/(1-\nu_{12}.\nu_{21}) \\ C_{66} &= G_{12} \end{aligned} \tag{5}$$

3. CALCULATION OF ELASTO-PLASTIC STRESSES

Various computational procedures have been used with success for a limit range of elasto-plastic problems utilising the finite element approach. Two main formulation appear. In this first, during an increment of loading, the increase of plastic strain is computed and treated as an initial strain for which the elastic stress distribution is adjusted. This approach manifastly fails in ideal plasticity is postulated or if the hardening is small. The second approach is that in which the stress-strain relationship every load increment is adjusted to take into account plastic deformation. With properly specified elasto-plastic matrix this incremental elasticity approach can successfully treat ideal as well as hardening plasticity.

From the computational point of view the incremental elasticity process has one serious disadvantage. At each step of computation the stiffness of the structure is changed and iterative process of solition are necessary to avoid excessive computer times. The initial stress method is developedly Zienkiewicz as an alternative approach to the incremental elasticity process. By using the fact that even in ideal plasticity increments of strain prescribe uniquely the stress system (while the reverse is not true forideal plasticity) an adjustment process is derived in which initial stresses are distributed elastically through the structure.

This approach permits the advantage of initial process (in which the basic elasticity matrix remains unchanged) to be retained. The process appears to be the most rapidly convergent. To start elasto-plastic stress analysis, this method uses one dimensional tensile specimen in elasto-plastic region, then moves on to the two and three dimensional stress case. For a tensile specimen loaded just over the elastic region ($\varepsilon_{total} = \varepsilon_1$). Stress σ_x is calculated linear elasticity, thus the stress σ_{f1} as shown in Figure.1 below is given by the following form;

$$\sigma_{01} = \sigma_x - \sigma_{f1} = \sigma_1 - \sigma_{f1} \tag{6}$$

By using σ_{01} one obtains increasing stress value

$$\sigma_2 = \sigma_x + \sigma_{01} \tag{7}$$

which corresponding to ε_2. The stress difference between σ_2 and real stress at ε_2 gives σ_{02}. σ_3 is obtained by replacing σ_{02} in Equation.7. The following analog iteration steps lead to the point corresponding to the elasto-plastic strain ε_n and stress σ_x. Where σ_{01} is the initial stress.

For calculation of stress in two dimensional cases equivalent is usually obtained according to Von Misses Criterion (Distortion Energy Theory). The equivalent stress in plane stress case is

$$\bar{\sigma} = \left\{0.5((\sigma_x - \sigma_y)^2 + \sigma_x{}^2 + \sigma_y{}^2) + 3\tau_{xy}{}^2\right\}^{1/2} \tag{8}$$

where σ_x, σ_y and τ_{xy} are the stress components.

Therefore initial stress can be obtained for plastic region is one dimensional case,

$$\sigma_0 = \bar{\sigma} - \sigma_f \tag{9}$$

where σ_f is obtained from σ, ε_{total} diagram a uniaxially loaded tensile specimen. But the initial stress cannot be exactly described as in Figure.1 in two dimensional case. It can be mathematically described as follows,

$$\{\sigma_0\} = \{\sigma_{0x}\ \sigma_{0y}\ \tau_{0xy}\} \tag{10}$$

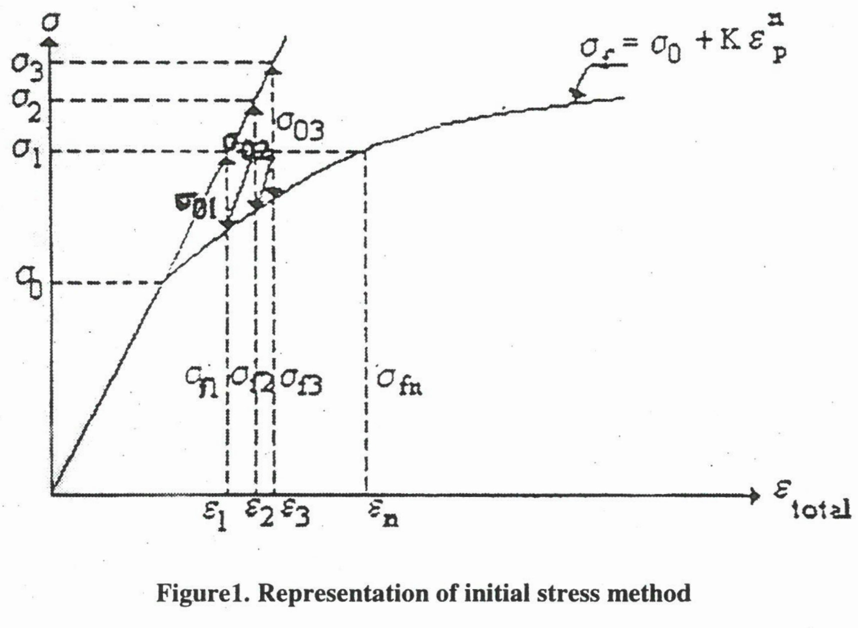

Figure1. Representation of initial stress method

where σ_{0x}, σ_{0y} and τ_{0xy} are components of the initial stress in plane stress case. By using the following formal, one obtains

$$\{\sigma_0\} = \{\sigma\}.\{\sigma_0 / \bar{\sigma}\} \tag{11}$$

where the component of $\{\sigma_0\}$ are proportional to elastically calculated stress. The related equivalent stress value is equal to $\{\sigma_0\}$obtained in one dimentional case, according to Equation.6.

$$\bar{\sigma} = \left\{ \frac{1}{2} \left[\left(\sigma_{0x} - \sigma_{0y}\right)^2 + \sigma_{0x}{}^2 + \sigma_{0y}{}^2 \right] + 3\tau_{0xy}{}^2 \right\}^{1/2} \tag{12}$$

The loading corresponding to the initial stress as follows;

$$\{F\} = \int_V [B]^T \{\sigma_0\} dV \tag{13}$$

First the solution vector is calculated for $\{F\}$, σ_{01}, mechanical loading in the first iteration step.

$$\{\delta\}_1 = [K]^{-1}.\{F\}_m \tag{14}$$

where $\{F\}_m = \{F\}_s + \{F\}_{\sigma_0}$ then the following iteration steps $\{\delta\}_i$, i=1,2,....,n are calculated until there is no difference between $\{\delta\}_i$ and $\{\delta\}_{i+1}$. Then the displacement vector is

$$\{\delta\}_n = [K]^{-1}.\{F\}_m \tag{15}$$

Finally the stress σ_n corresponding to $\{\delta\}_n$ in elasto-plastic region is calculated as

$$\{\sigma\}_n = [C][B]\{\delta\}_n \tag{16}$$

In the elasto-plastic region, residual stresses are found at the end of iteration as follows,

$$\{\sigma_{0i}\} = \{\sigma\}_L - \{\sigma\}_n \tag{17}$$

where $\{\sigma\}_L$ is the linear elastic stress obtained at the end of iteration. In the polar coordinates, residual stresses are written as,

$$\{\sigma_{0i}\}_{pe} = [T]\{\sigma_{0i}\} \tag{18}$$

where

$$[T] = \begin{bmatrix} Cos^2\theta & Sin^2\theta & 2Sin\theta.Cos\theta \\ Sin^2\theta & Cos^2\theta & -2Sin\theta.Cos\theta \\ -Sin\theta.Cos\theta & Sin\theta.Cos\theta & Cos^2\theta - Sin^2\theta \end{bmatrix} \tag{19}$$

is transformation matrix.

4. FINITE ELEMENT ANALYSIS

The geometry of plate is shown in Figure.2. In this application, the plates are loaded in various uniaxial tension and used orientation angles are 0^0, 30^0, 45^0, 60^0 and 90^0.

These loads are 70, 75, 80 MPa for 0^0, 45, 50, 55MPa for 30^0, 30, 35, 40MPa for 45^0, 20, 25, 30 MPa for 60^0 and 15, 20, 25 MPa for 90^0.

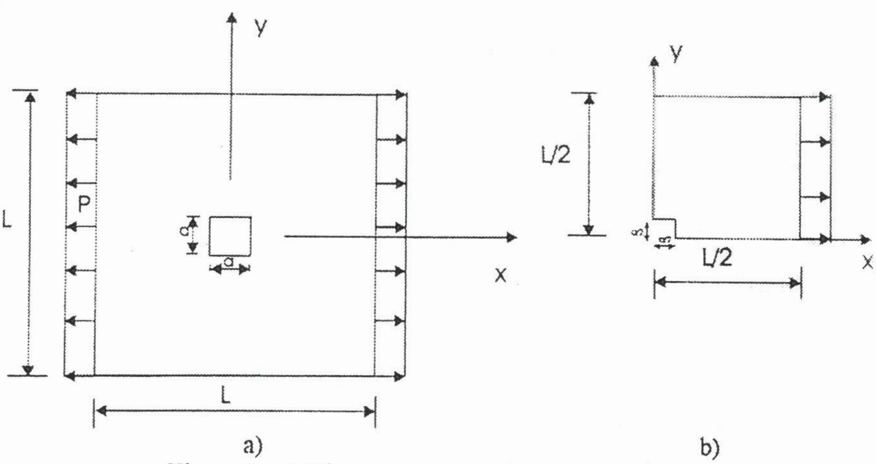

Figure 2. a) Whole square plate, b) A quarter plate

In the 0^0 and 90^0 angles because of symmetry with respect to x and y axis of shape, loading and material properties a quarter of the plate is taken finite element model. In the other angles, whole plate is used as finite element model.

These plates with a square hole are symbolically divided into finite elements as shown in Figure.3. These square holes are 20, 40 and 60mm.

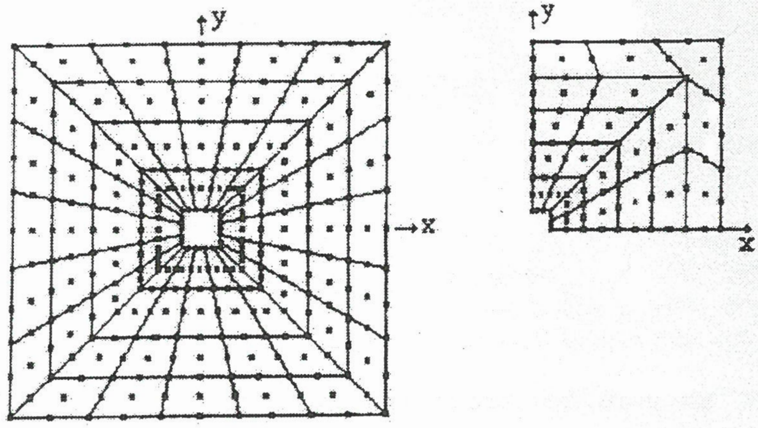

Figure3. Mesh generation whole and ¼ plate

In the solution of the problem, two dimensional isoparametric rectangular e-lement with nine nodes is used. The finite element model consist of 80 meshes and 360 nodal points.

5. RESULTS AND CONCLUSION

In this study, orientation of plastic regions and internal stresses for uniform tension loads of various square holes are shown. Stresses, which are caused by linear elastic events, are marked with opposite sign according to stresses which are composed within plastic regions after external forces are removed. So, in the condition of application of tension loads when after internal stresses has been occured, the stresses that neighbour to notch reduces the newest stresses and reduces the concentration of stresses that also neighbour to notch. Therefore, the plastic region becomes noncritical region. In this state, the critical region is the boundary region, because of the small amount of stress concentration the plate subject to more amount of loading according to the elastically plates loaded. By the way, the plate has more strength to higher amount of loads within using high strength materials.

If the compressive loads are applied after the formation of internal stresses, the stresses become dangerous due to internal and compressive load stresses have the same sign effect. As a result, if plate subjected to tension load, the internal stresses should be existed by tension loads and in the situation of compressive loading the internal stresses should be existed by compressive loads.

The following figures are draw by means of prepared computer program. The yielding points are pointed out and the area which contains these yielding points stayed in the plastic region. In the figures 4 and 5 because of symmetry, the figures show plastic regions on a quarter plate. In the figures 6, 7 and 8 the whole plate plastic region separation are shown.

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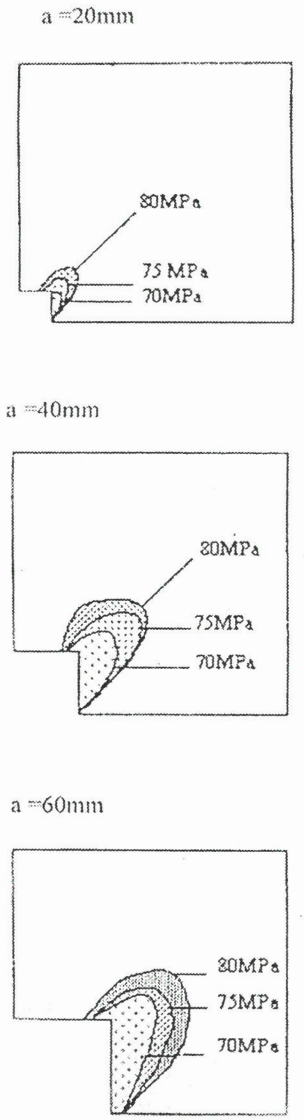


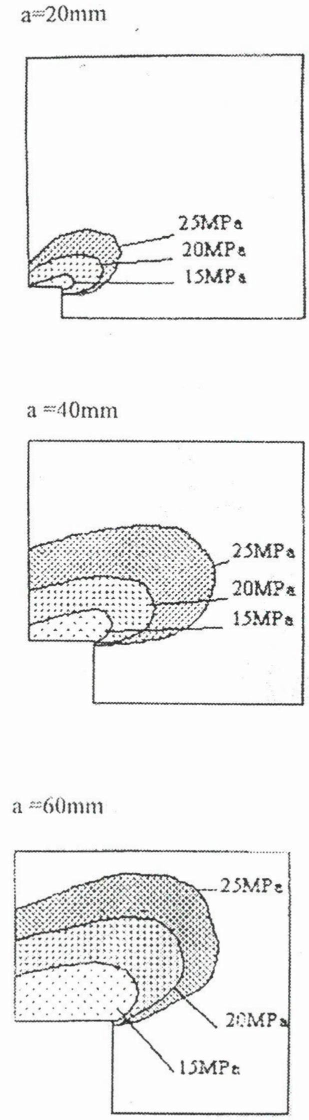
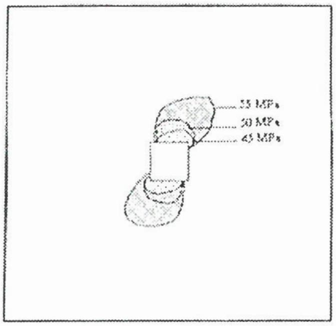


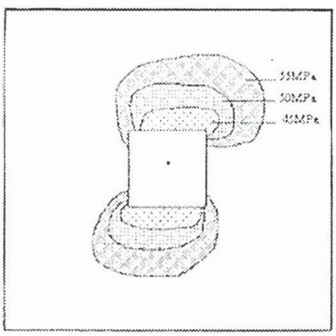
Figure 4. $\theta = 0^0$, Plastic regions on quarter plates, hole sizes 20x20 mm, 40x40 mm and 60x60 mm

Figure 5. $\theta = 90^0$, Plastic regions on quarter plates, hole sizes 20x20 mm, 40x40 mm and 60x60 mm

a=20mm



a=40mm



a=60mm

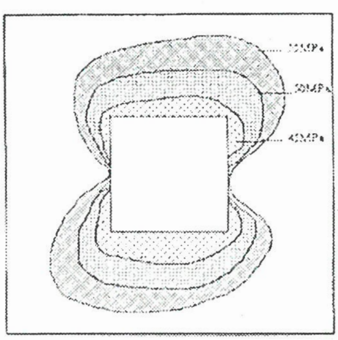
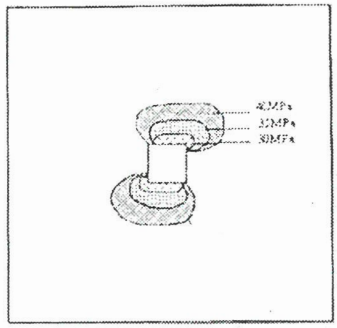
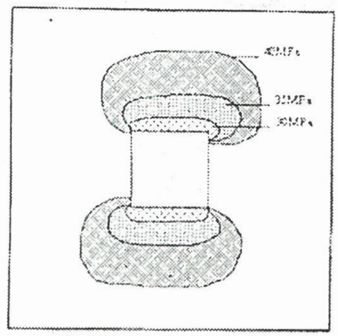


Figure 6. $\theta = 30^0$, Plastic regions on whole plates, hole sizes 20x20mm,40x40mm and 60x60mm

a=20mm



a=40mm



a=60mm

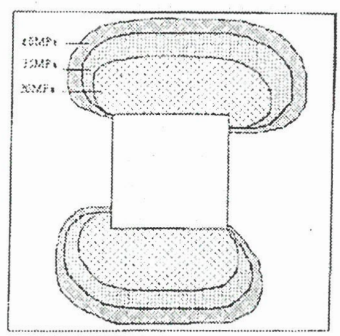


Figure 7. $\theta = 45^0$, **Plastic regions on whole plates, hole sizes 20x20mm,40x40mm and 60x60mm**

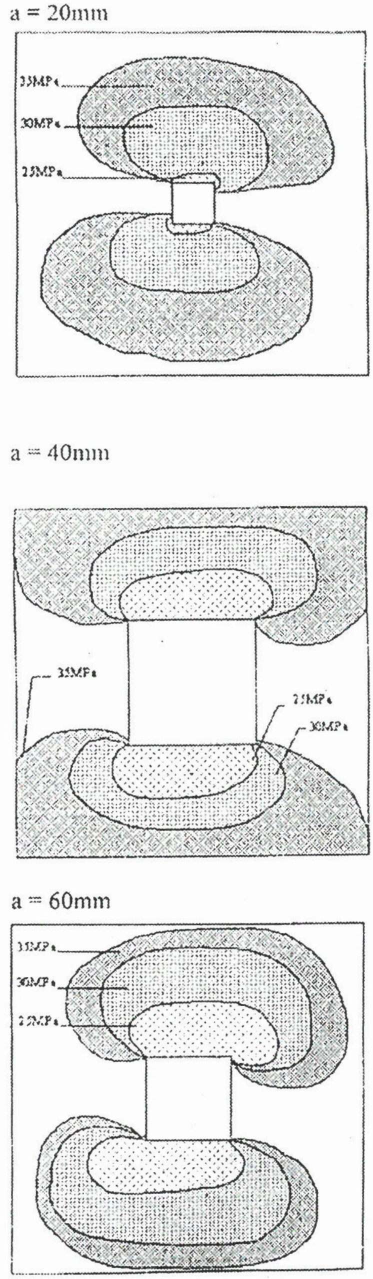


Figure 8. θ = 60^0, Plastic regions on quarter plates, hole sizes 20x20 mm, 40x40 mm and 60x60 mm