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Quasi-Periodic Photonic Structures: Fibonacci Fractal-Based Optical Filter Design Using ZnO/MgF_2

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Keywords

Fibonacci Photonic Crystals

ZnO

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Optical Filter

Abstract

This study investigates the optical properties of one-dimensional photonic crystals based on ZnO/MgF_2, designed using Fibonacci fractal sequences. The effects of structures based on standard Fibonacci (FFPC), inverted Fibonacci (IFFPC), and mirror symmetry Fibonacci (MSFFPC) sequences on photonic band gaps and light-matter interactions were theoretically analyzed. Calculations were performed using the Transfer Matrix Method (TMM). The analyses revealed that lower-order sequences offer broad and uniform transmission, while higher-order sequences exhibit complex optical resonances with narrower bandwidths. MSFFPC structures, characterized by their regular configurations and narrow bandwidths, are ideal candidates for applications requiring precise color selection, such as sensors and narrow-band optical filters. Conversely, IFFPC structures provide advantages for wide spectral applications due to their broad transmission bands. FFPC structures, offering a balanced performance, can be employed in both wide-band and narrow-band optical systems.

Cite

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1. INTRODUCTION

Photonics technology advancement and the integration with optoelectronic devices have led to the development of devices whose electrical and optical properties can be tuned. Therefore, innovative photonics-based device designs have a critical role in future technologies. Designing these structures with optimal parameters has become a significant requirement, particularly for precisely adjusting optical properties. Integrated photonic systems offer solutions for controlling optical characteristics based on the principle of "light management engineering" by altering the propagation of electromagnetic waves. Among these solutions, the deposition of dielectric mirror layers with one-dimensional (1D) photonic crystal properties stands out as an effective method. 1D photonic crystals are structures where the dielectric constant periodically varies in only one direction, forming a photonic band gap and providing optical reflection at certain wavelengths.

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Fibonacci photonic crystals, with their distinctive quasi-periodic structure, exhibit unique properties that differentiate them from conventional periodic photonic crystals. These structures, designed based on the Fibonacci sequence, offer exceptional advantages in fundamental optical processes such as light propagation and localization (Taherzadeh & Keshavarz, 2024). In addition to the diffraction and photonic band gaps observed in regular crystal arrangements, Fibonacci photonic crystals possess broad spectral sensitivity and a strong capacity for light confinement (Elsayed et al., 2024). These characteristics enable innovative solutions in various fields, including telecommunications, sensor technology, optical filters, and waveguides. For instance, the complexity offered by Fibonacci arrangements enhances light absorption at specific wavelengths, facilitating the development of efficient energy devices. Importantly, the optical properties of Fibonacci photonic crystals have been extensively examined through theoretical modeling and experimental studies. Research findings indicate that these arrangements have the potential to optimize light-matter interactions. Notably, the photonic band gaps achieved in aperiodic Fibonacci sequences provide an ideal platform for applications such as waveguides and sensors (Jiménez-Vivanco et al., 2024).

While knowledge about light wave propagation in completely ordered and disordered structures has rapidly advanced, the behavior of optical waves in the vast intermediate region between perfect order and disorder remains poorly understood. Fibonacci fractal photonic crystal (FFPC) structures, which can be described as non-Euclidean geometric configurations, exhibit fascinating optical properties. The transmission spectrum of Fibonacci systems includes forbidden frequency regions referred to as "pseudo band gaps," analogous to the band gaps of conventional photonic crystals (Capaz et al., 1990). Light waves are critically localized in the frequency regime outside these Fibonacci band gaps. In contrast to the completely disordered (Anderson-type) localization state, these critically localized states weaken non-exponentially, most likely by a power-law, and exhibit a rich, self-similar structure (Fujiwara et al., 1989). This makes these systems very interesting for light localization studies, as suggested by Kohmoto et al. (1987). Furthermore, FFPCs have been successfully utilized in the development of devices such as high-quality resonant microcavities (Escorcia-García & Mora-Ramos, 2009) and mirror with broad omni-directional photonic band gaps (Han & Wang, 2005; Srivastava et al., 2008). These applications underscore the versatility and potential of FFPCs in advancing photonic technologies (Tavakoli & Jalili, 2014).

ZnO/MgF_2 layered photonic crystals offer significant advantages in controlling light reflection and optimizing specific optical properties due to their multilayer structures. These systems are designed by alternately arranging MgF_2, with a low refractive index, and ZnO, with a high refractive index, resulting in a wide optical band gap. This characteristic makes them suitable for applications such as optical filters, sensors, and high-performance laser devices (Fujihara et al., 2001). The contrast between the low refractive index of MgF_2 and the high refractive index of ZnO creates wide omnidirectional photonic band gaps in these crystals. The layered structure is particularly optimized for anti-reflective coatings, creating a surface that minimizes energy loss and ensures high light transmission (Dai et al., 2006). MgF_2 provides thermal and mechanical durability, making it stable for high-temperature applications. In contrast, ZnO's high optical absorption coefficient enhances the system's photonic performance. The fabrication of ZnO/MgF_2 layered structures commonly involves techniques such as physical vapor deposition or sol-gel processes, which ensure the homogeneity and structural precision of thin films.

The optical properties of ZnO/MgF_2 photonic crystals offer an ideal solution for applications requiring wavelength-dependent light control. For instance, they can regulate light transmission at different wavelengths in optical filters, enhancing precision in spectral selection. In anti-reflective coatings, these structures reduce reflection losses over a wide spectral range, thereby improving the efficiency of optical devices. Optimizes light transmission at narrow bandwidths, especially high-performance lasers (Fujihara et al., 2001; Dai et al., 2006).

In this study, the optical properties of ZnO/MgF_2-based quasi-periodic Fibonacci fractal photonic crystal (Fibonacci Fractal Photonic Crystal, FFPC) structures were theoretically investigated with respect to different sequence parameters. In addition to the conventional Fibonacci sequence, Fibonacci fractal photonic crystals with inverted and mirror symmetry (IFFPC and MSFFPC) are analyzed. The effects of these structures on the photonic band gap properties, which can be tuned to the visible spectral region, were examined in detail.

2. MATERIAL AND METHOD

This study uses various Fibonacci-based sequences to construct different fractal structures for application in 1D photonic crystals. The optical Transfer Matrix Method (TMM) was utilized to calculate the optical properties of Fibonacci photonic crystal structures, including transmission, reflection, and absorption. The details of TMM calculations and its fundamental operation have been presented in our previous studies (Çetinkaya et al., 2021; Çokduygulular et al., 2023).

The FFPC structures created within the scope of this study are based on the classical Fibonacci sequence. The Fibonacci sequence can be defined as a particular ordering of binary digits (or symbols of any two-letter alphabet). Similar to Fibonacci numbers, the Fibonacci word is generated not through successive addition but by sequential concatenation. To describe this process, the initial sequences are set as S_0 = "A" and S_1 = "B". Here, A represents ZnO, and B represents MgF_2. Then, the sequence Sn is obtained by successive concatenation of the previous (S_{n-1}) and the preceding (S_{n-2}) sequences ($S_n = S_{n-1} + S_{n-2}$).

The IFFPC structures are based on the inverted Fibonacci sequence. The inverse code is simply the binary complement of a sequence, where all zeros are converted to ones and all ones to zeros. For example, if the binary representation of Sn is AAAABBAA, its inverted code, $(S_n)_{inv}$, becomes BBBBAABB. The inverted Fibonacci sequence is defined in this manner and is represented as a specific structural arrangement.

Lastly, MSFFPC structures are constructed based on the mirror-symmetry Fibonacci sequence. Mirror symmetry is a structure obtained by mirroring the binary code of a number and is constructed as follows: $S_n = S_{n-1} + (S_{n-2})_{ms}$. This approach generates distinct arrangement sets for FFPC, IFFPC, and MSFFPC, as provided in Table 1.

Table 1. *Sequence sets for FFPC, IFFPC and MSFFPC.*

FFPC		IFFPC		MSFFPC	
S_n	$S_{n-1} + S_{n-2}$	S_n	$S_{n-1} + (S_{n-2})_{inv}$	S_n	$S_{n-1} + (S_{n-2})_{ms}$
S_0	A	S_0	A	S_0	A
S_1	B	S_1	B	S_1	B
S_2	BA	S_2	BB	S_2	BA
S_3	BAB	S_3	BBA	S_3	BAB
S_4	BABBA	S_4	BBAAA	S_4	BABAB
S_5	BABBABAB	S_5	BBAAAAAB	S_5	BABABBAB
S_6	BABBABABBABBA	S_6	BBAAAAABAABBB	S_6	BABABBABBABAB

In the investigated FFPC, IFFPC, and MSFFPC systems, the central wavelength is set to the middle of the visible region, i.e., λ_0=550 nm. Each layer of the Fibonacci-based photonic crystal is designed with a quarter-wave thickness, which is expected to reveal quasi-periodic effects more prominently, i.e., $n_A d_A = n_B d_B = \lambda_0/4$.

3. RESULTS AND DISCUSSION

The transmission spectra of ZnO/MgF_2-based FFPCs at different orders were analyzed to investigate their optical behavior as a function of the sequence order. The transmission spectra were calculated using the Transfer Matrix Method. Figure 1 presents the transmission spectra of ZnO/MgF_2-based FFPCs constructed at various orders.

The spectrum exhibits a single wide and symmetric transmission band in the third-order sequence, with maximum transmittance observed around 550 nm. This indicates that lower-order sequences possess simpler optical properties and demonstrate a straightforward light transmission behavior. Moving up to the fourth order, a single passband is retained, but this band appears sharper. This suggests that the optical properties become more pronounced as the sequence order increases. The transmission spectra display a more complex structure with the transition to the fifth and sixth orders. Two distinct transmission bands are observed in the fifth order, while in the sixth order, these bands increase to three. This indicates that the structure incorporates more optical modes and exhibits increasingly complex interactions with light. The behavior observed across the sequence orders in FFPC highlights the influence of the quasi-periodic nature of the Fibonacci sequence on the optical properties.

The maximum transmittance is around 100% for certain modes in all cases, and at higher orders, the transitions are concentrated in narrower bandwidths. These features indicate that optical selectivity and mode diversity are enhanced as the sequence order increases. This demonstrates the ability of higher-order sequences to support more precise and complex light manipulation.

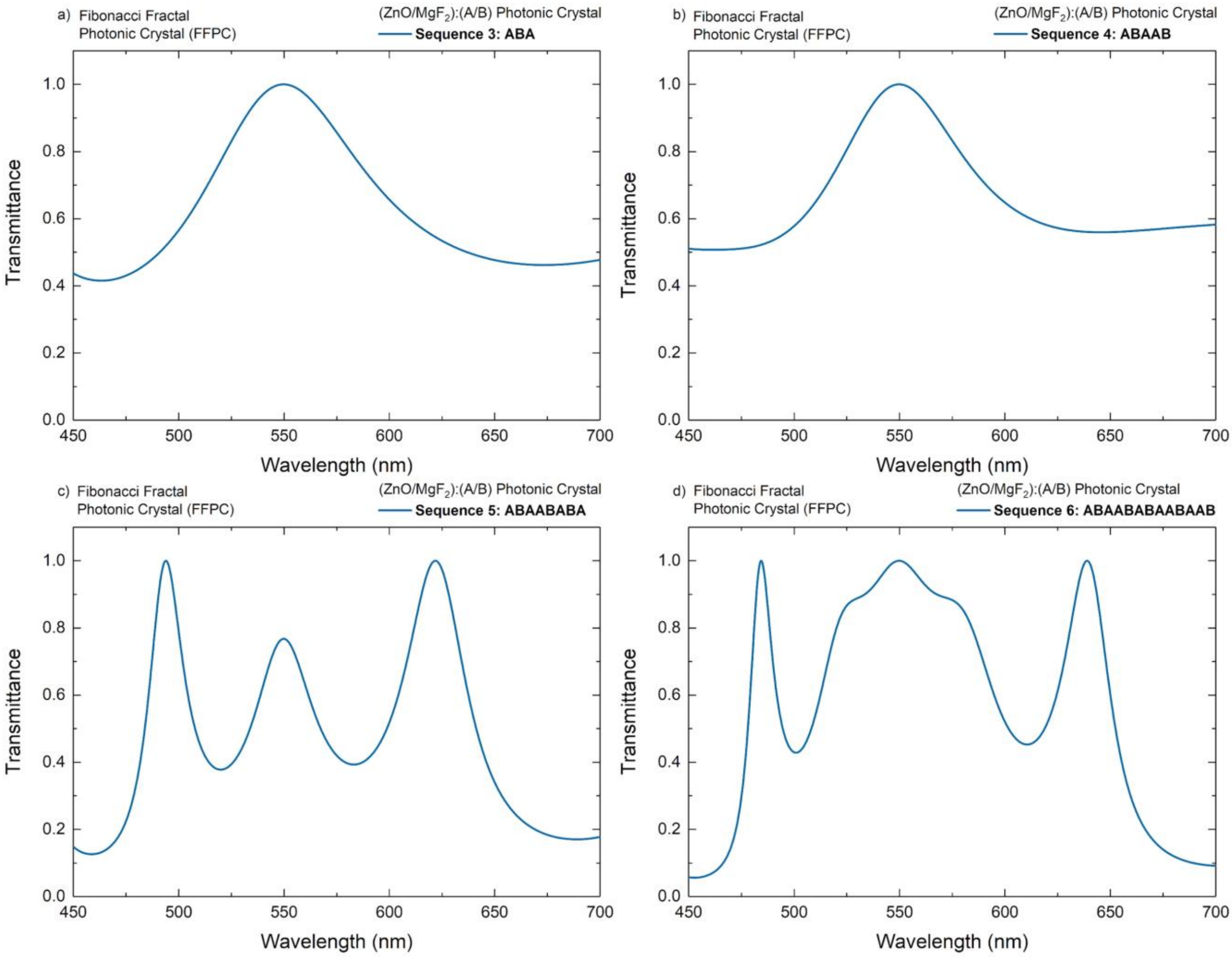

Figure 1. *Transmittance spectra of ZnO/MgF₂ based Fibonacci fractal photonic crystals with a) 3rd, b) 4th, c) 5^{th,} and d) 6th order alignments.*

The transmission spectra of ZnO/MgF_2-based IFFPCs at different orders were analyzed to investigate their optical behavior as a function of the sequence order. The transmission spectra were calculated using the TMM. Figure 2 presents the transmission spectra of ZnO/MgF_2-based IFFPCs constructed at various orders.

The third-order structures of the IFFPC sequence are characterized by a wide transition band in the transmittance spectrum, with the maximum transmittance observed at a wavelength of approximately 550 nm. This indicates that lower-order sequences exhibit simpler and more uniform optical behavior. When transitioning to the fourth order, more defined features emerge in the spectrum, although the overall width of the transition band remains largely preserved. In the fifth order, the spectrum becomes more complex. The transmission spectrum splits into multiple narrow-band transition peaks, indicating an increase in the optical complexity of the structure. This complexity becomes even more pronounced in the sixth order, with the spectrum displaying sharp and well-defined transition peaks. This behavior demonstrates that higher-order IFFPC sequences interact more strongly with light at specific wavelengths, exhibiting multiple optical modes and enhanced spectral selectivity.

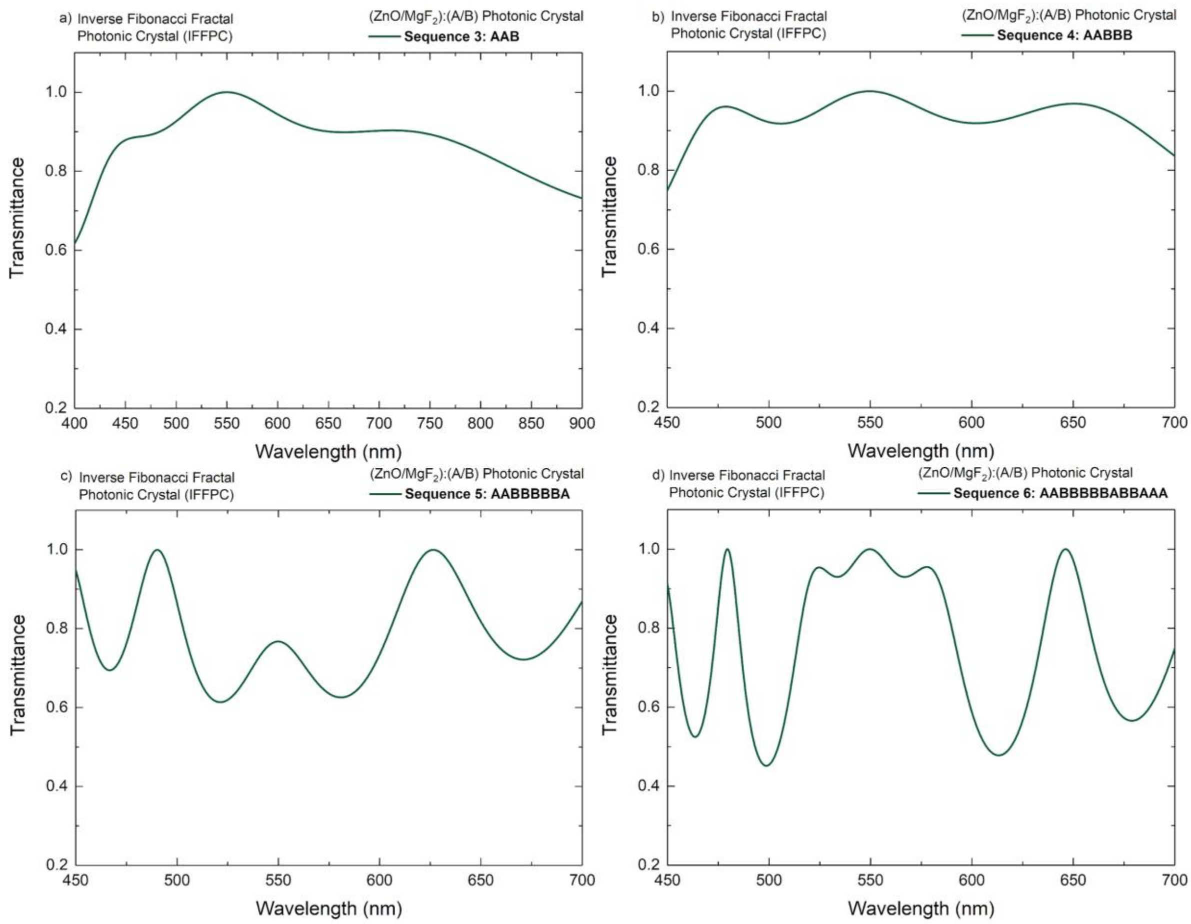

Figure 2. *Transmittance spectra of ZnO/MgF_2 based Inverse Fibonacci fractal photonic crystals with a) 3rd, b) 4th, c) 5th, and d) 6th order alignments.*

The transmittance spectra of ZnO/MgF_2-based MSFFPCs of different orders and their optical characteristics depending on the order of the sequence were investigated with the transmittance spectra calculated by TMM. Figure 3 shows the transmittance spectra of ZnO/MgF_2 based MSFFPCs of different orders.

For MSFFPC, a wide transition band is observed for third-order sequences, with the maximum transmittance occurring at a wavelength of about 550 nm. This indicates that lower-order sequences exhibit simpler and more fundamental optical behaviors. The spectrum transitions from a single broadband to two distinct transmission bands in the fourth order. This shift signifies that increasing the sequence order diversifies the optical modes and introduces more complex interactions between light and the structure. In the fifth order, this complexity increases further, with the spectrum displaying three primary transmission peaks. This demonstrates that higher-order sequences create more precise optical resonances. The transmission spectrum becomes highly intricate by the sixth order, with multiple narrow-band peaks emerging prominently. This highlights the growing impact of sequence order on the structure's optical properties, enabling more selective light guidance at specific wavelengths. Such photonic structures have significant potential in various applications, including wavelength-selective sensors, precision optical filters, and optoelectronic systems requiring advanced light management. High-order MSFFPCs generate complex optical resonances due to their increased diversity and density of optical modes, making them suitable for integration into advanced technologies.

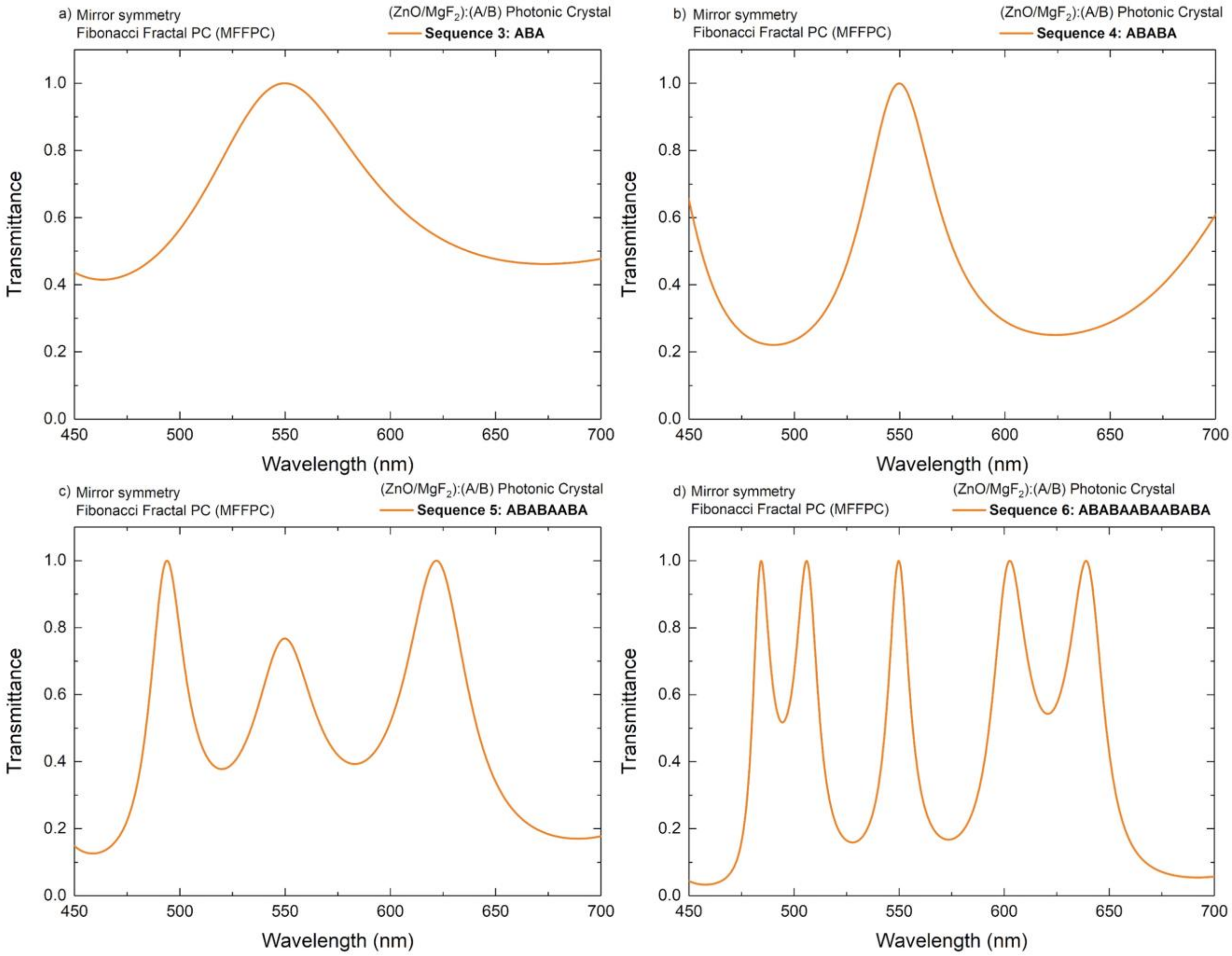

Figure 3. *Transmittance spectra of ZnO/MgF$_2$ based Mirror Symmetry fibonacci fractal photonic crystals with a) 3rd, b) 4th, c) 5^{th,} and d) 6th order sequences.*

The optical properties of ZnO/MgF$_2$-based FFPC, IFFPC, and MSFFPC structures exhibit significant variations depending on the type of sequence. These structures address a wide range of applications by exhibiting different optical behaviors. FFPC can be used in wavelength-selective optical filters and general optical applications. IFFPC, with its wide transmission spectrum, stands out for use in general-purpose optical devices and applications requiring wide spectral filtering. Its characteristics make it particularly beneficial for systems demanding energy efficiency. MSFFPC, on the other hand, is distinguished by its orderly structure and narrow-band transmission peaks, making it the most suitable configuration for color-selective sensors and optical filters requiring high precision. This structure provides maximum selectivity at specific wavelengths, making it ideal for precise optical control applications. FFPC offers versatility; IFFPC is for broadband systems, and MSFFPC is for precision and specific optical applications.

FFPC, IFFPC, and MSFFPC optical characteristics are highly effective, particularly for optical applications requiring single-mode light management. The 3rd and 4th order transmittance spectrum of FFPC can be used in optical filtering applications. IFFPC, with its broader bandwidth in the 3rd and 4th orders, is better suited for general-purpose optical systems and applications requiring wide spectral coverage. The single-mode optical characteristics with the narrowest half-width are achieved in third-order MSFFPC. This highly regular structure is ideal for precision color-selective sensors and laser systems, where high specificity and accuracy are critical. The color coordinates of the optical transitions corresponding to different orders of Fibonacci sequences (FFPC, IFFPC, MSFFPC) are presented in CIE color space in Figure 4. These graphs are particularly useful for visualizing how photonic crystals' optical transmittance influences the colors the human eye perceives.

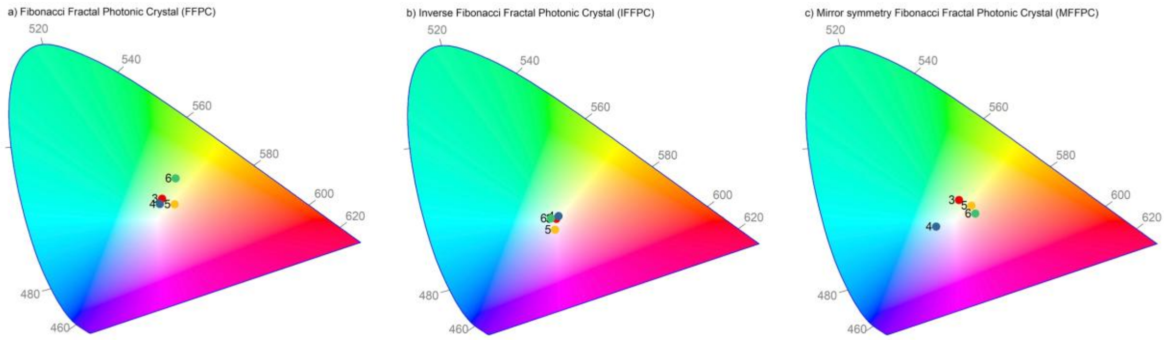

Figure 4. *Different order CIE color coordinates of a) standard, b) inverse, and c) mirror symmetry Fibonacci fractal photonic crystals based on ZnO/MgF$_2$.*

Although the color points corresponding to different Fibonacci sequence types are relatively close to one another, they exhibit slight shifts as the sequence order increases. For FFPC, the 3rd order is associated with warmer color coordinates. As the order increases, the color coordinates shift toward cooler tones (greenish-yellow). This behavior can be attributed to the more pronounced optical transitions at higher orders. In IFFPC structures, the 3rd and 4th orders concentrate around warm color tones (reddish-orange). By the 6th order, the coordinates shift toward more neutral tones (a green-yellow mix). This broader spread of colors in IFFPC is consistent with its wider transmission bandwidth.

For MSFFPC, the 3rd-order sequence (redpoint) indicates a warm color coordinate. A more regular transition is observed at higher orders (5th and 6th), with color points exhibiting less spread. This demonstrates that the mirror symmetry of the Fibonacci sequence results in more orderly optical behavior, offering more precise color selection. Among the Fibonacci-based structures, MSFFPC exhibits the most regular color distribution, making it the most suitable solution for color-selective applications. In contrast, IFFPC, with its broader color spectrum, is more appropriate for general-purpose wide-band applications.

4. CONCLUSION

In this study, the optical properties of one-dimensional photonic crystals based on ZnO/MgF$_2$, designed using Fibonacci fractal sequences, were thoroughly investigated. Analyses conducted on standard Fibonacci (FFPC), inverted Fibonacci (IFFPC), and mirror symmetry Fibonacci (MSFFPC) sequences revealed the significant effects of different sequence configurations and orders on photonic band gaps and optical light-matter interactions.

The results demonstrate that lower-order sequences exhibit wide and relatively simple transmission spectra, indicating their suitability for general optical filtering and wide spectral applications. However, spectral complexity was observed at higher sequence orders, characterized by the emergence of narrow-band transition peaks and enhanced selective optical behavior. Notably, MSFFPC sequences provide high spectral regularity and selectivity, making them ideal for precision applications such as color-selective sensors and narrow-band optical filters. IFFPC sequences, on the other hand, with their broad transmission bandwidths, offer advantages for wide spectral applications and systems requiring energy efficiency.

The results demonstrate that the optical properties of Fibonacci fractal photonic crystals can be effectively optimized through structural parameters such as sequence type and order selection. These findings open new opportunities for developing advanced photonic devices in telecommunications, optical sensing, color selection, and laser technologies. Furthermore, this research highlights the potential of Fibonacci sequences as a powerful tool for designing innovative photonic devices with high performance and customizable optical properties. In this context, photonic crystals based on Fibonacci sequences represent a significant research area with implications for fundamental scientific studies and industrial applications. These findings highlight the versatility of Fibonacci-based photonic crystal designs and their potential for tailored solutions across diverse optoelectronic and photonic technologies.

CONFLICT OF INTEREST

The author declares no conflict of interest.

REFERENCES

Capaz, R. B., Koiller, B., & de Queiroz, S. L. A. (1990). Gap states and localization properties of one-dimensional Fibonacci quasicrystals. Physical Review B, 42(10), 6402–6407. https://doi.org/10.1103/PhysRevB.42.6402

Çetinkaya, Ç., Çokduygulular, E., Kınacı, B., Güzelçimen, F., Özen, Y., Efkere, H. İ., Candan, İ., Emik, S., & Özçelik, S. (2021). Design and fabrication of a semi-transparent solar cell considering the effect of the layer thickness of MoO3/Ag/MoO3 transparent top contact on optical and electrical properties. Scientific Reports, 11(1), 13079. https://doi.org/10.1038/s41598-021-92539-8

Çokduygulular, E., Çetinkaya, Ç., Emik, S., & Kınacı, B. (2023). In-depth analysis on PTB7 based semi-transparent solar cell employing MoO3/Ag/WO3 contact for advanced optical performance and light utilization. Scientific Reports, 13(1), 7548. https://doi.org/10.1038/s41598-023-34507-y

Dai, X. F., Wang, H. Y., Chen, L. J., Duan, X. F., Chen, J. L., Wu, G. H., Zhu, H., & Xiao, J. Q. (2006). Growth and characterization of ferromagnetic shape memory alloy Co50Ni20FeGa29 single crystals. Journal of Crystal Growth, 290(2), 626–630. https://doi.org/10.1016/j.jcrysgro.2006.01.054

Elsayed, H. A., El-Sherbeeny, A. M., Nayak, S., Abukhadra, M. R., & Mehaney, A. (2024). Optical properties of the 1D quasiperiodic photonic crystals comprising gyroidal superconductor for THz applications. Europhysics Letters, 147(6), 65001. https://doi.org/10.1209/0295-5075/ad7758

Escorcia-García, J., & Mora-Ramos, M. E. (2009). Study of optical propagation in hybrid periodic/quasiregular structures based on porous silicon. PIERS Online, 5(2), 167–170.

Fujihara, S., Naito, H., & Kimura, T. (2001). Visible photoluminescence of ZnO nanoparticles dispersed in highly transparent MgF2 thin-films via sol–gel process. Thin Solid Films, 389(1–2), 227–232. https://doi.org/10.1016/S0040-6090(01)00893-8

Fujiwara, T., Kohmoto, M., & Tokihiro, T. (1989). Multifractal wave functions on a Fibonacci lattice. Physical Review B, 40(10), 7413–7416. https://doi.org/10.1103/PhysRevB.40.7413

Han, P., & Wang, H. (2005). Criterion of omnidirectional reflection in a one-dimensional photonic heterostructure. Journal of the Optical Society of America B, 22(7), 1571. https://doi.org/10.1364/JOSAB.22.001571

Jiménez-Vivanco, M. R., Lugo, E., Torres-Costa, V., Martín-Palma, R. J., Santana, M., & Herrera, R. (2024). Determination of the complex refractive index of free-standing porous silicon and oxidized porous silicon in the Visible and Ultraviolet range. Applied Physics A, 130(12), 952. https://doi.org/10.1007/s00339-024-08129-8

Kohmoto, M., Sutherland, B., & Iguchi, K. (1987). Localization of optics: Quasiperiodic media. Physical Review Letters, 58(23), 2436–2438. https://doi.org/10.1103/PhysRevLett.58.2436

Srivastava, R., Pati, S., & Ojha, S. P. (2008). Enhancement of omnidirectional reflection in photonic crystal heterostructures. Progress In Electromagnetics Research B, 1, 197–208. https://doi.org/10.2528/PIERB07102903

Taherzadeh, S., & Keshavarz, A. (2024). Optical Bistability in Vanadium Dioxide Photonic Crystals Engineered with One-Dimensional Fibonacci Sequences. Physics of Wave Phenomena, 32(6), 401–409. https://doi.org/10.3103/S1541308X24700407

Tavakoli, M., & Jalili, Y. S. (2014). One-dimensional Fibonacci fractal photonic crystals and their optical characteristics. Journal of Theoretical and Applied Physics, 8(1), 113. https://doi.org/10.1007/s40094-014-0113-0