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| Research Article / Araştırma Makalesi|

A Gifted High School Student's Abstraction Process of Divisibility Rules

Üstün Yetenekli Bir Lise Öğrencisinin Bölünebilme Kurallarını Soyutlama Süreci

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1.Divisibility
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Abstract

Purpose: Gifted students are often motivated by complex mathematical tasks. Mathematical abstraction allows access to gifted students' cognitive processes in knowledge construction. The question "How and why do the divisibility rules work?" evokes in them an intellectual need for constructing the working principle of a divisibility rule. Hence, this research focused on a gifted high school student's abstraction process of divisibility rules. By examining mathematical abstraction through observable actions, this study presents a deeper insight into the gifted student's thoughts, difficulties, and strategies regarding the working principle of divisibility rules.

Design/Methodology/Approach: The data was obtained from a 9th-grade gifted high school student through clinical interviews in a case study research design. The data were analyzed using the RBC+C abstraction theoretical framework's epistemic actions: *Recognizing, Building-with, Constructing*, and *Consolidating*.

Findings: The gifted student could recognize and use the necessary prior knowledge about divisibility to abstract the divisibility rules. In the construction process, the student explored the complex divisibility rules based on the place values of numbers with different digits.

Highlights: The student needed guidance in the process of creating more complex divisibility rules. With the researcher's help, the student could understand even more complicated divisibility rules and consolidate the cognitive way.

Öz

Çalışmanın amacı: Üstün yetenekli öğrenciler genellikle karmaşık matematiksel görevlerle motive olurlar. Matematiksel soyutlama, üstün yetenekli öğrencilerin bilgi alanındaki bilişsel süreçlerine erişim sağlar. "Bölünebilme kuralları nasıl ve neden çalışır?" sorusu, onlarda bir bölünebilirlik kuralının çalışma prensibini inşa etmek için entelektüel bir ihtiyaç uyandırır. Bu nedenle, bu araştırma üstün yetenekli bir lise öğrencisinin bölünebilme kurallarını soyutlama sürecine odaklanmıştır. Matematiksel soyutlamayı gözlemlenebilir eylemler aracılığıyla inceleyen bu çalışma, üstün yetenekli öğrencinin bölünebilme kurallarının çalışma prensibine ilişkin düşünceleri, zorlukları ve stratejileri hakkında daha derin bir bakış sunmaktadır.

Materyal ve Yöntem: Veriler, nitel araştırma deseninde dokuzuncu sınıf üstün yetenekli bir öğrenciden klinik görüşme yöntemiyle elde edilmiştir. Veriler, RBC+C soyutlama teorik çerçevesinin *Tanıma-Recognizing, Kullanma-Building with, Oluşturma-Constructing ve Pekiştirme-Consolidation* epistemik eylemleri kullanılarak analiz edilmiştir.

Bulgular: Üstün yetenekli öğrencinin bölünebilme kurallarını soyutlamak için bölünebilme ile ilgili gerekli ön bilgileri tanıyabilmiş ve kullanabilmiştir. Öğrenci, oluşturma sürecinde farklı basamaklı sayıların basamak değerlerinden yola çıkarak daha karmaşık bölünebilme kurallarını keşfetmiştir.

Önemli Vurgular: Öğrenci, daha karmaşık bölünebilme kurallarının soyutlanma sürecinde araştırmacı rehberliğine ihtiyaç duymuştur. Araştırmacının rehberliği ile öğrenci, bilişsel olarak daha karmaşık bölünebilme kurallarının altında yatan mantığı anlayabilmiştir.

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INTRODUCTION

Gifted students have the potential for showing, an exceptional level of performance in one or more areas of expression" (National Association for Gifted Children, 2005, p. 4). Especially high potential in mathematics is generally considered to be an understanding of mathematical ideas beyond arithmetic calculations (Miller, 1990). In this regard, gifted students in mathematics use their superior abilities by making associations and trying to understand the world (Sheffield, 1994). For this reason, most countries recognize and appreciate the unique characteristics of gifted students, and they have given significant attention to addressing and meeting their specific educational requirements (Mofield, 2020; VanTassel-Baska et al., 2021). However, gifted students follow the same curriculum at the same speed as other students in regular mathematics classes, without any changes to meet their individual needs (Diezmann & Watters, 2001; Dimitriadis, 2011). The standardized education primarily comprises memorization and similar messy activities, which fail to stimulate gifted students (Johnson, 2000; Maggio & Sayler, 2013).

The relevant literature shows that gifted students are often motivated by abstract and challenging contexts and materials (Perrone et al., 2010; Siegle et al., 2014). Such challenging tasks help them abstract mathematical knowledge by spending time and effort to find, apply and evaluate appropriate mathematical tools (Brousseau, 1997; Girit-Yıldız & Durmaz, 2021; VanTassel-Baska & Brown, 2007). Mathematical abstraction requires establishing relationships between concepts, understanding the source of information and knowing the areas of use. Since mathematical abstraction could be examined by observable actions, it is possible to reach gifted students' cognitive processes in knowledge construction processes (e.g., İnan, 2019). In other words, examining gifted students' knowledge construction processes with observable actions provides deeper information about what they think, where they have difficulties, and what strategies and concepts they use (Hershkowitz et al., 2001). However, like in many other countries, the educational system in Turkey fails to adequately support the needs of gifted students (e.g., Kanlı & Özyaprak, 2016). Leikin (2011) emphasizes the gap between mathematics education and gifted education studies. Therefore, there is a need for well-organized studies that will focus on gifted students' mathematical abstraction processes to meet their needs at primary and secondary education levels.

In the current study, we focused on the abstraction process of a ninth-grade student who was diagnosed as gifted during the problem solving process related to divisibility rules due to several reasons. Divisibility is a necessary and important concept for understanding the conceptual structure of number theory (Fitrianti & Suryadi, 2020; Shekatkar et al., 2015; Zazkis, 2008). The concept of divisibility is at the basis of many mathematical concepts, like arithmetic, the division algorithm, the greatest common divisor, polynomials, and the Euclidean algorithm from primary education to the end of undergraduate education (Potgieter & Blignaut, 2018). In recent years, the tendency to focus on real-life situations in mathematics education has revealed the danger of seeing divisibility rules as a result of division, an abstract and useless technique for memorization, or a trick (Zazkis, 1999; Zazkis et al., 2013). However, rather than memorizing divisibility rules, it is necessary to engage in mathematical reasoning by developing and testing conjectured rules or trying to understand how and why the rules work (Nahir, 2008; Zazkis et al., 2013). The question "How and why do the divisibility rules work?" evokes in learners an intellectual need for constructing and proving the working principle of a divisibility rule. One of the main goals of research in mathematics education is to better understand how students construct abstract mathematical knowledge (Dreyfus & Kidron, 2014). In such activities, learners create a new structure that gives a different perspective on previous knowledge (Tsamir & Dreyfus, 2002). Hence, they become aware of the mathematical structures and organize them to create a new structure to perform a mathematical task in the mathematical abstraction process. For this reason, the abstraction process of divisibility rules can provide opportunities to reveal gifted students' mathematical thinking through the analysis of observable epistemic actions. Therefore, this process can offer challenges and opportunities that align with the potential of gifted students. In this context, the current study seeks to address the following question: "What is a gifted high school student's process of abstraction regarding the divisibility rules?"

Significance of the study

In this study, we introduced the case of Alp (a pseudonym). He was a gifted ninth grader shown a willingness to openly and clearly communicate his ideas, thoughts, and uncertainties. He specifically inquired about the operational principles behind various divisibility rules. Alp, an exceptionally talented and self-reflective student, offered the researcher tremendous chances to closely study his cognitive processes while he participated in recognizing and constructing pre-established knowledge structures. Alp's desire to engage in open and honest communication with the interviewer, not only by giving accurate answers but also by voicing his doubts and views when he was unclear, provides valuable insights into the student's goals of knowledge and understanding. Dreyfus et al. (2015) also asserted that abstraction takes place when small groups of two to four students engage in classes. Nevertheless, even in a scenario with only two students, it is still conceivable for one student to take charge. Additionally, there may be situations where it is not feasible to distinguish between the epistemic actions of the students or gather sufficient knowledge about one of them. For this reason, this research was conducted with a single student to acquire comprehensive data on the process of abstraction.

In our context, examining the gifted students' abstraction processes can guide teachers in terms of developing high-level thoughts and meeting gifted students' needs by offering examples of a challenging context related to divisibility rules. Hence, teachers can possess a deep understanding of the cognitive processes occurring in the minds of their students, particularly in relation to how they grasp abstract mathematical concepts. This knowledge is crucial for creating effective learning environments that allow gifted students to express their unique mathematical ideas during lessons (Dreyfus et al., 2015) since addressing gifted students’ needs within the regular education setting is a vital challenge for experts and proponents of gifted education (Johnsen, 2021). The findings of the current study may guide researchers and teachers who want to benefit from differentiated educational opportunities parallel to gifted students’ cognitive needs in their classroom environments (Baykoç et al., 2014; Özdemir & Işıksal-Bostan, 2021). From a curricular standpoint, while there is currently no explicitly defined mathematics curriculum for use with Science and Art Centers (SACs), there is a significant desire to develop one in Turkey. The current study demonstrates how a gifted student in Turkey was able to construct knowledge of divisibility rules at different complexity levels. We expect the findings to be beneficial for task designers and curriculum creators working on SACs.

THEORETICAL BACKGROUND

Mathematical Abstraction Process

There is no common definition in the literature on the concept of abstraction. However, abstraction is expressed in its simplest form as raising from concrete to abstract or as a process that directs the concept to the product (e.g., Ozmantar & Monaghan, 2007). In the process of abstraction, which has a complex structure, the student becomes aware of the mathematical structures and organizes them to create a new structure to perform a mathematical task. The abstraction process cannot be observed directly because it contains details about cognitive processes (Dreyfus et al., 2001; Dreyfus, 2007; Hershkowitz et al., 2001). In this sense, researchers have defined observable epistemic actions, which include visualization of mental actions through students' verbal expressions or physical actions, in the RBC abstraction theory (Dreyfus 2007, Hershkowitz et al., 2001). These epistemic actions are R*ecognizing*, *Building-with*, *and Constructing*. *Recognizing* means the realization of a mathematical structure that is already familiar to the student as a result of earlier abstractions. If students do not recognize the structure, they cannot move on to the other stages. The second epistemic action, *building-with* involves using known pieces of mathematical elements with new content to solve a problem or achieve a goal. Since the process is in the form of combining old information with new information, the act of building-with also includes the recognition process. *Constructing* consists of combining and integrating previous structures with a *vertical reorganization to* produce a new structure. Vertical reorganization typically refers to a process in which previous mathematical structures within mathematics are rearranged and students construct a new abstract structure (Dreyfus et al., 2015). The creation of a structure can usually occur when the individual thinks intensely on this mathematical subject alone (Dreyfus et al., 2001). According to the theory, the act of constructing is not independent of recognizing and building-with, and includes both actions. The key difference between building-with and constructing is that the act of building-with involves using existing constructs to solve a problem or explain a situation, while the act of constructing involves establishing a mathematical generalization. The fragility of newly acquired structures in the abstraction process makes it difficult for this knowledge to become permanent (Monaghan & Ozmantar, 2006). Therefore, the need for *consolidation* has emerged as an essential and integral part of the abstraction for the permanence of abstracted information (Monaghan & Ozmantar, 2006). In this direction, the RBC model was updated as RBC+C by adding consolidation as a cognitive action.

Abstraction is handled from two perspectives, *cognitive* and *sociocultural.* Cognitive abstraction claims that learning abstraction will take place based on the similarities in the examples presented on the subject. According to the view that considers abstraction from a sociocultural perspective, there is an understanding that learning cannot occur independently of the environment, social interactions, and conditions (Hershkowitz et al., 2001). In this sense, Hershkowitz et al. (2001) proposed abstraction in context that evaluates the abstraction process in forming knowledge from a sociocultural perspective. Abstraction in context has a student-centered understanding that requires student-student, teacher-student, student-tool interactions due to its sociocultural nature. In gifted education, it is not enough for the student to be born as gifted, s/he needs support from the teacher or the program. In this sense, the teacher has a crucial role in supporting gifted students’ distinct needs (Brigandi et al., 2019; Özdemir & Isiksal-Bostan, 2021). In the present study, we decided to use the RBC+C model from the sociocultural perspective as an analytical framework to examine a gifted student’s abstraction process on divisibility rules, since we took into account the student-teacher interactions (Hershkowitz et al., 2001; Ozmantar & Monaghan, 2007). Furthermore, due to the challenge of distinguishing between the epistemic actions of two students in an intervention (Altun & Yılmaz, 2010; Dreyfus et al., 2015; Kobak-Demir & Gür, 2019; Tsamir & Dreyfus, 2002), we focused on a gifted student’s knowledge-construction processes.

National and international studies examining students' abstraction of mathematical knowledge based on the RBC or RBC+C model have focused on various mathematical concepts such as probability and statistics (Dreyfus et al., 2015; Katrancı & Altun; 2013); full and piecewise functions (Altun & Yılmaz, 2010); quadrilaterals (Butuner & İpek, 2023); coordinate systems and line equations (İlgün et al., 2018); linear relationships (Altun & Durmaz, 2013); limits (Sezgin-Memnun et al., 2017); infinite sets (Dreyfus & Tsamir, 2004; Tsamir & Dreyfus, 2002), fractions (Özçakır-Sümen, 2019) and parabola (Kobak-Demir & Gür, 2019) at various grade levels. However, the number of studies that aim at gifted students’ abstraction processes on any mathematical concept is quite low (e.g., Çıldır, 2014; İlgün et al., 2018). For example, Çıldır (2014) examined how students abstracted the concept of the equation with two gifted secondary school students. In another study, İlgün et al. (2018) gave various problem situations to a gifted high school student and discussed the process in which the student formed the line equation passing through the origin. The results of both studies showed that gifted students could perform observable cognitive epistemic actions using the RBC+C model. In the study, the gifted students could recognize and use the knowledge they had previously formed, and they formed and consolidated the targeted mathematical knowledge about the concept correctly at a certain level. We could not find any study that focuses on high school students' abstraction of the divisibility rules, although many concepts such as least/greatest common divisors, factorization, and polynomials in high school require knowledge of divisibility rules. Memorization of the divisibility rules can cause the information not to be fully structured in the mind (Zazkis & Campbell, 1996). As a result, students can stay at the level of remembering the information rather than recognizing and using the information. Therefore, examining gifted students' abstraction process of the divisibility rules will contribute to developing an understanding of their difficulties and needs in order to plan effective teaching approaches.

Literature on divisibility rules

The concept of divisibility has relations with division, multiplication, composing and decomposing of numbers, factorization, prime numbers, and divisibility rules (Zazkis & Campbell, 1996). Divisibility can be defined as follows: If $a, b \epsilon Z, b \neq 0$, we say a divides b and write $a|b$ means that $b = a \cdot k$ for some $k \epsilon Z$. This implies that b can be expressed as a multiple of a. In this sense, there is a strong relationship between divisibility and multiplicative structure of natural numbers. In early grades, students learn that a is a factor of b if a can divide into b without any remainder (The Ministry of Turkish National Education [MoTNE], 2018a). Divisibility rules can be employed to ascertain the primality of a number by examining if the number possesses any factors other than one and itself. By utilizing the divisibility rules, students can ascertain whether both the numerator and denominator of a fraction are divisible by the same integer, thus establishing if the fraction can be simplified. Students who possess knowledge of the divisibility laws will also possess the ability to employ a sound approach to determine the lowest common multiple or greatest common denominator of two or more non-zero numbers. In the curricula, it is recommended that students are motivated to memorize and apply the divisibility rules to enhance their mental calculating abilities (MoTNE, 2018a).

Early mathematics knowledge, especially about divisibility, is an important predictor of later mathematics achievement (Claessens & Engel, 2013; Siegler et al., 2012). However, many students see divisibility rules as abstract and useless (Zazkis, 1999), although some of them are useful in our daily lives (Chakraborty, 2007; Zazkis, 1999). For instance, the divisibility rules for 7 and beyond 10 are rarely discussed in middle or high school curricula although those rules provide opportunities to explore magical relationships between numbers (Eisenberg 2000; Zazkis 1999). In particular, divisibility by 7, 11, and 13 can be remembered as completely strange and unnecessary (Zazkis, 1999), although these rules reveal how mathematics is driven not only by usefulness but also by the desire to find patterns, beauty, and elegance (Eisenberg 2000; Zazkis 1999; Zazkis, Sincliar & Liljedahl, 2013). However, although many students and prospective teachers mostly remember some popular divisibility rules like divisibility by 2, 3, 4, 5, 9, and 10, they have difficulties proving them (Zazkis, 1999) or they often make the mistake of applying these rules too broadly or incorrectly (Zazkis & Campbell, 1996). Matz (1982) argued that these errors can be explained as students' unsuccessful attempts to adapt previously learned knowledge to a new situation. For this reason, it is necessary to engage in mathematical reasoning through developing and testing conjectured rules or trying to understand how and why the rules work (Zazkis, Sinclair & Liljedahl, 2013), since one of the main goals of research in mathematics education is to better understand how students construct abstract mathematical knowledge (Dreyfus & Kidron, 2014). For example, the divisibility by 3 states that a number is divisible by 3 if the sum of its digits is divisible by 3. However, the main point to be questioned should be why it should be so, rather than the sum of its digits must be a multiple of three for a number to be divisible by 3.

Gofer (1986) found that even fourth and fifth grade gifted students could discover different aspects of divisibility that they had not known previously when they were encouraged to reason and relate ideas. They were keen to think in an abstract way and in depth. In a recent study, Dinamit and Ulusan (2023) examined the proving processes of five 11th-grade Turkish science high school students who were diagnosed with giftedness from the Science and Art Education Centre. They found that gifted students could not complete the proof of divisibility by 3. They only memorized the divisibility rules, but they expressed a desire to acquire knowledge regarding the proof of the divisibility rule in the interview process. In the abstraction processes, Coşar and Keşan (2021) found that the gifted student utilized a cognitive rehearsal strategy to consider her prior structure of divisibility rules and associated it with the problem situation. These studies indicate the importance of abstraction process in understanding details of gifted students' mathematical knowledge related to divisibility rules. In this sense, our study can broaden the results of these studies since we focused on a gifted student's mathematical abstraction processes regarding divisibility rules at different complexity levels. We think that the results of this study are important for the design and shaping of teaching tasks related to divisibility rules for both gifted and non-gifted students in terms of designing effective discourse in a classroom environment. Moreover, we could not find any study that focuses on high school students' abstraction of the divisibility rules, although many concepts such as least/greatest common divisors, factorization, and polynomials in high school require knowledge of divisibility rules. Memorization of the divisibility rules can cause the information not to be fully structured in the mind. As a result, students can stay at the level of remembering the information rather than recognizing and using the information. Therefore, examining the students' abstraction process of the divisibility rules will contribute to developing an understanding of the difficulties and needs of students in this process and to planning effective teaching approaches.

METHODS

In this study, we used a qualitative research approach to obtain detailed and rich information about the student's actions in the problem solving process related to divisibility. In particular, we utilized a singular-case approach (Yin, 2014) to analyze a gifted high school student's construction of mathematical understanding of divisibility rules.

The Case

This research focused on a gifted male student named Alp, who was in the ninth grade in a public science high school in Turkey. Science high schools are secondary educational institutions (grades 9–12) that admit students based on high performance in a central examination. They employ a more extensive specialized mathematics curriculum than other secondary education institutions, equipping students for careers in the natural sciences and mathematics. The weekly hours allocated to science and mathematics courses markedly increase, especially after the 10th grade (Güçyeter et al., 2017). Nonetheless, their identification method is inadequate as it exclusively depends on a statewide multiple-choice examination that basically evaluates intellectual achievement.

Alp was the sole student in his class identified as gifted in mathematics. In Turkey, the Ministry of National Education (MoTNE) identifies gifted students through the Science and Art Centers (SACs). SACs facilitate the education and advancement of gifted students in Grades 2–12 by providing supplementary instruction in science, mathematics, foreign languages, literature, music, visual arts, social studies, and computing after school. A three-phase evaluation process is utilized to select gifted students for SACs. Initially, educators recommend their kids by filling out an online form. In the following phase, a committee performs a collective evaluation of the nominees. The committee evaluates students who meet or exceed the designated criteria, analyzing their performance in the chosen talent domain (intellectual ability, art, and music) indicated in their application. Intellectual capacity is assessed using findings from standardized tests, such as the Wechsler Intelligence Scale for Children–Revised Form (WISC-R), together with evaluations by specialists, to identify talented individuals. Students scoring 130 or above on these IQ exams qualify for attendance at a SAC. Due of Alp's proficiency in mathematics, he enrolled in additional mathematics courses at the SAC after school. Furthermore, the mathematics teacher, who was both a researcher and performed the interviews in this study, observed that Alp possesses the ability to comprehend abstract concepts, exhibits accelerated learning compared to his peers and demonstrates exceptional academic aptitude. Consequently, he was a highly driven, talented, and accomplished student. Alp displayed a remarkable level of openness and expressiveness, as well as a high degree of introspection. He articulated not only his "excellent ideas," but also his misgivings and uncertainties. His clear expression provided valuable insights into his cognitive processes in a coherent and verifiable manner.

In Turkey, sixth-grade students are expected to memorize and apply the divisibility rules of 2-3-4-5-6-9-10 without using a calculator (MoTNE, 2018a). In high school, students at the ninth-grade level are expected to memorize and apply the divisibility rules of 2-3-4-5-6-8-9-10-11-12-15 in mathematical problems (MoTNE, 2018b). In the curricula, students are expected to memorize the divisibility rules as in Figure 1. The same divisibility rules are included in science high school curriculum objectives at the ninth-grade level (MoTNE, 2018c). The divisibility by 7 and 13 are not included in both middle and high school teaching curricula in Turkey. This study was conducted before Alp's mathematics teacher covered division rules based on the science high school program. Before data collection, the teacher asked him what he knew about divisibility rules. Based on Alp's explanation, he learned the rules of divisibility for 3, 4, 5, 8, 9, and 10 by memorizing them in school beforehand. He learned the rules of divisibility for 7 and 11 from the internet, without knowing the underlying mathematical reasons.

Divisor	Rule
2	The last digit must be divisible by 2
3	The sum of the digits must be divisible by 3
4	The number formed by the last two digits must be divisible by 4
5	The last digit must be 0 or 5
6	The number must be divisible by both 2 and 3
8	The number formed by the last three digits must be divisible by 8
9	The sum of all digits must be divisible by 9

Figure 1. Summary of the divisibility rules in the textbooks

Data Collection

We gathered data using the teaching interview methodology, which enables the interviewer to delve deeply into the learner's thought process and didactical intents (Hershkowitz et al., 2001; Tsamir & Dreyfus, 2002). The teaching interview approach enables the researcher to thoroughly explore the learning process of a student when the learner exhibits constraints in their current cognitive patterns. Before starting the study, a preliminary interview was conducted with Alp to give information regarding the study. This meeting included information regarding the volunteer participant form and the family consent form. The University Ethics Committee and the Ministry of National Education provided the necessary approvals. The researcher and the teacher collaboratively devised the data collection process. The teacher acquired the interview data due to Alp's prolonged interaction with them. Data were obtained through two clinical interviews. We arranged the interviews based on the complexity of divisibility rules. We chose divisibility by 3 as the initial conceptual milestone because of the student's prior familiarity with this rule during their middle school years. For the first interview, we prepared a worksheet (see Figure 2) that was derived from prior studies (Zazkis & Campbell, 1996; Zazkis et al., 2013). When creating the task, we considered three factors: (i) the questions should keep him in problem-solving efforts, (ii) he should use his prior knowledge as much as possible, and (iii) the abstraction should take place in the process. In this worksheet, we included problems related to divisibility by 3. We asked the first question to understand how he made a relationship between the digits of a number divisible by 3. The problems aimed to observe the processes of recognizing and building-with the information required for the relationship between the divisibility rule and place values of the digits of the number. After creating the numbers that can be divided by 3, he was expected to solve the problem by recognizing (e.g., division) and building-with (e.g., place value analysis) structures. The second and third questions were required to investigate the divisibility of a three-digit number (2a4 and 2aa) whose digits are given as unknown. We asked the fourth and fifth questions to find what kind of relationship there should be between the divisibility of the numbers of $2aa^2$ and 2a0b by 3. We used these questions to determine how the newly created mathematical structure can be used in complex situations. In addition, in the first interview, after the study of divisibility by 3, we examined how he approached other divisibility by 9, 11, 7, and 13, respectively. Following the analysis of divisibility by 3, we examined his construction of divisibility by 11 and divisibility by 7 during the initial interview. After evaluating the previous interview, we did not prepare a worksheet for the second interview. In this interview, we asked him to examine the rule of divisibility by 13, which was not in the Science High School Curriculum. He had no prior knowledge about divisibility by 13. In the interview, we expected that he generated this divisibility rule based on the information he had previously formed. In this context, it was aimed for him to *recognize, build-with, construct* and *consolidate* the information abstracted in the previous problem.

Worksheet

1. If a number is divisible by 3, what kind of relationship is there between the digits of the number?

a) Write numbers that are divisible by 3.

b) Write numbers that are not divisible by 3.

2. If the three-digit number 2a4 is divisible by 3, what is the relationship between the digits of the number and a?

3. If the three-digit number 2aa is divisible by 3, what is the relationship between the digits of the number and a?

4. If the three-digit number $2aa^2$ is divisible by 3, what is the relationship between the digits of the number and a?

5. If the four-digit number 2a0b is divisible by 3, what is the relationship between the digits, a and b?

Figure 2. Worksheet used in the first interview

The student was not subject to any time constraints throughout the interviews. Each interview lasted approximately 30 minutes. There was a one-day gap between the first and second interviews. During the interviews, Alp voiced his thoughts about his solution and also provided written explanations if he wanted to. In this sense, mathematical explanations and solutions written by the student while addressing the problems in the interview were also used as data sources. We recorded both interviews on audio with the permission of the participant and then converted them into written transcripts. Alp's written explanations and mathematical operations were included in the appropriate sections of the transcribed document.

In this study, the two researchers had similar and different roles. The first author of this study was the mathematics teacher of the gifted student Alp. She performed the interviews in this study. During the teaching interviews, the interviewer takes on the simultaneous responsibilities of both a teacher and a researcher. She posed questions to Alp with two main objectives: (i) to prompt Alp to elucidate his actions and their underlying rationale, and (ii) to stimulate Alp to contemplate his actions, thereby facilitating progress beyond his individual capacity. In particular, the first researcher allowed the student to construct knowledge based on his prior knowledge, provided guidance and hints, provided discussion environments, asked the student to explain his answers, questioned the student's answers, encouraged him to find alternative ways to solve problems, and supported him to use multiple representations. Moreover, in cases where Alp remained silent or did not explain his solution, the interviewer asked, "How did you come to this conclusion?", "I don't understand, can you explain how you did it?" and "I understood this situation like this, did I understand it correctly?" Since the interviewer was also the mathematics teacher of the gifted student Alp, the strong dialogue between the student and the teacher helped to ensure teacher-student interaction in the study. On the other hand, the second author of the study took an active role in constructing and analyzing the interview process according to abstraction theory.

Data Analysis

In this qualitative study, we examined the data (audio recordings, interviews, and written answers) based on the framework, the RBC+C model. According to the content analysis, we often included direct quotations in the findings. In the data analysis, firstly, we translated the audio recordings into written text to reveal Alp's actions in the abstraction process. Then, we added Alp's written answers to the relevant parts of the interview data. After transcription of the data, we examined the document carefully several times and divided it into meaningful sections based on the RBC+C model. In the light of the theoretical framework, we examined Alp's cognitive epistemic actions in the abstraction process under the themes of recognition, build-with, construction and consolidation. In Table 1, we presented an example coding of data related to the abstraction of divisibility by 11 based on

RBC+C model. To deepen the student's abstraction process, the findings were supported by direct quotations from the student's written and verbal explanations.

Table 1. Example of coding data

Name	Utterance	Epistemic actions
Researcher	Do you know the divisibility by 11?	
Alp	Yes, from the internet, I recall the rule being related to +,–,+,– and the number's digits. But I am not sure…	Recognizing
Researcher	What does it mean for a number to be divisible by 11?	
Alp	I can use the rule. +,–,+,–… [He is writing]	
Researcher	without breaking the rules?	
Alp	If a number is divisible by 11, it can be written as 11.k, like 11, 22, 33,... 99.	Building with
Researcher	For a three-digit number, abc, how can you explain why it is divisible by 11?	
Alp	Should I decompose it according to the place values?	Recognizing
Researcher	Do it if you want.	
Alp	Yes, 100a+10b+c= abc. We can write 100a as 99a+a. For 10b, what can I do? It would be great if this was 11b... [he pondered]. Should we write it like this? 10b+b? ... [Silence]	Building with
Researcher	Think about the place value of b in abc. What is it?	
	10b. It has to be 11b-b=10b. [He planned] abc=99a+11b+a-b+c. 99a and 11b are divisible by 11. The digits of numbers and signs are important for divisibility by 11.	Building with Constructing

This coding example in Table 1 illustrates how Alp constructed divisibility by 11 based on knowledge of divisibility by 3 that he had previously constructed. Alp recognized with the rule of divisibility by 11 on the internet as a remembered component (recognizing). The interviewer’s question prompted him to acknowledge the relevance of multiplication and divisibility by 3 in the given setting (building with). Alp was in the early stages of abstracting for divisibility principles that rely on place value (building with). After remembering the role of place value in divisibility rules, Alp constructed the divisibility by 11 (constructing). The student's interview data and written solutions were cross-checked by two different researchers in line with the epistemic actions in the RBC+C abstraction model.

The validity of the research was first ensured by the variety of data. The interviews with the student, the researcher teacher’s written notes, and the student's written solutions were used as data sources (Bogdan & Biklen, 2006). After the audio-recorded interview data were transcribed, the written texts were confirmed by the participant to ensure accuracy and consistency between their statements and what was written and to make peer assessments (Creswell, 2007). In this study, the gifted student was informed by the researcher teacher that these interviews were conducted only to understand what he was thinking in order to avoid grade anxiety. In this way, the validity and reliability of the study were tried to be increased by ensuring that the student clearly said what he thought. Moreover, to ensure validity and reliability in the study, a long-term interaction was formed between the participant and the researcher. Furthermore, we carried out the coding for the abstraction process in a separate and individual manner. In this study, we implemented observable epistemic actions, specifically recognizing, building-with, and constructing (RBC), for each divisibility rule in the sequence mentioned in the interview. The interrater reliability was computed using the formula developed by Miles and Huberman (1994). We have attained a level of consensus amounting to 83%. Subsequently, we reviewed the codes to achieve a comprehensive consensus.

RESULTS

Using Memorized Divisibility Rule through Numerical Reasoning

The researcher first asked Alp to give examples that are divisible by 3 and are not exactly divisible by 3. Meanwhile, Alp read the problem and made the following statements:

Alp: If a number is divisible by 3, when we add the digits of that number it should be a multiple of 3 like 21 or $321.$ If it is a very large number, we can find its divisibility by 3 if only the sum of its digits is divisible by 3.

While explaining the divisibility rule for 3, Alp focused on the sum of the digits of the number. At this point, Alp has not yet associated an expanded notation for the number with divisibility. On the contrary, he used a memorized rule that he knew from his previous lessons. The researcher asked Alp to find the possible values of a so that three-digit numbers $2a4$ and $2aa$ with unknown digits are divisible by 3. Alp made the following explanations:

Alp: Here, the sum of these $[2 + a + 4$ and $2 + a + a]$ must be a multiple of 3… When we add 2 and 4, we get 6. If it $[2 + a + 4]$ is a multiple of 3, here a could be 3, could be 6, or could be 9. It would be 3, 6, or 9 because a specifies a digit.

Alp carried out the operations without using pen and paper to find the divisibility of $2a4$ and $2aa$ by 3. He found all possible numbers like 3, 6, and 9 except 0. Alp reached an answer without difficulty using the memorized divisibility rule that he was familiar with. Therefore, Alp was not asked the third and fourth questions in Figure 2, which required Alp to apply the same process. In the interview, the researcher asked Alp the reason why the divisibility rule by 3 is related to the sum of the digits of the number. The following dialogue took place between the researcher and Alp:

Researcher	If you did not know the divisibility rule for 3 in any number or if I said as your teacher, “No, Alp, you are wrong, for a number to be divisible by 3, the sum of its digits does not have to be a multiple of 3”, Do you give me an answer?
Alp	So… Can I think about it?
Researcher	Of course… Here is the pen and paper**.** You can write numbers or something if you want. Did you understand me?
Alp	Yes, I get it [he thinks]…
Researcher	You just said, “there are some numbers that are divisible by 3.” For example, you said 21.
Alp	… [Silence]

As seen in the interview excerpt, although Alp correctly applied the rule he knew in the questions, he could not yet question the underlying meaning of this rule. At this point, the researcher asked again what it means for a number to be divisible by 3 to understand how Alp distinguishes numbers that are divisible by 3. The researcher also wanted to activate the questioning capacity of the student with this question. With this question, the researcher pushed the student to question the mathematics in the divisibility rule.

Researcher	What does it mean when a number is exactly divisible by 3?
Alp	It means a multiple of 3…
Researcher	What exactly does a multiple of 3 mean? For example, if we examine ab as a two-digit number or abc as a three-digit number… How will we decide whether these numbers are divisible by 3?
Alp	Is ab the number like a multiple of 3?
Researcher	a and b can be represented by various numbers.
Alp	ab must be a multiple of 3. So, they [a and b] must be certain digits. For example, 72, 75. For ab, we can write numbers that are multiples of 3 from 12 to 99 (see **Figure 3**).

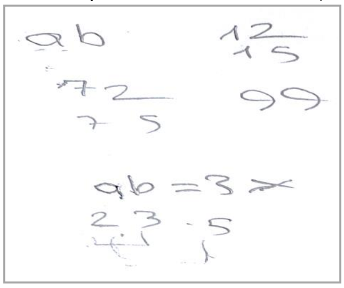


Figure 3. Alp’s examples of numbers divisible by 3

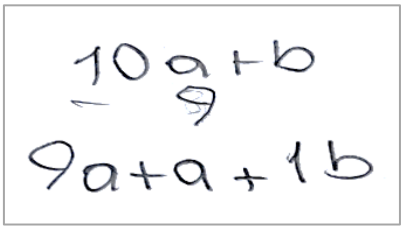
The interview excerpt revealed that Alp recognized all two-digit numbers that are divisible by 3 because he focused on all two-digit numbers that are multiples of 3 instead of a few examples. This shows that he recognized his previous knowledge elements regarding the multiplicative structure of numbers and divisibility. Although he concentrated on specific, carefully selected examples as a result of his awareness of the need for generalization, he could not associate the divisibility rule for 3, which he had memorized yet, with the place value analysis (an expanded form of the number).

Abstraction of Divisibility Rule through Place Value Analysis

Since Alp could not establish a relationship between the expanded form of ab and its divisibility for 3, the researcher asked Alp whether he remembered how a number can be written as an expanded form. This question helped Alp relate place value to divisibility. In this way, he expressed the number in the form of $10a + b$ using the place values of a number ab. Then, Alp wrote the number as $9a + a + b$, thinking that for a number to be divisible by 3, it must be a multiple of 3. Thus, Alp discovered that the remaining number $a + b$ is critical in dividing by 3, as seen in the interview section below.

Alp	We have ab as a two-digit number. We do not know whether it is divided by 3 or not.
Researcher	We can focus on a and b. For example, is ab 3 times the sum of its digits?
Alp	The number can be a multiple of 3.
Researcher	Ok. Do you remember how to expand a number based on its digits?
Alp	[He writes an expanded form of the two-digit number ab on the paper] (see Figure 4-a).
Researcher	Is $10a$ a multiple of 3?
Alp	10 is not a multiple of 3. It depends on a.
Researcher	Well, can you make the number a multiple of 3?
Alp	If we add something like 3 next to it. We can write it as $9a + a + 1b$. Here, $9a$ is always divisible by 3. The second addend ($a + b$) is the sum of the digits. Therefore, the number is divisible by 3 only if the sum of the digits is divisible by 3.

According to the interview excerpt, Alp used the knowledge of place value analysis with the guidance of the researcher. Then, he grouped the expanded form of the number as a multiple of 3 and focused on the remainder $a + b$. Thus, he explored the divisibility rule for 3 conceptually. One of the most important indicators of the formation of conceptual understanding was that Alp decided to use the place value analysis to question the division of a three-digit number abc by 3 (see Figure 4-b). This situation shows that Alp needed to construct a new structure to reveal the divisibility rule. This attempt of Alp was an action that showed that he was trying to abstract the structure he has constructed. After examining the divisibility of two and three-digit numbers by 3, Alp made sense of the rule conceptually that he previously memorized as "if a number is divisible by 3, the sum of the digits must be divisible by 3”.

(a) 

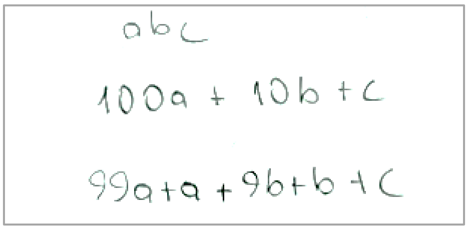
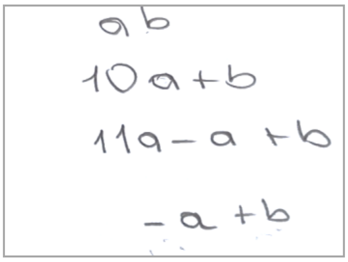
(b) 

Figure 4. (a) Two-digit and (b) three-digit number analysis for divisibility by 3

Extending the Abstracted Structure to Other Divisibility Rules

During the interview process, Alp wondered whether he could create the divisibility rule for 11, as he had learned the divisibility rule by 3 conceptually. For the divisibility rule for 11, Alp decided to first use a two-digit number ab and then a three-digit number abc. Figure 5-a and Figure 5-b show Alp's operations while questioning the divisibility rule for 11. Alp's written explanations showed that he tried to generalize the divisibility of a number by 11, not just from a few examples, but using a two-digit number ab and then a three-digit number abc. At this point, Alp grouped the numbers he obtained through place value analysis to be a multiple of 11. Then, he made the rule of divisibility by 11 meaningful, which he stated that he remembered with symbols such as + and –. Thus, for Alp, the procedural processes used in dividing a number by 11 (e.g., (i) writing the $+, -, +, -, \ldots$ signs under the digits of that number, respectively, starting from the ones digit, (ii) adding the numbers in the plus groups among themselves and adding the numbers in the negative groups among themselves, (iii) taking the difference of these two sums and (iv) dividing this sum by 11.

(a) 

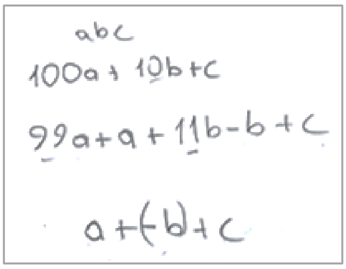
(b) 

Figure 5. (a) Two-digit and (b) three-digit number analysis for divisibility by 11

In the interview, after divisibility by 11, the researcher asked Alp about the rule of divisibility by 7, which is not in the science high school curriculum. Alp said that he had looked at the rule on the internet before, but he did not remember it exactly. At this point, Alp decided to use a three-digit number abc by performing a place value analysis to examine the divisibility rule for 7, as in Figure 6.

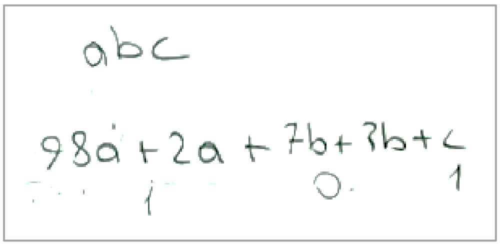


Figure 6. Alp's representations for the divisibility rule for 7

After Alp wrote the expression $98a + 7b + 2a + 3b + c$ and separated the multiplies of 7, he focused on the remaining $2a + 3b + c$. He discovered that the number abc is related to the coefficients $+1,\ +3, +2$, respectively. Alp generalized the situation with a pattern as in Figure 7 by using a number with a large number of digits to examine the situation.

$abcdefghı$ digits of the number	a	b	c	d	e	f	g	h	$ı$
Coefficients corresponding to digits	+2	+3	+1	-2	-3	-1	+2	+3	+1
The number in the digit is multiplied by the relevant coefficient and added. If the number found is divisible by 7, the number is divisible by 7.	$2a + 3b + c - 2d - 3e - f + 2g + 3h + ı$								

Figure 7. Working principle of the divisibility rule for 7

The Student's Abstraction of a Divisibility Rule Never Encountered Before

The second interview focused on the process of abstracting the divisibility rule for 13, which is not included in the science high school curriculum and Alp has no prior knowledge. How Alp handled the divisibility rule for 13 by using the expanded form of the number is presented below:

Researcher	In our previous meeting, we talked about divisibility by 3, 9 and 11 and 7. Now, we can focus on the divisibility rule for 13. If a number is divisible by 13, what do you say about this number?
Alp	I don't know anything about the divisibility rule for 13.
Researcher	Ok, now think about it!
Alp	We can examine it in the same way.
Researcher	Please. I give you as much time as you want.
Alp	Well. I try it for abc.I've grouped numbers here too. $97a$ [thinking]…
Researcher	Is $97a$ multiple of 13?
Alp	I will divide.
Researcher	Ok.
Alp	It will be $91a$. It is $91a + 9a$ (see Figure 8). Then, $10b$ is not divisible by 13. Let's write it as $13b - 3b$s. Also, c is not divisible by 13. If we write $13c$…No…wait a minute…$13c$…I mixed it. I couldn't remember how I did these in other divisibility cases.

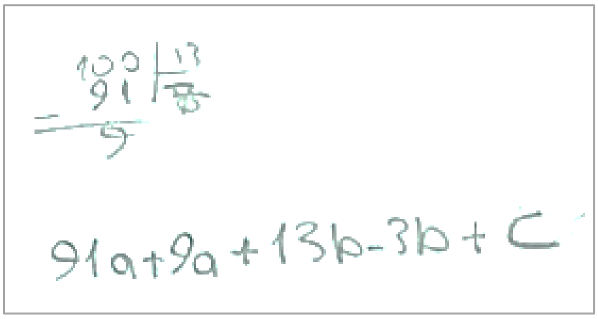


Figure 8. Alp's analysis of divisibility by 13 using a three-digit number

Alp tried to find the divisibility rule for 13 from the way he had learned in the first interview. Therefore, he used his knowledge of place value analysis again. However, after grouping the numbers as the multiples of 13 (see Figure 8), he was confused because he needed to create a new structure for the divisibility rule for 13. Alp sensed that a three-digit number does not reveal the mathematical structure of divisibility by 13. This blockage forced Alp to remember the previous divisibility rule. For this, he decided to examine the divisibility rule for 11 again. In this way, the consolidation process emerged by repeating the structure acquired in previous learning in Alp's cognitive actions.

Alp	For the divisibility rule for 11, $99a + 11b + a - b + c$ [he wrote] 100a + 10b + c 99a+a + 11b-b + c
Researcher	When we looked at the divisibility by 13, what happened?
Alp	$9a - 3b + c$
Researcher	Ok. This is for a three-digit number, so how would we generalize if the number was four, five, or six digits? It's also divisible by 11 $+, -, +, -$. Well, what do you think comes in here? $+1, -3, +9, +1, -3, +9$ does it come? Or something else?
Alp	I think it could be like this. $+1, -3, +9$ then -27…though not so.
Researcher	Shall we try?
Alp	It can be four digits. $1000a$ [he divided 1000 by 13]. Then, $988a + 12a + 91b + 9b + 13c - 3c + d$ [he wrote]. $12a + 9b - 3c + d$
Researcher	Is a pattern forming?
Alp	Here…
Researcher	Can you write something instead of $12a$? Here are the coefficients $1, -3, 9$.
Alp	They were three times each other. They became -3, then $+3$. Then…

In this process, it was seen that Alp needed to examine how he found the previous divisibility rule for 11 while dealing with divisibility by 13. This behavior of Alp exemplifies the 'fragile' structure of newly created knowledge. Because the structure, that was just constructed, was not consolidated it caused difficulties in using this structure in future activities. Although Alp discovered that divisibility by 13 was different from divisibility by 11, he had difficulty in searching for the rule for divisibility by 13 with a three-digit number. At this point, the researcher offered Alp a clue to think about numbers with different digits. Thus, Alp realized that he needed to focus not only on a three-digit number but also on a four-digit number. Alp focused on the four-digit number $abcd$ (see Figure 9-a). Although Alp does a step analysis, he has not yet reached a clear rule for divisibility by 13. Meanwhile, Alp sensed that he had to go beyond the usual trials. After Alp's attempts failed, the researcher introduced a new guiding question and encouraged Alp to make a new attempt. Alp decided to focus on a five-digit number $abcde$ to investigate its divisibility by 13 (see Figure 9-b). In the interview section below, it is seen how Alp abstracted the mathematics underlying the divisibility rule for 13.

Researcher	Shall we try another number?
Alp	[He considered the five-digit $abcde$ as in see Figure 9-b]. Divide 10000 by 13. We have 3 left. Then $1, -3,\ 9,\ 12,\ 3$. The first thing that happened. There are-3 and $+3$. Plus a…
	You can write $13b - b$ instead of $12b$.
Researcher	Yes… If we divide $1000b$ by 13, the remainder is $12b$. We can write $12b$ as $13b - b$. Then, we can write $-9a$ after we
Alp	divide 10000a by 13.
Researcher	Shall we arrange them?
Alp	Then $e - 3d + 9c - 1b + 3d - 9a$ [he wrote].
Researcher	How about the rule?
Alp	The numbers in the rule repeat every three digits, but first + then -, with opposite sign… $+1, -3, +9$ then $-1, +3, -9$ it happens.

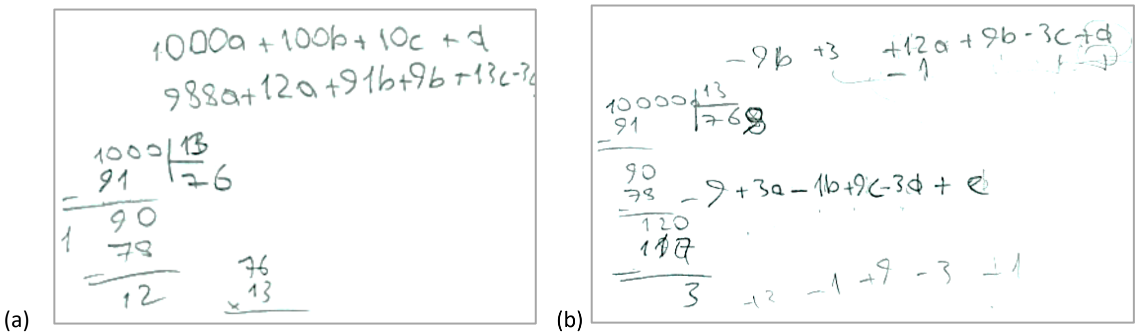


Figure 9. Alp's (a) four-digit and (b) five-digit number analysis for the divisibility by 13

The Student's Attempt to Consolidate New Knowledge

Alp needed the guiding questions of the researcher while abstracting the divisibility rule for 13. After Alp constructed the divisibility rule for 13, he felt that the information he created needed to be verified. He decided to consolidate newly constructed knowledge. For this reason, Alp performed operations by taking a three-digit and a seven-digit number, which can easily be seen to be a multiple of 13, to show the correctness of the structure he found through the examples (see Figure 10-a and Figure 10-b).

Figure 10. Alp's test of the rule with (a) 3-digit number and (b) 7-digit number

In addition, when the researcher asked Alp what he thought about finding whether any number is divisible by 13 or not, Alp stated that he was sure of the mathematical correctness of the newly formed structure as follows:

Researcher	If I tell you a number, can you find out if that number is divisible by 13 using the rule you found?
Alp	In this way, we can find the rule for almost all of them. If we know how that number is resolved, we do not divide each number for no reason. Everything has an explanation. If I know [the divisibility rule], I can easily find the divisibility condition according to the rule and we gain practicality.

Alp tried to reconstruct the divisibility rules that he knew by heart before he started the abstraction process, based on his previous knowledge. In addition, Alp's progress in the abstraction process by determining a strategy showed that he was in the process of using the information. At this stage, Alp investigated the accuracy of the hypotheses he produced regarding the solution of the problem in the process of using it. Hence, Alp could consolidate the conceptual structure he learned by considering the divisibility rules in the curriculum (divisibility by 3, 9, and 11) and not in the curriculum (divisibility by 7 and 13). With the changes he made to the structures he knew, Alp was able to construct new meanings about the rules of divisibility by realizing the processes of creating and arranging the new structure required for each divisibility.

To conclude, Alp tried to reconstruct the divisibility rules that he had memorized before the abstraction process. In addition, Alp's progress in the abstraction process by determining a strategy (e.g., examining two-digit, three-digit numbers, if necessary, four- and five-digit numbers, grouping them according to the dividend number) showed that he was in the process of recognizing and building-with the information. At this stage, Alp investigated the accuracy of the hypotheses he produced regarding the solution of the problem. As a result, Alp could consolidate the conceptual structure he learned by considering the divisibility rules (3-9-11) in the high school curriculum and (7 and 13) not in the high school curriculum. Thanks to the changes Alp made on the structures he knew, he could construct and arrange the new structure required for each divisibility. Thus, he could construct new meanings about divisibility rules.

DISCUSSION AND CONCLUSIONS

In this study, we examined a gifted high school student's abstraction process regarding divisibility rules. The results indicated that there are two important stages for abstraction: (i) the need for a new structure for abstraction to begin, and (ii) the emergence and consolidation of the new structure. In the first stage, Alp remembered the divisibility rules that he had memorized to solve the given problems. In the problem situation, when all the numbers in the digits were expressed with letters (e.g., abc), he had difficulty using the conceptual meaning of the rule of divisibility by 3 due to his rote learning. This finding supported Özçakır-Sümen's (2019) view that rote-based learning causes learning difficulties, that students cannot abstract the concepts they have memorized, and that not only unsuccessful students but also successful students experience these difficulties.

When Alp was asked about the reason for the divisibility rule by 3, he started to search for a new structure. This situation coincides with the assumption of Dreyfus et al. (2015) that the birth of abstraction begins with the need for a new structure. In addition, Alp's discovery that he can reach a general conclusion based on numbers in the problem without difficulty supports the conclusion that gifted students see generalizable conditions easier in problem solving (Girit-Yılmaz & Durmaz, 2021; İlgün et al., 2018). Abstraction arises from a need, and this is a mental process in the class that spontaneously comes true, difficult (Dreyfus et al., 2015). At this point, teachers' questions contributed to the student's need for abstraction. The researcher's prompts throughout the interviews aided the student's advancement in mathematical abstraction. Certain investigations asserted that students can generate knowledge when educators furnish the requisite assistance (Kobak-Demir & Gür, 2019; Williams, 2007). The findings of the current study corroborated the notion that the teacher plays significant and essential functions in the process of knowledge abstraction (Schwarz et al., 2009).

The results also indicated that Alp experienced uncertainties in the abstraction process of the divisibility rule by 3. However, it is not unexpected for students to give ambiguous, incomplete or incorrect answers while constructing knowledge (Bozkurt & Polat, 2018). Because in an unfamiliar situation, the need to act and uncertainty is high, and the capacity to pay attention to and remember the objects, features and events in the situation may be limited (Ozmantar & Monaghan, 2007). In such situations, it is not easy for the student to get out of this uncertainty without help. At this point, the researcher's questions have functions not only to evaluate the accuracy of the student's answers, but also to reveal the ideas of the students, support their thinking, enable them to share their thoughts verbally and aloud, and help them develop their conceptual understanding. Within the scope of this study, it has been observed that Alp, who is gifted, does not have difficulty in performing algebraic operations in the abstraction process, and even in some cases, he does operations without using pen and paper. However, in this process, the student needed the teacher's guidance as he had difficulty remembering and using some information. The teacher has an important role in the initiation and progression of the abstraction process. Thus, Alp's progress by using the given clues in structuring new knowledge has also been a result that supports the studies in the literature. Some studies also claimed that students can create knowledge when teachers provide necessary guidance (Kobak-Demir & Gür, 2019; Williams, 2007). The results of the present study supported that the teacher has important and necessary roles in the process of abstracting information (Ozmantar, 2004; Schwarz et al., 2009).

In the second interview, we found that although the student structured the knowledge through various divisibility rules, he could not easily transfer this knowledge to a new situation (e.g., divisibility by 13). This may be related to the student's inability to consolidate the newly formed knowledge (İlgün et al., 2018). However, since the student correctly formed the divisibility rule by 11, he could easily use this information while finding the divisibility rule by 13. This showed that he recognized the correct information while abstracting a divisibility rule that he had never known. This result supported the view that the student's previous learning affects the knowledge construction process (Dreyfus, 2007). Furthermore, students may feel the need to verify a newly discovered mathematical fact or idea (Dreyfus et al., 2015). Researchers state that for the abstraction of new knowledge, consolidation must occur (e.g., Monaghan & Ozmantar, 2006) to use the new knowledge in a new situation. In this study, Alp needed to show the correctness of the structure he created through examples after abstracting the divisibility rule by 13. Eventually, we observed that epistemic actions (recognizing, building-with, constructing, and consolidating) in the abstraction process are not independent of each other as mentioned in the results of some studies (Dreyfus, 2007; Hershkowitz et al., 2001; Kobak-Demir & Gür, 2019; Monaghan & Ozmantar, 2006).

In this study, the student's memorization of information created difficulties in remembering and using the information. For this reason, teachers should provide students with an opportunity to construct their own knowledge by guiding them rather than providing ready-made information. For this, in an inquiry-based learning environment, students focus on why and how questions. At this point, gifted students can be encouraged to question information more easily than other students. Gifted students can be allowed to construct knowledge through more challenging tasks. Gifted students are expected to work on difficult, interesting, challenging, and high-level tasks and questions (Aydemir-Özdemir & Işıksal-Bostan, 2021). This study also revealed that one of the factors that negatively affects the abstraction process was the lack of prior knowledge required for abstraction or memorized information. Therefore, before passing the abstraction processes, students' previous learning on the concepts can be checked. If deficiencies are detected in the preliminary analysis, a preliminary study should be carried out to eliminate these deficiencies. In this study, we studied with only one gifted student. In future studies, researchers may examine how gifted and non-gifted students navigate the abstraction process together. They can analyze the effects of peer learning on the abstraction process among gifted and non-gifted students by analyzing the nature of the interaction between them.

LIMITATIONS AND RECOMMENDATIONS FOR FUTURE STUDIES

While focusing only on a gifted student's abstraction process was a limitation because it restricts the ability to formulate generalizable conclusions, the results possess considerable significance for educators teaching gifted students in conventional classroom environments and those operating in SACs in Turkey. This study assists educators in creating assignments that consider the distinct needs of intellectually advanced children, with the goal of improving the quality of mathematics teaching in specialized academic centers (SACs). This study focused on a gifted student's knowledge construction processes about divisibility rules at different complexity levels. Despite the varying complexity levels of these divisibility rules, the methods used to uncover the underlying concept of each rule are similar. This similarity and the student's strong content understanding in algebra posed challenges in drawing a generalizable conclusion about the knowledge construction processes. Therefore, it is essential to analyze students' knowledge construction on tasks from different contents in mathematics. In the current study, the researcher plays a crucial role in the abstraction of knowledge. Future studies could investigate learners' knowledge construction processes both with and without the involvement of the interviewer to understand the role of the interviewer's guidance and support.

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Statements of publication ethics

We hereby declare that the study has not unethical issues and that research and publication ethics have been observed carefully.

Researchers' contribution rate

The study was conducted and reported with equal collaboration of the researchers.

Ethics Committee Approval Information

Ethical approval for the current study was taken from the Social Sciences & Humanities Ethics Committee at the University of Kastamonu (02/02/2022).

REFERENCES

Altun, M. & Durmaz, B. (2013). A case study of process in generating knowledge of linear relationship. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 26*(2), 423–438.

Altun, M. & Yılmaz, A. (2010). High school students' processes of constructing and reinforcing piecewise function knowledge. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 23*(1), 311–337.

Aydemir-Özdemir, D., & Işıksal-Bostan, M. (2021). Mathematically gifted students' differentiated needs: What kind of support do they need? *International Journal of Mathematical Education in Science and Technology. 52*(1), 65–83.

Baykoç, N., Aydemir, D., & Uyaroğlu, B. (2014). Inequality in educational opportunities of gifted and talented children in Türkiye. *Procedia-Social and Behavioral Sciences*, *143*, 1133-1138.

Bogdan, R. C. & Biklen, S. K. (2003). *Qualitative research for education: An introduction to theory and methods* (3rd ed.). Boston: Allyn & Bacon.

Bozkurt, A. & Polat, S. (2018). An examination of the teacher's questions for revealing students' mathematical thinking. *Turkish Journal of Computer and Mathematics Education*, *9*(1), 72–96.

Brigandi, C. B., Gilson, C. M., & Miller, M. (2019). Professional development and differentiated instruction in an elementary school pullout program: A gifted education case study. *Journal for the Education of the Gifted*, *42*(4), 362-395.

Brousseau, G. (1997). *Theory of didactical situations in mathematics.* Kluwer Academic.

Butuner, N., & Ipek, J. (2023). Examination of the Abstraction Process of Parallelogram by Sixth-Grade Students According to RBC+C Model: A Teaching Experiment. *European Journal of Educational Sciences*, *10*(2), 101-125.

Chakraborty, S. (2007). Divisibility by 4 and 8 for smaller natural numbers. *International Journal of Mathematical Education in Science and Technology*, *38*(5), 699–701.

Claessens, A., & Engel, M. (2013). How important is where you start? Early mathematics knowledge and later school success. *Teachers College Record*, *115*(6), 1-29.

COŞAR, M. Ç., & KEŞAN, C. (2021). Self-regulation behaviours of a gifted student in mathematical abstraction process. *Turkish International Journal of Special Education and Guidance & Counselling, 10*(2), 152-168.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five traditions* (2nd ed.). SAGE.

Çıldır, M. (2014). A special case study on the concept of equation with two gifted students. *Procedia - Social and Behavioral Sciences 116*, 650–654.

Diezmann, C. M., & Watters, J. (2001). The collaboration of mathematically gifted students on challenging tasks. *Journal for the Education of the Gifted*, *25*(1), 7–31.

Dreyfus, T. (2007). Processes of abstraction in context the nested epistemic actions model. *Journal for Research in Mathematics Education*, *32*, 195–222.

Dimitriadis, C. (2011). Developing mathematical ability in primary school through a “pull-out” programme: A case study. Education 3-13, 39, 467–482.

Dinamit, D., & Ulusan, S. (2023). Investigation of Gifted Students' Mathematical Proving Processes. *Acta Didactica Napocensia*, *16*(2), 49-69.

Dreyfus, T., Hershkowitz, R. & Schwarz, B. (2001). Abstraction in context II: The case of peer interaction. *Cognitive Science Quarterly*, *1*(3/4), 307–368.

Dreyfus, T., Hershkowitz, R. & Schwarz, B. (2015). The nested epistemic actions model for abstraction in context: theory as methodological tool and methodological tool as theory. *Encyclopedia of Mathematics Education* (pp.9–13), *Springer Science+Business Media Dordrecht.*

Dreyfus, T., & Kidron, I. (2014). Introduction to abstraction in context (AiC). *Networking of theories as a research practice in mathematics education*, 85-96.

Dreyfus, T., & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *The Journal of Mathematical Behavior*, *23*(3), 271-300.

Eisenberg, T. (2000). On divisibility by 7 and other low valued primes. *International Journal of Mathematical Education in Science and Technology*, *31*(4), 622-626.

Fitrianti, Y. & Suryadi, D. (2020). Analysis of difficulties for pre-service mathematics teacher in problem solving of division and divisibility based on theory of action, process, object, and schemes. *Journal of Physics: Conference Series*, *1521*(3), 032007. IOP Publishing.

Girit-Yildiz, D. & Durmaz, B. (2021). A gifted high school student’s generalization strategies of linear and nonlinear patterns via Gauss’s approach. *Journal for the Education of the Gifted*, *44*(1), 56–80.

Gofer, D. (1986). Mathematics for Gifted Third and Fourth Graders. *Gifted Education International*, *4*(1), 59-61.

Güçyeter, Ş., Kanlı, E., Özyaprak, M., & Leana-Taşcılar, M. Z. (2017). Serving gifted children in developmental and threshold countries—Turkey. *Cogent Education*, *4*(1), 1332839.

Hershkowitz, R., Schwarz, B. & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education*, *32*(2), 195–222.

İlgün, Ş., Altıntaş, E., Şimşekler, Z. H. & Ezentaş, R. (2018). A case study on mathematical abstraction of gifted students, *Turkish Studies*, *13*(4), 707–727.

İnan, E. (2019). An activity sample of differentiated maths programme for the gifted students. *Bilim Armonisi Dergisi*, *2*(2), 15–23.

Johnsen, S. K. (2021). Definitions, models, and characteristics of gifted students. In *Identifying gifted students* (pp. 1-32). Routledge.

Johnson, D. T. (2000). Teaching mathematics to gifted students in a mixed-ability classroom. Reston, VA: Eric Clearinghouse.

Kanlı, E. & Özyaprak, M. (2016). Stem education for gifted and talented students in Turkey. *Journal of Gifted Education Research*, *3*(2), 1–10.

Katrancı, Y. & Altun, M. (2013). Constructing and consolidating process of the second level students of primary schools for their knowledge of probability. *Kalem Eğitim ve İnsan Bilimleri Dergisi, 3*(2), 11–58.

Kobak-Demir, M. & Gür, H. (2019). The effect of teachers on constructing parabola knowledge process of high school students. *Journal of Theoretical Educational Science, 12*(1), 151–184.

Leikin, R. (2011). The education of mathematically gifted students: Some complexities and questions. *The Mathematics Enthusiast*, *8*(1), 167–188.

Maggio, M. R., & Sayler, M. (2013). Trying out acceleration for mathematically talented fifth graders. *Gifted Child Today*, *36*(1), 20-26.

Matz, M. (1982). Towards a process model for high school algebra errors. In D. Sleeman & J. S. Bro (Eds.), Intelligent tutoring systems (pp. 25-50). London: Academ

Miles, M. B. (1994). Qualitative data analysis: An expanded sourcebook. *Thousand Oaks*.

Miller, A. (1990). *The untouched key: Tracing childhood trauma in creativity and destructiveness*. Doubleday.

Ministry of National Education [MoNE] (2018a). *Middle school (5-8th graders) mathematics curriculum*.http://mufredat.meb.gov.tr/Dosyalar/201813017165445MATEMAT%C4%B0K%20%C3%96%C4%9ERET%C4%B0M%20PROGRAMI%202018v.pdf

Ministry of National Education [MoNE] (2018b). *High school (9-12th graders) mathematics.*http://mufredat.meb.gov.tr/Dosyalar/201821102727101OGM%20MATEMAT%C4%B0K%20PRG%2020.01.2018.pdf

Ministry of National Education [MoNE] (2018b). *Science high school (9-12th graders) mathematics.*https://mufredat.meb.gov.tr/Dosyalar/201821102457808OGM%20FEN%20L%C4%B0SES%C4%B0%20MATEMAT%C4%B0K%20PRG%2020.01.2018.pdf

Mofield, E. L. (2020). Benefits and barriers to collaboration and co-teaching: Examining perspectives of gifted education teachers and general education teachers. *Gifted Child Today*, *43*(1), 20-33.

Monaghan, J., & Ozmantar, M. F. (2006). Abstraction and consolidation. *Educational Studies in Mathematics*, *62*(3), 233–258.

Nahir, Y. (2008). On divisibility tests and the curriculum dilemma. *Sutra International Journal of Mathematical Science Education. Technomathematics Research Foundation*, *1*(1), 16–29.

National Association for Gifted Children (US), & Council of State Directors of Programs for the Gifted. (2005). *State of the States: A Report by the National Association for Gifted Children and the Council of State Directors of Programs for the Gifted*. National Association for Gifted Children..

Ozmantar, M. F. (2004). Scaffolding, abstraction, and emergent goals. *Proceedings of the British Society for Research into Learning Mathematics, 24*(2), 83–89.

Ozmantar, M. F. & Monaghan, J. (2007). A dialectical approach to the formation of mathematical abstractions. *Mathematics Education Research Journal*, *19*(2), 89–112.

Özçakır-Sümen, Ö. (2019). Primary school students' abstraction levels of whole-half-quarter concepts according to RBC theory. *Journal on Mathematics Education, 10*(2), 251–264.

Özdemir, D. A., & Işiksal Bostan, M. (2021). Mathematically gifted students' differentiated needs: what kind of support do they need?. *International journal of mathematical education in science and technology*, *52*(1), 65-83.

Perrone, K. M., Wright, S. L., Ksiazak, T. M., Crane, A. L., & Vannatter, A. (2010). Looking back on lessons learned: Gifted adults reflect on their experiences in advanced classes. *Roeper Review*, *32*(2), 127–139.

Potgieter, P. & Blignaut, P. (2018). The effect of learners' knowledge of the divisibility rules on their gaze behaviour. *African Journal of Research in Mathematics, Science and Technology Education 22(3),* 351–362.

Schwarz, B. B., Dreyfus, T., & Hershkowitz, R. (2009). The nested epistemic actions model for abstraction in context. In B. B. Schwarz, T. Dreyfus, & R. Hershkowitz (Eds.), *Transformation of knowledge through classroom interaction* (pp. 11–41). Routledge.

Shekatkar, S. M., Bhagwat, C., & Ambika, G. (2015). Divisibility patterns of natural numbers on a complex network. *Scientific Reports*, *5*(1), 1-10.

Sezgin-Memnun, D., Aydın, B., Özbilen, Ö., & Erdoğan, G. (2017). The abstraction process of limit knowledge. *Educational Sciences: Theory & Practice, 17*, 345–371.

Sheffield, L. J. (1994). *The development of gifted and talented mathematics students and the National Council of Teachers of Mathematics standards*. National Research Center on the Gifted and Talented.

Siegle, D., Rubenstein, L. D., & Mitchell, M. S. (2014). Honors students' perceptions of their high school experiences: The influence of teachers on student motivation. *Gifted Child Quarterly*, *58*(1), 35–50.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., ... & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological science*, *23*(7), 691-697.

Tsamir, P., & Dreyfus, T. (2002). Comparing infinite sets—a process of abstraction: The case of Ben. *The Journal of Mathematical Behavior*, *21*(1), 1–23.

VanTassel-Baska, J. & Brown, E. F. (2007). Toward best practice: An analysis of the efficacy of curriculum models in gifted Education. *Gifted Child Quarterly*, *51*(4), 342–358.

VanTassel-Baska, J., Hubbard, G. F., & Robbins, J. I. (2021). Differentiation of instruction for gifted learners: Collated evaluative studies of teacher classroom practices. *Handbook of giftedness and talent development in the Asia-Pacific*, 945-979.

Williams, G. (2007). Abstracting in the context of spontaneous learning. *Mathematics Education Research Journal 19*(2), 69–88.

Yin, R., 2014. Case Study Research. Sage Publications.

Zazkis, R. (2008). Divisibility and transparency of number representations. In M. Carlson and C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics* (pp. 81-92), Washington, DC: The Mathematical Association of America.

Zazkis, R. (1999). Divisibility: A problem solving approach through generalizing and specializing. *Humanistic Mathematics Network Journal, 21,* 34–38

Zazkis, R., & Campbell, S. (1996). Divisibility and multiplicative structure of natural numbers: Preservice teachers' understanding. *Journal for Research in Mathematics Education*, *27*(5), 540–563.

Zazkis, R., Sinclair, N., & Liljedahl, P. (2013). *Lesson play in mathematics education: A tool for research and professional development*. Springer.