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Research Article

The Effect of Argumentation-Based Teaching on Conceptual Understanding in Transformation Geometry*

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Abstract – This study examines the effect of argumentation-based teaching on pre-service teachers' conceptual understanding of transformation geometry. The mixed research design was used in this study. In the quantitative dimension of the study, a quasi-experimental design with pretest and post-test control groups was used, in which the existing classes were randomly assigned as experimental and control groups. The qualitative dimension of the research is a case study. The study participants consisted of 43 secondary school pre-service math teachers who studied in the third grade at the education faculty of a state university in Turkey and took the Analytical Geometry course in the fall semester of the 2019-2020 academic year. In line with the purpose of the study, the Transformation Geometry Achievement Test (TGAT) was used as a data collection tool, and interviews were conducted with pre-service teachers. As a result, it has been concluded that argumentation-based teaching positively affects pre-service teachers' academic achievements and conceptual understanding of transformation geometry. In line with this result, it can be said that examining the effects of this teaching practice on academic success and conceptual understanding in other areas of mathematics will contribute to the field.

Keywords: Argumentation, transformation geometry, pre-service teachers, conceptual understanding.

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Introduction

Transformation geometry is an area where learners have learning difficulties (Clements & Burns, 2000; Kandaga et al., 2022; Leikin et al., 2000; Sevgi & Erduran, 2020). It is known that not only students but also teachers struggle with interpreting and applying geometric transformations such as reflections, rotations, and translations (Bansilal & Naidoo, 2012; Lee & Boyadzhiev, 2020; Luneta, 2015). For example, studies show that pre-service elementary mathematics teachers make errors in understanding the algebraic meaning of rotation and translation transformations (Ada & Kurtuluş, 2010; Portnoy et al., 2006; Thaqi et al., 2011; Yanık & Flores, 2009; Yanık, 2011).

Transformation geometry helps students learn about different concepts, such as function, congruence, and similarity, and discover the relationships between these concepts (Hollebrands, 2003; Portnoy et al., 2006). Numerous patterns and structures in daily life have been formed as a result of reflection, translation, and rotational transformations. Examples of geometric transformations can be seen in the shapes of wallpapers, historical places, and works of art. Representing geometric transformations through matrices is essential today in robot technology and motion theory (Selig, 2013; Zembat, 2013). In view of the above, it is extremely important to teach transformation geometry effectively for students to realize the vital link between daily life and mathematics (Çetin et al., 2015).

Due to the importance of transformation geometry, adequate measures should be taken to teach the subject effectively and prevent misconceptions (Kandaga et al., 2022; Leikin et al., 2000; Luneta, 2015). One of the practical teaching approaches to develop students' critical and analytical thinking skills is argumentation (Inglis et al., 2007). Argumentation can be defined as a discussion that includes the processes of putting forward and sharing thoughts on any subject, clarifying these thoughts, and explaining the basic reasons underlying the thought (stating the reasons) (Cross, 2009). Argumentation includes using different forms of representation to express assumptions, hypotheses, mathematical ideas, understanding the perspective of others, and analyzing mathematical expressions. Not all mathematical activities in the classroom are formal activities, and many of these activities include argumentation (Reid & Knipping, 2010). Information on the argumentation approach in mathematics education is included under the title of theoretical background.

Theoretical Background

Transformation Geometry

Transformation geometry studies how each point of a geometric shape corresponds to a point in a new shape formed by transformations such as translation, reflection, and rotation (Argün et al., 2014; Martin, 2012; Solomon, 2014). Translation is a transformation that moves all the points that make up the geometric shape in a specific direction and direction (Sevgi & Erduran, 2020). Reflection transformation takes place with the help of a reflection axis so that the perpendicular distance of the new shape obtained as a result of the transformation to the axis is the same as the perpendicular distance of the reflected shape to the axis (Argün et al., 2014; Martin, 2012). Rotational transformation moves the geometric shape around a certain point (rotation center) by a certain angle in the desired direction (Argün et al., 2014; Martin, 2012; Solomon, 2014).

In transformation geometry, which has an essential place in secondary and high school mathematics curricula, the geometry standards determined by NCTM (2000) also emphasize that students should be made to think about reflection, translation, and rotation transformations. Transformation geometry emphasizes the importance of understanding and teaching geometric transformations (Kandaga et al., 2022; Leikin et al., 2000). Transformation geometry enriches students' geometric experiences, imagination, and thinking abilities and develops three-dimensional thinking skills (Hollebrands, 2003; Lee & Boyadzhiev, 2020). For students to be successful in advanced mathematics, they must have sufficient knowledge of geometric transformations (Carraher & Schlieman, 2007; NCTM, 2000). However, research has shown that students need help understanding transformations (Clements & Burns, 2000; Olson et al., 2008; Rollick, 2009).

Argumentation Approach and Mathematics Education. Argumentation discusses discourses regarding a concept or situation (Fiallo & Gutierrez, 2017). In other words, argumentation is a process that requires individuals to base their thoughts on a cognitive filter and defend them with appropriate data (Naylor et al., 2007). Toulmin (2003), who conducted the first studies on the argumentation approach, revealed an argumentation model widely used in education (Rumsey & Langrall, 2016). This argumentation model, created by Toulmin (2003), is not unique to a particular field. This model is used for the analysis of arguments for various purposes in research on different fields (Erduran et al., 2004; Knipping & Reid, 2015). The basic argumentation model created by Toulmin (2003) includes three components:

data, warrant, and claim, each of which has a separate function in the argument generation process (Metaxas et al., 2016).

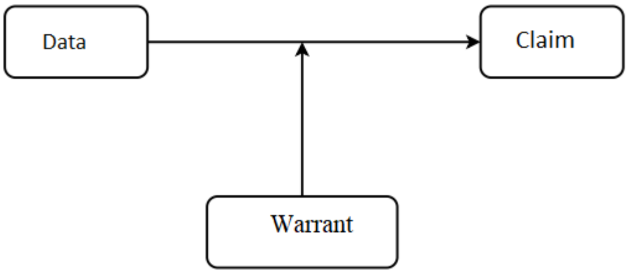


Figure 1 Toulmin Basic Argumentation Model (Toulmin, 2003)

This three-component argumentation structure constitutes the basic structure of argumentation (Pesen, 2018). Data refers to the reason for the claim and the information supporting it. Claims are thoughts based on data, the results that the individual wants to achieve based on the data. Warrants explain how the claim is reached and the relationship between data and the claim (Conner et al., 2014; Driver et al., 2000; Toulmin, 2003).

Many researchers consider argumentation necessary for mathematics learning (Reid & Knipping, 2010). The argumentation approach is one of the learning approaches that positively affects students' scientific thinking and logical reasoning skills (Kosko et al., 2014; Yackel & Cobb, 1996). With the realization of the importance of communication and social processes in mathematics education, interest in the argumentation approach is increasing daily (Erduran et al., 2004; Reid & Knipping, 2010). According to Eemeren and Grootendorst (2010), during the argumentation process, students are encouraged to make new claims and to support these claims by providing evidence while appropriately evaluating the claims made by their friends.

In the mathematical argumentation process, teachers undertake the task of helping students actively participate in the learning process (Ayalon & Hershkowitz, 2018). There should be a suitable learning environment for an effective argumentation process in which students can collaborate (Douek, 1999; Şengül & Tavşan, 2019). In this study, transformation geometry teaching was carried out with an argumentation-based teaching approach. GeoGebra dynamic geometry software was used in the activities during the teaching process. During the course process, technology is only a supporting component of the teaching process with the argumentation approach. Drawing is essential in teaching transformation geometry. Therefore, GeoGebra program was used to use time efficiently in teaching transformation geometry and to provide a learning environment suitable for the argumentation approach. By using GeoGebra program, it was aimed for students to be able to obtain data in the argumentation process, to present a claim based on this data, and to perform justification and generalization steps quickly and effectively.

Boero (1999) evaluates the argumentation process that occurs in mathematical activities in six stages: Hypothetical production, formulation of an expression, investigation of the content of the assumption, creation of a deduction chain with the help of arguments, turning the relevant arguments into a proof and approaching a formal proof. The main goal in learning environments where the argumentation approach is adopted in mathematics education is for students to make claims based on data, to strengthen these claims by basing them on justifications, to evaluate their ideas by comparing them with the ideas put forward by others, and to obtain mathematical knowledge at the end of this process (Brown & Redmond, 2007; Yackel & Cobb, 1996).

Studies revealed that the argumentation approach positively affected students' mathematics achievement (Cross, 2009; Semana & Santos, 2010) and desire to learn (Brown & Redmond, 2007; Civil & Hunter, 2015; Mueller & Yankelewitz, 2014); on the contrary, Hollebrands et al. (2010) stated in their research on geometry that argumentation skill was not related to academic achievement. Sanchez and Uriza (2008) determined that students created arguments directly without any influence from teachers. Brown and Reeves (2009) stated that the argumentation approach positively affects active participation in the learning process in mathematics lessons. In their study, Mueller and Yankelewitz (2014) stated that the argumentation approach improves mathematical discourse and contributes to a practical argumentation skill. Pesen (2018) concluded in his study that misconceptions are one of the factors affecting the quality of the argument.

Purpose of the Study

One of the main components of a teaching process is the teacher. Students' learning is directly related to teachers' teaching styles. Teachers will use the argumentation approach effective (Hollebrands, 2003; Leikin et al.,2000; Metaxas et al., 2016). Considering this situation, pre-service teachers, who are the future teachers, need to be well-trained in what they will teach and how they will teach (Ball et al., 2008). Transformation geometry is a subject included in curricula and is closely related to other key concepts in mathematics. Therefore, teaching transformation geometry effectively becomes more meaningful. Based on this idea, it is thought that this study, which is about teaching transformation geometry with an argumentation-based learning approach, is necessary. Therefore, this study examines the effect of argumentation-based teaching on pre-service teachers' conceptual understanding of transformation geometry. The main research question is, "How does argumentation-based teaching affect pre-service mathematics teachers' conceptual understanding of transformation geometry?". The sub-problems of the study are as follows;

1. What is pre-service teachers' knowledge in the experimental and control groups about the transformation geometry before argumentation-based teaching?

2. Is there a statistically significant difference between the academic achievements of pre-service teachers in the experimental and control groups on transformation geometry after argumentation-based teaching?

3. How does argumentation-based teaching affect pre-service teachers' conceptual understanding of transformation geometry in control and experimental groups?

Method

Research Design

The mixed research design was used in this study. Events in life are not unidimensional. They are more complex and related. Mixed studies are deeper and more comprehensive than studies in which qualitative and quantitative research approaches are preferred (Johnson & Christensen, 2004; Creswell, 2012). This is the main reason for using mixed methods. In order to sufficiently explain the situations to be investigated in the field of social sciences, it is recommended to use more than one research method. The mixed research approach is based on the idea that a situation has both qualitative and quantitative dimensions. Therefore, in order to discuss our research problem with a more in-depth and holistic view, a mixed research approach was used.

In the mixed research process, the data obtained with different approaches and techniques used in data collection are verified. The results obtained from the research become more acceptable. This is another important reason for using mixed methods in this study. In addition, some results obtained with a single data collection technique may need to be explained by results obtained with another data collection technique. In this context, it was thought that the triangulation and complementary functions of the mixed method would make a significant contribution to the research (Johnson and Onwuegbuzie, 2004; Yıldırım and Şimşek, 2016).

The mixed research design used in this study is the explanatory embedded design. The aim of this design is to explain, support or generalize the results obtained from one method with the results obtained from the other method (Creswell & Plano Clark, 2007). In this study, qualitative data were collected after the experimental implementation process and it was aimed to better explain the quantitative data.

In the quantitative dimension of the study, a quasi-experimental design with a pre-test and post-test control group was used. The research was conducted at a state university. The faculty administration created two different classes in which the participants would study when they enrolled at the university. At the beginning of the research, it was randomly determined which of these classes would be the experimental group and which would be the control group. The qualitative dimension of the research is a case study. Case studies require a comprehensive review to understand complex facts or events (Johnson & Christensen, 2004; Timans et al., 2019).

Participants

The study participants consist of 43 pre-service secondary school mathematics teachers who studied in the third grade at the education faculty of a state university in Turkey and took the Analytical Geometry-I course in the fall semester of the 2019-2020 academic year. Accordingly, 21 pre-service teachers in the experimental group and 22 in the control group participated in this study.

Table 1 Distribution of Teacher Candidates Participating in the Research

Groups	Gender		
	Female	Male	Total
Experiment	15	6	21
Control	19	3	22
Total	34	9	43

For the interviews in the qualitative dimension of the research, four pre-service teachers were selected, two pre-service teachers from each group. These pre-service teachers were determined according to their pre-test and post-test scores. From each group, one pre-service teacher whose pre-test and post-test scores differed and one pre-service teacher whose pre-test and post-test scores did not differ were selected.

Data Collection Tools

Transformation Geometry Achievement Test (TGAT)

In the study, we prepared a Transformation Geometry Achievement Test (TGAT) that consisted of 20 open-ended questions. Mathematics books used at high school and higher education levels were used to select questions (Altun, 2018; Balcı, 2007; Emin et al., 2018; Kemancı et al., 2018; MoNE, 2017; Ünlü and Er, 2015). The questions in these books were either adapted for the study or used directly. Care was taken to ensure that the questions were clearly articulated and could be understood equally by everyone. Regarding the questions, the opinions of three academicians who are experts in mathematics education were also consulted. Consensus has been reached by experts on the point that TGAT consists of questions suitable for the research. The content validity of TGAT was ensured with some partial corrections and changes made in line with expert evaluations. Accordingly, out of the twenty questions in the test, six are about translation, six are about reflection, and eight are about rotation transformation. TGAT was finalized after expert evaluations and opinions received from pre-service teachers within the scope of the pilot study. TGAT was applied to 47 pre-service teachers before starting the main study to test the achievement test's reliability. This way, the reliability coefficient (Cronbach α) of 0.74 was calculated. According to this value, it can be said that TGAT is reliable (Streiner, 2003; Büyüköztürk, 2014).

Interviews

The study conducted semi-structured interviews with the pre-service teachers from both the experimental and control groups to understand pre-service teachers' written solutions. The aim here is to better understand their written explanations and provide data combinations. Interviews were conducted with two participants from each group. They were determined according to their pretest and post-test scores. Interviews were conducted with one pre-service teacher whose pre-test and post-test scores showed a difference and one pre-service teacher whose pre-test and post-test scores showed no difference from each group. The first author conducted all interviews with these four volunteer pre-service teachers. The interviews were recorded with a tape recorder. Then, the recorded speeches were listened to, and transcripts were prepared. Transcripts prepared for each interview were shown again to the interviewed pre-service teacher, and their approval was obtained.

Pilot Study, Experimental Implementation Process, and Data Collection

A pilot study was carried out in the spring semester of the 2018-2019 academic year with a single group consisting of 17 pre-service teachers at the same university in the upper class of the main study participants. Since the study requires active use of the GeoGebra program, the lessons were conducted in the computer lab. All pre-service teachers in the pilot study use the Geogebra program proficiently. In the current study, Geogebra was preferred because it is easy to use with its simple interface, and it is free.

In the experimental study, the lessons were taught by the study's first author in the experimental group and by the responsible lecturer in the control group. The lessons are planned to last four weeks, 3 hours per week. Translational transformation was discussed in the first week; reflection transformation in the second and third weeks; and rotational transformation in the fourth week. The TGAT, which is a data collection tool applied as a pre-test and post-test in the study, was applied piecemeal in line with the results obtained from the pilot application. The questions about translation in TGAT were applied as a pre-test and post-test before and after the first week's lesson, in which the translation transformation was explained. The questions about reflection were administered before the second week's lesson, where the reflection transformation was introduced, and after the third week's lesson, in which the topic was completed. Lastly, the questions about rotation in TGAT were applied before and after the fourth week's lesson, in which the rotation transformation was explained.

In the experimental group, the lessons were taught with argumentation-based teaching. A total of 16 activities were used in the study. To prepare appropriate activities, textbooks used in mathematics classes in high schools and education faculties and related literature were examined. Regarding the activities prepared, the opinions of three field experts were consulted. The evaluation determined that the activities were suitable for transformational geometry learning and argumentation-based teaching.

The activities were projected onto a screen that all pre-service teachers could see clearly. Thus, all participants could follow the instructions the researcher gave. The researcher voiced the instructions for the activities. In addition, the researcher applied the process steps of the activity on his computer. After the GeoGebra applications, an argumentation process was created within the framework of the questions in the activity. The information, relations, and generalizations that the pre-service teachers reached as a result of the argumentation were noted on the board on the other side of the laboratory. An example of the argumentation process in the study is given below with "the translation of a point along the vector."

First of all, the transformation process is shown in GeoGebra. Then, the participants were asked to transform the arbitrary two points along the two vectors they determined. The participants noted the coordinates of the new points obtained as a result of the transformations. Then, they were asked to compare the coordinates of the new points and the shifted points. Thus, they created the data they can use in the argumentation process. With questions such as "What kind of change in the coordinates of the shifted points draws your attention?" and "How can you mathematically generalize the coordinates of the new points obtained by the translation of a point along the vector in the analytic plane?", the participants were enabled to put forward their claims on the subject. Thus, as Toulmin (2003) stated in the argumentation model, the participants realized the process of creating claims based on the data. After evaluating the answers, they were asked to give a few examples showing the correctness of the generalizations and justify their claims. In addition, considering the results obtained from other activities, they were asked to put forward new claims regarding the issue. Then, the participants were asked to give their reasons for their claims. During the activity, the researcher took the role of guiding the participants and giving them instructions. Other activities were carried out, like the exemplary activity process.

In the control group, teaching was mainly carried out by presentation. The same questions were solved in the control and experimental groups. No digital technology was used in the control group, and the drawings made on the classroom board were used to present the information directly.

After the lessons in the experimental and control groups were completed, TGAT (as explained above) was applied as a post-test. After the solutions of the pre-service teachers for the questions in TGAT in the pre-test and post-test were evaluated qualitatively, interviews were conducted with two pre-service teachers from each group in order to examine the effect of the teaching practices on their conceptual understanding of transformation geometry. Interviews were conducted with one pre-service teacher whose pre-test and post-test scores showed a difference and one pre-service teacher whose pre-test and post-test scores showed no difference from each group.

Data Analysis

The answers given by the participants to the questions in the TGAT were evaluated separately by the researcher and a field expert. If a student gave a mathematically correct and complete answer to a question on the TGAT, this answer was considered a correct answer. It was scored with 2 points. If the answer had missing aspects, it was considered partially correct and was scored with 1 point. If the answer was irrelevant or completely wrong, it was coded as incorrect, and if no answer was given to the question, it was coded as blank. Both cases were scored with 0 points.

The percentage of agreement between the evaluations was calculated with the formula expressed by Miles and Huberman (1994) as the agreement percentage = [(Agreement / (Agreement + Disagreement)] x100 and was found to be 94%. According to Miles and Huberman (1994), if the percentage of agreement is more significant than 70%, the analysis is reliable. The evaluations made in this direction are reliable. The evaluations made in this direction are reliable. 812 of the 860 cases that were independently examined were evaluated in the same way. Researchers and field experts came together for 48 cases where different evaluations were made, exchanged views on the reasons for these evaluations, and reached a consensus.

The study's quantitative data were analyzed using the licensed SPSS 22 package program at a 95% confidence level (p = 0.05). In the study, it was first examined whether the data followed a normal distribution. Skewness and kurtosis coefficients were examined to determine whether the data followed normal distribution.

Table 2 TGAT Skewness and Kurtosis Coefficients

	Groups		Mean	SD	Skewness	Kurtosis
TGAT	Control	Pre-test	12.18	4.95	-0.257	-1.316
		Post-test	13.32	6.27	0.004	-0.915
	Experiment	Pre-test	11.95	4.14	0.467	-0.373
		Post-test	22.62	8.01	-0.770	0.319

It is seen that the skewness and kurtosis coefficients presented in Table 2 are between -2 and +2 values. It can be said that the data are distributed normally if the skewness and kurtosis coefficients are between -2 and +2 (George & Mallery, 2020). Accordingly, it was decided to use parametric analysis techniques for data analysis. Independent groups t-test was used to compare pre-service teachers' academic achievements in the experimental and control groups. In addition, in cases where a statistically significant difference was detected due to the t-tests, the effect size (Cohen d) was calculated to interpret this significance (Leech et al., 2008; Myors et al., 2010). Effect size is interpreted regardless of sign (Timans et al., 2019). According to Cohen (1988), an effect size smaller than 0.2 is considered a “weak effect” and an effect larger than 0.8 is considered a “strong effect” (Myors et al., 2010). Leech et al. (2008) stated that an effect size of 1 and above could be interpreted as a “very strong effect”.

In the study, those in the experimental group were coded from E1 to E21, and those in the control group from C1 to C22, and these codes were used instead of real names. For the sub-problem related to conceptual understanding, the solutions made in the pretest and post-test were evaluated qualitatively, and semi-structured interviews were conducted with two pre-service teachers from each group. As a result of the evaluations made with the field experts, it was decided to conduct the interviews with the solutions of the fourth and twentieth questions in TGAT. Interviews were conducted with pre-service teachers coded as E1, E2, C1 and C2. Pre-service teachers coded as E1 and C1 represent participants whose TGAT pre-test and post-test scores differ, while pre-service teachers coded as E2 and C2 represent participants whose TGAT pre-test and post-test scores do not differ.

With the permission of the pre-service teachers, the speeches were recorded with a tape recorder. The data obtained here were transcribed and analyzed. The descriptive analysis technique was used to analyze qualitative data from solutions and interviews. Findings have been described with direct quotations. Since the interviews with four pre-service teachers would be pretty long to be reported in this article, only one interview with one pre-service teacher (E1) was included in the findings section.

Validity and Reliability

In experimental studies, various factors threaten validity, such as the selection of the participants, the background of the participants, the data collection tool, and the loss of participants (Büyüköztürk et al., 2016). During the research process, it was assumed that events other than experimental conditions had a similar effect on the participants. The data were collected from the experimental and the control groups with the same data collection tools, and the same people made the evaluations. Since no pre-service teachers left the study group, there was no effect on the difference between the two groups due to the loss of participants.

In the study, the role of the researcher in the research process was clearly explained, the study group, the research process, and the social environment where the data were collected were clearly defined, and the data collection and analysis methods were explained in detail (Timans et al., 2019). In addition, the triangulation strategy was used in data collection and analysis using different methods (Merriam, 2009). At each stage of the research, such as the preparation of data collection tools, data analysis, and the writing of the research report, the confirmation analysis strategy was used, and expert evaluation was applied.

To ensure the internal reliability of the study, as LeCompte and Goetz (1982) suggested, descriptions were enriched with direct quotations, and the findings obtained from written data collection tools were tried to be confirmed with the data obtained from the interviews. However, the data was also analyzed by another evaluator, and in this way, it was aimed to confirm the results. Participant confirmation strategy was used for internal validity. The data collected from the interviews were summarized and shared with the interviewed pre-service teachers. Thus, it was confirmed whether their explanations were understood correctly or not. To ensure the external validity of the study, as suggested by Miles and Huberman (1994), the analysis of the data and the findings and results obtained from these analyses are presented in detail. In addition, the purposeful sampling method, another way to increase external validity, was used in qualitative research. Interviews were conducted with pre-service teachers whose pretest and post-test scores differed from both groups.

Findings

Findings Related to First Sub-Problem

In this section, the findings regarding the first sub-problem of the study are presented. The analysis results of the data collected to answer the question "What is the knowledge of the pre-service teachers in the experimental and control groups about transformation geometry before the teaching process?" are given. The averages of the TGAT pretest scores of the pre-service teachers in the experimental and control groups are presented in Table 3.

Table 3 T-test Results Regarding the TGAT Pre-test Scores

	Groups	N	Mean	SD	df	t	p
TGAT pre-test	Control group	22	12.18	4.95	41	0.164	0.870
	Experimental group	21	11.95	4.14			

When Table 3 is examined, it is seen that there is no statistically significant difference between the pretest scores of the experimental and control groups ($t = 0.164$; $p > 0.05$). According to the t-test results, the experimental and control groups are equal regarding academic achievement in transformation geometry before the teaching process. Considering the maximum score that can be obtained from the TGAT (a maximum of 40 points can be obtained) and considering that pre-service teachers have encountered this issue in middle school and high school before, it can be said that the candidates' knowledge about transformation geometry is at a low level.

Findings Related to Second Sub-Problem

In order to examine whether argumentation-based transformation geometry teaching affected academic achievement, the post-test scores of the experimental and control groups were compared. The results of the independent groups t-test conducted for this purpose are presented in Table 4.

Table 4 T-test Results Regarding the TGAT Post-test Scores

	Groups	N	Mean	SD	df	t	p
TGAT post-test	Control group	22	13.32	6.27	41	-4.250	0.000
	Experimental group	21	22.62	8.01			

Table 4 shows that the statistically significant difference between the post-test averages is in favor of the experimental group ($t = -4.250$; $p < 0.05$). The mean score of the control group is 13.32, whereas the mean score of the experimental group is 22.62. Considering the pretest averages, the increase in the experimental group is much more significant than in the control group.

As a result of the independent groups t-test regarding TGAT post-test scores, it was determined that technology-supported argumentation-based teaching affected academic achievement in transformation geometry. In order to interpret the significance of this finding more effectively and to determine the level of effect of technology-supported argumentation-based teaching on academic achievement, the effect value (Cohen d) was calculated.

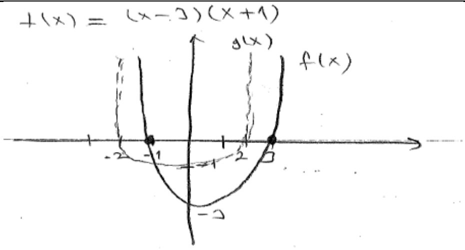
$$d = t.\sqrt{\frac{N_1 + N_2}{N_1.N_2}} = (-4.250).\sqrt{\frac{22 + 21}{22.21}} = -1.296$$

The value of d=-1.296 calculated in the equation above shows that technology-supported argumentation-based teaching has a very high effect on academic achievement.

Findings Related to Third Sub-Problem

This sub-problem aims to examine how teaching practices affect the conceptual understanding of transformation geometry. For this, the solutions given by a selected pre-service teacher (E1) to the questions in the TGAT were examined qualitatively in depth. Then, interviews were conducted on the pre-service teacher's solutions to the fourth question, which is a translation question, and the twentieth question, which includes rotation and reflection transformation. The solutions of E1 for the fourth question in TGAT are presented in Table 5.

Table 5 E1's Solutions for the Fourth Question

Pre-test	Post-test
$f(x) = (x-3)(x+1)$  $g(x) = a(x+2)(x-2)$ $(0,-1)$ denklemi sağlar. $-1 = a(2)(-2)$ $-1 = a(-4)$ $\frac{1}{4} = a$ $\boxed{g(x) = \frac{1}{4}(x^2-4)}$	$(x,y) \to (x+1, y-2)$ $y-2 = (x+1)^2 - 2(x+1) - 3$ $y-2 = x^2+2x+1-2x-2-3$ $y = x^2+\cancel{2x}-\cancel{2x}-4+2$ $y = x^2-2$

The fourth question in TGAT is "Find the equation of the parabola formed by translating the parabola f(x) = x^2-2x-3 in the negative direction by 1 br along the x-axis and in the positive direction by 2 br along the y-axis." When the solutions were examined, it was seen that E1 could not reach the correct answer in the pretest, but the solution was correct in the post-test. This question involves the problem of translating the parabola along the axes. In the solution in the pretest, it is seen that the participant first factors the parabola equation. As understood from his drawing, he determined the points where the parabola intersected the x-axis in this way. Although he did not show it by processing, he also determined where the parabola intersected the y-axis and drew the parabola. It can be seen from the parabola expressed by g(x) that the points where f(x) intersect the x-axis are shifted by 1 unit in the negative direction. However, it can be seen that he translates two units in the positive direction along the y-axis. Using the new points he obtained this way, he found the equation of g (x) by using the points where the axes intersected. It can be seen that E1 did not make any drawings in his solution for the same question in the final test. In this solution, it is seen that E1 has written (x + 1) instead of x and (y-2) instead of y in the parabola equation given in the question. Then, he edited the equation and got the correct answer. In the solution, E1 applied the transformations specified in the question (as points on the x-axis or y-axis) without distinguishing between points on the parabola.

The interview with E1, who offers different solutions in the pretest and post-test for the same question, is given below.

Researcher: “What did you think about this solution in the pretest?”

E1: “First of all, I needed to learn the formula for translation. That is why I drew graphics. I tried to comment on the graph. I proceeded through the points where the function intersects the axes on the graph. After that, because it says in the negative direction along the x-axis, for example, I came from point 3 to 2, the other from -1 to -2. Since it says along the y-axis in the question, I brought the point -3, which intersects the y-axis to -1.”

Researcher: “Then?”

E1: “I used this formula to write the equation for the graphic I just created. This way, I found the equation for the new graph.”

Researcher: “… Now, looking at the last test, your answer is correct. How did you come up with the solution here?”

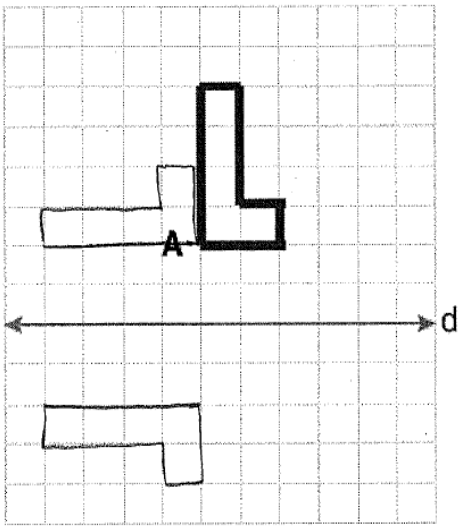
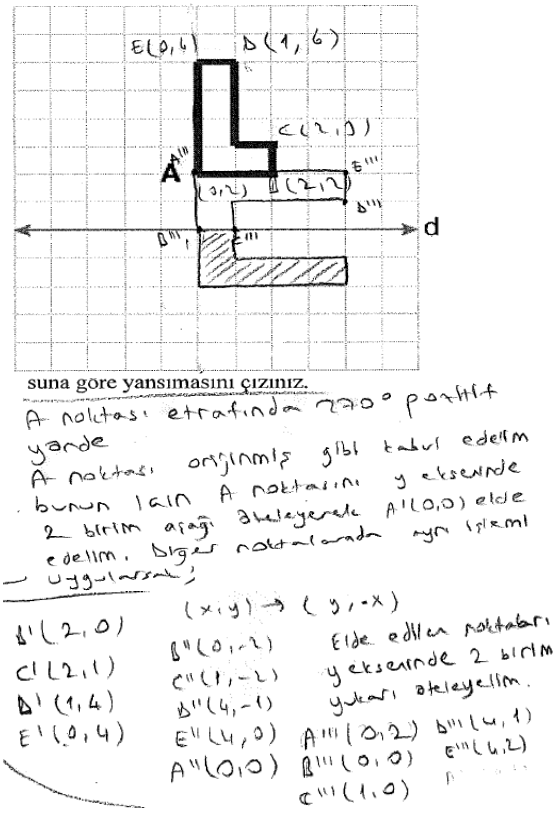
E1: “I learned the formula after the GeoGebra application. So, for example, we need to write (x + 1) for a 1-unit unfavorable translation on the x-axis. When I followed this rule, I could find the new equation immediately.”

Researcher: “Well, the last thing I want to ask about this question is:…. When you compare these two solutions, what could be the problem with the thinking in the pretest? ….”

E1: “I previously thought, for example, that for a unit displacement in the negative direction, the graph would shift one unit to the left on the x-axis. That is why I came to such a wrong conclusion.”

As a result of the interview, it was understood that in the pretest process, E1 tried to create a new function by shifting the points where the function intersected the axes according to the instructions. In the post-test, it is seen that he gave up the idea of considering the points on the axes separately. Here, it is seen that he makes use of the solution approach he named the "translation formula" regarding translation transformations. The statement of E1, "I learned the formula after GeoGebra application," shows that he was affected by the "generalization" process, which is the last step of the activities used in the argumentation-based teaching process. E1's solutions for the twentieth question in the TGAT are presented in Table 6.

Table 6 E1's Solutions for the Twentieth Question

Pre-test	Post-test
	

The twentieth question in DGBT was "Draw the reflection of the given figure on line d after rotating it 90 degrees in the negative direction about point A." The twentieth question involved the resultant of transformations (rotation and reflection transformation). When the solutions of E1 were examined, it was seen that he did not have a problem with reflection transformation in the pretest, but he made a mistake in the rotation process. When looking at the solution in the post-test, it was seen that E1 did not make an error in the rotation process. In addition, it was seen that he made the reflection transformation correctly in the post-test. A part of the interview with E1 regarding this question is given below.

Researcher: “Regarding the twentieth question, your answer in the pretest is wrong, and your answer in the post-test is correct. Where do you think you went wrong in the pretest?”

E1: “In the pretest, I realized I was doing the rotation movement against what was wanted.”

Researcher: “Well, can you explain your answer in the post-test?”

E1: “I took the given d-line as the x-axis. According to this line, I gave positive values for the upper part and negative values for the lower part. First of all, I made the rotation according to point A. This time, I did it by taking 270 degrees in the positive direction instead of 90 degrees in the negative direction. I saw this in your class. I determined the coordinates of the new points on the figure. Then, I reflected on the new shape according to the d-line. I redefined the coordinates of the corners on the newly formed shape according to this reflection transformation. It is easier to make such transformations when the figure is given on a scaled ground."

It is understood from this interview that E1 does not know that the negative rotation movement must be clockwise. Therefore, it is seen that he answered the twentieth question incorrectly in the pretest. However, after the teaching process, he corrected his misinformation and gave a correct answer to the question in the post-test. Especially in the post-test, it was seen that he moved the points to their position after the transformation operations. This situation shows that he acted consciously in the solution steps. In this sense, the argumentation processes carried out in the experimental group contributed to E1's creation of knowledge.

Conclusion, Discussion, and Suggestions

Within the scope of the first sub-problem of the study, the knowledge levels of pre-service teachers in the experimental and control groups about the transformation geometry before the teaching process were examined. As a result of the research, there is no statistical difference between the knowledge levels of the pre-service teachers, and the pretest mean scores of both groups are low. Participants in the study have recently encountered geometry transformation in the 11th grade of high school. It is not surprising that the participants have forgotten the relevant topic in the last 3-4 years and their level of knowledge on this subject is low.

In the analysis for the second subproblem of the study, a statistically significant difference was found between the post-test averages of the participants. This difference was found to be in favor of the experimental group. It has been determined that the effect of argumentation-based teaching on the occurrence of this difference is significant. In addition, this type of teaching affects the increase in the average achievement of the participants in the experimental group because argumentation-based teaching offers pre-service teachers the opportunity to share their mathematical thoughts while creating new mathematical knowledge (Cross, 2009). This situation makes them aware of their misconceptions and contributes to effective learning. In the relevant literature, some studies examine argumentation-based teaching and teaching separately and show that these teaching approaches increase academic achievement (Cross, 2009; Güven & Kaleli Yılmaz, 2012; Marshman & Brown, 2014; Sanchez & Uriza, 2008; Semana & Santos, 2010; Shadaan & Leong, 2013).

Contrary to this study, Can et al. (2017) concluded that the argumentation-based learning approach does not affect academic achievement. Can et al. (2017) listed this situation as the students' unwillingness to participate in the argumentation process, individual characteristics such as being afraid to speak in public, negative attitude, and the classroom environment in which the study is conducted is unsuitable for group work. In this study, the argumentation process was adequate for the participants because the experimental group students were open to exchanging ideas. In addition, the teaching practice of the experimental group was carried out in the computer laboratory. The fact that the laboratory is U-shaped made it easy for the participants to be included in the argumentation process.

Studies on teaching transformation geometry with GeoGebra have reached a similar result as in this study (Campbell & Zelkowski, 2020; Hollebrands et al., 2010; Sinclair et al., 2016). Technology is enjoyable for the participants in this study, and the concretization of abstract situations is among the factors that explain the success. Shadaan and Leong (2013) also stated that in a teaching process using dynamic geometry software, students enjoy learning more and can make influential associations between their learning. In summary, even in studies where the argumentation approach or teaching was used alone, there was an increase in academic achievement. In this study, where both the argumentation approach and technology were used together, the increase in the academic achievement of pre-service teachers regarding transformation geometry is a natural result.

Within the scope of the third sub-problem of the study, the effect of the teaching practices in the experimental group on pre-service teachers' conceptual understanding of transformation geometry was examined. The answers to the fourth question in the TGAT before the teaching practices and the interviews showed that the pre-service teachers had an incomplete or incorrect conceptual understanding that all points on a curve should be shifted holistically. After the teaching process, it was observed that only E1 reached the correct answer to this question, and the other three participants needed a better conceptual understanding. In line with his solution and his explanations in the interview, it was understood that E1 was affected by the "generalization" process, which is the last step of the activities used in argumentation-based teaching practice. Research shows that learners can make mathematical generalizations by using GeoGebra in teaching geometry (Campbell &

Zelkowski, 2020; Hollebrands et al., 2010; Santos-Trigo & Cristóbal-Escalante, 2008; Shadaan & Leong, 2013).

The answers to the twentieth question in TGAT showed that pre-service teachers needed to learn about the direction of rotation before teaching. As a result of the solutions and interviews after the teaching process, it was seen that E1 knew what he did and why and used his geometric thinking skills effectively. Based on the explanations made by E1, argumentation-based teaching affects this change. It is understood that E2 has the correct knowledge about the direction of rotation, but due to carelessness, E2 moves in the wrong direction in the solution. It was determined that C1 and C2 also corrected their knowledge about the direction but needed help to make the correct drawing due to their conceptual problems about the center of rotation.

When the research findings are evaluated in general, it can be said that argumentation-based transformation geometry teaching positively affects E1's geometric thinking skills and conceptual understanding of the subject. Although it is seen that there is no significant change in the conceptual understanding of E2, his thoughts on the solution were slightly affected. On the other hand, teaching transformation geometry with the current teaching method does not significantly affect the conceptual understanding of C1 and C2. Studies revealing the positive effect of teaching with the argumentation approach and dynamic geometry software on conceptual understanding support the results of this research (Gürbüz & Gülburnu, 2013; Hollebrands et al., 2010; Jackiw, 2003; Oldknow & Tetlow, 2008). It has also been stated that argumentation-based teaching encourages students' participation in the lesson and allowing them to discuss positively contributes to their mathematical abilities and argumentation levels (Civil & Hunter, 2015; Mueller & Yankelewitz, 2014; Rumsey & Langrall, 2016).

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

No conflict of interest.

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CRediT author statement

The first and the second author jointly carried out the processes of conceptualization of the theoretical framework, the determination of the research questions and the design of the method. The first author carried out the processes of data collection, data analysis and discussion of the results under the supervision of the second author. The first author wrote a Turkish draft of the manuscript on which both authors worked in cooperation. The second author edited the English version. The first author applied the article template before submission, and submitted the manuscript to the journal.

Research involving Human Participants and/or Animals

We hereby declare that the study has not unethical issues and that research and publication ethics have been observed carefully. The study was produced from the first author's doctoral thesis. The research process started in 2017 and the data were obtained before 2020 (2019-2020 Fall Semester). For this reason, the research is among the studies that do not require ethics committee approval.

Argümantasyon Tabanlı Öğretimin Dönüşüm Geometrisinde Kavramsal Anlayış Üzerine Etkisi

Özet:

Bu çalışmada, argümantasyon tabanlı öğretimin öğretmen adaylarının dönüşüm geometrisi konusundaki kavramsal anlayışlarına etkisi incelenmektedir. Çalışmada karma araştırma deseni kullanılmıştır. Çalışmanın nicel boyutunda, mevcut sınıfların deney ve kontrol grubu olarak rastgele atandığı, ön test ve son test kontrol gruplu yarı deneysel desen kullanılmıştır. Araştırmanın nitel boyutu ise bir durum çalışmasıdır. Çalışmanın katılımcılarını, Türkiye'deki bir devlet üniversitesinin eğitim fakültesinde üçüncü sınıfta öğrenim gören ve 2019-2020 eğitim öğretim yılı güz döneminde Analitik Geometri dersini alan 43 ortaokul öğretmen adayı oluşturmuştur. Çalışmanın amacı doğrultusunda, Dönüşüm Geometrisi Başarı Testi (DGBT) veri toplama aracı olarak kullanılmış ve öğretmen adaylarıyla görüşmeler yapılmıştır. Sonuç olarak, argümantasyon tabanlı öğretimin, öğretmen adaylarının akademik başarılarını ve dönüşüm geometrisi kavramsal anlamalarını olumlu yönde etkilediği sonucuna varılmıştır. Bu sonuç doğrultusunda, bu öğretim uygulamasının matematiğin diğer alanlarındaki akademik başarı ve kavramsal anlama üzerindeki etkilerinin incelenmesinin alana katkı sağlayacağı söylenebilir.

Anahtar kelimeler: Argümantasyon, dönüşüm geometrisi, öğretmen adayları, kavramsal anlayış.

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