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Assessing the applicability of the MCRAT, RAPS, RAMS and RATMI methods in the optimization of a dam geodetic control network

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Abstract

Only a few studies have so far addressed the applicability of Multi-Criteria Decision-Making (MCDM) methods in the optimization of geodetic control networks. For instance, the methods such as Višekriterijumsko Kompromisno Rangiranje/Rešenje (VIKOR) (in english: Multi-Criteria Optimization and Compromise Ranking/Solution), Analytic Hierarchy Process (AHP), Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Multi-Attributive Border Approximation Area Comparison (MABAC), Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA), Combined Compromise Solution (COCOSO), Range of Value (ROV) can be mentioned. Among them, the VIKOR method, it can be said, proved to be the most suitable tool in the mention area of engineering geodesy as it allows treating alternative solutions identified in a specific task in a desired way by using different preferential approaches represented by sets of criterion weights. Thus, the VIKOR method is used as a reference herein, and methods whose applicability is tested are four novel methods that have been used in some other areas, different from geodesy. It is about Multiple Criteria Ranking by Alternative Trace (MCRAT), Ranking the Alternatives by Perimeter Similarity (RAPS), Ranking by Alternatives Median Similarity (RAMS) and Ranking by Alternatives Using the Trace to Median Index (RATMI). The subject of this study is optimization of a 2D geodetic control network established to provide information regarding horizontal movements of a dam. The dam is located in Montenegro. The optimization is performed considering several pre-set constraints related to precision and reliability in the corresponding geodetic network. It turned out the four tested MCDM methods provided exactly the same results that were highly correlated with those obtained using the selected reference method. This fact led to the conclusion those four novel methods could be equally used in tasks similar to the one presented herein.

1. Introduction

In earlier studies, there were many words regarding old, classical, geodetic optimization methods and their application in designing geodetic networks. Namely, as already known, there are four orders of geodetic network design and those are the following [1,2]:

- Zero-Order Design (ZOD) – the datum problem;
- First-Order Design (FOD) – the configuration problem;
- Second-Order Design (SOD) – the weight problem;
- Third-Order Design (THOD) – the optimal improvement of an existing geodetic network.

However, such a classification is conditional due to disadvantage that is reflected in the fact that each of the listed tasks is solved individually, or by partially solving another task. For instance, in the SOD the design matrix varies, which partially solves the FOD, etc. [3].

The classical geodetic methods, based on the previously mentioned four optimization tasks, are not used herein. On the contrary, a MCDM-based approach is applied. It is about a completely different approach to optimizing special-purpose geodetic networks that involves solving a complex geodetic optimization task and provides the optimal solution for designing a geodetic network by considering multiple conflicting requirements simultaneously.

A great number of articles deal with the application of MCDM methods in various scientific fields. Some of the most recent ones are related to environmental fields (e.g. [4], [5], [6], [7]), photogrammetry (e.g. [8]), land consolidation (e.g. [9]), etc. However, only a few scientific articles presenting the application of MCDM methods in engineering geodesy have been published so far. As far as the author is aware, those articles, chronologically ordered, are the following: [10], [11], [3] and [12].

In [10] the VIKOR method was used in a geodetic task for the first time. It was applied on a bridge geodetic network represented by four alternatives, three of which involved different observation plans consisting of both horizontal directions and lengths, and one that included only measured lengths. In the research, only one geodetic instrument (total station) was used. The article [3] deals with using the VIKOR method, but this time in finding the optimal solution for a special-purpose trilateration geodetic network. The network was established to ensure the detection of deviation of the realized position of a structure pillar centre relative to the corresponding projected one for each of six evenly distributed pillars. The geodetic network design was established fourfold by considering four alternatives based on different number and location of reference points. In this study, same-precision length measurements were also considered. It was confirmed once again that the VIKOR method was an effective mathematical tool in such type of geodetic tasks.

On the other hand, the application of the AHP, PROMETHEE and TOPSIS methods was demonstrated in [11]. The author of this paper also proposed a new method as an alternative to the AHP and PROMETHEE and called it PROTERRA (PROcessing TEchnique of Ratings for Ranking of Alternatives) method. The entire process was based just on ranking the alternatives established by defining the reference system with assumption of fixed location of a specified pair of network points.

When it comes to the recent research presented in [12], it deals with the multi-criteria optimization of a tower geodetic micro-network. This research aimed to check applicability of the MABAC, MAIRCA, COCOSO and ROV methods with geodetic networks. It turned out that the MABAC, MAIRCA and ROV methods provided exactly the same results, while the results obtained using the COCOSO strategy were slightly different. However, in the final conclusion it was ascertained that all the four examined methods can also be successfully used in optimization of geodetic networks due to their effectiveness in handling a greater number of conflicting requirements.

Given that this study is the first research that deals with the application of the MCRAT, RAPS, RAMS and RATMI methods in engineering geodesy, due to its novelty, it will make a significant contribution to the current literature. Namely, this paper presents the performance of these four recent methods in optimizing a special-purpose geodetic network, whereby several key requirements in terms of precision and reliability are considered simultaniously.

The results obtained will serve as a valuable dataset for comparing the performance of the four methods in such a task, that is, to conclude whether there are differences in their application in this or similar engineering geodesy tasks.

A reader interested in application of these methods (or hybrid models that include some of them) in other fields, different from geodesy, can see, for instance, publications: [13], [14], [15], [16], [17], [18], [19], [20].

2. Descrition of the system to be optimized in the study

The system to be optimized in this study is a dam geodetic control network (hereinafter: geodetic network). It is about a concrete arch dam located in Montenegro. and discretized by eight control points, denoted in the text by N1, N2, ..., N8 (see subsection 2.1.2). All reference points are materialized by concrete pillars. Each pillar has a built-in forced-centring device on the top, so miscentring, i.e. centring error at a pillar, regardless of whether it is an instrument (total station) or a signal (prism), is assumed to be negligible and, therefore, it is not considered in horizontal directions and lengths measured between reference points. In addition, as there is a built-in forced-centring device at each control point as well, there is no impact of signal miscentring on horizontal direction measurements, which are performed from the pillars to the control points.

Six variants of the geodetic network, with varying number of reference points, are established for the purpose of the study. Each of them represents a particular acceptable solution, i.e. alternative which fulfills certain pre-set requirements related to precision and reliability (see subsection 2.1.2). The six alternatives are subjected to multi-criteria optimization, being ranked using five different MCDM methods (MCRAT, RAPS, RAMS, RATMI), one of which (VIKOR) is elected as a reference. Considering the ranking results, the applicability of the first four methods in optimizing geodetic control networks will be evaluated. The alternatives are presented in detail in subsection 2.1.4.

2.1.1. Mathematical background of geodetic network adjustment in a nutshell

The mathematical tool used in performing the adjustment of the geodetic network is the widely known Least Squares method. Thus, it is briefly presented herein, but a reader who needs more details can find a detailed description in [21].

Calculations start with establishing the design matrix ($\mathbf{A}$) and weight matrix ($\mathbf{P}$) in a well-known way. Then, using those matrices, the singular normal equation coefficient matrix is calculated as $\mathbf{N}=\mathbf{A}^{\mathrm{T}}\mathbf{PA}$. For further procedure performing, it is necessary to calculate the pseudoinverse $\mathbf{N}^{+}$ (for more details, see e.g. [3,12]), that, in fact, represents the cofactor matrix for the estimates of unknowns, i.e. $\mathbf{Q}_{\hat{\mathbf{x}}}=\mathbf{N}^{+}$. After that, for this study, two more cofactor matrices are obtained: cofactor matrix for the estimates of values of measured lengths and horizontal directions, $\mathbf{Q}_{\hat{\mathbf{l}}}=\mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}$, and cofactor matrix for the estimates of corrections of measured lengths and horizontal directions, $\mathbf{Q}_{\hat{\mathbf{v}}}=\mathbf{P}^{-1}-\mathbf{Q}_{\hat{\mathbf{l}}}$. Then, the redundancy matrix is calculated as $\mathbf{R}=\mathbf{Q}_{\hat{\mathbf{v}}}\mathbf{P}$.

2.1.2. Pre-set constraints introduced for extrication of acceptable geodetic network design solutions

Let's first introduce the following denotations:

- a – alternative ($a \in \{\mathrm{A}_1, \dots, \mathrm{A}_6\}$);
- r, c – reference point and control point, respectively, whereby $r \in \{\mathrm{R1}, \dots, \mathrm{R}n_{r,a}\}$, $c \in \{\mathrm{N1}, \dots, \mathrm{N}n_c\}$, with $n_{r,a} \in \{4,5,6,7,8\}$ and $n_c = 8$;
- $n_{m,a}$ – total number of measurements in alternative a, i.e. the sum of the number of measured horizontal directions ($n_{hd,a}$) and lengths ($n_{l,a}$);
- $A_{r(c)}$ and $B_{r(c)}$ – semi-major and semi-minor axis of the standard error ellipse in reference (control) point r (c), respectively;
- i – measurement ($i \in \{1, \dots, n_{m,a}\}$);
- σ_i – standard deviation of measurement i;
- σ_0 – square root of an adopted variance coefficient (*a priori*);
- r_{ii} – redundancy coefficient of measurement i, i.e. i^{th} diagonal element of the redundancy matrix $\mathbf{R}$;
- r_{tr} – redundancy coefficient threshold value (in this study $r_{tr} = 0.20$);
- $(1-\beta_0)$, α_0 – test power and significance level in one-dimensional statistical hypotheses, respectively;
- $t_{1-\beta_0}$ and $t_{1-\alpha_0/2}$ – normal distribution quantiles in the *data snooping test*;
- $|G_i^*|$ – marginal gross-error that can be detected in measurement i (obtained adopting $1-\beta_{0;i} \equiv 1-\beta_0 = 0.80$ and $\alpha_{0;i} \equiv \alpha_0 = 0.01$ herein);
- $|G_{i,tr}^*|$ – gross-error threshold value for measurement i (obtained adopting $1-\beta_{0;tr} \equiv 1-\beta_0 = 0.80$, $\alpha_{0;tr} \equiv \alpha_0 = 0.01$ and $r_{ii} \equiv r_{tr}$ herein);
- $1-\beta_{0;G_{i,tr}^*}$ – test power in detecting the gross-error limit value in measurement i.

Now, the pre-set constraints can be listed as follows:

$$\max_r (A_r/B_r) < 3.5 \quad (1)$$

$$\max_c (A_c/B_c) < 3.5 \quad (2)$$

$$r_{ii} \geq r_{tr} = 0.20 \quad (3)$$

$$|G_i^*| < |G_{i,tr}^*| \quad (4)$$

$$1-\beta_{0;G_{i,tr;a}^*} \geq 0.80 \quad (5)$$

The semi axes appearing in Inequality (1) and (2) are obtained in the following way:

$$A_{r(c)} = \sigma_0 \sqrt{0.5\left(S_{r(c)} + R_{r(c)}\right)\chi^2_{1-\alpha;2}} \quad (6)$$

$$B_{r(c)} = \sigma_0 \sqrt{0.5\left(S_{r(c)} - R_{r(c)}\right)\chi^2_{1-\alpha;2}} \quad (7)$$

whereby $\chi^2_{1-\alpha;2}$ is the Chi-square distribution quantile (obtained in the study for significance level $\alpha = 0.05$ and two degrees of freedom) and

$$S_{r(c)} = Q_{\hat{x}_{r(c)}\hat{x}_{r(c)}} + Q_{\hat{y}_{r(c)}\hat{y}_{r(c)}} \quad (8)$$

$$R_{r(c)} = \sqrt{\left(Q_{\hat{x}_{r(c)}\hat{x}_{r(c)}} - Q_{\hat{y}_{r(c)}\hat{y}_{r(c)}}\right)^2 + 4Q_{\hat{x}_{r(c)}\hat{y}_{r(c)}}} \quad (9)$$

The Inequality (4) involves the detectable marginal gross-error and its threshold value that are calculated as:

$$|G_i^*| = \left(t_{1-\beta_{0;i}} + t_{1-\alpha_{0;i}/2}\right)\sigma_i/\sqrt{r_{ii}} \quad (10)$$

$$|G_{i,tr}^*| = \left(t_{1-\beta_{0;tr}} + t_{1-\alpha_{0;tr}/2}\right)\sigma_i/\sqrt{r_{tr}} \quad (11)$$

The test power from Inequality (5) is calculated based on the equation:

$$|G_i^*| = |G_{i,tr}^*| \quad (12)$$

from which, after fixing $1-\beta_{0;tr} = 0.80$ and $\alpha_{0;i} = \alpha_{0;tr} = 0.01$, and considering $r_{tr} = 0.20$, the following equality is obtained:

$$1-\beta_{0;G_{i,tr}^*} = \mathrm{normsdist}\left(7.64\sqrt{r_{ii}} - t_{1-\alpha_{0;i}/2}\right) \quad (13)$$

2.1.3. Criteria for ranking the alternatives

In this study, the author introduces eight criteria. They are represented by the following eight functions ($a \in \{\mathrm{A}_1, \dots, \mathrm{A}_6\}$, $i_{hd} \in \{1, \dots, n_{hd,a}\}$, $i_l \in \{1, \dots, n_{l,a}\}$, $i \in \{1, \dots, n_{m,a}\}$, $c \in \{\mathrm{N1}, \dots, \mathrm{N}n_c\}$):

First criterion function:

The mean standard positional error obtained for control points

$$f_1 = \bar{m}_{p,a} = \sum_{c=\mathrm{N1}}^{\mathrm{N}n_c} m_{p,c;a}/n_c \quad (14)$$

where standard positional error of control point c is obtained using the square root of an adopted variance coefficient and the corresponding diagonal elements of the cofactor matrix for the estimates of unknowns, i.e.

$$m_{p,c;a} = \sigma_{0,a}\sqrt{Q_{\hat{y}_c\hat{y}_{c,a}} + Q_{\hat{x}_c\hat{x}_{c,a}}} \quad (15)$$

Second criterion function:

The mean ratio of semi axes of the standard error ellipse obtained for control points

$$f_2 = \overline{(A/B)}_a = \sum_{c=\mathrm{N1}}^{\mathrm{N}n_c}\left(A_{c,a}/B_{c,a}\right)/n_c \quad (16)$$

Third criterion function:

The difference between mean redundancy coefficient that is calculated for all measurements in the geodetic network and the optimal redundancy coefficient ($r_{opt} = 0.40$ herein)

$$f_3 = \bar{r}_a - r_{opt} = \left(\sum_{i=1}^{n_{m,a}} r_{ii,a}/n_{m,a}\right) - 0.40 \quad (17)$$

Fourth criterion function:

The sum of the numerical values of marginal gross-errors that can be detected obtained for all horizontal directions and lengths in the geodetic network

$$f_4 = \sum_{i_{hd}=1}^{n_{hd,a}} |G_{i_{hd},a}^*|/'' + \sum_{i_l=1}^{n_{l,a}} |G_{i_l,a}^*|/\mathrm{mm} \quad (18)$$

Fifth criterion function:

The mean test power in detecting the gross-error limit value calculated for all measurements in the geodetic network

$$f_5 = \overline{1-\beta_{0;G_{tr;a}^*}} = \sum_{i=1}^{n_{m,a}}\left(1-\beta_{0;G_{i,tr;a}^*}\right)/n_{m,a} \quad (19)$$

Sixth criterion function:

The mean Cook-Perović's distance that is obtained for all measurements in the geodetic network

$$f_6 = \overline{CP}_a = \sum_{i=1}^{n_{m,a}} CP_{i,a}/n_{m,a} \quad (20)$$

whereby ($\alpha_0 = 0.01$ is used in the study)

$$CP_{i,a} = t^2_{1-\alpha_0/2}\left(1-r_{ii,a}\right)\mathrm{r}\left(\mathbf{Q}_{\hat{\mathbf{x}},a}\right)^{-1} r_{ii,a}^{-1}/n_{m,a} \quad (21)$$

Seventh criterion function:

The sum of the influence coefficients on adjusted measurements calculated for all measurements in the geodetic network

$$f_7 = \mathrm{tr}(\mathbf{P}_a\mathbf{Q}_{\hat{\mathbf{l}},a}\mathbf{P}_a)/n_{m,a} \quad (22)$$

Eighth criterion function:

The mean value of the minimal movements that can be detected in each control point between two epochs using the figure (network) congruence test

$$f_8 = \overline{dp_a} = \sum_{c=\mathrm{N1}}^{\mathrm{N}n_c} dp_{c,a} / n_c \quad (23)$$

where the minimal movement in control point c between two epochs is calculated as follows:

$$dp_{c,a} = \sigma_{0,a}\sqrt{\lambda_a} \Big/ \sqrt{\mathbf{c}_{c,a}^{\mathrm{T}} \mathbf{Q}_{\mathbf{d},a}^{+} \mathbf{c}_{c,a}} \quad (24)$$

whereby λ_a is the non-centrality parameter (the value read from the corresponding table, given, for instance, in [21], for test power of 0.80, significance level of 0.05 and $f_a = 2(n_{r,a} + n_c) - 3$ degrees of freedom), $\mathbf{Q}_{\mathbf{d},a}^{+}$ is the pseudoinverse of the cofactor matrix $\mathbf{Q}_{\mathbf{d},a} = 2\mathbf{Q}_{\hat{\mathbf{x}},a}$ (the same observation plan is assumed for both epochs, i.e. $\mathbf{Q}_{\hat{\mathbf{x}},a(\mathbf{1})} = \mathbf{Q}_{\hat{\mathbf{x}},a(\mathbf{2})} \equiv \mathbf{Q}_{\hat{\mathbf{x}},a}$), and

$$\mathbf{c}_{c,a}^{\mathrm{T}} = (0 \quad \cdots \quad \sin\theta_{c,a} \quad \cos\theta_{c,a} \quad \cdots \quad 0) \quad (25)$$

with trigonometric terms at the places that correspond to unknowns $\hat{y}_{c,a}$ and $\hat{x}_{c,a}$ and $\theta_{c,a}$ denoting the standard error ellipse azimuth angle obtained from the following equation:

$$\tan 2\theta_{c,a} = 2Q_{\hat{x}_{c,a}\hat{y}_{c,a}} / \left(Q_{\hat{x}_{c,a}\hat{x}_{c,a}} - Q_{\hat{y}_{c,a}\hat{y}_{c,a}}\right) \quad (26)$$

Only the fifth criterion function represents a loss, which is why it should be minimized. On the other hand, the remaining ones are to be maximized, as they represent gains.

This choice of criteria was made because the quality of a special-purpose geodetic network is primarily determined based on its precision and reliability indicators, as appropriate quality measures.

2.1.4. Six alternatives to be ranked

This section provides a detailed review of the six alternatives (previously denoted by A_1, A_2, A_3, A_4, A_5 and A_6) that are to be ranked in the study.

In establishing these six alternatives, the author's intention was to provide different network configurations. Namely, by the chosen alternatives, design solutions with different number of reference points, horizontal directions and lengths are given. As already pointed out in subsection 2.1.2 (see the second bullet point of the first paragraph), eight reference points were used to identify the alterbatives.

Varying the (number of) reference points and measurements led to different geometry and, consequently, different values representing the precision and reliability in the network. That way, a variety of datasets of values related to the introduced criterion functions were available for the analysis conducted herein.

The geometrical representation for each alternative is given in the continuation, in figures 1-6.

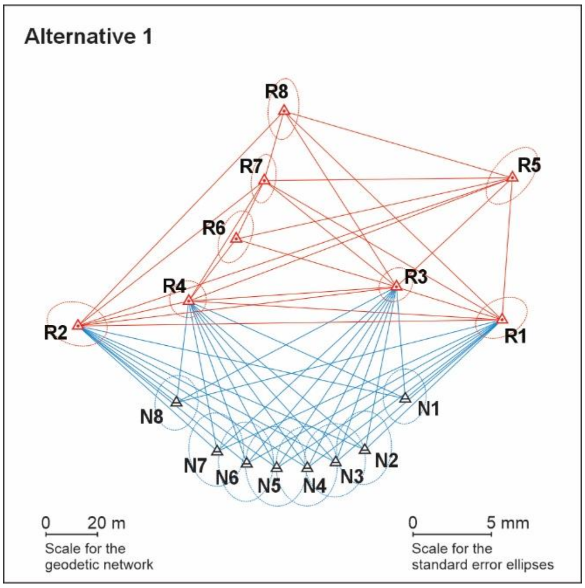

Figure 1. Geodetic network configuration with standard error ellipses in all reference and control points (A_1)

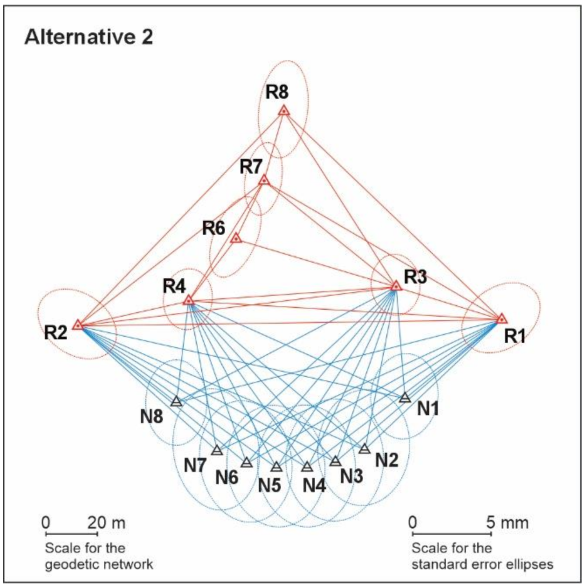

Figure 2. Geodetic network configuration with standard error ellipses in all reference and control points (A_2)

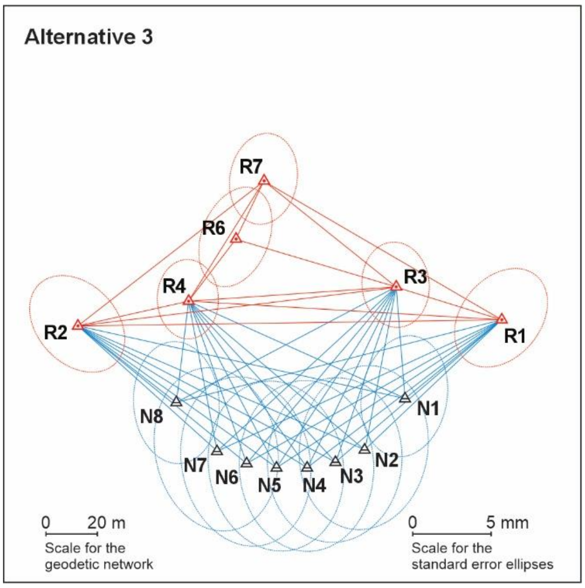

Figure 3. Geodetic network configuration with standard error ellipses in all reference and control points (A_3)

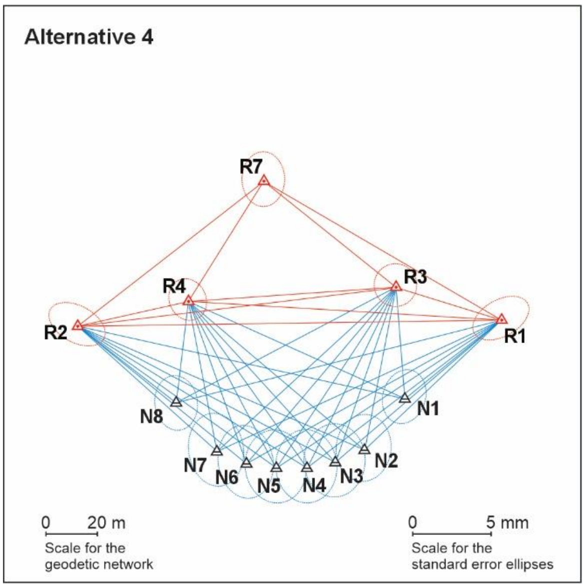

Figure 4. Geodetic network configuration with standard error ellipses in all reference and control points (A_4)

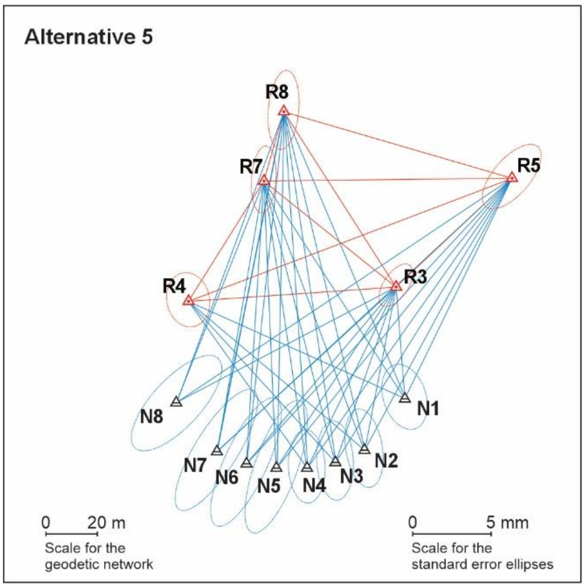

Figure 5. Geodetic network configuration with standard error ellipses in all reference and control points (A_5)

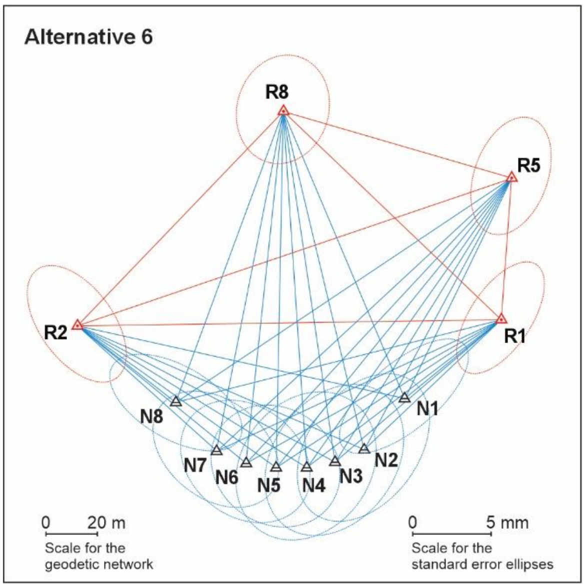

Figure 6. Geodetic network configuration with standard error ellipses in all reference and control points (A_6)

Tables 1-6 include the information regarding observation plan (twice measured quantities are marked by '(×2)' that follows after a target station label).

Table 1. Alternative 1 observation plan

Instrument station	Target station	
	Horizontal direction	Length
R1	R2, R3, R4, R5, R7, R8 N1, N2, N3, N4, N5, N6, N7, N8	R2
R2	R1, R3, R4, R5, R7, R8 N1, N2, N3, N4, N5, N6, N7, N8	
R3	R1, R2, R4, R5, R6, R7, R8 N1(×2), N2, N3, N4, N5, N6, N7, N8	R1, R2, R4, R5, R6, R7, R8
R4	R1, R2, R3, R5, R6, R7 N1, N2, N3, N4, N5, N6, N7, N8(×2)	R1, R2, R5, R6, R7
R5	R1, R2, R3, R4, R6, R7, R8	R1, R2, R6, R7, R8
R6	R3, R4, R5, R7	R7
R7	R1, R2, R3, R4, R5, R6, R8	R1, R2
R8	R1, R2, R3, R5, R7	R1, R2

Table 2. Alternative 2 observation plan

Instrument station	Target station	
	Horizontal direction	Length
R1	R2, R3, R4, R7, R8 N1, N2, N3, N4, N5, N6, N7, N8	R2
R2	R1, R3, R4, R7, R8 N1, N2, N3, N4, N5, N6, N7, N8	
R3	R1, R2, R4, R6, R7, R8 N1(×2), N2, N3, N4, N5, N6, N7, N8	R1, R2, R4, R6, R7, R8
R4	R1, R2, R3, R6, R7 N1, N2, N3, N4, N5, N6, N7, N8(×2)	R1, R2, R6, R7
R6	R3, R4, R7	R7
R7	R1, R2, R3, R4, R6, R8	R1, R2, R8
R8	R1, R2, R3, R7	R1, R2

Table 3. Alternative 3 observation plan

Instrument station	Target station	
	Horizontal direction	Length
R1	R2, R3, R4, R7 N1, N2, N3, N4, N5, N6, N7, N8	R2
R2	R1, R3, R4, R7 N1, N2, N3, N4, N5, N6, N7, N8	
R3	R1, R2, R4, R6, R7 N1(×2), N2, N3, N4, N5, N6, N7, N8	R1, R2, R4, R6, R7
R4	R1, R2, R3, R6, R7 N1, N2, N3, N4, N5, N6, N7, N8(×2)	R1, R2, R6, R7
R6	R3, R4, R7	R7
R7	R1, R2, R3, R4, R6	R1, R2

Table 4. Alternative 4 observation plan

Instrument station	Target station	
	Horizontal direction	Length
R1	R2, R3, R4, R7 N1, N2, N3, N4, N5, N6, N7, N8	R2
R2	R1, R3, R4, R7 N1, N2, N3, N4, N5, N6, N7, N8	
R3	R1, R2, R4, R7 N1(×2), N2, N3, N4, N5, N6, N7, N8	R1, R2, R4, R7
R4	R1, R2, R3, R7 N1, N2, N3, N4, N5, N6, N7, N8(×2)	R1, R2, R7
R7	R1, R2, R3, R4	R1, R2

Table 5. Alternative 5 observation plan

Instrument station	Target station	
	Horizontal direction	Length
R3	R4, R5, R7, R8 N1, N2, N3, N4, N5, N6, N7, N8	R4, R5, R7, R8
R4	R3, R5, R7 N1, N2, N3, N4(×2)	R5, R7
R5	R3, R4, R7, R8 N1, N2, N3, N4, N5, N6, N7, N8	R7, R8
R7	R3, R4, R5, R8 N1, N2, N3, N4, N5, N6, N7, N8	R8
R8	R3, R5, R7 N1, N2, N3, N4, N5, N6, N7, N8	

Table 6. Alternative 6 observation plan

Instrument station	Target station	
	Horizontal direction	Length
R1	R2, R5, R8 N1(×2), N2, N3, N4, N5, N6, N7, N8	R2, R5, R8
R2	R1, R5, R8 N1, N2, N3, N4, N5(×2), N6(×2), N7(×2), N8(×2)	R5, R8
R5	R1, R2, R8 N1, N2, N3, N4, N5, N6, N7, N8	R8
R8	R1, R2, R5 N1, N2, N3, N4, N5, N6, N7, N8	

For each of the six alternatives the geodetic network adjustment is done by defining the reference system (datum) by the minimum trace of the covariance matrix of the unknown coordinate estimates ($\mathrm{tr}(\mathbf{K}_{\hat{\mathbf{x}}}) = \sigma_0^2\mathrm{tr}(\mathbf{Q}_{\hat{\mathbf{x}}}) \rightarrow$ minimum) for all reference and control points in the geodetic network. The number of unknown coordinates is calculated as $u_a = 2n_{r,a} + 2n_c = 2n_{r,a} + 2 \cdot 8 = 2(n_{r,a} + 8), a \in \{\mathrm{A}_1, \dots, \mathrm{A}_6\}$.

The weight matrix ($\mathbf{P}$) is calculated based on the variances of measurements (horizontal directions and lengths) that differ between alternatives. The reason for this is reflected in the fact that total stations of varying precision are used, for both horizontal directions and lengths.

Table 7 lists the standard deviation values from manufacturers' declarations and numbers of sets (one set includes observations in both total station telescope face positions) used herein.

Table 7. Declared standard deviations and numbers of sets in measuring the horizontal directions

Alternative	Standard deviation		Number of sets
	Horizontal direction	Length	
A_1	7″	3 + 2 ppm	3
A_2	5″	3 + 2 ppm	1
A_3	7″	2 + 2 ppm	3
A_4	5″	1.5 + 2 ppm	2
A_5	3″	3 + 2 ppm	2
A_6	3″	2 + 2 ppm	1

The extreme values of the precision and reliability indicators for the geodetic network are shown in Table 8.

Table 8. Values of the main geodetic network quality indicators for all six alternatives

Indicator	Alternative					
	A_1	A_2	A_3	A_4	A_5	A_6
	min max	min max	min max	min max	min max	min max
A_r/B_r	1.20 1.83	1.17 1.95	1.15 1.61	1.18 1.49	1.27 3.21	1.22 2.10
A_c/B_c	1.27 1.40	1.26 1.42	1.25 1.43	1.23 1.40	1.46 3.42	1.12 2.25
$m_{p,r}$[mm]	0.76 1.18	1.02 1.44	0.81 1.16	0.74 0.97	0.87 1.53	1.21 1.32
$m_{p,c}$[mm]	1.20 1.72	1.46 2.11	1.16 1.70	1.00 1.47	1.12 2.04	1.35 1.65
r	0.20 0.94	0.20 0.89	0.20 0.83	0.20 0.80	0.20 0.93	0.23 0.71
$\lvert G_{hd}^*\rvert$[″]	15.0 30.7	18.8 38.0	15.4 30.8	13.5 27.0	8.2 16.1	12.4 21.6
$\lvert G_l^*\rvert$[mm]	10.9 12.5	11.4 13.2	8.0 9.6	6.3 7.6	10.8 13.3	9.0 9.9
CP_{hd}	0.03 0.71	0.04 0.77	0.05 0.85	0.06 0.95	0.07 0.93	0.12 0.91
CP_l	0.01 0.04	0.03 0.07	0.04 0.10	0.07 0.11	0.02 0.10	0.11 0.16
$1-\beta_{0;G_{tr}^*}$	0.81 1.00	0.81 1.00	0.80 1.00	0.80 1.00	0.81 1.00	0.85 1.00

The author made this selection of alternatives with the aim of providing a multi-criteria analysis that includes geodetic network configurations with different numbers of reference points and different numbers of measured quantities.

3. Methods

This section briefly presents all the methods used in this study.

3.1. The MCRAT method

This is a nine-step procedure that, according to [13], involves the next steps:

Step 1. Evaluating each of m alternatives (A_1, A_2, ..., A_m) against each of n criteria (C_1, C_2, ..., C_n), i.e. obtaining values x_{ij}, with $(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\}$, as the elements of the so-called decision matrix ($\mathbf{X}$), written as follows:

$$\mathbf{X}_{m\times n} = \left(x_{ij}\right)_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \quad (27)$$

Step 2. Calculation of the normalized decision matrix ($\mathbf{R}$) in the following way:

$$\mathbf{R}_{m\times n} = \left(r_{ij}\right)_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \quad (28)$$

where the elements r_{ij}, $(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\}$, are obtained as:

$$r_{ij} = x_{ij}/\max_i\left(x_{ij}\right) \quad (29)$$

if the criterion C_j is maximized, i.e.

$$r_{ij} = \min_i\left(x_{ij}\right)/x_{ij} \quad (30)$$

if it is minimized.

Step 3. Calculating the weighted normalized decision matrix ($\mathbf{U}$) as follows:

$$\mathbf{U}_{m\times n} = \left(u_{ij}\right)_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & \ddots & \vdots \\ u_{m1} & \cdots & u_{mn} \end{pmatrix} \quad (31)$$

with the elements $u_{ij} = w_j r_{ij}$, $(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\}$, whereby w_j is the weight of the criteria C_j, obtained herein using the SWING method [22], performed through the following steps:

Step 3.1. Establishing a n-tuple $(p_1, p_2, \dots, p_n)$ by assigning a number of points to each of the criteria C_1, C_2, ..., C_n according to a preference that is introduced to express their individual importance (this way, the most important and least important criteria are, respectively, assigned the greatest and least number of points);

Step 3.2. Calculation of the criterion weight values in the following way:

$$w_j = p_j/\textstyle\sum_{j=1}^{n} p_j, \; j \in \{1,2,\dots,n\} \quad (32)$$

Step 4. Determination of the 'ideal' alternative as follows:

$$Q = \{q_1, q_2, \dots, q_n\} \quad (33)$$

with elements of the set obtained as:

$$q_j = \max_i\left(u_{ij}\right) = q_{j,\max} \quad (34)$$

if C_j is the criterion to be maximized, i.e.

$$q_j = \max_i\left(u_{ij}\right) = q_{j,\min} \quad (35)$$

if it is minimized, with $(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\}$.

Step 5. Rearranging the q_j-values so that the first k of them are those that correspond to the criteria to be maximized and the remaining $n-k=h$ are related to the criteria to be minimized. Then, decomposition of the set Q is performed as follows:

$$Q = \left\{q_{1,\max}, \dots, q_{k,\max}\right\} \cup \left\{q_{1,\min}, \dots, q_{h,\min}\right\} \quad (35)$$

Step 6. Rearranging the u_{ij}-values in the matrix $\mathbf{U}_{m\times n}$ by rows in a way analogous to that shown in *Step 5*. Then, for each alternative, the following decomposition is obtained:

$$U_i = \left\{u_{i1,\max}, \dots, u_{ik,\max}\right\} \cup \left\{u_{i1,\min}, \dots, u_{ih,\min}\right\} \quad (37)$$

with $i \in \{1,2,\dots,m\}$.

Step 7. Calculating magnitudes for each component of the 'ideal' alternative, which is to say:

$$Q_k = \sqrt{q_{1,\max}^2 + q_{2,\max}^2 + \cdots + q_{k,\max}^2} \quad (38)$$

$$Q_h = \sqrt{q_{1,\min}^2 + q_{2,\min}^2 + \cdots + q_{h,\min}^2} \quad (39)$$

as well as

$$U_{ik} = \sqrt{u_{i1,\max}^2 + u_{i2,\max}^2 + \cdots + u_{ik,\max}^2} \quad (40)$$

$$U_{ih} = \sqrt{u_{i1,\min}^2 + u_{i2,\min}^2 + \cdots + u_{ih,\min}^2} \quad (41)$$

whereby $i \in \{1,2,\dots,m\}$.

Step 8. Creating the following matrix for each alternative:

$$\mathbf{T}_i = \begin{pmatrix} Q_k & 0 \\ 0 & Q_h \end{pmatrix}\begin{pmatrix} U_{ik} & 0 \\ 0 & U_{ih} \end{pmatrix} = \begin{pmatrix} t_{11,i} & 0 \\ 0 & t_{22,i} \end{pmatrix} \quad (42)$$

and calculating the corresponding trace as:

$$t_i = \operatorname{tr}(\mathbf{T}_i) = t_{11,i} + t_{22,i}, \; i \in \{1,2,\dots,m\} \quad (43)$$

Step 9. Ranking the alternatives according to descending order of $\operatorname{tr}(\mathbf{T}_i)$-values. So, the alternative with the maximum value of $\operatorname{tr}(\mathbf{T}_i)$ is declared the optimal (best) solution.

3.2. The RAPS method

This method also carried out in nine steps which are the following (after [13]):

Step 1 to 7. The same as *Step 1* to *7* presented for the MCRAT method (see subsection 3.1).

Step 8. Calculating the perimeter of the 'ideal' alternative (it is expressed as the perimeter of the right-angle triangle, whereby components Q_k and Q_h represent the base and perpendicular side of that triangle, respectively) and perimeter of each alternative. These calculations are performed as follows:

$$P = Q_k + Q_h + \sqrt{Q_k^2 + Q_h^2} \quad (44)$$

$$P_i = U_{ik} + U_{ih} + \sqrt{U_{ik}^2 + U_{ih}^2} \quad (45)$$

Then, using Equation (44) and (45), the perimeter similarity value is obtained in the following way:

$$PS_i = P_i/P, \; i \in \{1,2,\dots,m\} \quad (46)$$

Step 9. Ranking the alternatives according to descending order of PS_i-values. Thus, the optimal (best) alternative (solution) is the one that the maximum value of PS_i corresponds to.

3.3. The RAMS method

In [15] the RAMS method was introduced as an extension to the previously developed RAPS method. The procedure is performed in nine steps as follows:

Step 1 to 7. The same as *Step 1* to *7* presented for the RAPS method (see subsection 3.2).

Step 8. Calculating the median of the 'ideal' alternative, expressed as the median of the right angle used for the RAPS, i.e.

$$M = \sqrt{Q_k^2 + Q_h^2}/2 \quad (47)$$

and the median of each alternative

$$M_i = \sqrt{U_{ik}^2 + U_{ih}^2}/2 \quad (48)$$

as well.

Now, using Equation (47) and (48), the median similarity value is calculated as:

$$MS_i = M_i/M,\ i \in \{1,2,\dots,m\} \tag{49}$$

Step 9. Ranking the alternatives according to descending order of MS_i-values. The alternative that the maximum value of MS_i corresponds to is declared the optimal (best) alternative (solution).

3.4. The RATMI method

The second method proposed in [15] is the RATMI method. It was based on a majority index and the concept of the VIKOR method (see subsection 3.5). The RATMI procedure entails an eleven-step algorithm involving the following steps:

Step 1 to 8. The same as *Step 1* to *8* presented for the MCRAT method (see subsection 3.1).

Step 9. The same as *Step 8* presented for the RAMS method (see subsection 3.3).

Step 10. Calculating the majority index between the strategies of the MCRAT and RAMS methods, i.e.

$$E_i = vE_{t,i} + (1-v)E_{MS,i} \tag{50}$$

with $i \in \{1,2,\dots,m\}$, whereby v and $1-v$ represent the weights of the MCRAT and RAMS strategies, respectively, and

$$E_{t,i} = (t_i - t^*)/(t^- - t^*) \tag{51}$$

$$E_{MS,i} = (MS_i - MS^*)/(MS^- - MS^*) \tag{52}$$

whereby

$$t_i = \mathrm{tr}(\mathbf{T}_i),\ t^* = \min_i \mathrm{tr}(\mathbf{T}_i),\ t^- = \max_i \mathrm{tr}(\mathbf{T}_i) \tag{53}$$

$$MS^* = \min_i MS_i,\ MS^- = \max_i MS_i \tag{54}$$

The v-value fulfills condition $0 < v < 1$, and herein $v = 0.5$ is adopted.

Step 11. Ranking the alternatives according to descending order of E_i-values. The the optimal (best) solution is represented by the alternative having the maximum value of E_i.

3.5. Reference method in the study – VIKOR

This well-known method involves the following algorithm (according to [23,24], [3,12]):

Step 1. The same as *Step 1* presented for the MCRAT method (see subsection 3.1).

Step 2. Calculation of the normalized decision matrix ($\mathbf{R}$) in the way analogous to that presented in *Step 2* of the MCRAT method, but now calculating its elements as follows:

$$r_{ij} = \left(x_{ij} - \max_i(x_{ij})\right)/\left(\min_i(x_{ij}) - \max_i(x_{ij})\right) \tag{55}$$

if the criterion C_j is maximized, i.e.

$$r_{ij} = \left(x_{ij} - \min_i(x_{ij})\right)/\left(\max_i(x_{ij}) - \min_i(x_{ij})\right) \tag{56}$$

if it is minimized, with $(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\}$.

Step 3. Calculation of the sum and extrication of the maximum of the weighted comparability sequence as follows:

$$S_i = \textstyle\sum_{j=1}^{n} w_j r_{ij} \tag{57}$$

$$R_i = \max_j\left(w_j r_{ij}\right) \tag{58}$$

with $(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\}$, where criterion weights w_j were already introduced in subsection 3.1 (see *Step 3*, Equation (32)).

When $R_i = \max_i R_i$ for two or more i-indices, then, using Equation (57) and (58), the modified value is computed as:

$$R_{i,\mathrm{mod}} = R_i + \left(S_i - \max_i R_i\right)/100 \tag{59}$$

Step 4. Calculation of the majority index which is the VIKOR method ranking based on as follows:

$$Q_i = vQS_i + (1-v)QR_i \tag{60}$$

with $i \in \{1,2,\dots,m\}$, whereby v represents the weight of the strategy of fulfilling most of the criteria (the value of 0.5 is assumed), and

$$QS_i = \left(S_i - \min_i S_i\right)/\left(\max_i S_i - \min_i S_i\right) \tag{61}$$

$$QR_i = \left(R_i - \min_i R_i\right)/\left(\max_i R_i - \min_i R_i\right) \tag{62}$$

Step 5. Forming the following three ranking lists:

- First ranking list, according to ascending order of QS_i-values;
- Second ranking list, according to ascending order of QR_i-values; and
- Third, compromise ranking list, according to ascending order of Q_i-values.

On all three ranking lists, the first-ranked alternative is the one having the least value of the measure which a particular list is based on.

Step 6. Checking which of the following conditions are met:

- Condition 1: The first-ranked alternative on the compromise ranking list, obtained for $v = 0.5$ (denoted by $a^{(1)}$), must have a 'sufficient advantage' over the second-ranked alternative (denoted by $a^{(2)}$), which means that
$$Q_{a^{(2)}} - Q_{a^{(1)}} \geq \min(0.25;\ 1/(m-1)) \tag{63}$$
- Condition 2: The first-ranked alternative on the compromise ranking list (for $v = 0.5$), must have a 'sufficiently stable' first position. Namely, at least one of the following requirements must be fulfilled: (1) alternative $a^{(1)}$ is first-ranked on the first ranking list; (2) alternative $a^{(1)}$ is first-ranked on the second ranking list; (3) alternative $a^{(1)}$ is first-ranked on the third ranking list, for $v = 0.25$ and $v = 0.75$.

Step 7. Decision making as follows:

- If $a^{(1)}$ fulfills both Condition 1 and Condition 2, it is considered the only and best solution;
- If $a^{(1)}$ does not fulfill only Condition 2, it is considered 'not sufficiently' better than $a^{(2)}$, and then a set of compromise solutions consisting of these two alternatives is formed;
- If $a^{(1)}$ does not fulfill only Condition 1 or both Condition 1 and Condition 2, it is considered 'not sufficiently' better than $a^{(2)}$ and any other alternative (denoted by $a^{(k)}$) that fulfills the following inequality:
$$Q_{a^{(k)}} - Q_{a^{(1)}} < \min(0.25;\ 1/(m-1)) \tag{64}$$
Then, a set of compromise solutions is formed. It now includes alternatives $a^{(1)}$, $a^{(1)}$ and each of the remaining alternatives for which Inequality (64) is valid.

4. Results and discussion

The results of the analysis conducted in the study are presented in two subsections. The first one is a presentation of the results obtained by applying the MCRAT, RAPS, RAMS and RATMI methods, and the second one is a review of what was obtained after the input data had been subjected to the VIKOR method.

Table 9 shows the results of the evaluation of the six alternatives according to each of the eight criteria. Based on those data, the ranking of the alternatives by each criterion is given in Table 10.

Table 9. Values of the criterion functions for all alternatives

Alternative	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	1.56	1.332	0.25	1814.2	0.9933	0.1292	1.0934	8.9
A_2	1.92	1.328	0.20	1836.8	0.9906	0.1699	0.4524	10.9
A_3	1.53	1.319	0.18	1287.1	0.9894	0.2009	1.5808	8.7
A_4	1.33	1.313	0.16	994.0	0.9893	0.2285	1.1713	7.5
A_5	1.60	2.594	0.16	692.0	0.9790	0.2623	0.8533	9.4
A_6	1.50	1.513	0.15	761.7	0.9914	0.2534	0.4843	8.8

Table 10. Ranking of the alternatives by each of the eight criteria

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
1	A_4	A_4	A_6	A_5	A_1	A_1	A_2	A_4
2	A_6	A_3	A_5	A_6	A_6	A_2	A_6	A_3
3	A_3	A_2	A_4	A_4	A_2	A_3	A_5	A_6
4	A_1	A_1	A_3	A_3	A_3	A_4	A_1	A_1
5	A_5	A_6	A_2	A_1	A_4	A_6	A_4	A_5
6	A_2	A_5	A_1	A_2	A_5	A_5	A_3	A_2

After looking at Table 10, what can be spotted is that alternative A_4 is the best for the criteria f_1, f_2 and f_8, but it takes the penultimate place when it comes to the criteria f_5 and f_7.

On the other hand, the worst alternative according to the criteria f_1, f_4 and f_8 is A_2, and also A_5 when it is about the criteria f_2, f_5 and f_6. However, the former is the best for the criterion f_7, and the latter for the criterion f_4.

As for the criterion weights, four different 8-tuples are used in applying all five MCDM methods herein. So, the analysis is carried out in four preferential approaches (let's label them as Approach I, II, III, IV, or abbreviated: A-I, A-II, A-III, A-IV), represented by the following 8-tuples of points (weight coefficients):

- For A-I:
  Points: (10, 8, 15, 4, 4, 4, 4, 12)
  Weight coefficients: (0.1639, 0.1311, 0.2459, 0.0656, 0.0656, 0.0656, 0.0656, 0.1967);
- For A-II:
  Points: (10, 6, 8, 4, 4, 4, 4, 15)
  Weight coefficients: (0.1818, 0.1091, 0.1455, 0.0727, 0.0727, 0.0727 0.0727 0.2727);
- For A-III:
  Points: (8, 6, 8, 6, 6, 6, 6, 10)
  Weight coefficients: (0.1429, 0.1071, 0.1429, 0.1071, 0.1071, 0.1071, 0.1071, 0.1786);
- For A-IV:
  Points: (10, 10, 10, 6, 8, 4, 6, 15)
  Weight coefficients: (0.1449, 0.1449, 0.1449, 0.0870, 0.1159, 0.0580, 0.0870, 0.2174).

Below, the author presents the main results of the analysis, separately for the methods being tested and the reference method.

4.1. Results obtained by applying the MCRAT, RAPS, RAMS and RATMI methods

The input data (given in Table 9) are the same for all five methods in the study, but the corresponding normalized data, shown in Table 11, are the same only for the MCRAT, RAPS, RAMS and RATMI methods, due to the use of the same normalization method that differes from the one used in the VIKOR method.

Consequently, the weighted values of the data included in Table 11 are also the same for these methods in all four preferential approaches (A-I, A-II, A-III and A-IV). Those values are shown in tables 12-15.

Table 11. Normalized criterion function values for all alternatives (MCRAT, RAPS, RAMS, RATMI)

Alternative	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.8507	0.9858	0.5796	0.3815	1.0000	1.0000	0.4137	0.8427
A_2	0.6939	0.9890	0.7273	0.3768	0.9973	0.7608	1.0000	0.6883
A_3	0.8671	0.9954	0.8295	0.5377	0.9961	0.6431	0.2862	0.8615
A_4	1.0000	1.0000	0.8951	0.6962	0.9960	0.5656	0.3862	1.0000
A_5	0.8329	0.5063	0.8951	1.0000	0.9856	0.4926	0.5301	0.7974
A_6	0.8893	0.8679	1.0000	0.9086	0.9981	0.5100	0.9341	0.8575

Table 12. Weighted normalized criterion function values for all alternatives (MCRAT, RAPS, RAMS, RATMI; for A-I)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.1395	0.1293	0.1425	0.0250	0.0656	0.0656	0.0271	0.1658
A_2	0.1138	0.1297	0.1788	0.0247	0.0654	0.0499	0.0656	0.1354
A_3	0.1421	0.1305	0.2040	0.0353	0.0653	0.0422	0.0188	0.1695
A_4	0.1639	0.1311	0.2201	0.0457	0.0653	0.0371	0.0253	0.1967
A_5	0.1365	0.0664	0.2201	0.0656	0.0646	0.0323	0.0348	0.1569
A_6	0.1458	0.1138	0.2459	0.0596	0.0655	0.0334	0.0613	0.1687

Table 13. Weighted normalized criterion function values for all alternatives (MCRAT, RAPS, RAMS, RATMI; for A-II)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.1547	0.1075	0.0843	0.0277	0.0727	0.0727	0.0301	0.2298
A_2	0.1262	0.1079	0.1058	0.0274	0.0725	0.0553	0.0727	0.1877
A_3	0.1577	0.1086	0.1207	0.0391	0.0724	0.0468	0.0208	0.2350
A_4	0.1818	0.1091	0.1302	0.0506	0.0724	0.0411	0.0281	0.2727
A_5	0.1514	0.0552	0.1302	0.0727	0.0717	0.0358	0.0386	0.2175
A_6	0.1617	0.0947	0.1455	0.0661	0.0726	0.0371	0.0679	0.2339

Table 14. Weighted normalized criterion function values for all alternatives (MCRAT, RAPS, RAMS, RATMI; for A-III)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.1215	0.1056	0.0828	0.0409	0.1071	0.1071	0.0443	0.1505
A_2	0.0991	0.1060	0.1039	0.0404	0.1069	0.0815	0.1071	0.1229
A_3	0.1239	0.1066	0.1185	0.0576	0.1067	0.0689	0.0307	0.1538
A_4	0.1429	0.1071	0.1279	0.0746	0.1067	0.0606	0.0414	0.1786
A_5	0.1190	0.0542	0.1279	0.1071	0.1056	0.0528	0.0568	0.1424
A_6	0.1270	0.0930	0.1429	0.0974	0.1069	0.0546	0.1001	0.1531

Table 15. Weighted normalized criterion function values for all alternatives (MCRAT, RAPS, RAMS, RATMI; for A-IV)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.1233	0.1429	0.0840	0.0332	0.1159	0.0580	0.0360	0.1832
A_2	0.1006	0.1433	0.1054	0.0328	0.1156	0.0441	0.0870	0.1496
A_3	0.1257	0.1443	0.1202	0.0468	0.1155	0.0373	0.0249	0.1873
A_4	0.1449	0.1449	0.1297	0.0605	0.1155	0.0328	0.0336	0.2174
A_5	0.1207	0.0734	0.1297	0.0870	0.1143	0.0286	0.0461	0.1733
A_6	0.1289	0.1258	0.1449	0.0790	0.1157	0.0296	0.0812	0.1864

The main results obtained in application of the MCRAT, RAPS, RAMS and RATMI methods are presented in tables 16-19, respectively.

The results are given for all preferential approaches. In doing so, the quantity denotations introduced in subsections 3.1, 3.2, 3.3 and 3.4 are used.

Table 16. The MCRAT method application main results (for A-I, A-II, A-III, A-IV)

Quantity	A_1	A_2	A_3	A_4	A_5	A_6
	A-I: $Q_k = 0.06557$; $Q_h = 0.39515$					
U_{ik}	0.06557	0.06540	0.06532	0.06531	0.06463	0.06545
U_{ih}	0.29937	0.29577	0.33309	0.36786	0.32040	0.36270
t_i	0.12260	0.12116	0.13590	0.14964	0.13084	0.14761
	A-II: $Q_k = 0.07273$; $Q_h = 0.39543$					
U_{ik}	0.07273	0.07253	0.07244	0.07243	0.07168	0.07259
U_{ih}	0.31997	0.28825	0.33250	0.37595	0.31351	0.34830
t_i	0.13182	0.11926	0.13675	0.15393	0.12918	0.14301
	A-III: $Q_k = 0.10714$; $Q_h = 0.34442$					
U_{ik}	0.10714	0.10685	0.10672	0.10671	0.10560	0.10694
U_{ih}	0.26560	0.25828	0.27099	0.30179	0.26687	0.30186

Quantity	A_1	A_2	A_3	A_4	A_5	A_6
t_i	0.10296	0.10041	0.10477	0.11537	0.10323	0.11543
	A-IV: $Q_k = 0.11594$; $Q_h = 0.35882$					
U_{ik}	0.11594	0.11563	0.11549	0.11548	0.11428	0.11572
U_{ih}	0.28633	0.27338	0.30054	0.33461	0.27809	0.31922
t_i	0.11619	0.11150	0.12123	0.13345	0.11304	0.12796

Table 17. The RAPS method application main results (for A-I, A-II, A-III, A-IV)

Quantity	A_1	A_2	A_3	A_4	A_5	A_6
	A-I: $Q_k = 0.06557$; $Q_h = 0.39515$; $P = 0.86127$					
P_i	0.67141	0.66408	0.73785	0.80679	0.71189	0.79671
PS_i	0.77956	0.77104	0.85670	0.93674	0.82656	0.92504
	A-II: $Q_k = 0.07273$; $Q_h = 0.39543$; P =0.87022					
P_i	0.72083	0.65802	0.74524	0.83124	0.70680	0.77668
PS_i	0.82834	0.75616	0.85639	0.95521	0.81221	0.89251
	A-III: $Q_k = 0.10714$; $Q_h = 0.34442$; $P = 0.81226$					
P_i	0.65913	0.64465	0.66895	0.72860	0.65948	0.72905
PS_i	0.81149	0.79365	0.82357	0.89700	0.81191	0.89757
	A-IV: $Q_k = 0.11594$; $Q_h = 0.35882$; $P = 0.85186$					
P_i	0.71119	0.68583	0.73799	0.80406	0.69303	0.77449
PS_i	0.83487	0.80511	0.86633	0.94389	0.81355	0.90918

Table 18. The RAMS method application main results (for A-I, A-II, A-III, A-IV)

Quantity	A_1	A_2	A_3	A_4	A_5	A_6
	A-I: $Q_k = 0.06557$; $Q_h = 0.39515$; $M = 0.20028$					
M_i	0.15323	0.15146	0.16972	0.18681	0.16343	0.18428
MS_i	0.76512	0.75624	0.84743	0.93275	0.81602	0.92013
	A-II: $Q_k = 0.07273$; $Q_h = 0.39543$; $M = 0.20103$					
M_i	0.16407	0.14862	0.17015	0.19143	0.16080	0.17789
MS_i	0.81613	0.73928	0.84639	0.95224	0.79988	0.88491
	A-III: $Q_k = 0.10714$; $Q_h = 0.34442$; $M = 0.18035$					
M_i	0.14320	0.13976	0.14562	0.16005	0.14350	0.16012
MS_i	0.79400	0.77493	0.80745	0.88744	0.79570	0.88786
	A-IV: $Q_k = 0.11594$; $Q_h = 0.35882$; $M = 0.18855$					
M_i	0.15446	0.14841	0.16098	0.17699	0.15033	0.16977
MS_i	0.81921	0.78715	0.85382	0.93870	0.79731	0.90044

Table 19. The RATMI method application main results (for A-I, A-II, A-III, A-IV)

Quantity	A_1	A_2	A_3	A_4	A_5	A_6
	A-I: $t^* = 0.12116$; $t^- = 0.14964$; $MS^* = 0.75624$; $MS^- = 0.93275$					
E_i	0.05033	0.00000	0.51713	1.00000	0.33935	0.92863
	A-II: $t^* = 0.11926$; $t^- = 0.15393$; $MS^* = 0.73928$; $MS^- = 0.95224$					
E_i	0.36152	0.00000	0.50373	1.00000	0.28544	0.68444
	A-III: $t^* = 0.10041$; $t^- = 0.11543$; $MS^* = 0.77493$; $MS^- = 0.88786$					
E_i	0.16934	0.00000	0.28918	0.99645	0.18594	1.00000
	A-IV: $t^* = 0.11150$; $t^- = 0.13345$; $MS^* = 0.78715$; $MS^- = 0.93870$					
E_i	0.21245	0.00000	0.44154	1.00000	0.06845	0.74866

According to the results from tables 16-19, i.e. the values obtained for t_i, PS_i, MS_i and E_i, with $i \in \{1, 2, 3, 4, 5, 6\}$, final standings can be established for the four tested methods. Table 20 shows the ranking lists for all four preferential approaches considered. It is obvious that the rankings for each of the four MCDM methods differ slightly between the four preferential approaches, but when compared to each other, it can be observed that all these methods produce exactly the same results for each of the approaches.

However, in some studies, not related to geodetic applications, e.g. [13] and [15], slightly different ranking lists emerged as outcomes.

Table 20. Final ranking lists after application of the MCRAT, RAPS, RAMS and RATMI methods (for A-I, A-II, A-III, A-IV)

Rank	MCRAT				RAPS				RAMS				RATMI			
	A-I	A-II	A-III	A-IV	A-I	A-II	A-III	A-IV	A-I	A-II	A-III	A-IV	A-I	A-II	A-III	A-IV
1	A_4	A_4	A_6	A_4	A_4	A_4	A_6	A_4	A_4	A_4	A_6	A_4	A_4	A_4	A_6	A_4
2	A_6	A_6	A_4	A_6	A_6	A_6	A_4	A_6	A_6	A_6	A_4	A_6	A_6	A_6	A_4	A_6
3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3	A_3
4	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1
5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5	A_1	A_5
6	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2

Alternative A_4 is declared the best, and is followed in the ranking list by A_6, in A-I, A-II and A-IV. However, in A-III the optimal one is A_6, followed by the second-ranked A_4.

So, in general, the MCRAT, RAPS, RAMS and RATMI methods consider both alternative A_4 and A_6 suitable solutions for the geodetic network, but A_4 should be preferred since it is the best solution in as many as three out of four approaches.

4.2. Results obtained by applying the VIKOR method

The normalized input data, obtained by using the corresponding normalization method are given in Table 21 and the weighted values of that data for the four preferential approaches are shown in tables 22-25, respectively. At last, the main results obtained in application of the VIKOR method and the final standings are given in tables 26 and 27, respectively.

Table 21. Normalized criterion function values for all alternatives (VIKOR)

Alternative	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.3980	0.0147	1.0000	0.9803	0.0000	0.0000	0.5680	0.4122
A_2	1.0000	0.0114	0.5171	1.0000	0.1898	0.3053	0.0000	1.0000
A_3	0.3475	0.0048	0.2833	0.5198	0.2741	0.5388	1.0000	0.3550
A_4	0.0000	0.0000	0.1616	0.2638	0.2802	0.7458	0.6371	0.0000
A_5	0.4547	1.0000	0.1616	0.0000	1.0000	1.0000	0.3553	0.5611
A_6	0.2824	0.1560	0.0000	0.0608	0.1312	0.9327	0.0283	0.3669

Table 22. Weighted normalized criterion function values for all alternatives (VIKOR; for A-I)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.0653	0.0019	0.2459	0.0643	0.0000	0.0000	0.0372	0.0811
A_2	0.1639	0.0015	0.1271	0.0656	0.0124	0.0200	0.0000	0.1967
A_3	0.0570	0.0006	0.0697	0.0341	0.0180	0.0353	0.0656	0.0698
A_4	0.0000	0.0000	0.0397	0.0173	0.0184	0.0489	0.0418	0.0000
A_5	0.0745	0.1311	0.0397	0.0000	0.0656	0.0656	0.0233	0.1104
A_6	0.0463	0.0205	0.0000	0.0040	0.0086	0.0612	0.0019	0.0722

Table 23. Weighted normalized criterion function values for all alternatives (VIKOR; for A-II)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.0724	0.0016	0.1455	0.0713	0.0000	0.0000	0.0413	0.1124
A_2	0.1818	0.0012	0.0752	0.0727	0.0138	0.0222	0.0000	0.2727
A_3	0.0632	0.0005	0.0412	0.0378	0.0199	0.0392	0.0727	0.0968
A_4	0.0000	0.0000	0.0235	0.0192	0.0204	0.0542	0.0463	0.0000
A_5	0.0827	0.1091	0.0235	0.0000	0.0727	0.0727	0.0258	0.1530
A_6	0.0513	0.0170	0.0000	0.0044	0.0095	0.0678	0.0021	0.1001

Table 24. Weighted normalized criterion function values for all alternatives (VIKOR; for A-III)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.0569	0.0016	0.1429	0.1050	0.0000	0.0000	0.0609	0.0736
A_2	0.1429	0.0012	0.0739	0.1071	0.0203	0.0327	0.0000	0.1786
A_3	0.0496	0.0005	0.0405	0.0557	0.0294	0.0577	0.1071	0.0634
A_4	0.0000	0.0000	0.0231	0.0283	0.0300	0.0799	0.0683	0.0000

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_5	0.0650	0.1071	0.0231	0.0000	0.1071	0.1071	0.0381	0.1002
A_6	0.0403	0.0167	0.0000	0.0065	0.0141	0.0999	0.0030	0.0655

Table 25. Weighted normalized criterion function values for all alternatives (VIKOR; for A-IV)

Rank	Criterion							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	0.0577	0.0021	0.1449	0.0852	0.0000	0.0000	0.0494	0.0896
A_2	0.1449	0.0017	0.0749	0.0870	0.0220	0.0177	0.0000	0.2174
A_3	0.0504	0.0007	0.0411	0.0452	0.0318	0.0312	0.0870	0.0772
A_4	0.0000	0.0000	0.0234	0.0229	0.0325	0.0432	0.0554	0.0000
A_5	0.0659	0.1449	0.0234	0.0000	0.1159	0.0580	0.0309	0.1220
A_6	0.0409	0.0226	0.0000	0.0053	0.0152	0.0541	0.0025	0.0798

Table 26. The VIKOR method application main results (for A-I, A-II, A-III, A-IV)

Quantity	A_1	A_2	A_3	A_4	A_5	A_6
	A-I: $S^* = 0.16609$; $S^- = 0.58734$; $R^* = 0.04891$; $R^- = 0.24590$					
S_i	0.49569	0.58734	0.35007	0.16609	0.51025	0.21454
R_i	0.24590	0.19672	0.06984	0.04891	0.13115	0.07218
QS_i	0.78245	1.00000	0.43676	0.00000	0.81701	0.11503
QR_i	1.00000	0.75035	0.10626	0.00000	0.41747	0.11815
$Q_i(0.50)$	0.89122	0.87517	0.27151	0.00000	0.61724	0.11659
$Q_i(0.25)$	0.94561	0.81276	0.18888	0.00000	0.51736	0.11737
$Q_i(0.75)$	0.83684	0.93759	0.35414	0.00000	0.71713	0.11581
	A-II: $S^* = 0.16364$; $S^- = 0.63973$; $R^* = 0.05424$; $R^- = 0.27273$					
S_i	0.44444	0.63973	0.37139	0.16364	0.53960	0.25229
R_i	0.14545	0.27273	0.09682	0.05424	0.15303	0.10007
QS_i	0.58981	1.00000	0.43637	0.00000	0.78967	0.18620
QR_i	0.41747	1.00000	0.19489	0.00000	0.45216	0.20975
$Q_i(0.50)$	0.50364	1.00000	0.31563	0.00000	0.62091	0.19797
$Q_i(0.25)$	0.46056	1.00000	0.25526	0.00000	0.53654	0.20386
$Q_i(0.75)$	0.54673	1.00000	0.37600	0.00000	0.70529	0.19208
	A-III: $S^* = 0.22954$; $S^- = 0.55670$; $R^* = 0.07991$; $R^- = 0.17857$					
S_i	0.44079	0.55670	0.40396	0.22954	0.54774	0.24612
R_i	0.14286	0.17857	0.10714	0.07991	0.10714	0.09993
QS_i	0.64571	1.00000	0.53314	0.00000	0.97260	0.05068
QR_i	0.63801	1.00000	0.27601	0.00000	0.27601	0.20293
$Q_i(0.50)$	0.64186	1.00000	0.40458	0.00000	0.62431	0.12680
$Q_i(0.25)$	0.63993	1.00000	0.34030	0.00000	0.45016	0.16486
$Q_i(0.75)$	0.64378	1.00000	0.46886	0.00000	0.79846	0.08874
	A-IV: $S^* = 0.17748$; $S^- = 0.56557$; $R^* = 0.05540$; $R^- = 0.21739$					
S_i	0.42898	0.56557	0.36447	0.17748	0.56104	0.22033
R_i	0.14493	0.21739	0.08696	0.05540	0.14493	0.07977
QS_i	0.64807	1.00000	0.48183	0.00000	0.98832	0.11044
QR_i	0.55266	1.00000	0.19479	0.00000	0.55266	0.15040
$Q_i(0.50)$	0.60036	1.00000	0.33831	0.00000	0.77049	0.13042
$Q_i(0.25)$	0.57651	1.00000	0.26655	0.00000	0.66158	0.14041
$Q_i(0.75)$	0.62421	1.00000	0.41007	0.00000	0.87941	0.12043

Based on the insight into the standings given in Table 27, it can be noted that alternative A_4 is ranked first in all the preferential approaches. However, having regard to the results shown in Table 26, one can conclude that A_4 is 'not sufficiently' better than alternative A_6 in any of the four approaches. Thus, the VIKOR method declares both A_4 and A_6 to be suitable solutions for the geodetic network, i.e. it is about existing the compromise solution represented by the set consisting of alternatives A_4 and A_6.

Table 27. Final ranking lists after application of the VIKOR methods (for A-I, A-II, A-III, A-IV)

Rank	VIKOR			
	A-I	A-II	A-III	A-IV
1	A_4	A_4	A_4	A_4
2	A_6	A_6	A_6	A_6
3	A_3	A_3	A_3	A_3
4	A_5	A_1	A_5	A_1
5	A_2	A_5	A_1	A_5
6	A_1	A_2	A_2	A_2

4.3. MCRAT, RAPS, RAMS and RATMI outcomes vs. VIKOR outcomes

With the intention of evaluating the advantage of one alternative over another, especially A_4 over A_6 (for A-I, A-II and A-IV) and vice versa (for A-III), the author uses the following relative distances as appropriate measures:

$$\delta t_{a^{(1)},a^{(k)}} = \Delta t_{a^{(1)},a^{(k)}} / W_t \quad (64)$$

$$\delta PS_{a^{(1)},a^{(k)}} = \Delta PS_{a^{(1)},a^{(k)}} / W_{PS} \quad (65)$$

$$\delta MS_{a^{(1)},a^{(k)}} = \Delta MS_{a^{(1)},a^{(k)}} / W_{MS} \quad (66)$$

$$\delta E_{a^{(1)},a^{(k)}} = \Delta E_{a^{(1)},a^{(k)}} / W_E \quad (67)$$

where $a^{(1)}$ and $a^{(k)}$ denote the first-ranked and the k-ranked alternative in the list, with $k \in \{2, 3, 4, 5,6\}$, and

$$\Delta t_{a^{(1)},a^{(k)}} = t_{a^{(1)}} - t_{a^{(k)}} \quad (68)$$

$$\Delta PS_{a^{(1)},a^{(k)}} = PS_{a^{(1)}} - PS_{a^{(k)}} \quad (69)$$

$$\Delta MS_{a^{(1)},a^{(k)}} = MS_{a^{(1)}} - MS_{a^{(k)}} \quad (70)$$

$$\Delta E_{a^{(1)},a^{(k)}} = E_{a^{(1)}} - E_{a^{(k)}} \quad (71)$$

whereby

$$W_t = t_{a^{(1)}} - t_{a^{(6)}} \quad (72)$$

$$W_{PS} = PS_{a^{(1)}} - PS_{a^{(6)}} \quad (73)$$

$$W_{MS} = MS_{a^{(1)}} - MS_{a^{(6)}} \quad (74)$$

$$W_E = E_{a^{(1)}} - E_{a^{(6)}} \quad (75)$$

Using the values for t_i, PS_i, MS_i and E_i, given in tables 16-19, the relative distances (64), (65), (66) and (67) are calculated and presented in tables 28-31.

Table 28. Relative distances between first-ranked and k-ranked alternatives in the MCRAT method (for A-I, A-II, A-III, A-IV)

Relative distance	MCRAT			
	A-I	A-II	A-III	A-IV
$\delta t_{a^{(1)},a^{(2)}}$	7.1%	31.5%	0.3%	25.0%
$\delta t_{a^{(1)},a^{(3)}}$	48.2%	49.5%	71.0%	55.7%
$\delta t_{a^{(1)},a^{(4)}}$	66.0%	63.8%	81.2%	78.7%
$\delta t_{a^{(1)},a^{(5)}}$	95.0%	71.4%	83.0%	93.0%
$\delta t_{a^{(1)},a^{(6)}}$	100.0%	100.0%	100.0%	100.0%

Table 29. Relative distances between first-ranked and k-ranked alternatives in the RAPS method (for A-I, A-II, A-III, A-IV)

Relative distance	RAPS			
	A-I	A-II	A-III	A-IV
$\delta t_{a^{(1)},a^{(2)}}$	7.1%	31.5%	0.5%	25.0%
$\delta t_{a^{(1)},a^{(3)}}$	48.3%	49.6%	71.2%	55.9%
$\delta t_{a^{(1)},a^{(4)}}$	66.5%	63.7%	82.4%	78.6%
$\delta t_{a^{(1)},a^{(5)}}$	94.9%	71.8%	82.8%	93.9%
$\delta t_{a^{(1)},a^{(6)}}$	100.0%	100.0%	100.0%	100.0%

Table 30. Relative distances between first-ranked and k-ranked alternatives in the RAMS method (for A-I, A-II, A-III, A-IV)

Relative distance	RAMS			
	A-I	A-II	A-III	A-IV
$\delta t_{a^{(1)},a^{(2)}}$	7.1%	31.6%	0.4%	25.2%
$\delta t_{a^{(1)},a^{(3)}}$	48.3%	49.7%	71.2%	56.0%
$\delta t_{a^{(1)},a^{(4)}}$	66.1%	63.9%	81.6%	78.8%
$\delta t_{a^{(1)},a^{(5)}}$	95.0%	71.5%	83.1%	93.3%
$\delta t_{a^{(1)},a^{(6)}}$	100.0%	100.0%	100.0%	100.0%
$\delta t_{a^{(1)},a^{(6)}}$	100.0%	100.0%	100.0%	100.0%

Table 31. Relative distances between first-ranked and k-ranked alternatives in the RATMI method (for A-I, A-II, A-III, A-IV)

Relative distance	RATMI			
	A-I	A-II	A-III	A-IV
$\delta t_{a^{(1)},a^{(2)}}$	7.1%	31.6%	0.4%	25.1%
$\delta t_{a^{(1)},a^{(3)}}$	48.3%	49.6%	71.1%	55.8%
$\delta t_{a^{(1)},a^{(4)}}$	66.1%	63.8%	81.4%	78.8%
$\delta t_{a^{(1)},a^{(5)}}$	95.0%	71.5%	83.1%	93.2%
$\delta t_{a^{(1)},a^{(6)}}$	100.0%	100.0%	100.0%	100.0%

In the VIKOR method, when it comes to checking the advantage of the first-ranked alternative over the k-ranked one, the limit value for six alternatives, obtained according to Inequality (64), is 0.20. If the same threshold is adopted for the comparison in the MCRAT, RAPS, RAMS and RATMI methods, then one concludes alternative A_4 has not 'sufficient advantage' over A_6 for A-I, and vice versa for A-III. On the other hand, A_4 has 'sufficient advantage' over A_6 in A-II and A-IV.

5. Conclusion

According to the outcome of the research conducted in the study, a sensitivity of the MCRAT, RAPS, RAMS and RATMI algorithms to weight change was observed. This claim is supported by the fact that alternatives A_4 and A_6 swapped their positions in one (A-III) out of four rankings. As a consequence, unlike the VIKOR outcome, not the same triplet of alternatives emerged on the top of the ranking list every time. In contrast to that, the VIKOR strategy produced the same triplet (A_4, A_6, A_3) in each preferential approach.

The advantage of the VIKOR procedure over the procedures based on the MCRAT, RAPS, RAMS and RATMI algorithms lies in the fact that VIKOR evaluates the significance of the advantages between alternatives as well as the stability on the first position in the ranking list, and extricates a set consisting of all alternatives that could smoothly assume the role of the optimal one. In this study, it was always the compromise solution set consisting of alternatives A_4 and A_6.

Anyway, the MCRAT, RAPS, RAMS and RATMI methods can also serve as an appropriate mathematical tool in finding the optimal solution in the tasks similar to the one being the subject of the analysis in this study. This claim is valid since it turned out that in each of the four preferential approaches all four methods produced the optimal solution that was part of the corresponding two-membered compromise solution set extricated in the application of the VIKOR method.

In addition, since the four methods are open to changing preferences, as demonstrated by the possibility of assigning different weights to the introduced criteria, the practical applicability of these methods is obvious. Namely, depending of what the main goal is in a specific geodetic task, i.e. what is more desirable, higher precision or reliability or both, regardless of which of the MCRAT, RAPS, RAMS and RATMI methods is used, the best solution for a geodetic network can be provided in a short time. This is especially important in practice, as time also plays an important role. Otherwise, optimizing the geodetic network is a complex task, and this statement is particularly reflected in the fact that each individual civil engineering structure, due to its specific geometry, size, position and purpose, determines a specific shape and configuration of the accompanying geodetic control network.

At the very end, the author suggests a direction for further research. Namely, it could, for instance, be based on a comparison of the performance of the MCRAT, RAPS, RAMS and RATMI methods with the performance of some other MCDM methods that all use the same normalization method for data as the one included in the algorithms of the first four. Such methods are e.g. Simple Additive Weighting (SAW) [25], Preference Selection Index (PSI) [26], Measurement Alternatives and Ranking According to Compromise Solution (MARCOS) [27], Combinative Distance-Based Assessment (CODAS) [28]. For instance, with the aim of assessing the effectiveness of the methods applied, it could be interesting to expand some analyses to multi-criteria decision-making. Such analyses are e.g. those that were already presented in [29], [30], [31] and [32].

Conflicts of interest

The author declares no conflicts of interest.

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