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Pre-service Teachers' Conceptions of Rectangle–Parallelogram Hierarchy via Erroneous Examples

Seda ŞAHİN[1]

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Abstract
This case study investigates how pre-service mathematics teachers conceptualize the hierarchical relation between rectangles and parallelograms when confronted with an erroneous example. Participants were 90 second and third-year students enrolled in an elementary mathematics education programme at a Turkish state university. Data consisted of (i) the participants' written evaluations of the erroneous example and (ii) alternative tasks they designed for the same learning outcome; both sets were analysed through content analysis. Findings indicate that only 13 % of the participants possessed sufficient conceptual knowledge to identify the error and propose a valid task. A further 64 % failed to detect the error, and 59 % of these reproduced tasks containing similar misconceptions. The study demonstrates that erroneous examples can serve not only as a pedagogical tool in teacher education but also as a diagnostic lens for revealing concept-image versus concept-definition gaps. Most participants' understanding that "a rectangle is a special parallelogram" was grounded in visual perception rather than formal properties.

Keywords: erroneous examples, quadrilaterals, rectangle, parallelogram

[1] Asst. Prof. Dr., Kilis 7 Aralık University, Kilis, Türkiye; e-mail: seda.sahin@kilis.edu.tr ; ORCID: 0000-0003-3202-8852

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INTRODUCTION

Errors, which have a deep history in educational research, are seen as a natural part of the learning process and the role of error in this process is handled in different ways. In the perspective dominated by the behavioral theory, error is considered as a factor that should be avoided and prevents learning. According to this theory, errors should be avoided as much as possible since learning is a process of acquiring new behaviors, not new ideas, and errors only lead to wrong answers. Although errors are not allowed to occur in the learning process, if students make mistakes, these mistakes should be eliminated by repeating the rules for the correct answer without creating a discussion environment (Skinner, 1958 cited in Santagata, 2005). In the constructivist approach, errors have an important place in the construction of knowledge (Ding, 2007; Nesher, 1987; Santagata, 2005; von Glasersfeld, 1989). It is important to use errors as a tool to understand how students think and to intervene correctly when errors are encountered (Ding, 2007; von Glasersfeld, 1989). The basis of correct intervention is questioning student errors, examining students' perceptions of concepts, and criticizing their and their peers' thinking (Borasi, 1994).

When the studies on student errors in mathematics teaching are examined, it is seen that they mostly focus on detecting errors, determining the types and causes of errors, and eliminating them. In the studies that consider error as a learning tool, student errors encountered in classroom practices and teacher reactions and interventions to these errors were examined (e.g. Hansen, 2011; Gardee & Brodie, 2022; Santagata, 2005; Sahin, 2011). Teachers may avoid or ignore errors because they do not see errors as important tools for learning, or fear that errors may be 'contagious' (Swan, 2002, p. 151). Most of the time, teachers correct errors in order to guide students to the correct information. Teachers who own up to the errors, on the other hand, use errors as a special tool to produce knowledge. According to this approach, error plays an important role in the construction of knowledge. Error exploration is

important for teachers who try to understand how students understand and think, and these teachers investigate errors to uncover the underlying understandings (Brodie & Shalem, 2011; Gardee & Brodie, 2015). However, in order to turn an error into an opportunity, the error needs to occur. However, in order to use error as an effective learning tool, it is not enough just to wait for it to occur. In the literature, errors are referred to by various terms depending on researchers' preferences, such as *erroneous examples* (Tsovaltzi et al., 2010), *mistake-handling tasks* (Heinze, 2005), or *error-based tasks* (Gürbüz et al., 2021). Despite these different labels, they share the same conceptual meaning. Erroneous examples refer to a completed solution or evaluation that contains one or more mistakes (Tsovaltzi et al., 2010). The primary aim is to enhance students' conceptual understanding by encouraging them to identify and correct the error(s) in the worked examples (Tsovaltzi et al., 2010; Heinze, 2005). On the contrary, students should be consciously confronted with erroneous examples. Researchers (e.g. Akkuşçi, 2019; Heinze, 2005) who advocate this view and conduct studies in this field focus on the realization of the learning process through well-organized erroneous examples. More effective learning can be achieved by providing students with an environment where they can analyze and discuss the error. Errors can stimulate students' curiosity by creating a contrast with the expected situation and allow them to think about different alternatives. The analysis of errors enables the discussion of more abstract topics, concepts or content, making them more concrete and therefore accessible to students (Borasi, 1989). In addition, truly learning a concept or topic requires knowing what is wrong as well as what is right (Heinze, 2005). From this point of view, mistakes are not only opportunities that support learning but also a part of the learning process (Heinze, 2005). In fact, studies have shown that creating a discussion environment on mistakes in a planned way rather than discussing individual errors in the classroom provides a better learning opportunity (Heinze, 2005). Including erroneous examples as well as correct examples in teaching contributes positively to students' conceptual knowledge (Durkin & Rittle-Johnson, 2012; Hansen, 2011; Gürbüz et al., 2021). Durkin and Rittle-Johnson (2012) found that students who were asked to compare correct and incorrect examples while placing decimals on the number line in the fourth and fifth grades understood the subject better than the group that was taught only with correct examples. In their experimental study, Gürbüz et al. (2021) concluded that error-based tasks had an effective role in teaching measures of central tendency and line graphs to seventh grade students. In studies in which students' prior knowledge is considered to be a determining factor for the effective use of error-based examples (e.g. Große & Renkl, 2007; Gürbüz et al., 2021), it is argued that students whose prior knowledge is weak or inadequate do not know how to interpret error-based examples and therefore error-based examples are not an effective tool for the conceptual development of these students. Durkin (2012) states that misconceptions, rather than incomplete prior knowledge, prevent students from interpreting and analyzing errors.

Erroneous examples are not only a learning opportunity for students, they also provide opportunities for teachers and pre-service teachers. Watson and Jason (2007) point out the important role of examining and analyzing text-based tasks in particular to examine pedagogical and didactic issues and to stimulate thinking. Moreover, tasks involving student errors are an effective pedagogical tool for teachers and pre-service teachers to focus on student thinking and possible sources of errors (Biza et al., 2007; Zaslavsky & Sullivan, 2011). Teachers' pedagogical perspectives can be determined through tasks consisting of scenarios involving errors made by students (Ma, 1999; Biza et al., 2007; Bütün, 2005, 2011). The pre-service teachers' exposure to error scenarios before moving to the real classroom environment will enable them to be prepared for possible student difficulties and gain pedagogical knowledge that will determine the way to handle the error (Biza et al., 2007; Mason, 1998). Watson and Jason (2007) reviewed 111 studies on the role of tasks in teacher education (submitted for a special issue of the journal they edited) and found that the purposes of using tasks were concentrated under three headings: a. Understanding students' mathematical knowledge; b. Developing teachers' mathematical awareness (rethinking their views of mathematics or re-experiencing mathematics learning); c. Considering the different mathematical opportunities offered by different ways of teaching. In the studies where teachers were asked to examine or compare different tasks or different pedagogical strategies, it can be argued that these studies provide more information about the importance of the role of tasks in teacher education than the first two purposes.

Teachers face complex situations in their classroom practices that require them to make decisions and choices (Zaslavsky & Sullivian, 2011). Students' difficulties are one of the complex situations that teachers have to deal with and teachers should know how to intervene when they encounter such situations (Ma, 1999; Zaslavsky & Sullivian, 2011). The right intervention is directly related to the teacher's subject matter knowledge and pedagogical content knowledge. For teachers to acquire these competencies on the basis of student difficulties, it is important for them to encounter situations that create uncertainty and even confusion, just like students (Leinhardt, 2001; Zaslavsky & Sullivian, 2011). Such situations have the power to make teachers question their knowledge and provide productive explanations (Leinhardt, 2001). Scenarios involving student errors are frequently used for these purposes (e.g., Ma, 1999; Bütün, 2011).

Scenarios involving student errors are used in determining the content knowledge or pedagogical content knowledge of teachers and pre-service teachers or in the process of providing these competencies to pre-service teachers (e.g., Ma, 1999; Bütün, 2011). In such studies, teachers/pre-service teachers are asked to identify student difficulties by giving scenarios with student responses taken from the real classroom environment or designed in a realistic way, and their opinions about how they can intervene in such a situation or what can be done to prevent these difficulties are asked. When the related literature was examined, it was found that there was no use of the erroneous example scenarios for a teacher's practice. In this study, the erroneous example is not a student task but a teacher's in-class practice. Such teacher practice tasks provide an opportunity to determine both the knowledge and pedagogical approaches of teachers and prospective teachers.

It suggests that the studies in the literature may be consistent within themselves, but more studies are needed for generalizable results. In general, it can be said that the research on errors as a learning tool remains in the background compared to other subject areas (such as prevention, detection, intervention). Therefore, more studies are needed in this field to determine the place and effect of "error" in mathematics teaching.

The Relationship between Geometric Shapes and Conceptual Properties of Shapes

Geometry learning is considered a challenging process for many students as it requires the use of abstract thinking skills such as spatial reasoning, proof making and problem solving. One of the common results of the studies examining the difficulties experienced in defining geometric shapes and determining their distinctive features is the dominance of prototype shapes for the concept in students' perceptions (Ulusoy & Çakıroğlu, 2017; Fischbein, 1993; Tall & Vinner, 1981). The inability to establish a healthy spatial and logical relationship between shape and concept in geometry is the basis of these difficulties (Tall & Vinner, 1981; Fujita & Jones, 2007; Duval, 1995).

Knowing the definition of a concept does not always mean that the concept is understood (Vinner, 1991; de Villiers, 1998). For example, a student may not accept a square, rectangle and rhombus as a parallelogram even though he/she defines parallelogram correctly. Tall and Vinner (1981) emphasize concept image in explaining this situation. Concept image is defined as the nonverbal expressions formed in our minds about a concept. These expressions can be a visual representation, impression or experiences related to the concept (Tall & Vinner, 1981). According to them, the reason why a student does not accept the quadrilaterals mentioned above as parallelograms is that these quadrilaterals do not match the concept image of parallelogram in his/her mind. Although, according to Vinner (1983), an individual considers the concept image rather than the definition when dealing with concepts, one of the reasons for the failure to establish a relationship between geometric concepts is that definitions are not given enough importance. Students who do not know the formal definition produce intuitive answers according to the concept images in their minds (Ubuz & Gökbulut, 2015). Although students are not expected to provide the formal definition directly, they can be guided to the definition by exemplifying the logical relationships between the situations related to the concept (de Villiers, 1998; Ubuz & Gökbulut, 2015). Therefore, it is important for students to be as active in the process of defining concepts as they are in learning tasks such as problem solving, predicting, and proving (de Villiers, 1998). Concept images have a decisive role in the formation of conceptual infrastructure (Vinner, 1983). The standardized representation of geometric shapes in the lessons causes students' concept images to be limited and to have a limited perception of the concept. Therefore, this situation prevents the desired flexibility in the process of defining the concept.

In many studies examining pre-service teachers' understanding of geometric shapes, conceptual properties of shapes, and the hierarchical relationship between shapes, participants are usually first asked to define quadrilaterals (such as square, rhombus, rectangle, and parallelogram). One of the common results obtained from these studies is that although the participants correctly identified quadrilaterals, they failed to identify parallelograms in terms of shape. Sahin (2023) examined the relationship between pre-service mathematics teachers' conceptual definitions and formal representations of rectangles and parallelograms and found that the number of participants who were able to combine the formal definition of the concepts and the appropriate formal representation was quite low. Toluk, Olkun, and Durmuş (2002), in their study in which they tried to determine the geometry levels of pre-service teachers, found that although most of the pre-service primary school teachers saw the relationship between rectangle and parallelogram, they could not decide which one was higher in the hierarchical structure. The researchers stated that such a difficulty was experienced because the mathematics program emphasized the names and definitions of geometric shapes and did not include enough tasks for establishing relationships between shapes. Similarly, Fujita and Jones (2006) found that pre-service primary school teachers had difficulty in establishing the hierarchical relationship between quadrilaterals although they could define them conceptually. They stated that the result they obtained was due to the gap between individuals' personal concepts (shape knowledge) and formal definitions (concept knowledge). This result is generalizable for similar studies conducted with the participation of students, pre-service teachers or teachers. In general, individuals' shape knowledge (concept image) has a dominant role in making formal definitions.

In studies examining how individuals perceive the definition of geometric shapes and the relationship between these shapes, it is seen that the formal and informal knowledge of the individual is generally examined through direct questions (e.g. Fujita & Jones, 2006; Gal & Lew, 2008; Horzum, 2018; Karakuş & Erşen, 2016; Toluk et al. 2002). Horzum (2018) asked pre-service mathematics teachers to demonstrate their knowledge about quadrilaterals and the relationship between them by creating a concept map. Toluk et al. (2002), while questioning the participants' knowledge about the relationship between geometric shapes, asked them, "What kind of relationship is there between an isosceles triangle and an equilateral triangle? Which one includes which one? Why? Fujita and Jones (2006) asked pre-service teachers questions such as Is a square a rectangle? Is a rectangle a trapezoid? They asked them to explain their answers with reasons.

As in the studies discussed above, it is possible to reveal participants' views on the relationship between polygons by using concept maps and open-ended questions. However, the results obtained from such studies may not be a direct indicator of how the participants will evaluate when they encounter a new situation, especially an erroneous example. Considering the role of erroneous examples in conceptual learning, the purpose of this study was determined as "To examine pre-service mathematics teachers' understanding of rectangle as a special case of parallelogram through erroneous examples". In line with this purpose, the study has three sub-research questions:

- Pre-service mathematics teachers emphasize that "Rectangle is a special case of parallelogram." How do they evaluate the task with the erroneous example for the outcome?
- Pre-service mathematics teachers emphasize that "Rectangle is a special case of parallelogram." What kind of alternative task do they offer for the outcome?
- What is the relationship between pre-service mathematics teachers' evaluation of the erroneous example and the tasks they suggest?

The first sub-research question aimed to determine how the pre-service teachers interpreted the erroneous example, the second question aimed to determine how they would propose a task for the related outcome in line with the answer they gave to the first question, and the third question aimed to determine whether the tasks they proposed were consistent with the results of their evaluation of the erroneous example.

Although previous research has predominantly focused on the impact of erroneous examples on students' conceptual understanding, there has been limited investigation into how pre-service teachers analyze such examples or what tendencies they exhibit when constructing similar tasks. In this context, it is essential not only to examine how pre-service teachers evaluate erroneous examples, but also to explore how they generate alternative tasks in response to these examples. Investigating the consistency between these two processes would address a significant gap in the literature.

METHOD

This study is a case study conducted with the participation of 90 (63 female; 27 male) students studying in the Department of Elementary Mathematics Teaching at a state university in Türkiye. Case studies are a research strategy in which researchers conduct an in-depth investigation of an event, process, or one or more individuals (Creswell, 2015). In the present study, the participants were second and third-year undergraduate students, and the data were collected at the beginning of the fall semester. Therefore, none of the participants had yet taken a course on geometry instruction at the undergraduate level. The selection of participants was not purposive; this information is provided solely to support an accurate interpretation of the data. To ensure confidentiality, participants were coded as S1, S2, and so on.

The data collection tool used in the study was a two-stage form that required the evaluation of the erroneous example and the design of a task for the relevant outcome (Figure 1).

The Grade 6 task prepared using dynamic geometry software for the outcome "*It is emphasized that the rectangle is a special case of the parallelogram."* is as follows:

The teacher gives the students a parallelogram like the one below (Figure 1) and asks them which geometric shape it is. He then asks them to cut the small piece and add it to the other side of the large piece (Figure 2). The students discover that it is a rectangle. In this way, students understand why a rectangle is a special case of a parallelogram.

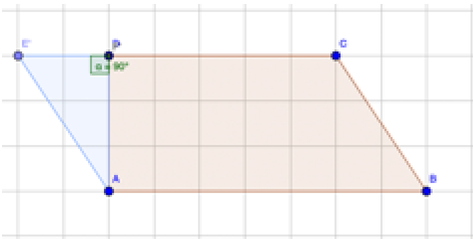


Figure 1.

Figure 2.

Evaluate the above practice in terms of the related outcome. How would you like to implement the same learning outcome?

Figure 1. Erroneous example task

The pre-service teachers were given an erroneous example aiming to show the relationship between a rectangle and a parallelogram, and they were asked to evaluate this task in terms of the related outcome and suggest a task for the related outcome. The data was collected in writing were analyzed by content analysis method. The data analysis framework regarding the criteria for evaluating the task with the erroneous example and the data analysis framework used for task suggestions are given in Table 1 and Table 2.

Table 1. Data Analysis Framework for the Erroneous example

Codes	Description	Example Situation
Erroneous example	Opinions that the example was inaccurate when evaluating the task	The task is not suitable for the outcome. We cannot take a shape and say that it is a special case of another shape. A parallelogram can be formed from two trapezoids, but it is clear that a parallelogram is not a trapezoid. (S11)
Correct example inadequatetask	While evaluating the task, opinions that the example was correct but the task was not sufficient	It is a very good example to concretize, but an association could have been made between the properties of the rectangle and parallelogram, not only in terms of shape. (S15)
The right example	Opinions that the example was correct when evaluating the task	The student realized that the parallelogram contains a rectangle within itself. (S10)
No response	No opinion expressed	

Table 2. Data Analysis Framework for Efficiency Evaluation

Codes	Description	Example Situation
Properties of Quadrilaterals	Tasks comparing the properties of rectangle and parallelogram and emphasizing that the rectangle has all the properties of the parallelogram	If I were to design a task, I would focus on the properties of these two quadrilaterals. I ask students to write the properties of both rectangles and compare them. Then I would ask questions about the relationship between them. One of these questions would be "Does the rectangle satisfy all the properties of the parallelogram?" (S48)
Area Conservation	Tasks in which the idea that the area of a parallelogram and a rectangle with the same height and base length is equal is used as a reference to show that a rectangle is a special case of a parallelogram	I give students a rectangle (paper or cardboard) and ask them to form two equal triangles. I ask them to form parallelograms with these triangles. (S46)

Figurative representation	Tasks based on the idea that both rectangles and parallelograms can be formed by moving the sides of a quadrilateral designed with movable vertices through concrete materials or dynamic geometry software	I would bring a moving parallelogram material like the one in the figure below. By moving the sides, it would be seen that it is a rectangle when the angles are 90° . In this way, since a rectangle was obtained from a parallelogram, students understood that a rectangle was a special case of a parallelogram. (S28)

After the data were coded according to the above analysis frameworks, the results of the evaluation of the erroneous example and the task suggestions were examined and discussed in relation to each other.

RESULTS

In this section, the findings obtained as a result of the pre-service teachers' evaluation of a task containing an erroneous example and the analysis of the examples they presented as alternatives are presented and then this process is evaluated holistically.

Evaluation of the Erroneous Example and Task Suggestions

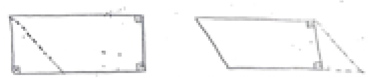
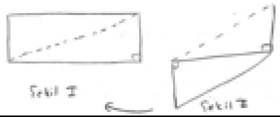
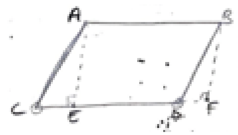
The pre-service teachers' evaluation of the erroneous example in the given task and the examples in the task they presented as an alternative were examined and the results were obtained. As can be seen in Table 3, the proportion of participants who realized that the example in the task about rectangle being a special case of parallelogram was 18% (n=16). The proportion of pre-service teachers who thought that the erroneous example was correct was the highest (n=40). The rate of participants who did not evaluate the task and/or the erroneous example was 11% (n=10). 27% of the pre-service teachers thought that the example in the task was correct but the task was inadequate (n=24). This result shows that the majority of the pre-service teachers were inadequate in analyzing the erroneous example. Only 18% of the pre-service teachers made a correct evaluation and stated that the example was erroneous. When the reasons of these pre-service teachers for why the erroneous example is not correct are analyzed, two different situations emerge: (1) Properties of quadrilaterals, (2) Different shapes can be created with cut-and-paste. For example, "It is not suitable for the learning outcome. There is a logic error. What is done here is to turn a parallelogram into a rectangle. It may also cause misconceptions. It is necessary to emphasize the properties of both geometric shapes and think about their common features." (S50), the participant drew attention to the deceptive aspect of the process performed in the task and stated that the important thing is to focus on the properties of rectangles. For example, it is seen that the pre-service teachers who think that the example used for area conservation is not suitable for showing the hierarchical relationship between parallelograms and rectangles, while others focus on the fact that different geometric shapes can be created by cut-and-paste that are not triangles, isosceles trapezoids or special polygons. One of the views of the participants in this group is as follows:

"It can lead to misconceptions. Any geometric shape can be transformed into another geometric shape by cutting any part of it and combining it in a different way. A rectangle can also be obtained by cutting a triangle." (S44)

When the opinions of the pre-service teachers are examined, it can be said that they realized that the example given in the task was erroneous, but rather than explaining why it was erroneous, they exemplified that the given situation would not always be valid.

Table 3. Pre-service teachers' evaluations and task suggestions

Task Evaluation	Sample Testimonials	Event Proposal Content	Sample Task Suggestions
Erroneous example (n=16)	The task is not suitable for the outcome. We cannot take a shape and say that it is a special case by obtaining another shape from it. A parallelogram can be formed from two congruent trapezoids, but it is clear that a parallelogram is not a trapezoid. S11	Properties of quadrilaterals (n=12)	"If I were to design a task, I would focus on the properties of these two quadrilaterals. I would ask students to write the properties of both quadrilaterals and compare them. Then I ask questions about the relationship between them. One of these questions is "Does the rectangle satisfy all the properties of a parallelogram?"." S48
		Area Conservation (n=2)	"If I were to prepare a task, I would cut and paste the triangle using GeoGebra so that the students can see that it is pasted correctly. I ensure that each student sees the application effectively by making them use it themselves." S52
		Figural representation (n=2)	"I would connect the juice straws and form a parallelogram that can be moved in all directions. Then I would ask the students to form a rectangle from this parallelogram." S31
Correct example but inadequate task (n=24)	It is a very good example to concretize, but it is not enough to show it only figuratively. An association could have been made between the properties of rectangle and parallelogram. S15	Properties of quadrilaterals (n=16)	"If it were me, I would divide the board into thirds and write the properties of the square, then the properties of the rectangle, and finally the properties of the parallelogram on the board together with the students. Thus, the students would see the differences and similarities themselves. Finally, I would draw the shapes on the board and let them see the relationship between them." S35
		Area Conservation (n=4)	"I would create a parallelogram by giving a rectangle and subtracting two equal right triangles from the rectangle. I would make them learn together by doing a group task." S46
		Figural representation (n=4)	"I give students a parallelogram and two right triangles whose hypotenuse lengths are the same length as the sides of the parallelogram. Then I make them realize that this is a rectangle by matching the sides of the parallelogram with these triangles. In this way, they discover that a rectangle is obtained from a parallelogram and that all the angles of a rectangle are right." S24
Correct example (n=40)	The student realized that the parallelogram contains a rectangle within itself. S10	Properties of quadrilaterals (n=10)	"I would do a task like this: I would show the students a parallelogram and a rectangle on paper. The children already know both shapes. Then I would ask them about the properties of these quadrilaterals in turn. In line with the answers given, I make them realize that the rectangle has all the properties of a parallelogram. They understand that a rectangle is a parallelogram." S17
		Area Conservation (n=23)	

		 "I would give them a rectangle and ask them to cut the small triangle piece and add it to the other side of the shape. I ask what the resulting shape is. The students discover that it is a parallelogram. Then I get a rectangle from a parallelogram and students discover that a rectangle is a special case of a parallelogram." S10
	Figural representation (n=8)	"I would bring a parallelogram made with straws to the classroom. First, I would ask the students about the shape in my hand. Then I would move the straws so that the corners were perpendicular. When I asked them this time, they would say that it was a rectangle. Then I would say: "Well, wasn't it a parallelogram just now? See, just by changing the angle between the sides, we got a rectangle from a parallelogram." Students would thus discover that a rectangle is also a parallelogram." S19
No answer (n=10)	Properties of quadrilaterals (n=4)	"Before going to class, I prepare a task sheet with two columns, one with the properties of a parallelogram and the other with the properties of a rectangle. I ask students to write the properties of quadrilaterals and compare them. Seeing that all the properties of a parallelogram are also properties of a rectangle, students discover that a rectangle is a special case of a parallelogram." S25
	Area Conservation (n=4)	"By cutting the rectangle from the marked diagonal, we get two equiangles. We can form a  parallelogram by connecting these two congruent triangles as in Figure II." S47
	Figural representation (n=2)	"I would create a rectangle out of wooden or plastic sticks with movable joints (corners). I could create shapes at different angles by moving the corners of the rectangle. I would stick  the material I prepared on the board as a parallelogram with the help of a magnet. Then I would have the students measure its height by lowering the perpendiculars from corners A and B to the sides. I would move the bottom two corners (C and D) and bring them to points E and F. I would have them measure the interior angles of the resulting shape and show that it is a rectangle. In this process, I would make them realize that only the angles change and the side lengths do not change and show that the rectangle is a special parallelogram." S5

All of the pre-service teachers who thought that the erroneous example was correct argued that the area relation of the parallelogram could be used to show that the rectangle is a special parallelogram with the cut-and-paste method, as in a task for exploring with the help of the area relation of the rectangle. The number of participants who associated the principle of invariance of area with the properties of quadrilaterals was 62. While most of them (n=40) believed that the example was correct, others (n=22) stated that the example was correct but the task was inadequate. The common view of the participants who thought this way was that this example could prevent possible misconceptions and that it was a remarkable task suitable for the learning outcome. Some of the participant opinions regarding the correctness of the example are as follows:

"It is a good technology-supported application for discovery. It also prevented the formation of misconceptions." (S19)

"Since students have a concept image for each shape, they experience misconceptions in special quadrilaterals. With this task, the misconception is corrected." (S23)

Along with the above opinions, it was found that the explanations made by two pre-service teachers who thought that the example was correct were inconsistent within themselves. One of these opinions belongs to S3:

"This example is correct. At the same time, the trapezoid can also be obtained by inverting and adding the hatched area. From this it can be seen that the trapezoid is a special case of the parallelogram. From both results it can be concluded that an isosceles trapezoid is a special case of a rectangle."

As can be understood from this statement, the pre-service teacher experiences the cognitive difficulty that the students will experience by inferring that an isosceles trapezoid can also be obtained with cut-and-paste based on this example and that the isosceles trapezoid will also be a rectangle. Similarly, S10 said, "The student realized that the parallelogram contains a rectangle within itself. When the operation is done in reverse, it can be seen that the parallelogram is a rectangle." His opinion is again aimed at making a wrong generalization.

Two pre-service teachers who thought that the given task was appropriate for the outcome tried to explain the erroneous example with the concept of area conservation. For example; S40 "It is a task with a high probability of retention. The student understands that the areas of the parallelogram and rectangle in the figure are the same. Because there is no decrease on the figure." With his statements, he associated the reason why the rectangle is a parallelogram with the fact that the areas are equal.

Although the pre-service teachers who thought that the example in the given task was correct but the task was inadequate provided similar justifications, some of them pointed out that it was necessary to emphasize the properties of quadrilaterals; some of them pointed out that different geometric shapes could be obtained if they were not careful in the cut-and-paste process and that the task was inadequate in this respect. For example, "The student understands that there is a relationship between a rectangle and a parallelogram and sees that a rectangle is a parallelogram. But for this, the student needs to know the properties of rectangle and parallelogram and compare the similarities and differences between them." (S46); S22 stated that the example given was appropriate to show that the rectangle was a parallelogram, but it was also important how to position the cut piece, "With this task, it was shown that the rectangle is a special form of a parallelogram. Since the angles of the parallelogram are not 90° , students cannot comprehend that it is a rectangle. They understand that it is a rectangle when they take a perpendicular from one corner of the parallelogram and put it on the other side. The task is good for them to see the rectangle, but it is also important how to put the triangle. They can also put it in such a way that no rectangle is formed." His opinion represents the second group.

One of the findings of the study is that 11% of the pre-service teachers did not evaluate the task. However, none of these participants left the question completely unanswered. Each of them answered the second part of the question which asked them to suggest an alternative teaching practice.

When the findings related to the question that asked the prospective teachers to suggest a task for the same outcome after evaluating the given task were analyzed, it was determined that the alternative tasks of the prospective teachers had three different themes: (1) Properties of Quadrilaterals, (2) Area Conservation, (3) Figurative Representation. The first of these themes, Properties of Quadrilaterals, includes tasks that compare the properties of rectangle and parallelogram and emphasize that rectangle has all the properties of parallelogram. The task suggestions collected under this theme were evaluated as correct. Conservation of area includes tasks in which it is tried to show that a rectangle is a special case of a parallelogram by referring to the idea that the area of a parallelogram and a rectangle with the same height and base length is equal. Therefore, the examples in this theme were evaluated as incorrect. The last theme, figural representation, is based on the idea that both rectangles and parallelograms can be formed by moving the sides of a quadrilateral whose vertices are designed to be movable through concrete materials or dynamic geometry software. Since the focus here was only on the shapes of quadrilaterals, not on their properties, these examples were also considered as incorrect. As can be seen in Table 3, all of the pre-service teachers who recognized the erroneous example, found it correct but inadequate, thought that the example was correct and did not comment on it presented examples for the three themes identified in the task suggestions. Figure 2 shows the relationship between the pre-service teachers' evaluation of the erroneous example and the themes of the task suggestions more clearly.

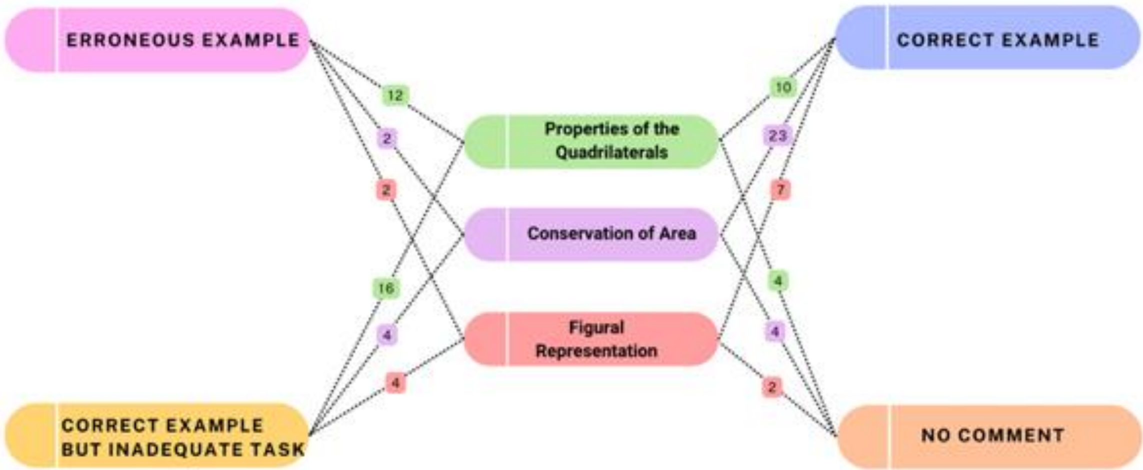


Figure 2. The relationship between task evaluations and the themes of the task suggestions

47% of the pre-service teachers (n=42) suggested a task in which the properties of rectangle and parallelogram were listed and the hierarchical relationship between these quadrilaterals was explored by determining their common properties. Among these pre-service teachers, 16 of them found the example correct but the task inadequate, and 12 of them realized the erroneous example. Explaining why the erroneous example was not correct, S48 presented the following task:

"If I were to design a task, I would focus on the properties of these two quadrilaterals. I would ask students to write the properties of both quadrilaterals and compare them. Then I would ask questions about the relationship between them. One of these questions would be "Does the rectangle fulfill all the properties of the parallelogram?".

S2, who saw the erroneous example and alternatively suggested a task in which students compare the properties of a rectangle and a parallelogram by listing the properties of the rectangle and the parallelogram, added, "I can make them realize why an application like the one above is erroneous." He stated that he could use the erroneous example appropriately in his lesson.

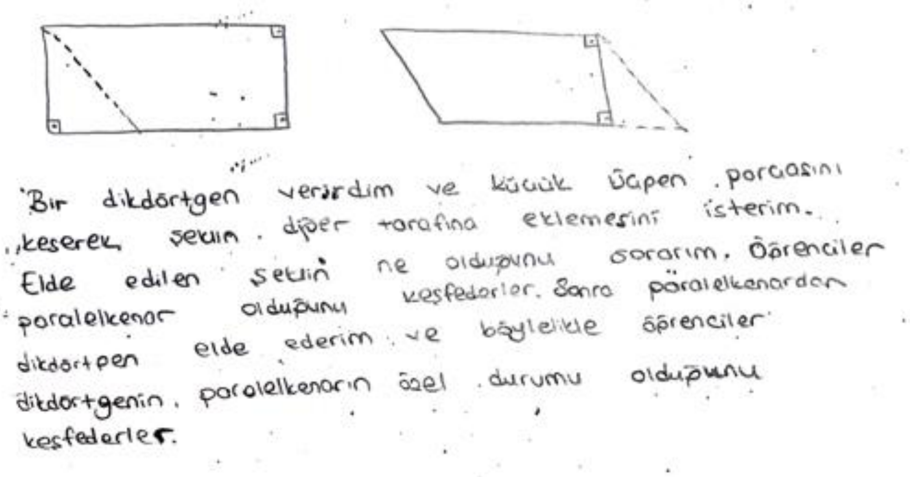
The task suggestion of S35, one of the participants who thought that the example in the given task was correct but insufficient, is as follows:

"If it were me, I would divide the board into thirds and write the properties of the square, then the properties of the rectangle, and finally the properties of the parallelogram on the board together with the students. Thus, the students would see the differences and similarities themselves. Finally, I would draw the shapes on the board and let them see the relationship between them."

S37, one of the participants who thought that the example in the given task was correct but insufficient, stated that she would apply the same task and in addition, she would ask questions to compare the common features and differences of parallelograms and rectangles.

The findings show that 37% (n=33) of the pre-service teachers tried to show that a rectangle is a parallelogram based on area conservation. Under this theme, there are task suggestions belonging mostly to the pre-service teachers who thought that the example was correct (n=23) and least to the pre-service teachers who evaluated the example incorrectly (n=2). In the tasks under this theme, there is a transition from parallelogram to rectangle and from rectangle to parallelogram by cut-and-paste. In addition, one pre-service teacher presented examples showing that a rectangle can be obtained from a trapezoid and a trapezoid can be obtained from a parallelogram with the cut-and-paste method.

The pre-service teachers who thought that the example in the task was correct tried to reach different quadrilaterals with the same area by cutting the quadrilaterals into pieces with the cut-and-paste method, as in the erroneous example. For example, the task suggested by S10 is as follows:



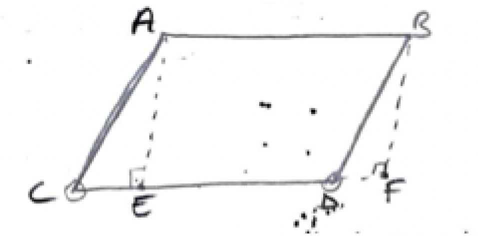
(Translation: "I give them a rectangle and then ask them to cut out a small triangle and add it to the other side of the shape. I ask what the resulting shape is. The students discover that it is a parallelogram. Then I get a rectangle from a parallelogram, and students discover that a rectangle is a special case of a parallelogram.")

As can be clearly seen in this example, the pre-service teacher tries to establish a direct relationship between area conservation and the properties of polygons. The participant thinks that first making a transition from rectangle to parallelogram and then applying the reverse is appropriate for the acquisition. It was determined that the other 32 pre-service teachers who had the same idea as S10 presented the same or similar examples.

It is seen that the two pre-service teachers who analyzed the erroneous example correctly gave examples that contradicted their thoughts. Both of them stated that the erroneous example could lead to misconceptions, but they suggested a task in which a rectangle was created from a parallelogram.

The task suggestions of the pre-service teachers in which they thought that rectangles and parallelograms could be transformed into each other as in area conservation were analyzed under the code of formal representation. However, the focus here is neither the properties of quadrilaterals nor area conservation; it is emphasized that rectangles and parallelograms can be transformed into each other by moving the opposite sides, and 17% (n=15) of the preservice teachers suggested tasks based on this idea. For example, S31 "I would combine juice straws and create a parallelogram that can be moved in all directions. Then I would ask students to create a rectangle from this parallelogram." It was found that the other pre-service teachers suggested similar tasks using concrete materials or GeoGebra, a dynamic geometry software. However, two pre-service teachers wanted to support the shape they would create by drawing while using concrete materials. S5 explained his material as follows:

"I would create a rectangle out of wooden or plastic sticks with movable joints (corners). I could create shapes at different angles by moving the corners of the rectangle. I would stick the material I prepared on the board as a parallelogram with the help of a magnet. Then I would have the students measure its height by lowering the perpendiculars from corners A and B to the sides. I would move the bottom two corners (C and D) and bring them to points E and F. I would have them measure the internal angles of the resulting shape and show that it is a rectangle. In this process, I would make them realize that only the angles change, the side lengths do not change and show that the rectangle is a special parallelogram."



As can be understood from S5's explanations, it can be said that the prospective teacher is aware that the lengths of the sides and the height of the parallelogram are different and that there will be no change in the lengths when the sides are moved until they form a right angle. However, the visual representation she drew is inconsistent with her thoughts. In fact, it is not possible to move point C to point E and point D to point F with the concrete material he designed. Such a situation requires the length and height of the sides AC and BD of the parallelogram to be equal.

When the research findings were evaluated in general, it was determined that the most suggested task theme was the properties of quadrilaterals and half of the pre-service teachers (n=42) paid attention to this theme. In particular, 67% (n=16) of the pre-service teachers who thought that the erroneous example given was correct but inadequatetried to make them discover that rectangle is a special case of parallelogram based on the properties of quadrilaterals, which shows that they made the correct suggestion even though they did not realize the mistake. The findings reveal that 37% (n=33) of all participants suggested a task based on area conservation to show the relationship between two quadrilaterals. In line with the purpose of the study, the ideal situation is that the pre-service teachers realize that the example in the given task is incorrect and offer a correct task alternative for the related outcome. When the findings were evaluated in this respect, it was concluded that the ideal situation was realized by only 13% (n=12) of the participants.

The Relationship between Evaluation of Erroneous Sample and Alternative Task Suggestions

In this study, it was found that the pre-service teachers who thought that the erroneous example was correct tended to make inferences by focusing only on the visual without considering the formal definitions or properties of quadrilaterals. These participants thought that the example in a task used to explore the area formula of a parallelogram was also appropriate to show that a rectangle is a special case of a parallelogram. The findings of the study revealed that 71% (n=64) of the pre-service teachers made comments by focusing on the visual of the geometric shape. This situation should not be considered as a simple surprise. In fact, when the task suggestions are taken into consideration, it is seen that many preservice teachers (n=48) used similar arguments. However, two different situations were encountered. The task suggestions coded as area conservation and figural representation in the data analysis point to different visual perception processes. As explained above, area conservation is based on the transformation of a rectangle and a parallelogram with the same area into each other in terms of shape, and 33 pre-service teachers suggested tasks involving such examples. Figurative representation involves the process of constructing these two quadrilaterals with rods of the same size using drawings, dynamic geometry software, straws or moving rods. The focus is only on the shape and the properties of both quadrilaterals are not taken into account. Fifteen pre-service teachers with this view proposed such a task, and two of them argued that the example in the given task was incorrect.

Within the scope of this study, it was determined that the conceptual knowledge of the pre-service teachers who were able to explain that the example in the given task was incorrect and suggested a task with the correct example was at the forefront and reasoning was at the basis of the association process (n=12). The fact that 4 pre-service teachers suggested tasks based only on visual perception despite recognizing the erroneous example can be interpreted as that these participants could not use their conceptual knowledge effectively in the reasoning process. Similarly, the fact that 26 pre-service teachers suggested tasks for the properties of quadrilaterals even though they thought that the erroneous example was correct shows that they did not have a complete conceptual understanding. In this study, 44% (n=40) of the pre-service teachers remained in this gap. There is a contradiction between the evaluations and suggestions of these participants.

DISCUSSION

The results of this study, which was conducted to examine the pre-service teachers' evaluation of the hierarchical structure between a rectangle and a parallelogram through an erroneous example, unfortunately show that the participants were not good enough in this regard. It should be noted that the purpose of this study was not to determine whether the pre-service teachers were aware of the fact that a rectangle is a parallelogram. It was assumed that the participants were aware of this topic since it was included in the Fundamentals of Mathematics 2 course. However, this information was reinforced for the pre-service teachers by including the outcome "A rectangle is a special case of a parallelogram" at the beginning of the erroneous example task given for evaluation.

The present findings therefore reinforce the claim that pre-service teachers continue to rely predominantly on ‘concept images’ rather than formally articulated ‘concept definitions’ when classifying quadrilaterals—an analytical blind spot that has persisted across cohorts (Fujita & Jones, 2007; Tall & Vinner, 1981).

Therefore, the pre-service teachers were asked to evaluate the given task to determine their mathematical awareness and to suggest a task to see how they handle different mathematical opportunities offered by different teaching methods. The results obtained showed that the erroneous examples were effective in gaining pedagogical knowledge to be prepared for possible student difficulties that may be encountered in the real classroom environment and to determine the way to handle the error (Biza et al., 2007; Mason, 1998), as well as revealing how misconceptions in students can be pedagogically teacher-induced. In addition, this study also provided an opportunity to examine whether the knowledge of pre-service teachers, who were found to be inconsistent between identifying the erroneous example and the task suggestion, was based on formal or conceptual foundations. Duval (1995; 2006) defines the process of inferring geometric shapes by focusing only on their visuals without considering their formal definitions or properties as processing. Processing is not always a process that leads to incorrect results. The characteristic of this cognitive process is that visual perception is at the forefront and inferences are not made based on mathematical arguments.

Complementing these descriptive results, recent controlled studies on incorrect-worked examples have shown that their efficacy hinges on the careful orchestration of cognitive load and on prompting learners to articulate why the displayed argument is invalid (Ngu et al., 2025; Yap & Wong, 2024; Soncini et al., 2022). In our data, participants who identified the error but still proposed visually driven tasks illustrate that cognitive conflict, by itself, is rarely sufficient; guided inquiry questions appear necessary to convert conflict into conceptual growth.

The results of the study show that most of the pre-service teachers (n=64) are in this process. Although there is not always an incorrect result at the end of the processing process, in this study, the participants' comments based on visual perception are the comments that cause them to misevaluate the erroneous example and to suggest tasks that contain the erroneous example. The results of the study show that pre-service teachers have two different processing processes: (1) area conservation and (2) figural representation. Each of these codes can lead to different student difficulties. For example, carrying a task that involves transforming a rectangle and a parallelogram with the same area into each other by ignoring their properties may lead to a misconception such as "If we can infer that the rectangle is a parallelogram because we can get a rectangle from a parallelogram by cutting and pasting, then we can say that the parallelogram is a rectangle when we get a parallelogram from a rectangle with the same method". Similarly, through drawings, dynamic geometry software, pipettes or moving rods, the focus is on the shape only, neglecting the properties of these two quadrilaterals. One such erroneous example is "If we can turn a rectangle created with moving sticks into a parallelogram by changing only the angles, then we can turn a parallelogram into a rectangle. Then a parallelogram is also a rectangle." This may cause a misconception. In fact, from a different perspective, this practice raises the following question: "If this method is functional, why cannot it be used to show that a square is a rectangle?" In both applications, the erroneous examples are illusions that create a visual illusion. Although the reasons underlying the reaction of the preservice teachers to the erroneous example and the task suggestions containing erroneous examples cannot be fully understood, it can be said that their concept images are the reason why they are deceived by these illusions.

This pattern resonates with the ‘analytic habits’ dimension of geometric mental habits developed by Özdemir & Çekirdekci (2022), wherein learners oscillate between surface-level manipulations and property-based argumentation. The large proportion of participants who initially failed to flag the erroneous example suggests that these analytic habits are not yet automatised at the end of their teacher-education programme.

As stated in Fischbein's (1993) Figurational Concept Theory, conceptual knowledge and shape knowledge do not support each other in the minds of these participants. When the study is evaluated in terms of pre-service teachers' visualization of the relationship between rectangle and parallelogram, it can be said that the results support different studies in the literature (e.g., Toluk et al., 2002; Fujita & Jones; 2006; Sahin, 2023). Evaluations based on visual perception can be evaluated as the pre-service teachers' concept images or shape knowledge overriding their concept knowledge or the gap between these two types of knowledge. One of the reasons for such a result is thought to be the neglect of the relationship between the definitions, properties and formal representations of concepts in mathematics teaching (Toluk et al., 2002; Sahin, 2023).

When the task suggestions of the pre-service teachers for the hierarchical relationship between rectangle and parallelogram are analyzed, it is seen that 47% (n=42) of them use the conceptual knowledge of these quadrilaterals. However, when the results of the evaluation of the erroneous example are taken into consideration, it can be said that only 13% (n=12) of them have conceptual knowledge, while the others prioritize their formal knowledge. Such a result shows the power of the erroneous example in revealing formal and conceptual knowledge. This result can be considered as the most important result of the study. In fact, it was revealed through the erroneous example that although the pre-service teachers knew that a rectangle is a parallelogram, for most of them this knowledge was not based on conceptual foundations. In studies using error-based tasks, it is emphasized that one of the factors determining the effect of the erroneous example is the students' readiness level (e.g., Große & Renkl, 2007; Gürbüz et al., 2021). In these studies, it was concluded that students who do not have sufficient prior knowledge have difficulty in analyzing errors. Since this study was conducted with the participation of pre-service teachers, such a result was not obtained.

Taken together, these outcomes imply that embedding erroneous examples within micro-teaching simulations could function as a powerful design element for cultivating pedagogical content knowledge in geometry. A recent systematic review by Shimizu and Kang (2025) likewise shows that error-oriented instructional strategies help pre-service teachers anticipate, diagnose, and productively address student misconceptions.

Finally, the study highlights that the misalignment between concept image and concept definition is not confined to K–12 learners but persists into teacher education. To mitigate this risk, it is recommended that university courses adopt multi-representation tasks mediated by dynamic-geometry environments and explicitly aligned with the Van Hiele levels, an approach further supported by recent classroom studies using GeoGebra to scaffold inclusive quadrilateral definitions (Vízek & Samková, 2023; Avcu, 2023).

CONCLUSION AND RECOMMENDATIONS

The main purpose of using erroneous examples in mathematics teaching is to support conceptual learning by analyzing the error and discussing the concepts and content (Borasi, 1989; Durkin & Rittle-Johnson, 2012). With reference to these results, especially those obtained as a result of studies conducted with students, it is recommended that erroneous examples be used as a tool in mathematics teaching, taking into account their "instructive" aspect. For teachers or pre-service teachers to effectively incorporate erroneous examples into the learning environment requires them to be aware of erroneous examples and their importance. For this purpose, one of the studies to be conducted is to determine their perspectives on erroneous examples and the other is to examine how they analyze erroneous examples. Expanding the literature on this subject will reveal effective results in terms of the use of erroneous examples in mathematics teaching and teacher education. In this sense, this study is thought to contribute to the literature. Future studies may focus on determining the views of pre-service teachers and/or teachers on the use of erroneous examples in mathematics teaching, and examining the classroom environments in which erroneous examples are used in terms of different factors such as student achievement, motivation, teacher intervention types, and teacher-student interaction.

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