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# Comparison of performances of heteroskedasticity tests under measurement error

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Abstract. While measurement error has an impact on the unbiasedness of the ordinary least squares (OLS) estimator, the heteroskedastic error term causes inefficient OLS estimators and biased variance estimates. Although the econometric literature has answers to these two fundamental concerns, such as applying measurement error correction methods and heteroskedasticity-robust standard errors, they do not directly address testing heteroskedasticity. This paper investigates the power of the most commonly used heteroskedasticity tests in the presence of error-in-variables. Monte Carlo simulations under different heteroscedasticity forms and sample sizes show that since measurement error inflates the variance of the explanatory variable and the response variable, heteroskedasticity tests lose their power in detecting heteroskedasticity. Simulations also show that the Glejser test is the most powerful one while the White test is weak, and the other tests lie in between them.

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## 1. Introduction

Homoskedasticity assumption of classical linear regression models (CLRM) represents that conditional variance of unobservable error term on explanatory variable(s) are constant. This assumption, as in highlighted in Wooldridge [29], fails whenever the variance of the unobservables changes across different segments of the population, which are determined by the different values of the explanatory variables. Then, the usual ordinary least squares (OLS) standard errors are invalid and so hypothesis testing and constructing confidence intervals. A number of tests have been proposed to determine if error terms are homoskedastic or not. A significant part of these tests does not directly test whether the variance of the unobservable error term remains constant with respect to the values of the explanatory variable(s). For example, White [26] is an indirect heteroskedasticity test since it is based on the idea that the variance of the error terms will be constant if the squares of the errors are independent of the explanatory variables, the squares of the explanatory variables, and the products of the explanatory variables. Glejser [9] proposed a test that relates absolute values of the error terms of the model to explanatory variable(s) that might lead to heteroskedasticity. Heteroskedasticity tests employing the squares and/or absolute values of the error terms need the most exact estimation of the error term since error terms cannot be observed even in the population and must be indirectly estimated. If there are no endogenous explanatory variables or omitted variable bias, the parameters of a CLRM are unbiased even if the error terms are heteroskedastic.

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Another important assumption for unbiased parameter estimations is that the regression variables are measured without error. This assumption is mostly neglected in applied analysis by researchers.

As it is well-documented by the econometric theory, ordinary least squares estimators are biased when the explanatory variables are measured with error [8, 19, 27]. Therefore, when the variables in the regression model are observed with measurement error, the results of the heteroskedasticity tests may not be valid, even if the error term is homoskedastic. In addition, it can be expected that the co-existence of both measurement error variables and heteroskedastic errors will significantly affect the reliability of the test results. Small or large measurement error variation, as well as a small or large sample size, are critical factors that determine these expectations. As a result, in terms of applied analysis, mutual evaluation of the performances of heteroskedasticity tests under the presence of error-in-variables is a critical problem. The purpose of this paper is to conduct Monte Carlo simulations to compare the results of the most preferable heteroskedasticity tests in the presence of error-in-variables in the CLRM.

Recent econometric literature is lack of studies comparing performances heteroskedasticity tests[1]. For different sample sizes, Uyanto [24] investigated the performance of seven different heteroskedasticity tests. The author concluded that the Goldfelt-Quandt and Harrison-McCabe tests performed better than the others and the White test which is mostly used one has a weaker performance. Like Uyanto [24], Adamec [1] compared the performances of heteroskedasticity tests under different mathematical forms of heteroskedasticity. Results of Adamec [1] show that Bartlett, White, Harrison and Godlfelt-Quandt tests have better performances than the other tests. The power of heteroskedasticity tests increases with increasing sample size under different heteroskedasticity forms, according to Adamec [1].

Even though the recent literature did not pay attention in comparing the results of heteroskedasticity tests in the presence of measurement error in the explanatory and/or response variables, Wallentin and Agren [25] investigated the behavior of heteroscedasticity tests statistics in a simple regression model when the regressor is measured with error [2]. They calculated three different sets of residuals; the least squares residuals, residuals calculated by using observed regressor values and residuals calculated using fitted values of the regressor. Their simulations are for 50, 200 and 800 sample sizes. According to their results, while the White test is the weakest one, the Glejser [9] and the Pesaran and Taylor [22] tests perform better when they applied to three different residuals. Unlike Wallentin and Agren [25], we compared the results of six commonly used heteroskedasticity tests for different levels of measurement error variance under distinct forms of heteroskedasticity. We looked at their performance not just in terms of error in the explanatory variable, but also in terms of error in the response variable and error in both variables. As a result, this research offers a thorough comparison of heteroskedasticity tests.

According to the study described above, the Glejser test performs better when the regressor is measured with error. In our study, we also concluded that the Glejser test performs better in most cases. However, because the Glejser test allows us to apply alternative auxiliary regressions, we used different auxiliary regressions to expand our results.

In Monte Carlo simulations, random variables taken from normal distribution. In addition, we make assumptions about the mathematical form of heteroskedasticity, the degree of measurement error variance, and sample size. For six heteroskedasticity tests, Monte Carlo simulation study is performed for comparing the performances of these tests in the presence of heteroskedastic errors and variables with and without measurement error. According to Monte Carlo simulation results based on simple regression, the power of the tests is low in small samples under a linear heteroskedasticity form when the variables are not observed truly. The power of these tests increases as the sample size increases. The results reveal that measurement error in the explanatory variable impacts the power of the tests more than measurement

[1]Earlier studies comparing the performances of the heteroskedasticity tests are Harvey and Phillips [17] (the recursive test has a better performance than the Goldfelt-Quandt test), Griffiths and Surekha [13] (the Szroter test is more powerful one ), Lyon and Tsai [20] (the Koenker test is the powerful one). Results of Dufour et al. [7] confirms the results of Griffiths and Surekha [13], and the Szroter test seems to be the best choice in terms of power.

[2]In Econometric Theory journal, Wooldridge [28] demonstrated a theoretical conclusion for the asymptotic performances of heteroskedasticity tests in the presence of measurement error. The author defined the expected value of y (response variable) conditional on x (explanatory variable) without error is the product of expected value of x with measurement error (x*) and slope term (b) and showed that this is not related to derive the asymptotic properties of heteroskedasticity tests. According to the author, the consistently estimated parameter is g, which is the coefficient of the x* in the presence of an explanatory variable measured with error, rather than b, which is the coefficient of the true x. In addition to, Wooldridge [28] showed that nR2 statistics, for testing heteroskedasticity, calculated from a regression with mismeasured x does not have a finite chi-square distribution.

error in the response variable, and the behavior of the tests is also controlled by measurement error in the explanatory variable when both variables are measured with error. Simulation results show that the Glejser test has the highest power among the six popular types of heteroscedasticity tests, while the White test has the least power.

The remainder of this paper is organized as follows. Section 2 discusses measurement error and heteroskedasticity tests. Section 3 presents the results of Monte Carlo simulations. Finally, Section 4 provides the conclusion.

## 2. Heteroskedasticity Tests and Measurement Error

Even though estimating the parameters of the CLRM suppresses the error terms in the estimation procedure, testing the assumptions of it are based on the estimated error terms. There are several potential obstacles to obtaining accurately estimated parameters: endogenous explanatory variables, omitted variables and measurement error. These issues also affect hypothesis testing. Both econometrics and statistics have provided strategies to address these challenges such as using heteroskedasticity-robust standard errors for hypothesis testing and estimating the parameters with the use of techniques that adjust the estimates for endogenous explanatory variables and measurement error. However, among these robust-standard errors, in particular, should be employed in a variety of conditions; if the model is specified correctly or parameters are estimated consistently [30]. Therefore, it is important to test whether error terms are homoskedastic or not. Most of the heteroskedasticity tests are commonly based on auxiliary regressions using explanatory variables. As a result, the null hypothesis asserts that the coefficients of these explanatories are all equal to 0 simultaneously, reducing the auxiliary model to a constant term model. Intuitively, it can be said that the coefficients of the test regression are also biased if the variables are measured with error, resulting in inaccurate heteroskedasticity conclusions. However, it is important to show that which heteroskedasticity test has a better performance in the presence of mismeasured explanatory and/or response variables.

As a hidden assumption of the CLRM, no measurement error assumption states that differences between theoretical (correct or true) value of a variable and its observed value is the same. If there is difference between these two, then a mismeasured variable is a “substitute” measure of a true variable. Within the context of regression, Carroll et al. [5] noted that “the substitute variable is not merely mismeasured version of the explanatory variable but is a separate variable acting as a type of proxy for the explanatory variable”. In other words, measurement error is taken as a “surrogate variable” not as a “manifest variable” which is mostly used in the path analysis. In this paper, we use simple linear regression framework.

$$Y_i = \beta_0 + \beta_1 X_i + u_i \tag{1}$$

$u_i$ are error terms with zero mean and $\sigma_i^2$ variances which are heteroskedastic.

Suppose that we observe $X^*$ instead of $X$ as below:

$$X_i^* = X_i + \epsilon_i. \tag{2}$$

Substituting Equation (2) into Equation (1) yields

$$Y_i = \beta_0 + \beta_1 X_i + (u_i + \beta_1 \epsilon_i). \tag{3}$$

The error terms of Equation (3) (say $u_{i1}$) includes both heteroskedastic errors and measurement errors. Since most of the heteroskedasticity tests use the squared or absolute values of error terms as dependent variable, testing heteroskedasticity will result in inaccurate in terms of mismeasured $X$. When dependent variable is measured with error ($Y_i^* = Y_i + \epsilon_i^{'}$), Equation (1) will be

$$Y_i = \beta_0 + \beta_1 X_i + (u_i - \epsilon_i^{'}) \tag{4}$$

with the error term, $u_{i2}$. A similar situation will be observed if both Y and X have measurement error.

$$Y_i = \beta_0 + \beta_1 X_i + (u_i + \beta_1 \epsilon_i - \epsilon_i^{'}) \tag{5}$$

where the new error term is $u_{i3}$.

A number of tests for homoskedasticity have been proposed in the literature: some of them are Goldfeld and Quandt [12], Glejser [9], Park [21], Harvey and Phillips [17], Hedayat and Robson [18], Szroeter [23], Harrison and McCabe [15], Harvey [16], Bickel [3], Breusch and Pagan [4], and White [26]. The most employed heteroskedasticity tests in applied research are summarized in the following section based on the model given in Equation (1).

2.1. **Goldfeld-Quandt Test.** Goldfeld and Quandt [12] proposed a test that can be used if the variance of error is related to one of the explanatory variables in the main regression model. The underlying idea of the test is to sort the data according to the explanatory variable that is thought to be related to the variance of the error term, divide it into two separate parts after taking c observations from the middle, to reveal change in variances clearly. The main model is estimated for the first $\frac{n-c}{2}$ and the last $\frac{n-c}{2}$ observations, and the respective residual sums of squares (RSS) are obtained. These sums are used to compute the Goldfeld-Quandt test ratio which is expressed below:

$$\lambda = \frac{\frac{\mathrm{RSS}_{\text{the last obs}}}{df}}{\frac{\mathrm{RSS}_{\text{the first obs}}}{df}}. \tag{6}$$

The $\lambda$ statistics is distributed as $F$ with $df = \frac{n-c}{2} - k$ degrees of freedoms under the assumptions of normally distributed errors of the main regression. $k$ expresses the number of parameters in the model including the intercept term.

This test relies on correctly selecting the number of observations to need to be excluded (c) because of the power of the test. Goldfeld and Quandt recommended that c=8 when working with 30 observations in a two-explanatory model, and c=16 when working with 60 observations. These numbers correspond to approximately 27% of the sample size. The sample sizes in this paper are 30, 50 and 200 and we omitted observations accordingly following Goldfeld and Quandt's recommendation.

2.2. **Park Test.** Park [21] defines the form of auxiliary regression as follows, based on the assumption that error term variance is a function of explanatory variable(s):

$$\sigma_i^2 = \sigma^2 X_i^{\beta} e^{u_i}. \tag{7}$$

Equation (8) is obtained by taking natural logarithms of both sides of Equation (7):

$$\ln \sigma_i^2 = \ln \sigma^2 + \beta \ln X_i + u_i. \tag{8}$$

Since $\sigma_i^2$ is usually unknown, [21] replaces $\sigma_i^2$ by $\hat{u}_i^2$. Therefore, Equation (8) can be written as in Equation (9):

$$\ln \hat{u}_i^2 = \ln \sigma^2 + \beta \ln X_i + v_i. \tag{9}$$

If $\beta = 0$, then logarithm of squared error terms equals to $\ln \sigma^2$ that is a constant value (say $\alpha$). The Park test is testing the significance of the estimated $\beta$. A statistically significant $\beta$ points out heteroskedasticity in the error term. In a multiple regression, the Park test turns to completely significance of the auxiliary regression. Godlfeldt and Quandt [11] as cited in Gujarati [14] criticized this test in terms of the possibility of violations of the CLRM assumptions in the test regression.

2.3. **Glejser Test.** Glejser [9] proposed a test that relates absolute values of the error terms of the model to explanatory variable(s) that might lead to heteroskedasticity. This test relies on different mathematical forms of the relation between the error variance and the explanatory variable.

The default mathematical form is $|\hat{u}_i| = a_0 + a_1 X_i + v_i$ and we also examined test's performance by using the following forms:

$$|\hat{u}_i| = a_0 + a_1 \sqrt{X_i} + v_i, \tag{10}$$

$$|\hat{u}_i| = a_0 + a_1 \frac{1}{X_i} + v_i, \tag{11}$$

$$|\hat{u}_i| = a_0 + a_1 \frac{1}{\sqrt{X_i}} + v_i. \tag{12}$$

Godlfeldt and Quandt [11] pointed out that error terms of the auxiliary regression might not have zero mean, might be autocorrelated and/or heteroskedastic. Besides some of auxiliary regressions cannot be estimated by OLS method since they are nonlinear according to their parameters. Under the null hypothesis of homoscedasticity, the test statistic, $nR^2$, is asymptotically distributed as $\chi^2_{(k)}$ where $k$ is the number of variables of the estimated model. Here, $R^2$ is the coefficient of determination from the test regression. The Glejser test is only valid when the errors are conditionally symmetrical.

2.4. **Harvey Test.** The Harvey test assumes that error variance is an exponential function of explanatory variable(s) [16]. The auxiliary regression is

$$\sigma_i^2 = e^{a_0 + a_1 X_i + w_i}. \tag{13}$$

Estimating this model by OLS requires taking the logarithm of both sides of the equation which is expressed below:

$$\ln u_i^2 = a_0 + a_1 X_i + w_i. \tag{14}$$

The test statistic, $nR^2$, is distributed as $\chi^2_{(k)}$ where $k$ is the number of variables of the estimated model.

2.5. **Breusch-Pagan Test.** According to Breusch and Pagan [4], error variance can be a function of some or all explanatory variables. Variance of error term of Equation (1) can be written as follows:

$$\hat{\sigma}_i^2 = a_0 + a_1 X_i. \tag{15}$$

Under $a_1 = 0$ assumption, variance of the error term of the model given in Equation (1) is constant. To test the null of the error terms being homoskedastic, $\hat{\sigma}^2 = \frac{\hat{u}_i^2}{n}$ is defined. The test regression is

$$p_i = a_0 + a_1 X_i + v_i \tag{16}$$

where $p_i = \frac{\hat{u}_i^2}{\hat{\sigma}^2}$.

This test is a Lagrange multiplier test, and the test statistic is distributed $\chi^2$ with $p-1$ degrees of freedom under the assumption of error term $u_i$ being normal and homoskedastic. $p$ is the number of explanatory variables in the model of interest.

2.6. **White Test.** White's test [26] is based on a regression of the squared residuals from the original regression on the original $X$ variables or regressors, their squared values, and the cross product(s) of the regressors if there are more than one regressor. The auxiliary regression of Equation (1) for the White test is given by Equation (17):

$$\hat{u}_i^2 = a_0 + a_1 X_i + a_2 X_i^2 + v_i. \tag{17}$$

Unlike the other tests for a simple regression, the null hypothesis is $\alpha_1 = \alpha_2 = 0$. Under the null hypothesis test statistics is $nR^2$ that is distributed $\chi^2$ with 2 degrees of freedom for Equation (17). Although the White test is preferable because it is not based on the assumption of normality and is easy to apply, it has the disadvantage that it creates a degree of freedom problem because there are too many regressors in the auxiliary regression model. The reason for the rejection of the null hypothesis in the White test may not be that there is a heteroskedasticity problem in the model, but that there is a specification error. In other words, the White test can be used not only to test for heteroskedasticity but also model specification errors.

In sum, the logic behind the heteroskedasticity tests is similar. An auxiliary regression shows whether the error variance is a function of explanatory variable(s) of the model. Unlike the other tests, the test proposed in Goldfelt and Quandt [12] is based on ordering the observations beginning with the lowest $X$ value.

## 3. Monte Carlo Simulations and Performances of Heteroskedasticity Tests

Monte Carlo simulations are used to assess comparing the performances of heteroskedasticity tests in the presence of heteroskedastic errors and variables with and without error. R-project programming language is used for all statistical simulations and analyses. The variables of the simple regression model, $Y$ and $X$, are generated from a multivariate normal distribution. Sample sizes are chosen to be comparable to those common in the social sciences as 30, 50 and 200 [6, 10]. Two mathematical forms of heteroskedasticity are used: $\sigma_i^2 = \sigma^2 X_i$ and $\sigma_i^2 = \sigma^2 X_i^2$. In each loop, 10,000 repetitions are run. Standard deviations of measurement errors for $X$ and $Y$ are taken values in a sequence 0.5 to 5 in an increasing order by 0.5. We reported results when measurement error variance is 0.5 (small), 2.0 (medium) and 5 (large). Throughout the experiment, $b$ in Equation (1) was set at $\beta_0 = 1$ and $\beta_1 = 0.5$.

The power of each heteroskedasticity test is obtained in this study to examine the efficacy of the heteroskedasticity test techniques. As is well known, the power of a test is defined as the probability of rejecting the null hypothesis (homoskedastic errors) when it is false (i.e., in the presence of heteroskedastic errors). Hence, in both tables and figures, the ratios (%) of rejection of the null hypothesis as determined by the heteroskedasticity tests are utilized to assess the simulation findings in comparison to the nominal type I error rates ($\alpha = [0.01; 0.05; 0.10]$).

TABLE 1. Performances of heteroskedasticity tests with/without measurement error in $X$

| Name of the Test | Het. Form: Linear | | | Het. Form: Quadratic | | | Het. Form: Linear | | | Het. Form: Quadratic | | | Het. Form:Linear | | | Het. Form:Quadratic | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n = **30** | | | | | | n = **50** | | | | | | n = **200** | | | | | |
| | Without Measurement Error | | | | | | Without Measurement Error | | | | | | Without Measurement Error | | | | | |
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| Goldfeld-Quandt | 0.046 | 0.157 | 0.250 | 0.229 | 0.458 | 0.580 | 0.104 | 0.270 | 0.384 | 0.507 | 0.739 | 0.829 | 0.643 | 0.834 | 0.899 | 0.998 | 1,000 | 1,000 |
| Park | 0.042 | 0.135 | 0.211 | 0.149 | 0.319 | 0.440 | 0.063 | 0.186 | 0.282 | 0.275 | 0.516 | 0.639 | 0.317 | 0.586 | 0.708 | 0.945 | 0.990 | 0.996 |
| Glejser | 0.059 | 0.184 | 0.278 | 0.273 | 0.512 | 0.637 | 0.127 | 0.308 | 0.427 | 0.557 | 0.778 | 0.863 | 0.719 | 0.884 | 0.933 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.039 | 0.124 | 0.201 | 0.143 | 0.311 | 0.430 | 0.066 | 0.188 | 0.284 | 0.298 | 0.532 | 0.653 | 0.369 | 0.612 | 0.723 | 0.963 | 0.991 | 0.997 |
| Breusch-Pagan | 0.045 | 0.167 | 0.268 | 0.179 | 0.441 | 0.591 | 0.102 | 0.284 | 0.411 | 0.419 | 0.713 | 0.828 | 0.709 | 0.890 | 0.940 | 0.999 | 1,000 | 1,000 |
| White | 0.041 | 0.110 | 0.181 | 0.141 | 0.305 | 0.434 | 0.080 | 0.190 | 0.288 | 0.305 | 0.545 | 0.692 | 0.565 | 0.807 | 0.895 | 0.997 | 1,000 | 1,000 |
| | With Measurement Error (small) | | | | | | With Measurement Error (small) | | | | | | With Measurement Error (small) | | | | | |
| Goldfeld-Quandt | 0.047 | 0.156 | 0.253 | 0.229 | 0.459 | 0.579 | 0.103 | 0.267 | 0.380 | 0.507 | 0.739 | 0.829 | 0.636 | 0.827 | 0.895 | 0.999 | 1,000 | 1,000 |
| Park | 0.046 | 0.131 | 0.208 | 0.149 | 0.319 | 0.437 | 0.059 | 0.179 | 0.272 | 0.276 | 0.513 | 0.639 | 0.310 | 0.582 | 0.709 | 0.943 | 0.990 | 0.996 |
| Glejser | 0.059 | 0.181 | 0.275 | 0.273 | 0.511 | 0.637 | 0.124 | 0.303 | 0.422 | 0.557 | 0.778 | 0.862 | 0.713 | 0.880 | 0.931 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.040 | 0.122 | 0.195 | 0.143 | 0.311 | 0.430 | 0.067 | 0.181 | 0.276 | 0.298 | 0.530 | 0.652 | 0.358 | 0.607 | 0.719 | 0.963 | 0.992 | 0.997 |
| Breusch-Pagan | 0.044 | 0.165 | 0.265 | 0.179 | 0.441 | 0.591 | 0.100 | 0.281 | 0.411 | 0.419 | 0.713 | 0.829 | 0.700 | 0.886 | 0.939 | 0.999 | 1,000 | 1,000 |
| White | 0.042 | 0.110 | 0.180 | 0.141 | 0.305 | 0.434 | 0.080 | 0.189 | 0.284 | 0.305 | 0.545 | 0.693 | 0.560 | 0.804 | 0.893 | 0.997 | 1,000 | 1,000 |
| | With Measurement Error (medium) | | | | | | With Measurement Error (medium) | | | | | | With Measurement Error (medium) | | | | | |
| Goldfeld-Quandt | 0.046 | 0.150 | 0.248 | 0.229 | 0.459 | 0.580 | 0.098 | 0.259 | 0.371 | 0.507 | 0.738 | 0.829 | 0.620 | 0.819 | 0.888 | 0.999 | 1,000 | 1,000 |
| Park | 0.043 | 0.130 | 0.204 | 0.150 | 0.318 | 0.438 | 0.058 | 0.172 | 0.268 | 0.274 | 0.513 | 0.641 | 0.299 | 0.564 | 0.699 | 0.943 | 0.990 | 0.996 |
| Glejser | 0.056 | 0.177 | 0.273 | 0.272 | 0.512 | 0.637 | 0.118 | 0.296 | 0.410 | 0.556 | 0.778 | 0.862 | 0.695 | 0.869 | 0.925 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.041 | 0.117 | 0.194 | 0.144 | 0.312 | 0.430 | 0.060 | 0.174 | 0.268 | 0.298 | 0.532 | 0.651 | 0.345 | 0.594 | 0.713 | 0.964 | 0.991 | 0.996 |
| Breusch-Pagan | 0.042 | 0.160 | 0.263 | 0.179 | 0.441 | 0.590 | 0.099 | 0.271 | 0.402 | 0.420 | 0.712 | 0.828 | 0.682 | 0.877 | 0.933 | 0.999 | 1,000 | 1,000 |
| White | 0.041 | 0.107 | 0.177 | 0.141 | 0.305 | 0.435 | 0.078 | 0.184 | 0.280 | 0.306 | 0.545 | 0.693 | 0.544 | 0.792 | 0.883 | 0.997 | 1,000 | 1,000 |
| | With Measurement Error (large) | | | | | | With Measurement Error (large) | | | | | | With Measurement Error (large) | | | | | |
| Goldfeld-Quandt | 0.045 | 0.145 | 0.239 | 0.229 | 0.459 | 0.581 | 0.090 | 0.248 | 0.357 | 0.508 | 0.737 | 0.828 | 0.586 | 0.794 | 0.871 | 0.999 | 1,000 | 1,000 |
| Park | 0.042 | 0.120 | 0.201 | 0.148 | 0.317 | 0.438 | 0.060 | 0.170 | 0.261 | 0.274 | 0.510 | 0.642 | 0.273 | 0.536 | 0.669 | 0.944 | 0.989 | 0.996 |
| Glejser | 0.053 | 0.169 | 0.266 | 0.271 | 0.512 | 0.637 | 0.112 | 0.282 | 0.397 | 0.555 | 0.777 | 0.862 | 0.660 | 0.848 | 0.911 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.040 | 0.114 | 0.189 | 0.143 | 0.312 | 0.428 | 0.061 | 0.173 | 0.264 | 0.297 | 0.532 | 0.652 | 0.316 | 0.564 | 0.682 | 0.965 | 0.991 | 0.996 |
| Breusch-Pagan | 0.040 | 0.155 | 0.254 | 0.179 | 0.440 | 0.592 | 0.092 | 0.261 | 0.386 | 0.419 | 0.712 | 0.829 | 0.651 | 0.856 | 0.919 | 0.999 | 1,000 | 1,000 |
| White | 0.041 | 0.104 | 0.169 | 0.141 | 0.304 | 0.435 | 0.074 | 0.178 | 0.268 | 0.305 | 0.545 | 0.694 | 0.511 | 0.762 | 0.864 | 0.997 | 1,000 | 1,000 |

Table 1 shows the simulation results if the variable X is measured with and without error. When the type of heteroskedasticity is linear and the sample size is small, the power of the tests ranges between 4% and 28%. For example, at a 10% significance level, the Glejser test's performance increases from 28% to 43% for a sample size of 50. If the sample size is 200, the test's power increases to 86%. When heteroskedasticity is a nonlinear function of variable X, the power of the tests is substantially higher, even for small sample sizes. When the sample size is 200, all tests strongly reject the null hypothesis of homoskedastic errors, when it is false. When the variable X has a small measurement error, the test results change slightly from those obtained when there is no measurement error in the variables. If the type of heteroskedasticity is linear, however, there are significantly larger declines in the performance of the tests as the variance of the measurement error increases. In the nonlinear heteroskedasticity version, this decline is not as high as in the linear form. In other words, if the form of heteroskedasticity is squared and the significance level is kept high, the large measurement error variance in X decreases the test performance less. When the tests are ordered according to their performances, the Glejser test is followed by the Glodfelt-Quandt test and the Breusch-Pagan test. The White test is found to be weak compared to other tests in rejecting the false null hypothesis when the sample size is large, and variance of measurement error is high.

Table 2 summarized the simulation results when the response variable $Y$ is measured with error. The existence of an error-prone response variable with small size in the regression has almost no effect on the performances of the heteroskedasticity tests. Their power is very small even at the 10% significance levels when the form of heteroskedasticity is $\sigma^2 X_i$. While the power of the tests ranges between 14% and 27% at the 1% significance level, their powers significantly increase at the 5% and 10% significance levels when the form of heteroskedasticity is dependent on the square of $X$ and the measurement error variance of it is at a medium level. In brief, powers of the heteroskedasticity tests are lower for the small sample sizes when the error term is heteroskedastic. Their performances increase with the increase in the sample size and measurement error in variable $Y$ does not have a dramatic effect on the powers of the tests especially when the form of heteroskedasticity is $\sigma^2 X_i^2$.

We also evaluated how the tests would perform if the measurement error variances of $X$ and $Y$ were in the reverse direction. The graphics are used to present these findings for the sample sizes 30 and

TABLE 2. Performances of heteroskedasticity tests with/without measurement error in $Y$

| Name of the Test | Het. Form: Linear | | | Het. Form: Quadratic | | | Het. Form: Linear | | | Het. Form: Quadratic | | | Het. Form: Linear | | | Het. Form: Quadratic | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n = **30** | | | | | | n = **50** | | | | | | n = **200** | | | | | |
| | Without Measurement Error | | | | | | Without Measurement Error | | | | | | Without Measurement Error | | | | | |
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| Goldfeld-Quandt | 0.046 | 0.157 | 0.250 | 0.229 | 0.458 | 0.580 | 0.104 | 0.270 | 0.384 | 0.507 | 0.739 | 0.829 | 0.643 | 0.834 | 0.899 | 0.998 | 1,000 | 1,000 |
| Park | 0.042 | 0.135 | 0.211 | 0.149 | 0.319 | 0.440 | 0.063 | 0.186 | 0.282 | 0.275 | 0.516 | 0.639 | 0.317 | 0.586 | 0.708 | 0.945 | 0.990 | 0.996 |
| Glejser | 0.059 | 0.184 | 0.278 | 0.273 | 0.512 | 0.637 | 0.127 | 0.308 | 0.427 | 0.557 | 0.778 | 0.863 | 0.719 | 0.884 | 0.933 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.039 | 0.124 | 0.201 | 0.143 | 0.311 | 0.430 | 0.066 | 0.188 | 0.284 | 0.298 | 0.532 | 0.653 | 0.369 | 0.612 | 0.723 | 0.963 | 0.991 | 0.997 |
| Breusch-Pagan | 0.045 | 0.167 | 0.268 | 0.179 | 0.441 | 0.591 | 0.102 | 0.284 | 0.411 | 0.419 | 0.713 | 0.828 | 0.709 | 0.890 | 0.940 | 0.999 | 1,000 | 1,000 |
| White | 0.041 | 0.110 | 0.181 | 0.141 | 0.305 | 0.434 | 0.080 | 0.190 | 0.288 | 0.305 | 0.545 | 0.692 | 0.565 | 0.807 | 0.895 | 0.997 | 1,000 | 1,000 |
| | With Measurement Error (small) | | | | | | With Measurement Error (small) | | | | | | With Measurement Error (small) | | | | | |
| Goldfeld-Quandt | 0.047 | 0.156 | 0.253 | 0.229 | 0.459 | 0.579 | 0.103 | 0.267 | 0.380 | 0.507 | 0.739 | 0.829 | 0.636 | 0.827 | 0.895 | 0.999 | 1,000 | 1,000 |
| Park | 0.046 | 0.131 | 0.208 | 0.149 | 0.319 | 0.437 | 0.059 | 0.179 | 0.272 | 0.276 | 0.513 | 0.639 | 0.310 | 0.582 | 0.709 | 0.943 | 0.990 | 0.996 |
| Glejser | 0.059 | 0.181 | 0.275 | 0.273 | 0.511 | 0.637 | 0.124 | 0.303 | 0.422 | 0.557 | 0.778 | 0.862 | 0.713 | 0.880 | 0.931 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.040 | 0.122 | 0.195 | 0.143 | 0.311 | 0.430 | 0.067 | 0.181 | 0.276 | 0.298 | 0.530 | 0.652 | 0.358 | 0.607 | 0.719 | 0.963 | 0.992 | 0.997 |
| Breusch-Pagan | 0.044 | 0.165 | 0.265 | 0.179 | 0.441 | 0.591 | 0.100 | 0.281 | 0.411 | 0.419 | 0.713 | 0.829 | 0.700 | 0.886 | 0.939 | 0.999 | 1,000 | 1,000 |
| White | 0.042 | 0.110 | 0.180 | 0.141 | 0.305 | 0.434 | 0.080 | 0.189 | 0.284 | 0.305 | 0.545 | 0.693 | 0.560 | 0.804 | 0.893 | 0.997 | 1,000 | 1,000 |
| | With Measurement Error (medium) | | | | | | With Measurement Error (medium) | | | | | | With Measurement Error (medium) | | | | | |
| Goldfeld-Quandt | 0.046 | 0.150 | 0.248 | 0.229 | 0.459 | 0.580 | 0.098 | 0.259 | 0.371 | 0.507 | 0.738 | 0.829 | 0.620 | 0.819 | 0.888 | 0.999 | 1,000 | 1,000 |
| Park | 0.043 | 0.130 | 0.204 | 0.150 | 0.318 | 0.438 | 0.058 | 0.172 | 0.268 | 0.274 | 0.513 | 0.641 | 0.299 | 0.564 | 0.699 | 0.943 | 0.990 | 0.996 |
| Glejser | 0.056 | 0.177 | 0.273 | 0.272 | 0.512 | 0.637 | 0.118 | 0.296 | 0.410 | 0.556 | 0.778 | 0.862 | 0.695 | 0.869 | 0.925 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.041 | 0.117 | 0.194 | 0.144 | 0.312 | 0.430 | 0.060 | 0.174 | 0.268 | 0.298 | 0.532 | 0.651 | 0.345 | 0.594 | 0.713 | 0.964 | 0.991 | 0.996 |
| Breusch-Pagan | 0.042 | 0.160 | 0.263 | 0.179 | 0.441 | 0.590 | 0.099 | 0.271 | 0.402 | 0.420 | 0.712 | 0.828 | 0.682 | 0.877 | 0.933 | 0.999 | 1,000 | 1,000 |
| White | 0.041 | 0.107 | 0.177 | 0.141 | 0.305 | 0.435 | 0.078 | 0.184 | 0.280 | 0.306 | 0.545 | 0.693 | 0.544 | 0.792 | 0.883 | 0.997 | 1,000 | 1,000 |
| | With Measurement Error (large) | | | | | | With Measurement Error (large) | | | | | | With Measurement Error (large) | | | | | |
| Goldfeld-Quandt | 0.045 | 0.145 | 0.239 | 0.229 | 0.459 | 0.581 | 0.090 | 0.248 | 0.357 | 0.508 | 0.737 | 0.828 | 0.586 | 0.794 | 0.871 | 0.999 | 1,000 | 1,000 |
| Park | 0.042 | 0.120 | 0.201 | 0.148 | 0.317 | 0.438 | 0.060 | 0.170 | 0.261 | 0.274 | 0.510 | 0.642 | 0.273 | 0.536 | 0.669 | 0.944 | 0.989 | 0.996 |
| Glejser | 0.053 | 0.169 | 0.266 | 0.271 | 0.512 | 0.637 | 0.112 | 0.282 | 0.397 | 0.555 | 0.777 | 0.862 | 0.660 | 0.848 | 0.911 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.040 | 0.114 | 0.189 | 0.143 | 0.312 | 0.428 | 0.061 | 0.173 | 0.264 | 0.297 | 0.532 | 0.652 | 0.316 | 0.564 | 0.682 | 0.965 | 0.991 | 0.996 |
| Breusch-Pagan | 0.040 | 0.155 | 0.254 | 0.179 | 0.440 | 0.592 | 0.092 | 0.261 | 0.386 | 0.419 | 0.712 | 0.829 | 0.651 | 0.856 | 0.919 | 0.999 | 1,000 | 1,000 |
| White | 0.041 | 0.104 | 0.169 | 0.141 | 0.304 | 0.435 | 0.074 | 0.178 | 0.268 | 0.305 | 0.545 | 0.694 | 0.511 | 0.762 | 0.864 | 0.997 | 1,000 | 1,000 |

TABLE 3. Performances of heteroskedasticity tests with/without measurement error in $X$ and $Y$

| Name of the Test | Het. Form: Linear | | | Het. Form: Quadratic | | | Het. Form: Linear | | | Het. Form: Quadratic | | | Het. Form: Linear | | | Het. Form: Quadratic | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n = **30** | | | | | | n = **50** | | | | | | n = **200** | | | | | |
| | Without Measurement Error | | | | | | Without Measurement Error | | | | | | Without Measurement Error | | | | | |
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| Goldfeld-Quandt | 0.046 | 0.157 | 0.250 | 0.229 | 0.458 | 0.580 | 0.104 | 0.270 | 0.384 | 0.507 | 0.739 | 0.829 | 0.643 | 0.834 | 0.899 | 0.998 | 1,000 | 1,000 |
| Park | 0.042 | 0.135 | 0.211 | 0.149 | 0.319 | 0.440 | 0.063 | 0.186 | 0.282 | 0.275 | 0.516 | 0.639 | 0.317 | 0.586 | 0.708 | 0.945 | 0.990 | 0.996 |
| Glejser | 0.059 | 0.184 | 0.278 | 0.273 | 0.512 | 0.637 | 0.127 | 0.308 | 0.427 | 0.557 | 0.778 | 0.863 | 0.719 | 0.884 | 0.933 | 1,000 | 1,000 | 1,000 |
| Harvey | 0.039 | 0.124 | 0.201 | 0.143 | 0.311 | 0.430 | 0.066 | 0.188 | 0.284 | 0.298 | 0.532 | 0.653 | 0.369 | 0.612 | 0.723 | 0.963 | 0.991 | 0.997 |
| Breusch-Pagan | 0.045 | 0.167 | 0.268 | 0.179 | 0.441 | 0.591 | 0.102 | 0.284 | 0.411 | 0.419 | 0.713 | 0.828 | 0.709 | 0.890 | 0.940 | 0.999 | 1,000 | 1,000 |
| White | 0.041 | 0.110 | 0.181 | 0.141 | 0.305 | 0.434 | 0.080 | 0.190 | 0.288 | 0.305 | 0.545 | 0.692 | 0.565 | 0.807 | 0.895 | 0.997 | 1,000 | 1,000 |
| | With Measurement Error (small) | | | | | | With Measurement Error (small) | | | | | | With Measurement Error (small) | | | | | |
| Goldfeld-Quandt | 0.041 | 0.135 | 0.220 | 0.185 | 0.402 | 0.525 | 0.090 | 0.240 | 0.346 | 0.429 | 0.659 | 0.763 | 0.552 | 0.768 | 0.846 | 0.992 | 0.999 | 1,000 |
| Park | 0.042 | 0.122 | 0.197 | 0.123 | 0.285 | 0.397 | 0.053 | 0.163 | 0.254 | 0.221 | 0.453 | 0.577 | 0.260 | 0.517 | 0.646 | 0.892 | 0.974 | 0.990 |
| Glejser | 0.053 | 0.162 | 0.258 | 0.228 | 0.457 | 0.581 | 0.103 | 0.267 | 0.384 | 0.475 | 0.709 | 0.806 | 0.628 | 0.822 | 0.893 | 0.998 | 1,000 | 1,000 |
| Harvey | 0.039 | 0.112 | 0.185 | 0.123 | 0.280 | 0.396 | 0.056 | 0.160 | 0.253 | 0.247 | 0.469 | 0.595 | 0.308 | 0.542 | 0.660 | 0.925 | 0.980 | 0.991 |
| Breusch-Pagan | 0.039 | 0.146 | 0.246 | 0.145 | 0.383 | 0.528 | 0.086 | 0.245 | 0.368 | 0.344 | 0.628 | 0.756 | 0.609 | 0.823 | 0.896 | 0.994 | 0.999 | 1,000 |
| White | 0.037 | 0.103 | 0.164 | 0.120 | 0.265 | 0.380 | 0.068 | 0.165 | 0.256 | 0.251 | 0.465 | 0.610 | 0.465 | 0.723 | 0.832 | 0.980 | 0.998 | 1,000 |
| | With Measurement Error (medium) | | | | | | With Measurement Error (medium) | | | | | | With Measurement Error (medium) | | | | | |
| Goldfeld-Quandt | 0.028 | 0.106 | 0.183 | 0.124 | 0.304 | 0.417 | 0.060 | 0.181 | 0.272 | 0.289 | 0.508 | 0.620 | 0.361 | 0.596 | 0.705 | 0.939 | 0.981 | 0.991 |
| Park | 0.035 | 0.100 | 0.166 | 0.083 | 0.209 | 0.308 | 0.038 | 0.129 | 0.204 | 0.140 | 0.324 | 0.448 | 0.422 | 0.661 | 0.765 | 0.970 | 0.994 | 0.997 |
| Glejser | 0.037 | 0.126 | 0.207 | 0.151 | 0.342 | 0.462 | 0.070 | 0.195 | 0.297 | 0.322 | 0.557 | 0.675 | 0.157 | 0.380 | 0.509 | 0.710 | 0.893 | 0.946 |
| Harvey | 0.032 | 0.099 | 0.164 | 0.088 | 0.218 | 0.319 | 0.042 | 0.126 | 0.208 | 0.170 | 0.355 | 0.474 | 0.191 | 0.398 | 0.519 | 0.788 | 0.917 | 0.956 |
| Breusch-Pagan | 0.024 | 0.113 | 0.196 | 0.088 | 0.275 | 0.407 | 0.057 | 0.181 | 0.287 | 0.209 | 0.463 | 0.601 | 0.394 | 0.654 | 0.768 | 0.923 | 0.985 | 0.994 |
| White | 0.027 | 0.081 | 0.135 | 0.085 | 0.191 | 0.283 | 0.050 | 0.124 | 0.201 | 0.161 | 0.328 | 0.449 | 0.278 | 0.522 | 0.661 | 0.849 | 0.959 | 0.984 |
| | With Measurement Error (large) | | | | | | With Measurement Error (large) | | | | | | With Measurement Error (large) | | | | | |
| Goldfeld-Quandt | 0.023 | 0.087 | 0.153 | 0.084 | 0.222 | 0.322 | 0.041 | 0.129 | 0.208 | 0.180 | 0.362 | 0.474 | 0.192 | 0.397 | 0.518 | 0.751 | 0.886 | 0.925 |
| Park | 0.029 | 0.084 | 0.139 | 0.054 | 0.149 | 0.227 | 0.027 | 0.094 | 0.161 | 0.081 | 0.213 | 0.313 | 0.081 | 0.236 | 0.353 | 0.422 | 0.687 | 0.801 |
| Glejser | 0.023 | 0.094 | 0.167 | 0.090 | 0.242 | 0.347 | 0.041 | 0.138 | 0.223 | 0.187 | 0.392 | 0.510 | 0.226 | 0.445 | 0.568 | 0.828 | 0.939 | 0.966 |
| Harvey | 0.023 | 0.083 | 0.141 | 0.062 | 0.162 | 0.253 | 0.028 | 0.094 | 0.163 | 0.101 | 0.251 | 0.358 | 0.100 | 0.251 | 0.364 | 0.552 | 0.755 | 0.839 |
| Breusch-Pagan | 0.016 | 0.085 | 0.156 | 0.051 | 0.181 | 0.291 | 0.034 | 0.125 | 0.211 | 0.112 | 0.300 | 0.434 | 0.205 | 0.427 | 0.561 | 0.688 | 0.874 | 0.931 |
| White | 0.021 | 0.065 | 0.109 | 0.056 | 0.135 | 0.198 | 0.033 | 0.092 | 0.153 | 0.095 | 0.211 | 0.309 | 0.141 | 0.315 | 0.443 | 0.545 | 0.785 | 0.874 |

200. Figure 1 and 4 illustrate the results. Figures 1 and 2 show bar graphs in the following order: no measurement error in $X$ and $Y$, small measurement error in $Y$, medium measurement error in $Y$, and large measurement error in $Y$, where measurement error of $X$ is set at a small size that is 0.5. Figures 3 and 4 has the same order when measurement error of $Y$ is set this small size.

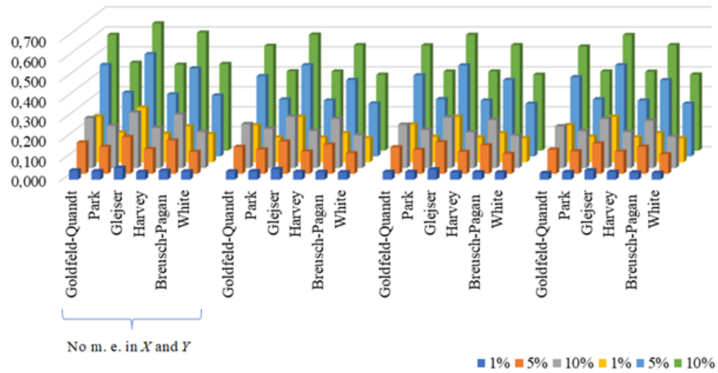


FIGURE 1. Performances of heteroskedasticity tests: M. E. in $Y$ is small, medium, and large while M. E. in $X$ is small (n = 30)

Figure 1[3] shows how adjusting the magnitude of the dependent variable's measurement error affects test results when the explanatory variable's measurement error is set to a small value with the results of simulations when both variables are at their true values. The tests reject the false null hypothesis (type II) in less than 10% of 10,000 trials under the conditions $n = 30$, $\alpha = 0.01$ and the assumption of $\sigma_i^2 = \sigma^2 X_i$. This result does not change when the variables take their error-free values. Even though the tests' power increases as type I error increases, it is still less than 40%. The results also reveal that when variable $X$ is adjusted to a small measurement error, the amount of the measurement error of $Y$ has no bigger effect on the power of the tests. As the previous simulation results showed, the power of the tests grows even with a small sample size when the assumption of $\sigma_i^2 = \sigma^2 X_i^2$. According to the results in Figure 1, the Glejser test outperforms rather than the others, it is followed by the Goldfelt-Quandt and the Breusch-Pagan tests.

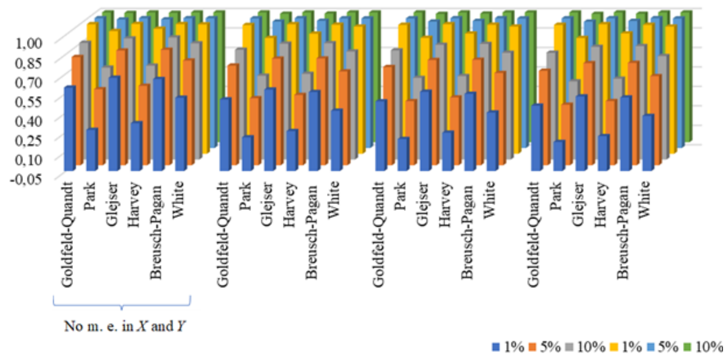


FIGURE 2. Performances of heteroskedasticity tests: M. E. in $Y$ is small, medium, and large while M. E. in $X$ is small (n = 200)

Figure 2 shows the simulation results under the identical conditions as in Figure 1 except for the sample size. In a larger sample size, performances of the tests increased in rejecting the false null hypothesis and this increase is greater when the form of heteroskedasticity is $\sigma_i^2 = \sigma^2 X_i^2$ than $\sigma_i^2 = \sigma^2 X_i$. The Glejser test and the Breusch-Pagan tests show the best performance if the sources of heteroskedasticity is $X_i$ instead of $X_i^2$. The Park and the Harvey tests are the weakest tests under measurement error in $Y$ and $X$.

[3]The values shown on the y-axis represent the ratios (%) of rejection of heteroskedasticity tests while x-axis represent the form of the heteroskedasticity.

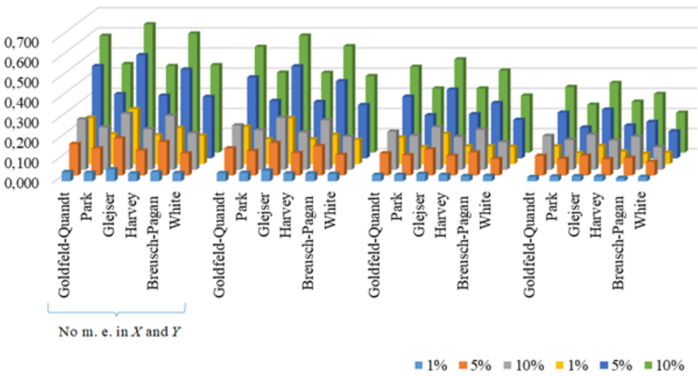


FIGURE 3. Performances of heteroskedasticity tests: M. E. in $X$ is small, medium, and large while M. E. in $Y$ is small (n = 30)

Figure 3 presents the results when variance of measurement error of $Y$ is set at a small value. In other words, the magnitude of measurement error variance of variable $X$ has been modified. Modifying the amount of the measurement error variance of $X$ has a higher influence on the power of the heteroskedasticity tests when $n = 30$, as seen in Figures 1 and 3. The performance of the tests decreases as the measurement error variance of $X$ increases, and this conclusion is independent of the form of heteroskedasticity. As can be seen from Figure 4, increasing the sample size does not improve the performances of the heteroskedasticity tests if the measurement error variance of the explanatory variable is large. It should be noted that even if $X$ and $Y$ are measured at their true values, the power of the tests is less than 70%. This result is consistent with the findings of previous studies. In brief, simulation results showed that the magnitude of measurement error variance of $X$ has the greatest impact on the power of the tests, independent of the magnitude of measurement error variance of $Y$.

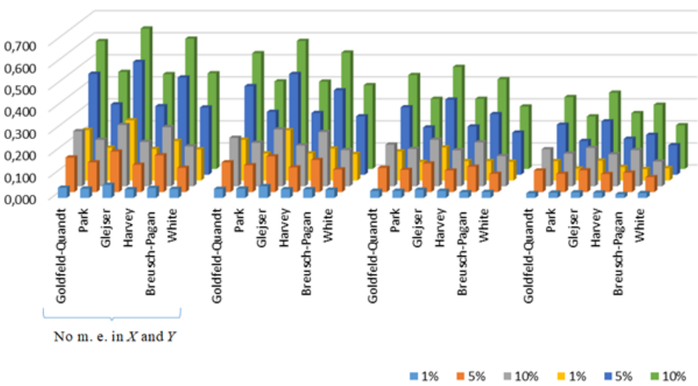


FIGURE 4. Performances of heteroskedasticity tests: M. E. in $X$ is small, medium, and large while M. E. in $Y$ is small (n = 200)

In this paper, we also looked at how well the heteroskedasticity tests performed when the heteroskedasticity was in the form of $\sigma_i^2 = E(Y_i)$. Simulation results were summarized in Table 4. If the sample size is 30 or 50 and the significance level is 1%, the power of the tests is roughly 4% and 8% when there is no measurement error in the variables. When the sample size is 200, their power is less than 52% at the same significance level. In comparison to the Glejser test, the Breusch-Pagan test performs better. The power of the tests rises as the Type I error increases, ranging from 57 percent to 86 percent in the large sample.

TABLE 4. Performances of heteroskedasticity tests with/without measurement error in X and Y when the heteroskedasticity form is $\sigma_i^2 = \sigma^2 E(Y_i)$

| | **n = 30** | | | **n = 50** | | | **n = 200** | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| | No measurement error | | | | | | | | |
| Goldfeld-Quandt | 0.034 | 0.125 | 0.207 | 0.069 | 0.197 | 0.304 | 0.453 | 0.690 | 0.787 |
| Park | 0.033 | 0.110 | 0.182 | 0.045 | 0.142 | 0.226 | 0.190 | 0.428 | 0.569 |
| Glejser | 0.041 | 0.142 | 0.229 | 0.080 | 0.227 | 0.339 | 0.520 | 0.755 | 0.843 |
| Harvey | 0.034 | 0.104 | 0.173 | 0.047 | 0.139 | 0.224 | 0.232 | 0.457 | 0.584 |
| Breusch-Pagan | 0.032 | 0.131 | 0.222 | 0.070 | 0.215 | 0.329 | 0.522 | 0.764 | 0.856 |
| White | 0.031 | 0.091 | 0.151 | 0.059 | 0.148 | 0.229 | 0.384 | 0.648 | 0.772 |
| | Small measurement error | | | | | | | | |
| Goldfeld-Quandt | 0.031 | 0.103 | 0.181 | 0.056 | 0.174 | 0.271 | 0.362 | 0.609 | 0.717 |
| Park | 0.033 | 0.102 | 0.170 | 0.038 | 0.127 | 0.209 | 0.152 | 0.376 | 0.510 |
| Glejser | 0.037 | 0.126 | 0.209 | 0.070 | 0.193 | 0.300 | 0.429 | 0.669 | 0.775 |
| Harvey | 0.032 | 0.093 | 0.158 | 0.041 | 0.126 | 0.201 | 0.190 | 0.392 | 0.520 |
| Breusch-Pagan | 0.027 | 0.113 | 0.203 | 0.058 | 0.188 | 0.291 | 0.424 | 0.684 | 0.784 |
| White | 0.029 | 0.085 | 0.139 | 0.051 | 0.127 | 0.207 | 0.304 | 0.555 | 0.688 |
| | Medium measurement error | | | | | | | | |
| Goldfeld-Quandt | 0.021 | 0.082 | 0.151 | 0.039 | 0.129 | 0.214 | 0.212 | 0.430 | 0.555 |
| Park | 0.029 | 0.087 | 0.148 | 0.029 | 0.102 | 0.174 | 0.093 | 0.267 | 0.395 |
| Glejser | 0.025 | 0.098 | 0.171 | 0.045 | 0.147 | 0.230 | 0.251 | 0.482 | 0.606 |
| Harvey | 0.024 | 0.082 | 0.143 | 0.029 | 0.100 | 0.168 | 0.112 | 0.271 | 0.387 |
| Breusch-Pagan | 0.018 | 0.091 | 0.164 | 0.040 | 0.138 | 0.224 | 0.249 | 0.486 | 0.618 |
| White | 0.021 | 0.068 | 0.117 | 0.037 | 0.099 | 0.163 | 0.170 | 0.367 | 0.499 |
| | Large measurement error | | | | | | | | |
| Goldfeld-Quandt | 0.017 | 0.069 | 0.130 | 0.026 | 0.097 | 0.167 | 0.105 | 0.261 | 0.370 |
| Park | 0.024 | 0.075 | 0.130 | 0.021 | 0.080 | 0.142 | 0.052 | 0.174 | 0.273 |
| Glejser | 0.018 | 0.073 | 0.137 | 0.026 | 0.101 | 0.174 | 0.121 | 0.286 | 0.404 |
| Harvey | 0.021 | 0.074 | 0.121 | 0.020 | 0.078 | 0.137 | 0.058 | 0.169 | 0.262 |
| Breusch-Pagan | 0.012 | 0.068 | 0.132 | 0.024 | 0.095 | 0.169 | 0.116 | 0.290 | 0.404 |
| White | 0.017 | 0.056 | 0.099 | 0.027 | 0.074 | 0.128 | 0.082 | 0.206 | 0.315 |

While the power of the tests is similar when the dependent variable has a small measurement error variance, a small measurement error in the explanatory variable lowers the power of the tests. Small measurement error in both variables has the same effect with small measurement error in the explanatory variable. The power of the tests grows as the sample size and Type I error increase, and the Breusch-Pagan test has a greater power. Although the tests have a better performance in the large sample, increases in measurement error variance in both variables reduces the power of the tests which is irrespective of the significance level. Under this circumstance, both Breusch-Pagan and Glejser tests perform better than the other tests. The Harvey and the Park tests are the weakest tests in rejecting the wrong null hypothesis if the measurement error variances are large. We only reported simulation results when both variables have small, medium, and large measurement error. All other results are available upon request.

Ali and Giaccotto [2] showed that the statistical power of the Glejser test depends on the true form of heteroscedasticity. However, the loss of power due to the type of auxiliary regression tends to be low especially in reciprocal values of $X$ and $\sqrt{X}$. In this paper, we also examined performance of the Glejser test under different types of auxiliary regressions with error-in-variables and reached relatively different results. The results were reported in Table 5 and Table 6. Even though the power of the Glejser test does not appear to be strongly dependent on the type of auxiliary regression for n = 30, rejection rates differ in both heteroskedasticity form and significance level when there is no measurement error in $X$ and/or $Y$ (Table 5). For example, while the test rejects the wrong null hypothesis 6% of 10,000 samples when it is $|\hat{u}_i| = \alpha_0 + \alpha_1 X_i)$ (default), the rejection rate becomes 4% if the auxiliary regression is $|\hat{u}_i| = \alpha_0 + \alpha_1 \frac{1}{X_i}$. The gap between the rejection rates is broader at the 5% significance level when the heteroskedasticity form is $\sigma^2 X_i$, compared to the default auxiliary regression. Interestingly, this similar result is obtained at the 1% significance level when the form of heteroskedasticity is $\sigma^2 X_i^2$

The gap between the rejection rates of the performances of the test under different type of auxiliary regression can be negligible at the 1% significance level under $\sigma^2 X_i$, heteroskedasticity form which is irrespective of the amount of measurement error variance of variable $X$, However, this gap is relatively extended when the significance levels is taken as 5% and 10%. Under the form of $\sigma^2 X_i^2$, power of the Glejser test has the lowest value among the others if the auxiliary regression is $|\hat{u}_i| = \alpha_0 + \alpha_1 \frac{1}{X_i}$ at the 1% significance level, and it decreases as the measurement error variance of $X$ increases. The power of the Glejser test does not strongly depend on the type of auxiliary regression neither measurement error in the response variable nor the amount of it. As it is shown before, power of the Glejser test decreases when both $X$ and $Y$ are measured with error. However, this power depends on the form of auxiliary regression. The test has always a better performance when it is $|\hat{u}_i| = \alpha_0 + \alpha_1 X_i$ since the estimated model is $Y_i = \beta_0 + \beta_1 X_i + u_i$.

As it was shown before, even though the power the Glejser test is higher under the nonlinear heteroskedasticity form than the linear form, its power decreases as measurement error variance of $X$ and $Y$ increase in a larger sample (n = 200). A similar result is obtained when the auxiliary regression is changed. As the form of heteroskedasticity is $\sigma^2 X_i^2$ and the significance level is 10%, the disparity between the powers of the test under different auxiliary regressions widens whenever compared to the default one (Table 6).

Table 5. Performance of the Glejser test under different type of auxilary regressions (n=30)

| | Het. Form: | | | Het. Form: | | | Het. Form: | Het. Form: | | | | | Het. Form: | | | Het. Form: | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Without Measurement Error | | | | | | Without Measurement Error | | | | | | Without Measurement Error | | | | | |
| Func. Form | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.052 | 0.169 | 0.269 | 0.231 | 0.477 | 0.611 | The results are the same. | | | | | | | | | | | |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.037 | 0.146 | 0.250 | 0.170 | 0.432 | 0.590 | | | | | | | | | | | | |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.043 | 0.154 | 0.255 | 0.193 | 0.452 | 0.602 | | | | | | | | | | | | |
| $\alpha_0+\alpha_1 X_i$ | 0.059 | 0.184 | 0.278 | 0.273 | 0.512 | 0.637 | | | | | | | | | | | | |
| | Small Measurement Error in ***X*** | | | | | | Medium Measurement Error in ***X*** | | | | | | Large Measurement Error in ***X*** | | | | | |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.046 | 0.153 | 0.243 | 0.183 | 0.412 | 0.550 | 0.032 | 0.117 | 0.201 | 0.111 | 0.293 | 0.411 | 0.021 | 0.088 | 0.160 | 0.058 | 0.194 | 0.299 |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.030 | 0.128 | 0.227 | 0.125 | 0.359 | 0.521 | 0.018 | 0.090 | 0.172 | 0.060 | 0.228 | 0.371 | 0.010 | 0.054 | 0.122 | 0.023 | 0.119 | 0.218 |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.035 | 0.139 | 0.235 | 0.148 | 0.381 | 0.536 | 0.023 | 0.103 | 0.184 | 0.079 | 0.256 | 0.392 | 0.012 | 0.066 | 0.137 | 0.031 | 0.144 | 0.252 |
| $\alpha_0+\alpha_1 X_i$ | 0.051 | 0.164 | 0.257 | 0.229 | 0.457 | 0.580 | 0.037 | 0.132 | 0.214 | 0.150 | 0.341 | 0.463 | 0.026 | 0.103 | 0.175 | 0.090 | 0.242 | 0.346 |
| | Small Measurement Error in ***Y*** | | | | | | Medium Measurement Error in ***Y*** | | | | | | Large Measurement Error in ***Y*** | | | | | |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.052 | 0.167 | 0.266 | 0.231 | 0.476 | 0.611 | 0.051 | 0.164 | 0.263 | 0.230 | 0.476 | 0.611 | 0.047 | 0.156 | 0.256 | 0.229 | 0.476 | 0.611 |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.037 | 0.143 | 0.249 | 0.170 | 0.432 | 0.590 | 0.035 | 0.139 | 0.246 | 0.169 | 0.431 | 0.590 | 0.034 | 0.133 | 0.236 | 0.168 | 0.430 | 0.590 |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.042 | 0.153 | 0.259 | 0.193 | 0.451 | 0.602 | 0.041 | 0.149 | 0.252 | 0.192 | 0.452 | 0.602 | 0.039 | 0.142 | 0.244 | 0.192 | 0.450 | 0.602 |
| $\alpha_0+\alpha_1 X_i$ | 0.059 | 0.181 | 0.275 | 0.273 | 0.511 | 0.637 | 0.056 | 0.177 | 0.273 | 0.272 | 0.512 | 0.637 | 0.053 | 0.169 | 0.266 | 0.271 | 0.512 | 0.637 |
| | Small Measurement Error in ***X* and *Y*** | | | | | | Medium Measurement Error in ***X* and *Y*** | | | | | | Large Measurement Error in ***X* and *Y*** | | | | | |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.044 | 0.151 | 0.243 | 0.182 | 0.410 | 0.611 | 0.031 | 0.112 | 0.199 | 0.110 | 0.294 | 0.421 | 0.019 | 0.084 | 0.153 | 0.057 | 0.194 | 0.299 |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.030 | 0.127 | 0.225 | 0.125 | 0.360 | 0.521 | 0.017 | 0.088 | 0.170 | 0.060 | 0.228 | 0.370 | 0.010 | 0.054 | 0.119 | 0.023 | 0.118 | 0.219 |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.035 | 0.136 | 0.233 | 0.148 | 0.380 | 0.535 | 0.021 | 0.098 | 0.181 | 0.079 | 0.256 | 0.391 | 0.013 | 0.063 | 0.133 | 0.031 | 0.145 | 0.252 |
| $\alpha_0+\alpha_1 X_i$ | 0.053 | 0.162 | 0.258 | 0.228 | 0.457 | 0.581 | 0.037 | 0.126 | 0.207 | 0.151 | 0.342 | 0.462 | 0.023 | 0.094 | 0.167 | 0.090 | 0.242 | 0.347 |

Table 6. Performance of the Glejser test under different type of auxiliary regressions (n = 200)

| | Het. Form: | | | Het. Form: | | | Het. Form: | Het. Form: | | | | | Het. Form: | Het. Form: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Without Measurement Error | | | | | | Without Measurement Error | | | | | | Without Measurement Error | | | | | |
| Func. Form | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.652 | 0.870 | 0.931 | 0.999 | 1,000 | 1,000 | The results are the same. | | | | | | | | | | | |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.621 | 0.867 | 0.883 | 0.999 | 1,000 | 1,000 | | | | | | | | | | | | |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.638 | 0.871 | 0.935 | 0.999 | 1,000 | 1,000 | | | | | | | | | | | | |
| $\alpha_0+\alpha_1 X_i$ | 0.719 | 0.884 | 0.933 | 1,000 | 1,000 | 1,000 | | | | | | | | | | | | |
| | Small Measurement Error in ***X*** | | | | | | Medium Measurement Error in ***X*** | | | | | | Large Measurement Error in ***X*** | | | | | |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.556 | 0.805 | 0.888 | 0.999 | 1,000 | 1,000 | 0.359 | 0.638 | 0.762 | 0.948 | 0.991 | 0.997 | 0.184 | 0.429 | 0.572 | 0.727 | 0.913 | 0.956 |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.510 | 0.792 | 0.885 | 0.994 | 0.999 | 1,000 | 0.271 | 0.582 | 0.724 | 0.906 | 0.982 | 0.992 | 0.075 | 0.279 | 0.422 | 0.445 | 0.731 | 0.830 |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.535 | 0.803 | 0.891 | 0.995 | 1,000 | 1,000 | 0.310 | 0.618 | 0.749 | 0.933 | 0.989 | 0.996 | 0.114 | 0.350 | 0.502 | 0.590 | 0.846 | 0.921 |
| $\alpha_0+\alpha_1 X_i$ | 0.633 | 0.829 | 0.896 | 0.998 | 1,000 | 1,000 | 0.439 | 0.678 | 0.779 | 0.970 | 0.994 | 0.997 | 0.253 | 0.482 | 0.601 | 0.827 | 0.939 | 0.967 |
| | Small Measurement Error in ***Y*** | | | | | | Medium Measurement Error in ***Y*** | | | | | | Large Measurement Error in ***Y*** | | | | | |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.643 | 0.866 | 0.928 | 0.999 | 1,000 | 1,000 | 0.623 | 0.852 | 0.921 | 0.999 | 1,000 | 1,000 | 0.572 | 0.588 | 0.829 | 0.999 | 1,000 | 1,000 |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.611 | 0.863 | 0.931 | 0.999 | 1,000 | 1,000 | 0.591 | 0.849 | 0.925 | 0.999 | 1,000 | 1,000 | 0.554 | 0.824 | 0.906 | 0.999 | 1,000 | 1,000 |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.630 | 0.866 | 0.933 | 0.999 | 1,000 | 1,000 | 0.610 | 0.854 | 0.926 | 0.999 | 1,000 | 1,000 | 0.502 | 0.830 | 0.908 | 0.999 | 1,000 | 1,000 |
| $\alpha_0+\alpha_1 X_i$ | 0.713 | 0.880 | 0.931 | 1,000 | 1,000 | 1,000 | 0.695 | 0.869 | 0.925 | 1,000 | 1,000 | 1,000 | 0.660 | 0.848 | 0.911 | 1,000 | 1,000 | 1,000 |
| | Small Measurement Error in ***X* and *Y*** | | | | | | Medium Measurement Error in ***X* and *Y*** | | | | | | Large Measurement Error in ***X* and *Y*** | | | | | |
| $\alpha_0+\alpha_1\sqrt{X_i}$ | 0.548 | 0.799 | 0.886 | 0.996 | 0.999 | 1,000 | 0.340 | 0.619 | 0.748 | 0.948 | 0.991 | 0.997 | 0.161 | 0.394 | 0.535 | 0.728 | 0.913 | 0.956 |
| $\alpha_0+\alpha_1\frac{1}{X_i}$ | 0.503 | 0.786 | 0.883 | 0.994 | 0.999 | 1,000 | 0.254 | 0.560 | 0.708 | 0.906 | 0.982 | 0.992 | 0.065 | 0.249 | 0.395 | 0.444 | 0.731 | 0.830 |
| $\alpha_0+\alpha_1\frac{1}{\sqrt{X_i}}$ | 0.528 | 0.799 | 0.888 | 0.995 | 1,000 | 1,000 | 0.297 | 0.596 | 0.737 | 0.933 | 0.989 | 0.996 | 0.102 | 0.321 | 0.472 | 0.589 | 0.845 | 0.920 |
| $\alpha_0+\alpha_1 X_i$ | 0.628 | 0.822 | 0.893 | 0.998 | 1,000 | 1,000 | 0.422 | 0.661 | 0.765 | 0.970 | 0.994 | 0.997 | 0.226 | 0.445 | 0.568 | 0.828 | 0.939 | 0.966 |

In multiple regressions, the Glejser test may not be favored since it necessitates determining the explanatory variable or variables that induce heteroskedasticity, as well as the auxiliary regression form. By keeping potential multicollinearity in mind, the findings produced here can be used as a reference for multiple regression.

## 4. Conclusion

In teaching basic econometrics, the learners are, in general, told that the homoskedasticity assumption of classical linear regression model will not most probably be satisfied in real time data applications, and therefore needs to be tested. They are also told that measurement error in the variables is an implicit assumption of it and generally is ignored in applications. Even though related literature provides different heteroskedasticity tests and correction methods for measurement error, performances of these tests under measurement error have not been comprehensively evaluated. In real-time data, even though we expect a sort of measurement error, we cannot exactly know the magnitude of it. Therefore, a simulation study helps us to compare performances of different heteroskedasticity tests in various scenarios of the magnitude of measurement error with the cases of measurement error in the explanatory and/or outcome variables.

In this paper, we evaluated performances of heteroskedasticity tests under measurement error by using Monte Carlo simulations. Simulations are done for the most used six tests with three sample sizes of 30, 50, and 200 at the 1%, 5%, and 10% significant levels. Power of the tests are evaluated under three mathematical forms of heteroskedasticity; error variance is a linear and nonlinear function of explanatory variable X and a linear function of expected value of the response variable.

There are several main conclusions obtained from the simulation results. First of all, when the true values of the variables are used, heteroskedasticity tests evaluated in this study are weak in failing the wrong null hypothesis when the form of heteroskedasticity is linear function of the explanatory variable and this result is irrespective of significance levels. However, power of the tests is higher when the error variance is a function of squared X and their power increases as sample size increases. The Glejser test and the Breusch-Pagan and/or the Goldfelt-Quandt test have the most power and the White test has the least power. The powers of the other tests are comparable and lie between the Glejser test and the White test. These results are compatible with the results of Uyanto [24] when there is no measurement error.

Simulation results show that power of the tests is lower when the variables are measured with error when the form of heteroskedasticity is the linear function of explanatory variable. Moreover, their power decreases as the measurement error variance increases and increases in sample size does not guaranty increases in tests' power. The results also show that measurement error in the explanatory variable has a greater impact on power of the tests than measurement error in the outcome variable. Measurement error in variables did not change the order of the tests according to their power. The Glejser test is the most powerful one while the White test is weak, and the other tests lie in between them. In brief, since measurement error inflates the variance of the explanatory variable and response variable, heteroskedasticity tests lose their power in detecting heteroskedasticity. Our results imply that using heteroskedasticity-robust inference after OLS estimation which weights the heteroskedastic errors by the explanatory variable, this type of inference might not solve the problem when the explanatory variable is measured with error. The methods correcting measurement error are suggested to use in the estimations; however, they might not be the final solutions for testing the heteroskedasticity.

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