



ENTROPY SQUEEZING OF A MULTI-PHOTON JAYNES-CUMMINGS ATOM IN THE PRESENCE OF NOISE

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ABSTRACT. In this work, we study the entropy squeezing of a two-level atom interacting with a single-mode quantum field by a multi-photon Jaynes-Cummings Model in the presence of the two-state random phase telegraph noise. We show that the entropy squeezing is very sensitive to the noise. It disappears in time quickly due to the strongly destructive effect of the noise.

The Jaynes-Cummings Model (JCM) [1, 2, 3] is the basic model for describing the interaction of a two-level atom with a single-mode cavity quantum field under the rotating-wave approximation. This model reveals crucial non-classical properties such as sub-Poissonian statistics, anti-bunching, squeezing and collapse and revival phenomena [4, 5]. Of the several interests to the model, one has been devoted to the squeezing properties of the atom [6, 7, 8, 9, 10]. In these works, the atomic squeezing properties were studied on the base of the Heisenberg uncertainty relation (HUR). But, HUR cannot provide sufficient information about the atomic squeezing in particular when the atomic inversion vanishes. As an alternative to the HUR, Hirschman [11] studied quantum uncertainty by using quantum entropy theory. And the limitations of the HUR have been overcome by using the entropic uncertainty relation (EUR) [12, 13]. Fang *et.al.* [14] found that EUR can be used as a general criterion for the squeezing of an atom. Accordingly, they proposed a measure of the squeezing of an atom the so-called squeezed in entropy in order to obtain sufficient information on atomic squeezing. The entropy squeezing of the atom has been studied extensively [15, 16, 17, 18, 19]. These works reveal that the entropy squeezing based on the EUR is more precise than the variance squeezing based on the HUR, as a measure of the atomic squeezing.

For the realistic situations, the JCM-type atom-field interactions should be considered with a decoherence mechanism. One consideration was formulated by Joshi *et.al.* [20, 21] in which the authors re-describe the JCM with the random telegraph noise. For the realization of this noise, the authors give some situations

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such as the source of the field or the instability in the atomic vapor production. This noise influences the dipole or the transverse relaxation of the interaction. The resulting decoherence mechanism conserves the energy of the system, but destructs the quantum coherence.

In this work, we study the entropy squeezing of a two-level atom interacting with a single-mode quantum field by a multi-photon JCM in the presence of the two-state random phase telegraph noise. We show that the entropy squeezing is very sensitive to the noise. It disappears in time quickly due to the strongly destructive effect of the noise.

The Hamiltonian of a multi-photon JCM with resonance between the atomic transition and the field frequency [22, 23] is given by ($\hbar = 1$)

$$H = \omega \frac{S_z}{2} + \omega a^\dagger a + g(S_+ a^k + S_- a^{\dagger k}) \quad (1)$$

where S_\pm, S_z are the spin-1/2 operators, a, a^\dagger denote the annihilation and the creation operators of the field, ω is the atomic transition frequency and the field frequency. g is the coupling coefficient which gives the interaction strength between the atom and the field and k represents the k -photon process. The experimental realization of the multi-photon process can be seen in a trapped ion [24].

In the case of the interaction with the random phase telegraph noise, the coupling coefficient is modified as [20]

$$g(t) = g_0 e^{-i\phi(t)} \quad (2)$$

where g_0 is the non-noisy coupling coefficient and $\phi(t)$ represents the random telegraph which fluctuates between two states of the noise denoted by (a) and $(-a)$. These random fluctuations obey the Poisson jump process. The fluctuations of $\phi(t)$ are also Markovian which allows one to take the average over the stochastic fluctuations. The average time between these jumps is called the mean dwell time.

The multi-photon JCM in the presence of the random phase telegraph noise becomes

$$H = \omega \frac{S_z}{2} + \omega a^\dagger a + g_0 (e^{-i\phi(t)} S_+ a^k + e^{i\phi(t)} S_- a^{\dagger k}) \quad (3)$$

For the initial state of the system, we assume for simplicity that the atom is in the excited state $|e\rangle$ and the field is in the Fock state $|n\rangle$. In this case, the initial state of the system is

$$\rho(0) = |n, e\rangle\langle n, e| \quad (4)$$

In order to find an exact solution to the system under the noise, we use the Burshtein equation [25, 26, 27] by the solution method in Ref. [28] in which we studied the entanglement of atom-field interaction by the JCM with two-state random phase telegraph noise. We also considered some other applications of the Burshtein

equation elsewhere for investigating entanglement dynamics in different atom-field systems with this noise [29]. The Burshtein equation is defined as

$$\frac{\partial}{\partial t} V_\alpha(t) = -iM(\alpha)V_\alpha(t) - \frac{1}{T} \sum_{\beta} [\delta_{\alpha\beta} - f(\alpha|\beta)] V_\beta(t) \quad (5)$$

where α and β represent the phase of the noise with the values (a) and $(-a)$, the function $f(\alpha|\beta)$ is the probability of $\phi(t)$ to change its state such that $f(a, -a) = f(-a, a) = 1$ and $f(a, a) = f(-a, -a) = 0$. The time-dependent element $V_\alpha(t)$ is the α -fixed state component of the vector $\hat{V}(t)$ which is the transpose of the matrix $[\rho_n^{11}(t), \rho_n^{22}(t), \rho_n^{12}(t), \rho_n^{21}(t)]$. $M(\alpha)$ is called the effective Liouville operator with the fixed α -state of the noise obtained from the equation $\hat{V}^k = -iM^{kl}\hat{V}^l$. T is the mean dwell time which determines the strength of the dephasing induced by the noise. The smaller T , the stronger noise. In the basis $|n, e\rangle$ and $|n+k, g\rangle$, the following expressions for the stochastic evolution of the elements of the density matrix of the system can be obtained from von Neumann-Liouville equation

$$\begin{aligned} \frac{d\rho_n^{11}(t)}{dt} &= ig_0 \sqrt{\frac{(n+k)!}{n!}} [e^{-i\phi} \rho_n^{12}(t) - e^{i\phi} \rho_n^{21}(t)] \\ \frac{d\rho_n^{22}(t)}{dt} &= ig_0 \sqrt{\frac{(n+k)!}{n!}} [e^{i\phi} \rho_n^{21}(t) - e^{-i\phi} \rho_n^{12}(t)] \\ \frac{d\rho_n^{12}(t)}{dt} &= ig_0 \sqrt{\frac{(n+k)!}{n!}} e^{i\phi} [\rho_n^{11}(t) - \rho_n^{22}(t)] \\ \frac{d\rho_n^{21}(t)}{dt} &= ig_0 \sqrt{\frac{(n+k)!}{n!}} e^{-i\phi} [\rho_n^{22}(t) - \rho_n^{11}(t)] \end{aligned} \quad (6)$$

where the diagonal elements are

$$\begin{aligned} \rho_n^{11}(t) &= \langle n, e | \rho(t) | n, e \rangle \\ \rho_n^{22}(t) &= \langle n+k, g | \rho(t) | n+k, g \rangle \end{aligned} \quad (7)$$

and the off-diagonal elements are

$$\begin{aligned} \rho_n^{12}(t) &= \langle n, e | \rho(t) | n+k, g \rangle \\ \rho_n^{21}(t) &= \langle n+k, g | \rho(t) | n, e \rangle \end{aligned} \quad (8)$$

By constructing the elements of the Burshtein equation from these expressions and by using the Laplace transformation techniques [28], one can obtain the following noise-averaged solution

$$\langle \rho_n^{11}(t) \rangle = \frac{1}{2} \left[1 + \sum_{j=1}^3 \frac{\lambda_j (\lambda_j + \frac{2}{T})}{\prod_{k \neq j} (\lambda_j - \lambda_k)} \exp(\lambda_j t) \right] \quad (9)$$

$$\langle \rho_n^{22}(t) \rangle = \frac{1}{2} \left[1 - \sum_{j=1}^3 \frac{\lambda_j (\lambda_j + \frac{2}{T})}{\prod_{k \neq j} (\lambda_j - \lambda_k)} \exp(\lambda_j t) \right] \quad (10)$$

$$\langle \rho_n^{12}(t) \rangle = ig_0 \cos a \sqrt{\frac{(n+k)!}{n!}} \sum_{j=1}^3 \frac{(\lambda_j + \frac{2}{T})}{\prod_{k \neq j} (\lambda_j - \lambda_k)} \exp(\lambda_j t) \quad (11)$$

and

$$\langle \rho_n^{21}(t) \rangle = \langle \rho_n^{12}(t) \rangle^* \quad (12)$$

λ_j s are the roots of the equation

$$\lambda_j^3 + \frac{2\lambda_j^2}{T} + 4g_0^2 \frac{(n+k)!}{n!} \lambda_j + \frac{8g_0^2(n+k)!}{Tn!} \cos^2 a = 0 \quad (13)$$

The noise-averaged density matrix of the system $\langle \rho(t) \rangle$ takes the form of

$$\begin{aligned} \langle \rho(t) \rangle &= \langle \rho_n^{11}(t) \rangle |n, e\rangle \langle n, e| + \langle \rho_n^{12}(t) \rangle |n, e\rangle \langle n+k, g| \\ &\quad + \langle \rho_n^{21}(t) \rangle |n+k, g\rangle \langle n, e| + \langle \rho_n^{22}(t) \rangle |n+k, g\rangle \langle n+k, g| \end{aligned} \quad (14)$$

The HUR for an atomic system is defined as

$$\Delta S_x \Delta S_y \geq \frac{1}{2} |\langle S_z \rangle| \quad (15)$$

The fluctuations in the components of the Pauli operators are squeezed if

$$V(S_k) = \Delta S_k - \sqrt{\frac{|\langle S_z \rangle|}{2}} < 0, \quad k = x \text{ or } y \quad (16)$$

where $\Delta S_k = \sqrt{\langle S_k^2 \rangle - \langle S_k \rangle^2}$. But, this definition of the variance squeezing can not give information when $\langle S_z \rangle = 0$. Fang *et.al.*'s definition for the squeezing the so-called the entropy squeezing is

$$E(S_k) = \exp(H(S_k)) - 2/\sqrt{\exp(H(S_z))}, \quad k = x \text{ or } y \quad (17)$$

where $H(S_k)$ denotes the information entropy of the component S_k

$$H(S_k) = - \sum_{i=1}^D P_i(S_k) \ln(P_i(S_k)), \quad k = x, y, z \quad (18)$$

where $P_i(S_k)$ represents the probability distribution of D possible measurement outcomes of the S_k component. It is given by $P_i(S_k) = \langle \psi_{ki} | \rho | \psi_{ki} \rangle$ for a quantum system ρ where $|\psi_{ki}\rangle$ is an eigenvector of the component S_k . So, they are the

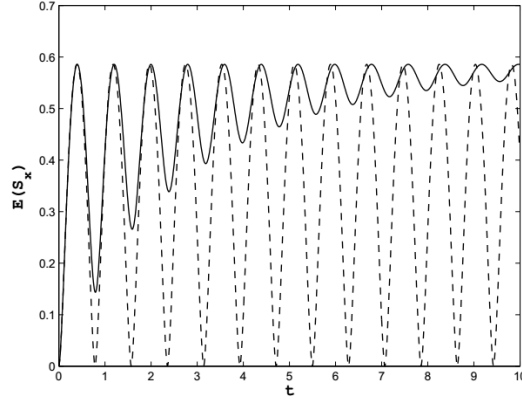


FIGURE 1. Entropy squeezing factor $E(S_x)$ as a function of time t . $n = 3$ and $k = 1$. The non-noisy case $a = 0$ and $T \rightarrow \infty$ for dash line and a noisy case $a = 0.4$ and $T = 1$ for solid line.

elements of projective measurements. In this definition, there exists a squeezing in the fluctuations of S_k , if $E(S_k) < 0$.

The probabilities are given as

$$\begin{aligned}
 P_1(S_x) &= 1/2(1 + 2\text{Re}\langle\rho_n^{12}(t)\rangle) \\
 P_2(S_x) &= 1/2(1 - 2\text{Re}\langle\rho_n^{12}(t)\rangle) \\
 P_1(S_y) &= 1/2(1 - 2\text{Im}\langle\rho_n^{12}(t)\rangle) \\
 P_2(S_y) &= 1/2(1 + 2\text{Im}\langle\rho_n^{12}(t)\rangle) \\
 P_1(S_z) &= \langle\rho_n^{22}(t)\rangle \\
 P_2(S_z) &= \langle\rho_n^{11}(t)\rangle
 \end{aligned} \tag{19}$$

Since the entropy squeezing factor $E(S_k)$ is more reliable in providing information for the squeezing of the atom than the variance squeezing $V(S_k)$, we will only deal with the analysis of the entropy squeezing factor.

We investigate the influence of the noise on the entropy squeezing factor of the atom by the following figures. (In these, we assume that the non-noisy coupling coefficient is unity $g_0 = 1$.) Figures (1)-(2) show that $E(S_x)$ oscillates periodically and has no negative values during the time-evolution of the system. This situation remains unchanged when taking into account the noise. So, there is no entropy squeezing in the S_x component at any time in the absence or in the presence of the noise. For the S_y component as shown in figures (3)-(4), there is an entropy squeezing. $E(S_y)$ oscillates periodically and achieves some negative values during the time-evolution of the system. But, the situation changes when the noise is

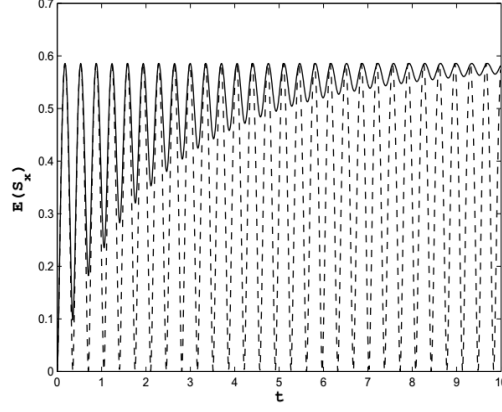


FIGURE 2. Entropy squeezing factor $E(S_x)$ as a function of time t . $n = 3$ and $k = 2$. The non-noisy case $a = 0$ and $T \rightarrow \infty$ for dash line and a noisy case $a = 0.4$ and $T = 1$ for solid line.

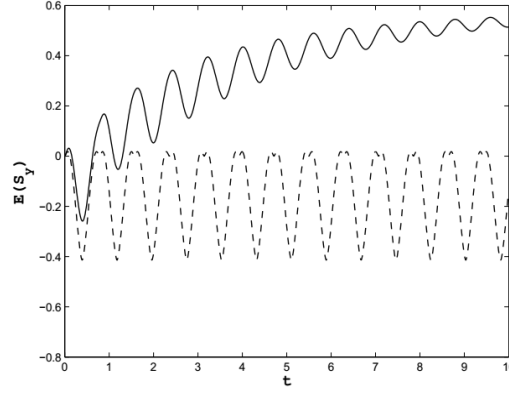


FIGURE 3. Entropy squeezing factor $E(S_x)$ as a function of time t . $n = 3$ and $k = 1$. The non-noisy case $a = 0$ and $T \rightarrow \infty$ for dash line and a noisy case $a = 0.4$ and $T = 1$ for solid line.

involved. The negative values of $E(S_y)$ disappear, as time passes. So, the noise obviously destructs gradually the existing squeezing in the S_y component during the time-evolution of the system. In the both components S_x and S_y , as the value of k increases, the decay of the entropy squeezing in these components occurs with a smaller period. Thus, the entropy squeezing is very sensitive to the noise. It disappears in time quickly due to the strongly destructive effect of the noise.

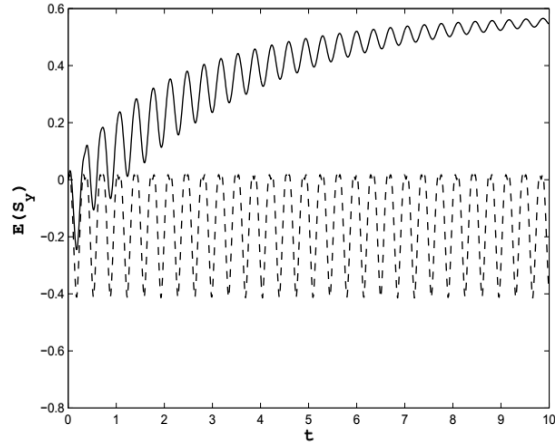


FIGURE 4. Entropy squeezing factor $E(S_y)$ as a function of time t . $n = 3$ and $k = 2$. The non-noisy case $a = 0$ and $T \rightarrow \infty$ for dash line and a noisy case $a = 0.4$ and $T = 1$ for solid line.

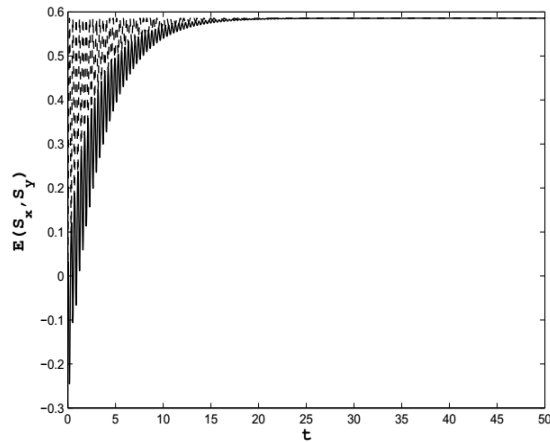


FIGURE 5. Entropy squeezing factor $E(S_x)$ for dash line and $E(S_y)$ for solid line as a function of time t . $n = 3$, $k = 2$, $a = 0.4$ and $T = 1$.

In Figure (5), we look at a longer-time behavior of the entropy squeezing factor for observing more clearly the decoherence effect of the noise. We see that both

$E(S_x)$ and $E(S_y)$ decay gradually and eventually reach the same stable value in time due to the destructive effect of the noise with $E(S_y) \leq E(S_x)$.

In summary, we have studied the entropy squeezing of a two-level atom interacting with a single-mode quantum field by a multi-photon Jaynes-Cummings Model in the presence of the two-state random phase telegraph noise. We have shown that the entropy squeezing is very sensitive to the noise. It disappears in time quickly due to the strongly destructive effect of the noise.

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