# Simulation of Entanglement Dynamics of Flying Qubits Through Photonic Fields

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***Abstract* —The capability of estimating and manipulating the dynamics of entanglement is a prerequisite to develop new quantum technologies. In this study an experimentally accessible model was developed in order to simulate the dynamics of entangled atomic pairs. The model depends on a modified version of a micromaser. In contrast to a standard micromaser set-up in which the atomic degrees of freedom are traced out and photonic field is the system of interest, this model is interested in the dynamics of one of the entangled atomic qubits injected into the successive photonic fields. The numerical simulations with realistic parameters of quantum dynamics show that the results are reliable and consistent with the previously published results. The model can be used to exactly quantify the entanglement dynamics of quantum systems not only for numerical simulations but also as an efficient photonic quantum simulator since it depends on modified version of a very well-known experimental architecture.**

## I. Introduction

THE idea of exploiting quantum resources in computation, communication and engineering is a fast developing research field over past decades. A quantum computer can perform computational tasks faster than a classical one, secure communication can be established between distant parties safer than any classical protocol by quantum cryptography [1] or quantum systems can reveal different thermodynamic properties [2]–[5] compared with their classical analogues.

The advantages of quantum technologies compared to their classical ones has revealed the need for more efficient use of quantum resources. Entanglement [6] is an indispensable quantum resource for quantum technologies enabling non-classical features which is not possible for classical cases. There are ongoing researches about entanglement distribution over long distances [7], [8]. Long distance quantum information carriers are so-called flying qubits and the common problem is to find suitable candidates for these qubits [9]. Unfortunately, the fragile nature of quantum correlations prevents to keep the valuable quantum information alive over long time or long distances. Entangled photon pairs propagating through fiber-optic cables [10]–[12] are one of the most studied tasks. Decay of entanglement [13]–[16] due to thermal effects and coupling to a Markovian or non-Markovian environment degrees of freedom is another considerable effort for fighting against decay of quantum information. To this end, realistic modelling and numerical simulations of the dynamics of quantum correlated systems appears to be extremely important for experimental purposes.

This study concerns with developing a generic model to examine the entanglement dynamics of qubit pairs in which one of them propagating through a thermal reservoir. The idea is based on a modified version of a micromaser model. A standard micromaser [17] deals with atom-field interactions from an atomic reservoir perspective. In the modified version of the model atom-field interactions investigated from a bosonic field reservoir perspective enables examination of dynamical evolution of entanglement of qubit pairs in the time domain. Efforts on prolonging qubit coherence time and suppressing noise effects are common goals of quantum information technologies [18]. Entangled two-level atoms considered as qubits and realistic parameters were used in order to perform faithful simulations. A Lindblad master equation was solved and decay of entanglement compared with the atomic lifetime in the time domain. Different decay characteristics observed due to different properties of bosonic reservoirs. Asymptotic decay or sudden death of entanglement [19] as different results for bosonic reservoirs with different parameters depicts the reliability of the developed model. Preparation of entangled states, suppression of entanglement decay or enlarging quantum correlation lifetime is out of the scope of this paper.

This paper organised as follows. In section II, first a standard micromaser is introduced and the system dynamics described. Also the developed model and its system hamiltonian introduced and the differences underlined. Section III presents the results of the simulations with realistic parameters. Conclusions depicted in Section IV.

## II. Model and System Dynamics

### A. Standard micromaser

Microscopic maser or micromaser consists a single-mode high-Q resonator and a beam of excited two-level atoms injecting into the cavity such that just a single atom interacts with the cavity field each time [17]. Atomic beam consisting velocity selected identical and uncorrelated atoms sent from an oven enter into the cavity. Atom-field interactions can be represented by Jaynes-Cummings Hamiltonian can be expressed as

$$H = \frac{\omega}{2}\sigma_z + \Omega\hat{a}^\dagger\hat{a} + g(\sigma_+\hat{a} + \sigma_-\hat{a}^\dagger) \quad (1)$$

in rotating wave approximation. Here, $\Omega$ is the single cavity mode frequency, $\omega$ is the atomic level frequency difference and $g$ is the atom-field coupling strength. Coupling strength $g \ll \omega, \Omega$ is small enough to safely neglect counter rotating terms in the Hamiltonian. Operators are $\sigma_z$ Pauli-z, $\sigma_{+,-}$ Pauli-raising and lowering operators and $\hat{a}$, $\hat{a}^\dagger$ are the bosonic annihilation and creation operators respectively. Planck constant divided by $2\pi$ was taken $\hbar = 1$ throughout the paper. Standard micromaser theory [17] solves the dynamics by a coarse-grained master equation by tracing out the atomic degrees of freedom. Hence, the single mode cavity field is the system of interest and the injected atoms behaves as an atomic reservoir.

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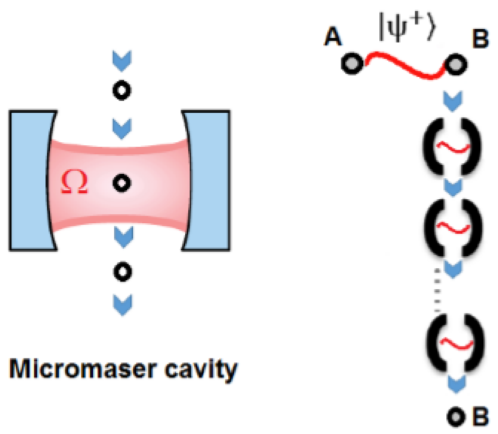


Fig. 1. Original (left panel) and modified (right panel) versions of micromaser models. In the original case identical atoms injected into the cavity field while in the developed model, a single two-level atom (Atom B here) which is initially entangled with A, enters the successive cavities with identical field parameters.

Typical relevant parameters are atom decay rate $\gamma$, cavity field decay constant $\kappa$, atom-field interaction time $\tau$ and injection rate $r$. Since cavity photon lifetime $1/\kappa \gg \tau$ cavity loss is negligible. The atom-cavity field interaction time $\tau < 1/r$ such that just one atom is inside the cavity for each injection. Since the dynamics of the system is exactly solvable, time evolution operator depicted as $U(t) = \exp(-iHt)$. At time $t_i$ when $i^{th}$ atom enters the cavity, the density operator describing the system is $\rho(t_i) = \rho_a(t_i) \otimes \rho_f(t_i)$ the tensor product of atomic and field density matrices respectively. When the $i^{th}$ atom leaves the cavity the cavity field can be calculated as

$$\rho_f(t_i + \tau) = \mathrm{Tr}_a \left[ U(\tau)\rho(t_i)U^{\dagger}(\tau) \right]. \tag{2}$$

Here, $\mathrm{Tr}_a$ stands for a partial trace operation over atomic degrees of freedom. By the injection of successive atoms, cavity field evolves at rate $r$ through a steady state with a mean photon number $\bar{n}$ as

$$\dot{\rho}_f \equiv \frac{r}{2}(\bar{n}+1)\mathcal{L}[\hat{a}] + \frac{r}{2}\bar{n}\mathcal{L}[\hat{a}^{\dagger}] \tag{3}$$

where $\mathcal{L}[x] = (2x\rho x^{\dagger} - x x^{\dagger}\rho - \rho x^{\dagger} x)/2$ is a Liouvillian superoperator in Lindblad form.

It's supposed that the initial field density matrix is diagonal in the number state basis and if the injected two-level atoms determine a well-defined temperature as T $= (\omega/k_B)ln(p_g/p_e)^{-1}$, then the cavity field will converge to a temperature dependent photon number as $\bar{n} = [\exp(\Omega/k_B T) - 1]^{-1}$ where $k_B$ is the Boltzmann constant [20]. Thus, the effective field temperature in the steady state becomes equal that of the injected atoms which is known as quantum thermalization [4]. The injected atoms carry information to the field about their initial parameters such as temperature hence the cavity field experiences an atomic reservoir with the parameters in which they have before they enter the cavity.

### B. Proposed model dynamics

In the proposed version of the model successive cavities which have a thermal single mode state was prepared. A single two-level atom (Qubit-B) which is initially in a maximally entangled state with Qubit-A passes through the cavities as in the right panel of Fig 1. In this case, the state of the injected atom is of interest and the successive thermal cavity fields behave as a bosonic reservoir to the atom. The main focus of the paper is to analyse the entanglement dynamics of two-qubit atomic states interacting with a bosonic reservoir with specific parameters. The initial state of the atoms is a maximally entangled state as $|\Psi^+\rangle = 1/\sqrt{2}(|ge\rangle + |eg\rangle)$ where $|g\rangle$ and $|e\rangle$ stand for ground and excited atomic states respectively. Initially entangled qubits A and B do not interact and B was sent into the cavity arrays while A discontacted from thermal reservoirs. In this scheme atom-field interaction can be written as

$$H = \frac{\omega_A}{2}\sigma_z^A + \frac{\omega_B}{2}\sigma_z^B + \Omega\hat{a}^{\dagger}\hat{a} + g(\sigma_+^B\hat{a} + \sigma_-^B\hat{a}^{\dagger}). \tag{4}$$

Atom-cavity interaction time is again $\tau$ and the spacing time between the cavities are assumed to be negligible. At time $t_i$ when atom B interacts with $i^{th}$ cavity, the density operator describing the system can be described as $\rho(t_i) = \rho_{AB}(t_i) \otimes \rho_f(t_i)$, tensor product of atomic and field density matrices respectively where $\rho_{AB}(0) = |\Psi^+\rangle\langle\Psi^+|$. In each interaction, Qubit-B is assumed to touch a fresh cavity field. Cavity decay time $1/\kappa >> \tau$ so that losses in fields of cavity arrays are neglected. On the other hand, since the number of cavities are assumed to be large enough for B to spend much time in cavity arrays, atomic decay should be taken into account. Then the relevant master equation for a single atom-cavity interaction is

$$\dot{\rho}_{AB}(\tau) = -i\mathrm{Tr}_f[H, \rho(\tau)] + \gamma \sum_{\nu} \mathcal{L}[\sigma_-^{\nu}] \tag{5}$$

where $\nu : A, B$ and $\mathrm{Tr}_f$ stands for a partial trace operation over field degrees of freedom. Evolution of atomic state $\rho_{AB}$ calculated by iterative solutions of Eq. 5 by successively inserting $\rho_{AB}$ into $\rho$. Typical microwave parameters used for the calculations. GHz, kHz, Hz and $\mu s$ ranges are relevant for micromaser cavity field $\Omega$, atom-field coupling $g$, atomic decay rate $\gamma$ and atom passage time $\tau$ from the cavity respectively [21].

Eq. 5 allows one to obtain the dynamical evolution of quantum correlated qubits $\rho_{AB}$ for each iteration which enables to calculate quantum entanglement dynamics. Concurrence was chosen as an entanglement measure in this study. A spin flipped state of $\rho_{AB}$ is defined as $\tilde{\rho}_{AB} = (\sigma_y \otimes \sigma_y)\rho_{AB}^*(\sigma_y \otimes \sigma_y)$ where $\rho^*$ is the complex conjugate of $\rho$. Thus concurrence defined as

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \tag{6}$$

where $\{\lambda_i\}$ are square roots of the eigenvalues of $\rho\tilde{\rho}$ in decreasing order. Note that though Qubit-A was assumed to be isolated from thermal environment, spontaneous emission decay $\gamma$ included for both qubits in Eq. 5 for the dynamical evolution. Master equation in Eq. 5 will also be calculated by neglecting spontaneous emission decay rates $\gamma$ for the qubits in order to compare the dynamical effects of thermal environment and vacuum noise as entanglement decay sources.

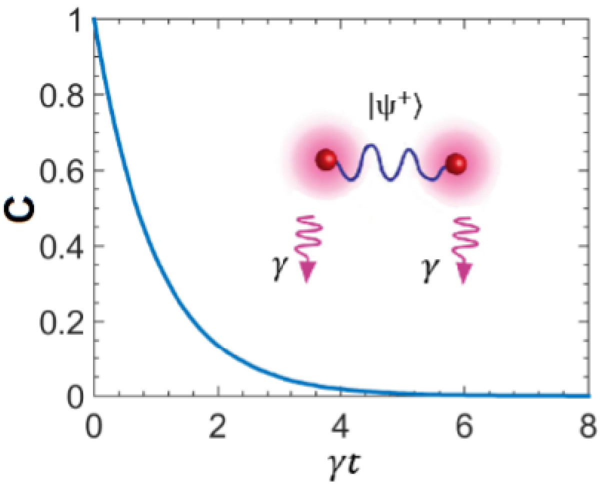


Fig. 2. Decay of entanglement of qubits A, B due to vacuum noise. Disentanglement time is infinite due to the asymptotic decay. Atomic energy level frequencies are $\omega_A = \omega_B = 50$ GHz and atomic decays are $\gamma = 33.3$ Hz. Time is dimensionless and scaled by $\gamma$.

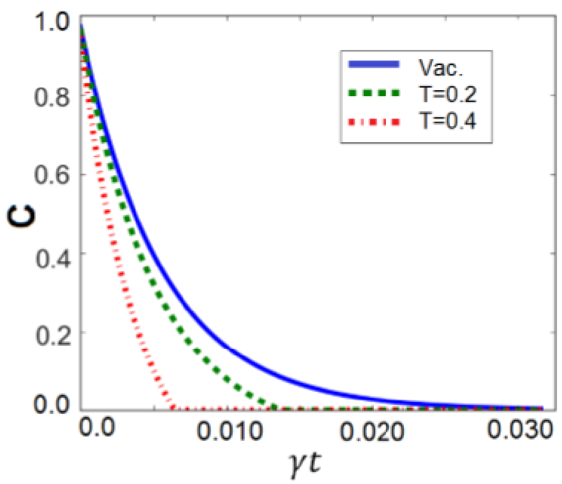


Fig. 3. Decay of entanglement of qubits A,B during qubit B propagating through cavity arrays. Identical cavity field frequencies are $\Omega = 50$ GHz, atom transit time $\tau = 9.5$ $\mu$s, atom-field coupling $g = 50$ kHz and atomic decays are $\gamma = 33.3$ Hz. Time is dimensionless and scaled by $\gamma$.

## III. Results

Numerical simulations performed on initially prepared state $\rho_{AB} \otimes \rho_f$ and the two-qubit atomic states updated in each iteration as depicted in the previous section. The cavity fields were investigated in two states: vacuum and a temperature dependent thermal state. A truncated Hilbert space used for $\rho_f$ in the calculations. A thermal field defined as

$$\rho_f = \frac{1}{Z} e^{-\beta H_f} \tag{7}$$

where $H_f = \Omega \hat{a}^\dagger \hat{a}$, $\beta = 1/(k_B T)$ and $Z = \mathrm{Tr} e^{-\beta H_f}$ is the partition function. Identical qubits are assumed to be resonant with the identical cavity fields ($\omega_A = \omega_B = \Omega$) in the microwave regime. Parameters of Rydberg atoms [22] are suitable for masing transitions were used in the calculations. In order to conduct a fair comparison between entanglement decay sources in the calculations, first the entanglement dynamics investigated only under vacuum noise as illustrated in Fig. 2. The master equation solved for this purpose is

$$\dot{\rho}_{A,B}(t) = -i[H_a, \rho_{AB}(t)] + \gamma \sum_{\nu} \mathcal{L}[\sigma_-^\nu] \tag{8}$$

where $H_a = 1/2\omega_A \sigma_z^A + 1/2\omega_B \sigma_z^B$. An exponential decay character of entanglement is apparent in Fig. 2 which states that only in the presence of vacuum noise, initially maximally entangled qubits become separable in the asymptotic limit. On the other hand, as the main objective of the paper, one of the initially quantum correlated qubits was sent into the cavity arrays. This scheme is reminiscent to the flying qubits which are the quantum information carriers propagating in contact with a thermal environment.

Two different entanglement dynamics are apparent in Fig. 3, asymptotic decay and decay of entanglement in finite time which is called entanglement sudden death [19]. Asymptotic decay of entanglement obtained when Qubit-B propagated through cavity fields in contact with vacuum state while entanglement sudden death obtained when the thermal cavity fields had finite temperatures in accordance with Eq. 7. In Fig. 3, solid line represents entanglement decay in presence of vacuum state and dashed and dashed-dotted lines represents decays of thermal states corresponding to $T = 0.2$ and $T = 0.4$ respectively in units of $\hbar\omega/k_B$. Entanglement lifetime in these cases is two orders of magnitude shorter than the case in Fig. 2. Though calculations were also repeated by neglecting the spontaneous emission decay rates in Eq. 5, same results appeared as in Fig. 3. This shows that for entanglement dynamics, time scale is too short the effect of $\gamma$ to appear. This confirms that entanglement decay due to coupling to the thermal environment is dominant over entanglement decay due to solely spontaneous emission even for weak coupling regime. Disentanglement time in this model can be described as $t_d = N_d \tau$ where $\tau$ is single cavity passage time and $N_d$ is the number of cavities passed by Qubit-B. Two disentanglement finite time curves were plotted for $N_d = 60$ corresponding to $t_d = 570$ $\mu$s obtained in Fig. 3. Thus, the developed model provides a reasonable result with previously published reports about the finite disentanglement time at finite temperature [23].

Since the atomic states are of interest in this paper, it's worth to consider the density matrices after the Qubit-B interaction. Fig. 4 depicts the density matrices after successive interaction of Qubit-B by the vacuum and thermal fields at the end of the disentanglement time. Though this time range is not sufficient to reach a stationary state for $\rho_{AB}$, it's very helpful to check the final two-qubit states in order to test the realiability of the proposed model. Here, $\rho^+_{AB}$ is the density matrix of one of the generic Bell states which is prepared for the proposed model. Coherences disappear after the vacuum state interaction as in $\rho^{00}_{AB}$. Atomic density matrices are $\rho^{02}_{AB}$ and $\rho^{04}_{AB}$ represents the atomic states after thermal field interactions corresponding to $T = 0.2$ and $T = 0.4$ respectively in units of $\hbar\omega/k_B$. All diagonal elements are not evident in $\rho^{02}_{AB}$ and looks more like

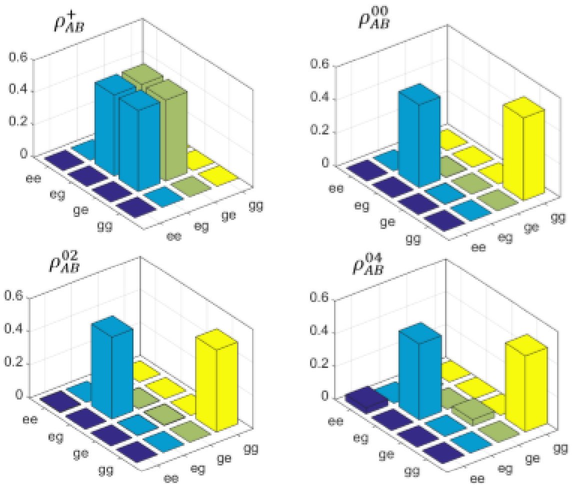


Fig. 4. Two-qubit density matrix representations in atomic basis. Density matrix of initial Bell state : $\rho_{AB}^{+}$, atomic density matrix after vacuum field interaction : $\rho_{AB}^{00}$, atomic density matrix after thermal field interaction for $T = 0.2 : \rho_{AB}^{02}$, atomic density matrix after thermal field interaction for $T = 0.4 : \rho_{AB}^{04}$.

the $\rho_{AB}^{00}$. On the other hand for a thermal field with small amount of more temperature, all diagonal elements are clearly evident in $\rho_{AB}^{04}$ which implies that thermal state of qubits starts to be built as a consequence of interaction with the thermal reservoir even for this short time scale.

## IV. Coclusions

In conclusion, a model to simulate entanglement dynamics of quantum information carriers depends on a modified version of a micromaser set up was developed. The study does not refer the developed model to as an efficient candidate for flying qubits but as a reliable model to estimate the decay of entanglement dynamics with exact quantification in contact with a thermal reservoir . The results with realistic experimental parameters show that the model is reliable and consistent with the previously published reports such as entanglement sudden death in the presence of a bosonic reservoir with finite temperature. Though a direct generalization of the developed model to the polarization entangled photon pairs is not trivial since polarization states are a different degrees of freedom, the model gives insight for the entanglement dynamics of the generic quantum information carriers.

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