# Implementation of Model Reference Adaptive Controller with Fractional Order Adjustment Rules for Coaxial Rotor Control Test System

G. Kavuran, B. B. Alagoz, A. Ates, C. Yeroglu

*Abstract*— **In this study, an experimental test platform of a low-cost coaxial rotors has been developed for implementation of Model Reference Adaptive Controller (MRAC) with Fractional Order Adjustment Rules by using MATLAB/Simulink. This paper provides a design method of MRAC with fractional order adjustment rule (FOAR-MRAC) that is implemented on a hardware based on ARM microcontroller. Setup of the test platform is presented and its performance is evaluated with real-time experimental measurements. This experimental study is useful to show the utilization of fractional order control systems on engineering applications by using low-cost hardware.**

*Index Terms*— **Adaptive PID controller, DC rotor control, fractional order integrator, model reference adaptive control, TRMS.**

## I. INTRODUCTION

COAXIAL rotors are composed of twin blade turning in opposite direction. The idea of coaxial rotors come from Mikhail Lomonosov who developed a small helicopter model with coaxial rotors in July 1754 and demonstrated it to the Russian Academy of Sciences [1]. Turning of twin blades in opposite direction provides symmetry of forces around the central axis. This is a main advantage of the system comparing to single blade rotor systems. This helps solution of problems of single rotor system such as rotate of main body in the direction opposite to the rotor blades, dissymmetry of lift. Main disadvantage of coaxial rotors is the complexity of mechanical parts.

MRAC was developed by Whitaker around 1960 [2] and Landau in late 1970 [3]. Later, it turned into a fundamental topic of adaptive control literature [4, 5]. MRAC was shown that it is capable of improving robust performance for parameter variations, noise and uncertain dynamics [6] and therefore it was widely used in practical applications. For instance, controlling hybrid tank systems with MRAC approach [7], the vector controlled induction motor control [8], controlling five-phase interior-permanent-magnet (IPM) motor [9]. MRAC structure was also utilized for distributed control applications [10, 11]. Recently, MRAC was considered for a class of uncertain dynamical systems with non-linear delayed state perturbations [12].

MRAC mainly used for the control applications where system dynamics alters by changing the environmental condition [13, 14]. Flight control requires dealing with complicated aircraft mechanisms, nonlinear and inaccessible aerodynamics and frequent variability in flight conditions, depending on altitude, payload and weather conditions. Reference model based adaptive control strategies allow auto-tuning of control systems for a desired system response and they can improve flight control performance under varying conditions [15]. Although metaheuristic methods can provide good solutions for unknown plants in simulations [15], they are not very effective for real time tuning application due to the requirement of repetitive set and trail sessions. However, real-time adaptation skill of MRAC can allow real-time convergence of feedback control system response to a desired response of theoretical reference models. It is very useful to robust performance of control system under varying conditions of real applications. In general, MRAC method is very effective for active control systems, of which gain parameters need to be adjusted when control dynamics changes. MRAC implements well known MIT rule that provides the descent of gradient of model approximation error during update of gain parameters. Essentially, this real-time self-tuning process leads to adaptation of control system.

During the last two decades, control engineering has been benefited from fractional calculus in system modeling and control problems. It was shown in many works that fractional order controller and system models can considerably improve control system performance [16-18].

Vinagre et al. modified the conventional MRAC structure by using fractional order integrator [5]. They theoretically demonstrated that fractional order adjustment rule MRAC (FOAR-MRAC) can improve the tracking performance.

In this paper, we developed an experimental test platform for coaxial rotors control and conduct an experimental adaptive control study by using for the FOAR-MRAC structure. The FOAR-MRAC structure is implemented on Arduino card by using embedding coding technique, and thus adaptive control operation can be per-formed real-time by low-cost Arduino

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Mega 2560 card. The FOAR-MRAC structure was designed in MATLAB/Simulink and then loaded to the Arduino Mega 2560, which control the coaxial rotors test platform. Coaxial rotors control test platform is composed of a coaxial rotors and a wooden shaft and it is fixed to container by means of incremental rotary encoder pin. We tested FOAR-MRAC structure with different fractional order integrator configurations in this test platform and discuss results.

## II. METHODOLOGY

### A. Foundation of Fractional Calculus

Fractional order derivative operator as an extension of integer order derivative and integrator were written in general form as [16],

$${}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \displaystyle\int_a^t (d\tau)^{(-\alpha)} & \alpha < 0 \end{cases} \tag{1}$$

where, the operator ${}_aD_t^\alpha$ denotes non-integer order derivative of fractional calculus. Parameters $a$ and $t$ are the lower and upper bounds of the operator, and $\alpha \in R$ denotes the fractional-order. The Caputo definition of fractional order differentiation was defined based on $\Gamma(.)$, namely Euler's gamma function, as follows [19],

$${}_aD_t^\alpha = \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau,\ n-1 \le \alpha < n \tag{2}$$

The equation (2) leads to very useful property on Laplace transform of fractional order derivative which was given as $L(D^\alpha f(t)) = s^\alpha F(s)$ for zero initial conditions.

Implementation of theoretical fractional order derivative is not practical because of its high computational complexity. Integer order approximations of fractional order models are widely preferred for the practical implementation of fractional order system models. In this study, we implemented the fourth order integer order approximate model of fractional order derivatives by means of CFE method [20].

### B. Theoretical Background for FOAR-MRAC Structure

Figure 1 illustrates basic blocks of conventional MRAC proposed for gain adaptive control applications. The adaptation mechanism of MRAC is based on gradient descent technique, which was also known as MIT rule [21]. MRAC method can perform adaptation under the assumption that system parameters deviate more slowly than the adaptation parameter of MRAC [5]. The model approximation error can be defined as,

$$e_d = y_r - y \tag{3}$$

where $y_r$ is the reference model output, and $y$ is the control system output. This error definition enforces the control system adapt itself according to a reference model. The reference model is commonly chosen a theoretical control system that represents a desired controller performance.

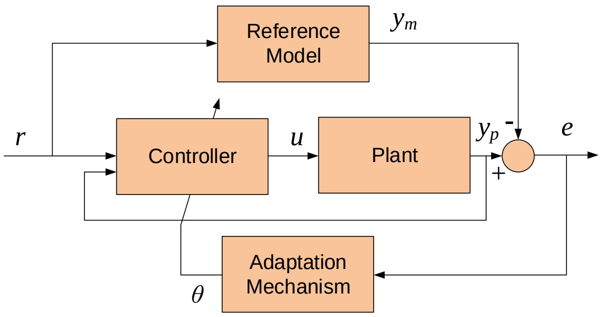

Fig. 1. Schematic diagram of MRAC

Whenever responses of reference model and control system differ, it causes increasing amplitude of $e_d(t)$ and MRAC updates the adaptation parameter $\theta(t)$ by minimizing a convex cost function $J$ via MIT rule:

$$J(\theta) = \frac{1}{2}e_d^2(\theta) \tag{4}$$

The adjustment rule of conventional MRAC for the change of adaptation parameter in the direction of the negative gradient of $J(\theta)$ was written [3, 5] as,

$$\frac{d\theta}{dt} = -\gamma\frac{dJ}{d\theta} = -\gamma e_d \frac{de_d}{d\theta} \tag{5}$$

Equation (5) provides evolution of adaptation parameter ($\theta$) in the direction reducing the cost function. Later, Vinagre et al. modified the conventional MRAC structure by using fractional order integrator and suggested FOAR-MRAC [5] as,

$$\frac{d^\alpha\theta}{d^\alpha t} = -\gamma\frac{dJ}{d\theta} = -\gamma\, e_d \frac{de_d}{d\theta} \tag{6}$$

For FOAR-MRAC structure, the adaptation parameter $\theta$ were solved by considering $\dfrac{\partial e_d}{\partial\theta} = y_m$ as,

$$\theta = D^{-\alpha}\left(-\gamma\, e_d\, y_m\right) \tag{7}$$

Then the control signal was scaled by adaptation parameter as,

$$u(t) = \theta\, u_c(t) \tag{8}$$

where, $\theta$ parameter is for feedforward adaptation parameter of the control system. The temporal evaluation of $\theta$ adaptation parameters minimizes cost function according to fractional order MIT rule [5] given by equation (8), Reference model is assumed to be a second order system in the form of $\frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$ and the control signal $u_c(t)$ is provided by PID controller in closed loop unity feedback system. Here, FOAR-MRAC provides adaptation by scaling control effort of closed-loop PID controller as illustrated in Figure 3. This approach comes out the advantage of easily transformation of conventional PID control system into adaptive PID control systems. In our experimental studies, we used PID coefficients $k_p = 0.6$, $k_i = 0.1$ and $k_d = 0.3$.

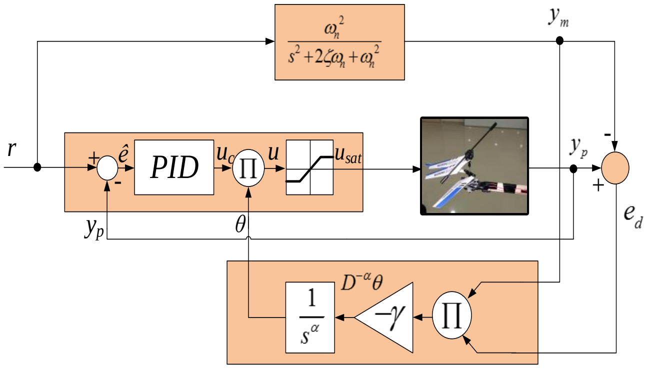

Fig. 2. Block diagram of the proposed FOAR-MRAC structure

## III. EXPERIMENTAL STUDY

### A. Coaxial Rotors Control Experimental Test Platform

Mechanical parts of coaxial rotors control experimental test platform is similar to well-known TRMS system produced by Feedback Instruments Inc. Coaxial rotors control experimental test platform composed of a coaxial rotor, a wooden shaft, incremental rotary encoder (ES5-0CCN 6942) and carton container. The shaft is made of light and flexible model woods that reduce transmission of high frequency vibration of coaxial rotors to rotary encoder. The carton container fixes rotary encoder holding the wooden shaft. Figure 3 depicts a prototype of experimental system.

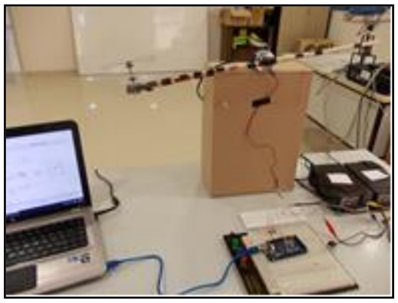

Fig. 3. A prototype of coaxial rotors controls experimental test platform

Coaxial rotors consist of twin blade turning in opposite direction. Turning of twin blades in opposite direction provides symmetry of forces around the central axis. This is useful for dealing with some complications of single rotor system such as rotate of main body in the direction opposite to the rotor blades, dissymmetry of lift. We used coaxial rotors and blades set of LS-222 Gyro 3.5 Channel model helicopter as shown in Figure 4.

Fig. 4. Close views of coaxial rotors and blades used in the experimental system.

For the control of test platform, we used Arduino Mega 2560 card. Arduino platforms are frequently used for experimental unmanned aerial vehicle (UAV) studies. The FOAR-MRAC structure was designed in MATLAB/Simulink and the design was loaded to the Arduino control card. This card receives the angle data of shaft from rotary encoder and drives two electrical motor of coaxial rotors by means of Darlington power amplification circuit. Figure 5 shows circuit diagram of electronic parts of the experimental system. Advantages of implementation of FOAR-MRAC by embedded programming on Arduino Mega 2560 card that it is a low-cost control card and it supports embedded programming with Matlab Simulink environment. MATLAB/Simulink design for FOAR-MRAC structure is illustrated in Figure 6.

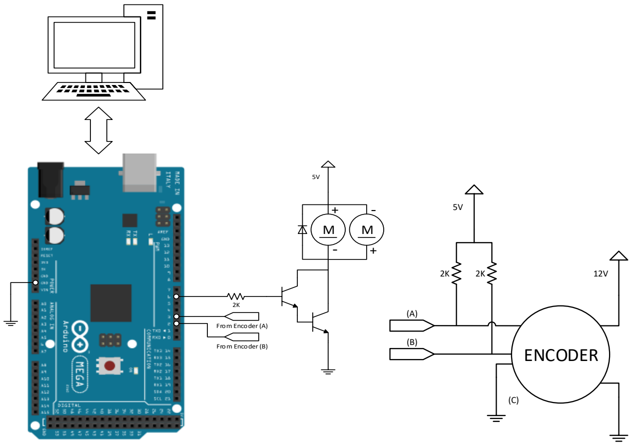

Fig. 5. Electronic circuit scheme of experimental system.

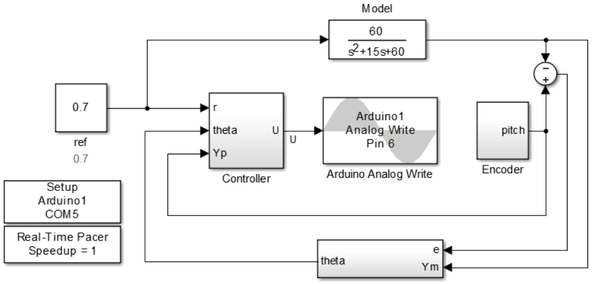

Fig. 6. MATLAB/Simulink design for FOAR-MRAC structure

### B. *Experimental Results*

In this section, test results obtained for FOAR-MRAC implementations are presented for the fractional order integration of 0.88 and 1.12. Figures 7 and 8 show the step responses of FOAR-MRAC implementations. For the fractional order integration of 0.88, it settles 0.7 radian without any overshooting in Figure 7 and this smooth settling is a desired response for flight control [15]. Figure 8 shows integration with the order of 1.12 provides faster rise but it gives slight overshoots and this response may result in disturbance for multi-rotors systems such as quadcopter. Because, all rotors are coupled with each other and overshoots in settling may cause swing of multi-rotor system.

Figures 9 and 10 show the change of adaptation parameter $\theta(t)$ during step responses. One can see that the change of $\theta(t)$ stabilizes in time. Because, it converges a local minima of the objective function $J(\theta)$ and thus the adaptation process of FOAR-MRAC takes places.

We observed that step responses are very smooth and ripples are very low. One of reasons is the materials used in test platform, which are light and flexible woods and carton container. They can reduce the transmission of mechanical vibration caused from blades and rotor motion.

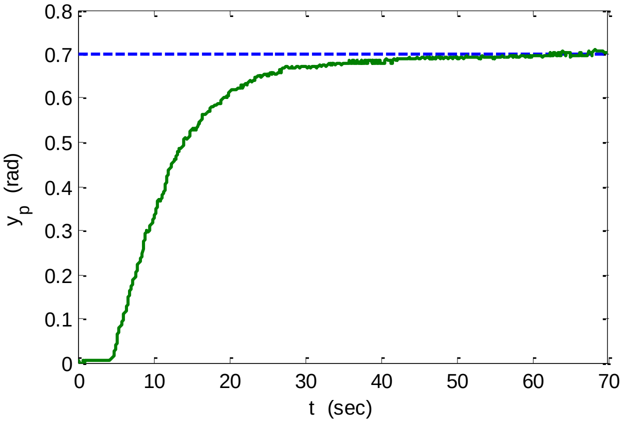

Fig. 7. Step response of FOAR-MRAC structure with the fractional order integration of 0.88

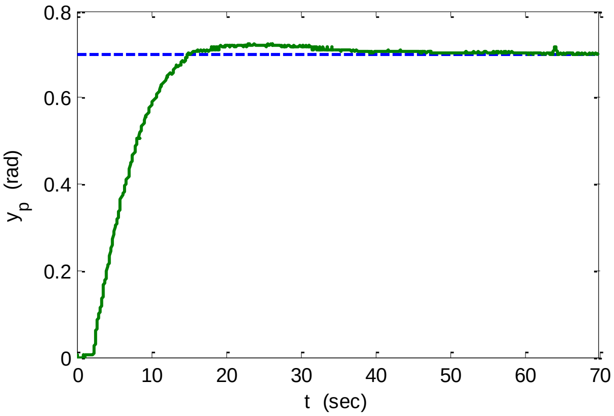

Fig. 8. Step response of FOAR-MRAC structure with the fractional order integration of 1.12

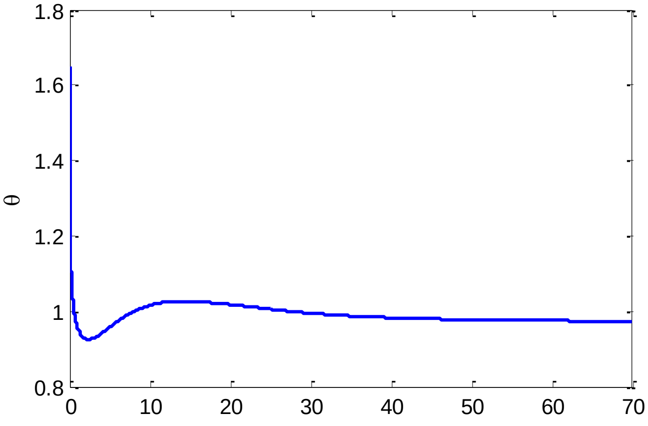

Fig. 9. Change of adaptation parameter $\theta(t)$ for FOAR-MRAC structure with the fractional order integration of 0.8

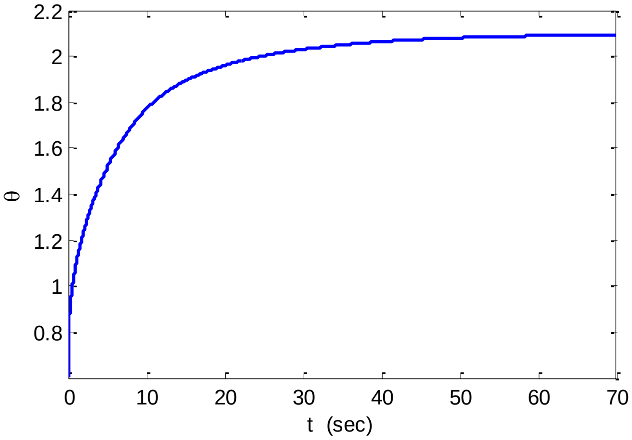

Fig. 10. Change of adaptation parameter $\theta(t)$ for FOAR-MRAC structure with the fractional order integration of 1.12

## IV. CONCLUSION

Rotor control application requires adaptive control techniques due to the changing dynamics of flight conditions. This study demonstrated an experimental study for implementation of FOAR-MRAC method for the propose of adaptive rotor control of coaxial rotors experimental test platform. The FOAR-MRAC structure suggested by Vinagre et al. was implemented on Arduino card. Indeed, FOAR-MRAC method was benefited to modify closed loop PID control system and thus turns the conventional PID control system into an adaptive PID control system with FOAR-MRAC support. Moreover, the coaxial rotors control experimental test platform developed was introduced in detail and experimental results were discussed briefly. In future study, we plan to add gyroscope sensors to the test platform and conduct experiments for adaptive coaxial rotors control of FOAR-MRAC structure by providing gyroscope sensors feedbacks.

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