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Time-Varying Beta Estimation: A Comparison of DCC-GARCH and Rolling-Window Methods in Turkish Industry Portfolios

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ABSTRACT

This study empirically compares the accuracy of two common methods for estimating time-varying betas in Turkish industry portfolios: rolling-window OLS regression and the DCC model. Using daily return from 2004 to 2024, the methods are evaluated based on their alignment with CAPM predictions, specifically the insignificance of Jensen's alpha and the significance of the market risk premium. Findings show that despite its complexity, the DCC model does not outperform the rolling-window approach. The rolling-window approach produces insignificant Jensen's alpha estimates for more industries and yields slightly higher mean and t-statistics for the market risk premium. These findings challenge the view that rolling-window estimators are inefficient due to assuming beta constancy within short windows and suggest that the DCC model's reliance on multiple constant parameters imposes a rigid structure that may hinder its adaptability to evolving market conditions. This study contributes to the literature by directly comparing these two widely used methods and highlighting the importance of carefully considering model assumptions when estimating time-varying betas.

Keywords: CAPM; Time-Varying Beta; DCC; Rolling-Window; Jensen's Alpha; Market Risk Premium.

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INTRODUCTION

Sharpe (1964), Lintner (1965), and Mossin (1966) independently developed the CAPM, which uses market beta to quantify an asset's exposure to market risk and estimate its expected return. This framework has been a crucial development in finance, significantly influencing capital budgeting, asset pricing, performance evaluation and academic research on market efficiency. However, Fama and French (1992) showed that market beta alone cannot explain return differences across size and book-to-market portfolios. Graham and Harvey (2001) showed that it continued to be widely used. Hence, empirical failure of the model lead to a search for improvements to the model instead of abandoning it.

Fama and French (2004) identified several potential explanations for the empirical failures of the basic CAPM and its multifactor extensions. These explanations include irrational stock pricing, the inadequacy of the model, and poor proxies for the market portfolio. Additionally, Lettau and Ludvigson (2001) and Adrian and Franzoni (2009) argued that the poor empirical performance could be attributed to neglecting the time-varying nature of conditional moments of returns as well as beta. Jagannathan and Wang (1996) were among the early studies to criticize the basic CAPM's implicit presumption of time-invariant beta, which underpinned the use of straight ordinary least squares (OLS) in earlier studies. There are concerns that a constant beta may not accurately reflect real-world conditions in which investors' expectations change over time.

Empirical investigations overwhelmingly confirm that beta is time-varying and this concept is well acknowledged in the literature. One of the earlier studies, Fabozzi and Francis (1977), showed that beta changes with bull and bear market conditions. Alexander and Chervany (1980) examined how time-variation of beta is associated with the portfolio formation approach and the size of the portfolios. Brooks, Faff and Lee (1992) compared varying-coefficient models for describing beta. Several other studies have introduced new methods for modeling the time-variation in beta, including the rolling-window OLS regression (Fama & MacBeth, 1973), multivariate GARCH models (Bollerslev, Engle & Wooldridge, 1988), state variable approach (Ferson & Harvey, 1991), Kalman filter (Shah & Moonis, 2003), regime-switching models

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(Korkmaz et al., 2010), and nonparametric models (Esteban & Orbe-Mandaluniz, 2010).

Among the many alternatives, using the rolling-window OLS regression is the oldest and simplest, yet a common method for estimating time-varying betas. This approach is commonly associated with Fama and MacBeth (1973), although they neither named nor specifically promoted it. They calculated betas from five years of monthly observations to use in the cross-sectional regressions carried out every month over the subsequent four years. At the end of each four-year period, they rolled the estimation window forward. Estimating beta using data from the latest five-year window has become a standard procedure, and Groenewold and Fraser (2000) referred to it as the "five-year rule of thumb". Agrrawal, Gilbert and Harkins (2022) questioned this rule and investigated the optimal return frequency and window length to produce betas that better forecast subsequent period's returns. Nonparametric models, such as the one developed by Baillie, Calonaci and Kapetanios (2022), are general versions of the rolling-window OLS regression. These models don't require determining a fixed window length but are not as commonly used as the rolling-window OLS regression approach.

A prominent alternative method for calculating time-varying betas is using one of the multivariate GARCH models. These models are formulated to address the time variation in volatility of return series and the clustering of that volatility. Multivariate versions of GARCH extend this capability by modeling co-movements in financial volatilities, providing conditional variances as well as covariances that can be used to calculate conditional betas. Silvennoinen and Teräsvirta (2009), as well as the earlier study by Bauwens, Laurent, and Rombouts (2006), carried out detailed surveys of multivariate GARCH models, highlighting the diversity within this family of models. The BEKK and DCC models are the most widely used among multivariate GARCH models, leading Caporin and McAleer (2012) to carefully select them for analytical comparison. Notably, the DCC model is the more recent of the two and avoids over-parameterization.

The Dynamic Conditional Correlations (DCC) model, a widely recognized framework in financial econometrics, was initially developed by Engle and Sheppard (2001). They designed it to provide strong estimation performance while allowing for ease of implementation across many assets by reducing the number of parameters that need to be estimated. Engle (2002) subsequently refined the model, enhancing its applicability and theoretical foundations, and found that the DCC model provides the most accurate correlation estimates among multivariate GARCH models in the two-asset case. Building on this work, Bali, Engle and Tang (2016) conducted an analysis and found that dynamic conditional betas estimated with the DCC model significantly predict the cross-sectional variation in daily stock returns. Shortly after, Engle (2016) developed a new approach based on the DCC model that allows for joint estimation of multiple covariance matrices, enabling the estimation of time-varying betas for multifactor CAPM. They applied it to industry portfolios and found lower significance for alphas, as predicted by the theoretical model, compared to traditional models other than the rolling-window approach.

A group of studies focused on comparing alternative methods using some in- and out-of-sample forecasting accuracy criteria on Australian industry portfolios (Brooks, Faff & McKenzie, 1998), European industry portfolios (Mergner & Bulla, 2008) and UK company stocks (Choudhry & Wu, 2008) However, they didn't include the rolling-window approach or DCC model. Nieto, Orbe and Zarraga (2014) conducted a more comprehensive comparison, including both methods, using time series and cross-section criteria on Mexican stock returns. They didn't provide conclusive evidence of better performance for the DCC model compared to the rolling-window approach. Recently, Aloy et al. (2021) found that the DCC model has unsatisfactory out-of-sample predictive performance on the US and European REITs data. These studies concluded that Kalman filter models are superior to other methods but the majority did not compare the relative performance of the DCC and the rolling-window approaches, despite their widespread use.

There is no consensus on whether the newer DCC model or the rolling-window approach is superior for estimating time-varying betas to be used in financial applications. The rolling-window approach is simple but has limitations, such as ignoring the behavior of return volatility and assuming a constant beta within each window. As noted by Adrian and Franzoni (2009), Aloy et al. (2021), and Lettau and Ludvigson (2001), an incomplete or inadequate treatment of time-variation in conditional moments can lead to model misspecification, resulting in both subpar empirical performance of the CAPM and suboptimal financial decisions. The DCC model attempts to mitigate these limitations by specifying a dynamic process for modeling conditional correlations, from which conditional moments are then calculated. However, the DCC model is inherently predicated on the assumption of constant parameters over the entire sample period, an overly simplifying assumption which may not hold for long horizons and may lead to misspecification either. The solution of Lewellen and Nagel (2006) to avoid the challenges of specifying conditioning information and beta instability over longer windows was estimating CAPM regressions over shorter windows.

Mixed findings on the DCC model's effectiveness and the enduring relevance of the rolling-window approach raise the question of whether increased model complexity necessarily leads to more accurate beta estimates. The author's review of the literature reveals that no study has directly compared these two specific methods within the context of industry portfolio beta estimation. This study aims to fill that gap by evaluating the estimation accuracy of the DCC model relative to the rolling-window approach, based on how each aligns with two key CAPM implications: (1) insignificant Jensen's alphas in time-series regressions, and (2) a significant positive relationship between betas and expected returns in cross-sectional regressions. Using Turkish industry portfolio data from 2004 to 2024 with daily returns, the study assesses which model produces time-varying betas that more faithfully reflect these CAPM predictions.

This study contributes to the time-varying beta literature by empirically comparing two widely used estimation methods and highlighting the importance of carefully considering the assumptions underlying these methods. It builds on recent work examining the time-varying nature of asset pricing model parameters. For instance, Çatık, Huyugüzel Kışla, and Akdeniz (2020) used OLS recursive regression betas, and Ustaoğlu (2022) employed DCC betas to demonstrate beta variability in Turkish industry returns. İlbasmış (2024) compared two versions of the DCC model for forecasting accuracy. In contrast, this study directly compares the basic DCC model with the traditional rolling-window approach, focusing on their consistency with CAPM implications.

The remaining sections are arranged as follows. Section 2 describes the data and outlines the methodology for obtaining time-varying betas using rolling-window regressions and DCC model covariance estimates. Section 3 presents the comparison results of the two estimation methods and examines their robustness to parameter instability. Finally, Section 4 ends the paper by discussing the findings.

DATA AND METHODOLOGY

The data is retrieved from the Financial Information News Network (Finnet) database, which contains sector and market index values for Turkish stocks listed on Borsa Istanbul (BIST), as well as index values for Turkish government bonds. BIST calculates sector and market indices using the free-float market values of the included stocks. It systematically identifies and selects the top 100 stocks based on specific criteria and designates their index as the main equity market index. This index was thus selected to function as the proxy for the market portfolio. The risk-free interest rate is proxied by the yield of the most actively traded Turkish government bond. The data available for the analysis spans the period from April 1, 2004 to August 2, 2024 at a daily frequency and covers 24 industry indices some of which are not mutually exclusive. The following subsections describe how the data are transformed into excess returns, detail the estimation procedures for time-varying betas using both the rolling-window and DCC approaches, and interpret summary statistics for excess returns and conditional betas.

Calculation of Excess Returns

The excess return for each industry index, along with that of the market, was derived by subtracting the risk-free rate from each portfolio's return. Specifically, the excess returns at day t for industry i and the market are calculated as $r_{it} = R_{it} - R_{ft}$ and $r_{mt} = R_{mt} - R_{ft}$, respectively. In this formulation, R_{ft} represents the daily risk-free rate expressed in percentage terms, obtained by dividing the annual bond yield at time by 365. R_{it} and R_{mt} denote the daily continuously compounded returns for the industry and market indexes, respectively, both expressed in percentage. R_{it} was computed as the log return using $R_{it} = \ln(P_{it}/P_{it-1})*100$, where P_{it} is the index value at time t, and R_{mt} was calculated from the market index analogously. Annualized values reported throughout the paper are calculated using the standard compounding formula $(1+r)^{251} - 1$, assuming 251 trading days per year, where r denotes the daily return, Jensen's alpha or risk premium.

Table 1 presents summary statistics for the excess returns of the industry indices and the market index. The transportation and storage industry recorded the highest average daily excess return at 0.0773% (21.4% annually), followed by the wholesale and retail trade industry at 0.0617% (16.7% annually), and the technology industry at 0.0591% (16.0% annually). In contrast, the property trusts industry had the lowest average return at 0.0125% (3.2% annually). These figures reflect the influence of government-led investments, sector-specific incentives and an inflation-related policy mix. Robust infrastructure projects such as the construction of

Table 1: Summary statistics of returns

Code	Index	Mean	Std. Dev.	Skew.	Kurtosis	J-B test	ADF test
XU100	BIST100 Turkish Market Index	0.0358	1.6754	-0.4223	6.9439	3461.54	-12.0320
XUSIN	Industrials	0.0441	1.4822	-0.8169	8.4557	6901.65	-11.8893
XGIDA	Food, Beverage & Tobacco	0.0386	1.7343	-0.5695	7.3482	4299.35	-12.8357
XTEKS	Textıle & Leather	0.0442	1.6895	-1.1315	9.5581	10241.59	-12.5415
XKAGT	Wood & Paper Products	0.0248	1.8357	-0.6366	6.9137	3604.32	-12.8238
XKMYA	Chemicals	0.0492	1.7679	-0.4578	6.8945	3405.78	-11.4833
XMANA	Basic Metal	0.0501	2.0928	-0.1981	6.5054	2648.07	-12.3458
XTAST	Mineral Products	0.0411	1.5831	-0.6011	8.2806	6241.19	-13.1292
XMESY	Fabrıcated Metal Products	0.0444	1.7408	-0.6234	7.5554	4746.60	-12.6506
XUHIZ	Services	0.0467	1.4908	-0.4500	7.1977	3921.85	-11.9935
XELKT	Electricity Gas & Steam	0.0210	2.0126	-0.4562	8.4065	6397.11	-12.4850
XTCRT	Wholesale & Retail Trade	0.0617	1.7038	-0.2333	9.1823	8179.27	-10.8475
XULAS	Transportation & Storage	0.0773	2.3371	-0.0888	5.8565	1742.98	-13.5450
XSPOR	Sports Activities	0.0187	2.4117	-0.3744	10.7372	12857.84	-9.6696
XTRZM	Hotels & Restaurants	0.0277	2.2413	-0.5200	7.2945	4154.54	-13.0898
XILTM	Telecommunications	0.0211	2.1163	-0.1347	6.2081	2205.48	-13.1623
XUTEK	Technology	0.0591	1.9272	-0.5680	7.5501	4680.17	-11.8573
XBLSM	Information Technology	0.0446	1.9435	-0.5330	7.5523	4651.56	-11.3383
XUMAL	Financial Institutions	0.0322	1.9319	-0.2629	6.2053	2245.09	-12.1105
XBANK	Banks	0.0338	2.2912	-0.0451	5.6902	1541.74	-12.9480
XSGRT	Insurance	0.0585	1.8321	-0.3197	7.7411	4870.19	-11.5079
XFINK	Leasing & Factoring	0.0381	2.4554	-0.2648	10.5740	12266.70	-12.3813
XHOLD	Holding & Investment	0.0306	1.7917	-0.4401	6.7655	3181.96	-12.1535
XYORT	Brokerage Houses	0.0195	1.7189	-0.8905	11.6020	16420.40	-11.3147
XGMYO	Property Trusts	0.0125	1.7474	-0.6899	6.9501	3725.40	-10.3424

Notes: The statistics are based on daily percentage excess returns for 24 Turkish industry indices and the market index, covering the period from April 1, 2004, to August 2, 2024. Each series includes 5,107 daily observations. All Jarque-Bera and Augmented Dickey-Fuller test statistics are significant at the 1% level, with ADF tests using MacKinnon p-values.

Istanbul Airport boosted the transportation and storage industry. The extension of Region 5 incentives to defense and aerospace investments nationwide bolstered the technology industry's returns. Low real interest rates encouraged short-term consumer spending, contributing to the strong performance of the wholesale and retail trade industry. Conversely, high nominal mortgage rates constrained housing affordability and limited long-term housing investments. This, coupled with restrictive measures such as the 25% cap on rental income growth, contributed to the underperformance of the property trusts industry.

Table 1 also reports the standard deviations of excess returns, highlighting substantial variation in return volatility across industries. For instance, the leasing and factoring industry shows a high standard deviation of 2.4554% despite a modest average daily excess return of 0.0381% (10% annually), suggesting that returns in this sector are highly volatile relative to their mean. In contrast, the industrials and services industries exhibit more stable performance, with standard deviations of 1.4822% and 1.4908%, respectively. This variation in the return volatility aligns with the number of constituent stocks in each industry index. The industrials and services indices exhibit lower return volatility due to greater diversification, each comprising more than a hundred stocks. In contrast, the leasing and factoring industry shows significantly higher volatility, reflecting its limited diversification, with fewer than ten constituent.

Table 1 further shows that all of the excess return series exhibit negative skewness, indicating longer left tails than would be expected under a normal distribution. Their kurtosis values exceed 3, suggesting a more peaked distribution compared to a normal distribution. As expected, the null of normality is rejected for these series with very high Jarque-Bera test statistics significant at the 0.01 level. These characteristics are compatible with the common observation that financial return series exhibit negative skewness and leptokurtosis. To ensure the validity of subsequent empirical analyses, testing for stationarity in the excess return series is essential to prevent spurious regression results arising from unit roots. The results of the stationarity check based on the ADF unit root test with a constant are displayed in the last column of Table 1. Unit roots are strongly and consistently rejected based on statistical evidence for all series at the 0.01 significance level using MacKinnon p-values. This confirms that the excess return series are stationary and therefore suitable for use in time-series modelling.

Estimation of Conditional Betas

The rolling-window approach estimates the conditional beta at each date by applying OLS regression to a fixed-size subsample of prior observations, referred to as the window. For period t, the window includes observations from t-1-w to t-1, where w is the window size. Within each window, the following market model is re-estimated to generate the rolling beta series:

$$r_{i\tau} = \alpha_{it} + \beta_{it}^{ROLL} r_{m\tau} + e_{i\tau}$$

where $r_{i\tau}$ and $r_{m\tau}$ are the excess returns of industry portfolio i and the market index, respectively, at each observation point τ within the window ending at time t-1; $e_{i\tau}$ denotes the regression residual; a_{it} is the intercept; and β_{it}^{ROLL} corresponds to the market beta estimated for time t.

Alternatively, the DCC model estimates conditional variances and covariances, which are subsequently employed to calculate conditional betas. Engle (2002) decomposed the conditional covariance matrix in the following form:

$$H_t = D_t R_t D_t$$

D_t is a diagonal matrix containing the conditional standard deviation σ_{it} on its ith diagonal, which is obtained from a univariate GARCH(1,1) model with variance equation $\sigma_{it}^2 = b_{i0} + b_{i1}u_{it-1}^2 + b_{i2}\sigma_{it-1}^2$ and mean equation $r_{it} = \mu_i + u_{it}$ where u_{it}~$N(0, \sigma_{it}^2)$. The matrix R_t contains the expected conditional correlations $\rho_{im,t}$ as its elements, defined as:

$$\rho_{im,t} = q_{im,t} / \sqrt{q_{ii,t} \bullet q_{mm,t}}$$

where $q_{im,t}$ denotes the quasi-correlation between industry i and market m. This normalization ensures that $\rho_{im,t} \in [-1,1]$.

The quasi-correlations $q_{im,t}$ are updated recursively according to:

$$q_{im,t} = \bar{\rho}_{im} + \lambda_1 \bullet \left(\varepsilon_{i,t-1}\varepsilon_{m,t-1} - \bar{\rho}_{im}\right) + \lambda_2 \bullet \left(q_{im,t-1} - \bar{\rho}_{im}\right)$$

where $\varepsilon_{i,t-1}$ and $\varepsilon_{m,t-1}$ are the standardized residuals from the GARCH models fitted to the excess returns of industry portfolio and the market index, respectively. The term $\bar{\rho}_{im}$ denotes the unconditional sample correlation between the two returns (or their standardized residuals). The scalars λ_1 and λ_2 govern the responsiveness of the quasi-correlations to new shocks and their persistence over time, implying an exponentially decaying weight on past observations. This updating mechanism gives greater weight to recent information, enabling the DCC betas to adapt more rapidly to changes in market conditions while still incorporating longer-term trends. Although this greater responsiveness can help capture sudden market movements, it may also lead to over-adjustment, causing short-term fluctuations in beta estimates that diverge from underlying systematic risk. By contrast, the rolling-window approach assigns equal weights to all observations within a fixed window, resulting in smoother beta dynamics that limit responsiveness to sudden shifts and yield a more stable relationship between beta and expected returns.

The bivariate DCC model described above involves a total of 11 parameters. These include two mean parameters (μ_i for the industry and market returns), six GARCH parameters (b_{i0}, b_{i1}, b_{i2} for each series), two DCC parameters (λ_1, λ_2), and one unconditional correlation $\bar{\rho}_{im}$. The number of estimated parameters directly determines the degrees of freedom used in the likelihood ratio tests for parameter stability, discussed later in the Parameter Stability of the DCC Model section. Since the model is specified in bivariate form, with industry and market returns as the two variables, the DCC parameters are estimated separately for each industry. However, this specification implicitly assumes that GARCH parameters and the DCC parameters remain constant over the estimation period. While the DCC model is designed to capture evolving conditional correlations, this constant-parameter assumption may limit its flexibility in adapting to structural changes in financial relationships over time. In contrast, the rolling-window approach re-estimates parameters over shorter intervals, potentially offering greater responsiveness to such changes. These considerations motivate the parameter stability tests presented in a later section.

Estimation of the DCC model involves finding the maximum of the log-likelihood function:

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log\,|D_t| + \log\,|R_t| + \varepsilon_t^{'} R_t^{-1}\varepsilon_t\right)$$

where ε_t denotes the vector of standardized residuals and T represents the number of observations. The DCC beta series associated with each industry portfolio is calculated by utilizing the entries of conditional covariance matrix H_t using the following formula:

$$\beta_{it}^{DCC} = \frac{\sigma_{im,t}}{\sigma_{m,t}^2}$$

where $\sigma_{im,t}$ denotes the conditional covariance between the market and i-th industry portfolio, and $\sigma_{m,t}^2$represents the conditional variance of the market return on day t. β_{it}^{DCC} corresponds to the market beta estimated for day t.

For brevity, market betas estimated by the methods described above are referred to as rolling betas and DCC betas. Both are calculated using the information available from the prior time period to provide one-period-ahead forecasts. The rolling-window approach uses a window size of 120 days, following prior literature (Nieto, Orbe & Zarraga, 2014). Unreported results indicate that among 60-, 120- and 180-day windows, the 60-day window produces rolling beta means closest to DCC beta means for the highest number of industries, while the 180-day window yields rolling beta standard deviations closest to those of DCC betas for the most industries. The 120-day window provides a balance between these two metrics, facilitating comparisons. Since rolling beta estimates are unavailable for the first 120 days, the first 120 DCC beta estimates are also excluded from further analysis to ensure consistency across two approaches. This limits the sample of beta estimates to the timeframe from September 22, 2004 to August 2, 2024. Note that although the DCC model provides beta estimates from the beginning of the sample, the first few estimates may contain biases due to the initial values assigned to the lagged factors used in DCC estimation. Excluding these early estimates also mitigates the issue.

Table 2 provides a summary of the means and standard deviations of time-varying beta estimates derived from the rolling-window approach and the DCC model for Turkish industry portfolios, along with comparisons of their means and standard deviations. The average beta values reflect meaningful variation in systematic risk exposure across industries. For example, based on rolling betas, the banks industry exhibits the highest average beta (1.2635), consistent with its high financial leverage and macroeconomic sensitivity. In contrast, the sports activities industry has the lowest (0.5157), reflecting an investor base driven by team loyalty, which dampens responsiveness to market-wide movements.

The DCC beta means are numerically close to the rolling beta means. The largest difference between two approaches appears in the mineral products industry, where the rolling beta exceeds the DCC beta by 0.0241, corresponding to approximately 3.5% of their average beta value. Assuming a daily market risk premium of 0.0358%, this beta difference translates into an expected return differential of about 0.00086% per day (0.0241×0.0358%), or 0.22% annually. This is economically small, as beta differences across industries can lead to larger variation in expected returns. However, t-tests reject the equality of means for most industries, suggesting statistically significant differences between the two estimation methods. This result confirms that, although the long-run averages of beta estimates appear similar, the two methods capture fundamentally different dynamics in beta evolution. Specifically, the rolling-window approach's equal weighting leads to smoother but slower adjustments to recent market shocks. In contrast, the exponential decay in the DCC model enables its betas to respond more quickly to new information. This difference in responsiveness likely explains the divergence in short-term beta estimates, even when their long-run means remain comparable.

The standard deviations of DCC betas are close to those of rolling betas, though they are slightly lower in most cases. The sports activities industry displays the largest difference in standard deviations between the two methods with the rolling beta has a standard deviation that is 0.0484 higher than the DCC beta, corresponding to approximately 9.6% of their average beta. This implies that, in this industry, the rolling-window approach attributes nearly 10 percent more day-to-day variation in systematic risk exposure than the DCC model. Assuming the same daily market risk premium of 0.0358%, this additional beta volatility translates into a swing of about ±0.0017% in daily expected returns. This amounts to 0.44% annually, which though small may still impact return forecasting and risk assessments. These comparisons depend on the chosen window length in the rolling-window approach. As discussed

Table 2: Summary statistics of conditional betas

Industry	Roll. β Mean	DCC β Mean	Mean Difference	Roll. β St. Dev.	DCC β St. Dev.	St. Dev. Difference
Industrials	0.7906	0.7895	0.0011	0.1090	0.0920	0.0170
Food, Beverage & Tobacco	0.6497	0.6582	-0.0085***	0.1603	0.1407	0.0196
Textile & Leather	0.7083	0.6943	0.0140***	0.2044	0.1729	0.0315
Wood & Paper Products	0.7749	0.7567	0.0182***	0.1738	0.1798	-0.0060
Chemicals	0.8073	0.8243	-0.0170***	0.1246	0.1200	0.0046
Basic Metal	0.8940	0.9013	-0.0073**	0.1449	0.1415	0.0034
Mineral Products	0.7000	0.6759	0.0241***	0.1758	0.1699	0.0059
Fabricated Metal Products	0.8245	0.8190	0.0055**	0.1258	0.1186	0.0072
Services	0.7639	0.7681	-0.0042**	0.0939	0.0899	0.0040
Electricity Gas & Steam	0.7951	0.7887	0.0064*	0.1687	0.1725	-0.0038
Wholesale & Retail Trade	0.6421	0.6567	-0.0146***	0.1326	0.1235	0.0091
Transportation & Storage	0.9950	0.9840	0.0110**	0.2394	0.2024	0.0370
Sports Activities	0.5157	0.4944	0.0213***	0.3000	0.2516	0.0484
Hotels & Restaurants	0.7770	0.7676	0.0094*	0.2565	0.2369	0.0196
Telecommunications	0.8268	0.8391	-0.0123***	0.1844	0.1649	0.0195
Technology	0.8053	0.7917	0.0136***	0.1985	0.1751	0.0234
Information Technology	0.7540	0.7440	0.0100***	0.1917	0.2008	-0.0091
Financial Institutions	1.1311	1.1355	-0.0044***	0.0820	0.0790	0.0030
Banks	1.2635	1.2627	0.0008	0.1677	0.1629	0.0048
Insurance	0.6766	0.6784	-0.0018	0.3125	0.2783	0.0342
Leasing & Factoring	0.7564	0.7461	0.0103**	0.2359	0.2642	-0.0283
Holding & Investment	0.9661	0.9694	-0.0033*	0.0922	0.0819	0.0103
Brokerage Houses	0.5705	0.5802	-0.0097**	0.2032	0.1741	0.0291
Property Trusts	0.8150	0.8080	0.0070**	0.1540	0.1447	0.0093

Notes: Mean differences are calculated as rolling beta mean minus DCC beta mean. Standard deviation differences are calculated as rolling beta standard deviation minus DCC beta standard deviation. Statistical significance of mean differences (based on t-tests) is indicated by asterisks: * (10%), ** (5%), *** (1%).

earlier, increasing the window size reduces the standard deviation of rolling betas, making them more comparable to those of DCC betas. Yet this adjustment comes at the cost of greater differences in mean estimates, which may complicate comparisons by obscuring whether observed performance differences reflect genuine predictive differences or simply shifts in beta levels.

COMPARISON OF CONDITIONAL BETAS

This section presents the comparison results of rolling and DCC betas based on two criteria. The first criterion, pertaining to time-series analysis, is the insignificance of Jensen's alpha. The second criterion, related to cross-sectional analysis, is the significance of the market risk premium.

Evaluation Based on Jensen's Alpha

Jensen's alpha estimated from conditional beta time-series serves as a metric for comparing beta estimation methods. In particular, it quantifies how much the realized return diverges from the expected under a given asset pricing model. For the CAPM, the model-implied expected excess return is a risk-adjusted premium calculated as an asset's beta multiplied by the market's excess return. Hence, Jensen's alpha is computed as the difference between the realized and model-implied excess returns, as follows:

$$\alpha_{it}^{J} = r_{it} - \widehat{\beta}_{it} r_{mt}$$

where α_{it}^{J} is Jensen's alpha at day t, r_{it} and r_{mt} denote the realized excess returns for industry portfolio i and the market index. $\widehat{\beta}_{it}$ refers to the estimated conditional beta at day t, estimated in the previous section using either rolling-window approach or the DCC model. The term $\widehat{\beta}_{it} r_{mt}$ represents the model-implied expected excess return.

Any insignificant deviations from the expected return can be considered random noise. However, a significant non-zero alpha can signal either superior performance, potentially due to skillful management as in the case of actively managed funds, or significant model misspecifications. These misspecifications might arise from unaccounted-for risk factors, such as industry-specific influences, unobserved risk dimensions, or market anomalies. When comparing rolling beta and DCC beta, the model that yields a smaller Jensen's alpha is considered more accurate in capturing the portfolio's systematic risk and, consequently, its true performance.

Table 3 presents the Jensen's alpha statistics calculated using both rolling beta and DCC beta, along with t-statistics for the mean tests and mean equality tests. The transportation and storage industry exhibits the highest mean alphas of 0.0474% and 0.0411%, based on rolling and DCC betas, respectively. The technology industry follows, with 0.0355% and 0.0389%. The property trusts industry has the smallest mean alphas, at -0.0170% and -0.0110%. In approximately three-quarters of the industries, mean Jensen's alpha values remain close to zero and statistically indistinguishable from zero. This finding reflects the success of the CAPM in the time-series context, suggesting that market beta alone, whether estimated via the rolling-window or DCC approach, adequately captures the variation in industry returns.

Table 3: Jensen's Alpha Means and t-tests

Industry	Roll. α Mean	Roll. α t-stat.	DCC α Mean	DCC α t-stat.	Mean Diff.	Mean Eq. t-stat.
Industrials	0.0113	1.3200	0.0139	1.6536*	-0.0026	-0.2206
Food, Beverage & Tobacco	0.0145	0.7646	0.0128	0.6796	0.0017	0.0631
Textile & Leather	0.0219	1.2899	0.0211	1.2699	0.0008	0.0356
Wood & Paper Products	-0.0024	-0.1275	0.0007	0.0400	-0.0031	-0.1187
Chemicals	0.0154	1.0034	0.0167	1.0927	-0.0013	-0.0587
Basic Metal	0.0084	0.4131	0.0190	0.9392	-0.0106	-0.3712
Mineral Products	0.0139	0.9321	0.0193	1.3044	-0.0054	-0.2581
Fabricated Metal Products	0.0108	0.7465	0.0108	0.7508	0.0000	-0.0004
Services	0.0193	1.8646*	0.0181	1.7549*	0.0011	0.0780
Electricity Gas & Steam	-0.0040	-0.1863	-0.0083	-0.3940	0.0044	0.1458
Wholesale & Retail Trade	0.0324	1.7510*	0.0319	1.7287*	0.0005	0.0189
Transportation & Storage	0.0474	1.9860**	0.0411	1.7163*	0.0062	0.1848
Sports Activities	0.0011	0.0340	0.0043	0.1349	-0.0032	-0.0710
Hotels & Restaurants	0.0086	0.3386	0.0136	0.5471	-0.0051	-0.1426
Telecommunications	-0.0103	-0.4711	-0.0095	-0.4349	-0.0008	-0.0245
Technology	0.0355	1.8712*	0.0389	2.0636**	-0.0034	-0.1286
Information Technology	0.0281	1.3613	0.0297	1.4515	-0.0016	-0.0551
Financial Institutions	-0.0047	-0.6941	-0.0057	-0.8499	0.0010	0.1065
Banks	-0.0062	-0.4344	-0.0079	-0.5557	0.0017	0.0819
Insurance	0.0301	1.6923*	0.0329	1.8537*	-0.0027	-0.1085
Leasing & Factoring	0.0013	0.0472	0.0068	0.2401	-0.0055	-0.1360
Holding & Investment	-0.0087	-0.8843	-0.0065	-0.6610	-0.0022	-0.1560
Brokerage Houses	-0.0043	-0.2196	-0.0041	-0.2091	-0.0002	-0.0077
Property Trusts	-0.0170	-1.1044	-0.0110	-0.7160	-0.0060	-0.2783

Notes: Jensen's alphas are calculated using either the rolling beta or the DCC beta, both estimated from daily returns. Roll. α t-stat. and DCC α t-stat. are the t-statistics testing whether each alpha mean differs from zero. Mean Eq. t-stat. is the t-statistic testing equality of mean alphas between the two methods. Significance levels are indicated by asterisks: * (10%), ** (5%), *** (1%).

A rough pattern emerges when comparing alpha rankings in Table 3 to the mean excess return rankings presented in Table 1, with industries exhibiting the highest (or lowest) excess returns also tending to outperform (or underperform) on a risk-adjusted basis. For instance, the transportation and storage industry, which recorded the highest average daily excess return, also shows the highest Jensen's alpha (12.6% annually). In practical terms, an alpha of this scale is attractive to active managers and can motivate sector rotation strategies seeking to exploit persistent excess returns. This elevated alpha likely reflects industry-specific factors, such as large-scale infrastructure investments during the sample period (as discussed earlier), that are not fully captured by market beta alone. It should be viewed as an exception rather than as evidence of a systematic failure of the CAPM.

To compare the estimation methods based on their Jensen's alphas, Table 3 also reports the differences in alpha values computed using DCC beta and rolling beta for each industry. These differences are generally small, and the mean equality test fails to reject the null hypothesis of equal means at conventional significance levels, suggesting no systematic advantage for either estimation method. However, rolling betas tend to produce alpha values closer to zero more often, indicating more accurate estimation of systematic risk under the CAPM framework. Specifically, in 15 out of 24 industries, the alpha based on rolling beta is lower. One illustrative case is the industrials portfolio, where the DCC alpha is statistically significant at 10% level ($t = 1.65$), while the rolling alpha is smaller and insignificant ($t = 1.32$). Although the overall differences are modest, this pattern points to lower beta estimation accuracy for the DCC model. Based on the Jensen's alpha criterion, the findings provide no evidence that the more sophisticated DCC model outperforms the traditional rolling-window approach in estimating time-varying beta.

Evaluation Based on Market Risk Premium

Estimating the market risk premium through cross-sectional regressions offers another way to compare beta estimation methods. Since the CAPM posits that an asset's model-implied expected excess return equals the market risk premium scaled by the asset's beta, a more accurate beta estimate should yield a market risk premium that is significantly different from zero and more closely aligned with the realized average excess return when tested in cross-sectional regressions.

Conducting a one-time cross-sectional regression analysis, wherein the average returns of assets are regressed upon their estimated betas, fails to produce correct t-ratios. Fama and MacBeth (1973) put forward a solution which involves performing cross-sectional regressions for each individual time period and conducting hypothesis tests on the average of the coefficient estimates. Following their methodology, a cross-sectional analysis is conducted by running the following regression for each day using the estimated conditional betas for each industry portfolio:

$$r_{it} = \lambda_{0t} + \lambda_{1t}\widehat{\beta}_{it} + e_{it}$$

where r_{it} is the realized excess return, $\widehat{\beta}_{it}$ is the conditional beta estimated either by the rolling-window approach or the DCC model and e_{it} refers to the residual for industry portfolio i. The intercept term, λ_{0t}, is expected to have a value close to zero as the market is the sole factor affecting the returns in the basic market model. The coefficient, λ_{1t}, is the estimated risk premium and is expected to have a value near the average excess return of the equity market because, theoretically, portfolios with a beta equal to one should generate same return as the market.

Table 4: Fama-MacBeth results

	Roll. β	DCC β
λ_0 Estimate	0.0275	0.0350
Std. Error	0.0234	0.0219
t-Statistic	1.1753	1.5984
p-Value	0.2399	0.1100
λ_1 Estimate	0.0133	0.0063
Std. Error	0.0306	0.0295
t-Statistic	0.4341	0.2143
p-Value	0.6643	0.8303
Adjusted R^2	0.0701	0.0714

Notes: Results are based on daily excess returns expressed in percentage terms. λ_0 is the time-series average intercept, λ_1 is the time-series average slope on conditional betas, and the adjusted R^2 is the time-series average from the cross-sectional regressions. Standard errors (and the resulting t-statistics and p-values) are based on the Fama-MacBeth procedure.

Table 4 reports the time-series averages of the intercept (λ_{0t}) and slope (λ_{1t}) coefficients from cross-sectional regressions of the industry portfolios' returns on their respective conditional market betas, estimated using either a rolling-window or a DCC approach. The associated Fama-MacBeth t-statistics assess the significance of these coefficients. For the rolling-window beta, the average intercept is 0.0275% and the average slope is 0.0133%. For the DCC beta, the intercept is slightly higher at 0.0350%, while the slope is lower at 0.0063%. All estimated coefficients are positive, but none are statistically significant at the 10% level. The adjusted values are 0.0701 for the rolling betas and 0.0714 for the DCC betas.

The slope estimates reported in Table 4 can be viewed as daily market risk premia. A one-unit increase in beta raises the model-implied expected excess returns by just 0.0133% per day (3.4% annually) for the rolling beta and 0.0063% per day (1.6% annually) for the DCC beta. Both estimated market risk premia are statistically insignificant and lie well below the realized average daily excess return of the BIST 100 Turkish Market Index, 0.0358% (9.4% annually; see Table 1). Moreover, the adjusted R^2 values, both around %7, indicate that the conditional betas explain only a small fraction of the cross-industry return variation. Although these betas produce alpha values close to zero in the time-series context, consistent with CAPM predictions, they are less informative in explaining return difference across industries. These results challenge a central implication of the CAPM that higher systematic risk reliably commands higher expected returns. For portfolio managers, they imply that beta-tilting strategies aimed at earning superior returns by overweighting high-beta industries may yield limited benefit for cross-industry allocation in the Turkish market.

When comparing the two estimation methods, the DCC model offers only marginal gains in explanatory power, as reflected by its slightly higher adjusted R^2. However, it yields a weaker empirical relationship between beta and expected return. Its higher average intercept and lower average slope imply a flatter securities market line, contrary to the CAPM's core prediction that higher beta should be associated with higher expected return. In contrast, the rolling-window approach produces a steeper, though still statistically insignificant, risk-return relation and a slope coefficient that is closer in magnitude to the observed market premium**.** While neither model generates a significant or economically meaningful risk premium, the smoother beta dynamics of the rolling-window approach appear to better preserve the theoretical pricing relation. The findings provide no compelling evidence that the DCC model offers a clear advantage over the simpler rolling-window approach in capturing cross-sectional return variation either.

Parameter Stability of the DCC Model

Given that the previous analysis is based on a 20-year sample, it is important to assess whether the DCC model's relatively weak performance stems from parameter instability. To investigate this, the stability of the DCC parameters is formally tested. Subsequently, model performance is re-evaluated on shorter subsamples to determine whether estimation over these intervals improves results according to both the Jensen's alpha criterion and the market risk premium criterion.

Because the DCC model is estimated via maximum likelihood, a likelihood ratio (LR) test is employed to evaluate parameter stability. In this procedure, the full sample is first divided into two equal halves. The DCC model estimated over the full sample is regarded as the restricted model because it imposes constant parameters. Estimating the DCC model separately on each subsample forms a composite model, regarded as the unrestricted model since it allows parameters to vary between subsamples. The restricted and unrestricted models are then compared using the LR test statistic, computed as

$$-2\ln\lambda = -2(\ln L_R - \ln L_U)$$

where $\ln L_R$ denotes the log-likelihood of the restricted model, and $\ln L_U = \ln L_1 + \ln L_2$ is the log-likelihood of the unrestricted model, obtained by summing the log-likelihoods from the two subsamples. Under the null hypothesis that the full-sample DCC parameters apply to each subsample, implying parameter stability, the test statistic follows a chi-squared distribution with degrees of freedom equal to the difference between the total number of parameters estimated in the composite model and those in the full-sample model.

Table 5 reports the results of likelihood ratio tests for parameter stability across three time spans: the most recent 20-year, 10-year, and 5-year periods of the dataset. The reported test statistics follow a chi-squared distribution with 11 degrees of freedom, calculated as $(11 + 11) - 11 = 11$. Under the null hypothesis, the test statistic is expected to fall within the typical range of this distribution. A high test statistic indicates that the composite model fits the data significantly better than the full-sample model, suggesting that the parameters differ between the two halves of the sample and the null hypothesis should be rejected. For the 20-year

Table 5: Results of Likelihood Ratio Tests for DCC Parameter Stability

Industry	χ^2 (20-Year)	χ^2 (10-Year)	χ^2 (5-Year)
Industrials	67.06***	77.32***	29.53***
Food, Beverage & Tobacco	29.19***	43.87***	11.97
Textıle & Leather	111.73***	48.23***	28.11***
Wood & Paper Products	13.12	85.70***	24.41**
Chemicals	5.53	83.67***	20.75**
Basic Metal	42.75***	71.68***	34.37***
Mineral Products	99.81***	173.61***	6.47
Fabrıcated Metal Products	20.62**	64.13***	25.29***
Services	53.77***	44.61***	20.02**
Electricity Gas & Steam	35.67***	52.63***	16.55
Wholesale & Retail Trade	4.76	45.09***	33.13***
Transportation & Storage	28.63***	45.97***	24.74***
Sports Activities	69.73***	71.75***	8.83
Hotels & Restaurants	14.58	47.60***	21.81**
Telecommunications	1.04	35.52***	32.19***
Technology	31.03***	35.99***	38.69***
Information Technology	40.61***	54.01***	7.14
Financial Institutions	27.24***	87.42***	24.50**
Banks	25.39***	69.67***	39.86***
Insurance	71.07***	70.19***	29.00***
Leasing & Factoring	65.89***	68.87***	3.91
Holding & Investment	13.77	89.99***	28.51***
Brokerage Houses	47.18***	93.57***	29.29***
Property Trusts	9.46	72.01***	17.59*

Notes: Each cell reports the likelihood ratio test statistic, which follows a chi-squared distribution with 11 degrees of freedom. The test compares a restricted DCC model estimated on the full sample (either the last 20, 10 or 5 years of the study period) to a composite model estimated separately on each half of the same sample. Significance levels are indicated by asterisks: * (10%), ** (5%), *** (1%).

sample, parameter stability is rejected at conventional significance levels in 17 out of 24 industries. In particular, industries such as textile & leather, mineral products, and sports activities exhibit especially large test statistics, implying substantial shifts in conditional dynamics over time and a clear violation of the constant-parameter assumption.

Given the widespread rejections in the 20-year sample, the test is repeated using more recent and shorter samples to examine whether parameter stability can be achieved over reduced time spans. This question is particularly relevant because instability in longer samples could undermine the earlier findings regarding the DCC model's underperformance in capturing time-varying betas. If, however, stability cannot be attained even in shorter periods, this would point to a more broad practical limitation of the model. In the 10-year sample, the null hypothesis of parameter stability is rejected for all industries, indicating that instability remains pervasive. Even in the 5-year sample, the null is rejected in 18 industries, suggesting that parameter shifts persist despite the shorter estimation span. The test is not applied to smaller samples, such as the last 2.5 years, because each half of that period contains fewer than 500 observations, a treshold recommended by Hwang and Valls Pereira (2006) to ensure convergence and reduce estimation bias in GARCH(1,1) models.

Following the parameter stability tests on the three samples, the performance of the DCC model is re-evaluated using the second halves of these samples, namely the most recent 10-year, 5-year, and 2.5-year periods, to assess whether shorter estimation spans yield improved results based on the Jensen's alpha criterion and the market risk premium criterion. In terms of the Jensen's alpha criterion, unreported results indicate that the relative underperformance of the DCC model persists or even worsens with shorter estimation periods. Specifically, in the 10-year sample, DCC betas produce higher alpha estimates than rollling betas in 14 out of 24 industries, slightly fewer than the 15 industries observed in the full sample. However, this number increases to 16 industries in the 5-year sample and further to 18 industries in the 2.5-year sample. This pattern suggests that as the estimation period shortens, the DCC model tends to generate more positive Jensen's alphas compared to the rolling-window approach, implying a weaker alignment with CAPM predictions.

Table 6: Fama-MacBeth results for DCC Betas over Shorter Subsamples

	10-Year	**5-Year**	**2.5-Year**
λ_0 Estimate	0.0905	0.2217	0.1430
Std. Error	0.0345	0.0579	0.0891
t-Statistic	2.6243	3.8267	1.6045
p-Value	0.0087	0.0001	0.1091
λ_1 Estimate	-0.0197	-0.0953	0.0526
Std. Error	0.0426	0.0695	0.1070
t-Statistic	-0.4624	-1.3705	0.4917
p-Value	0.6439	0.1707	0.6231
Adjusted R^2	0.0698	0.0697	0.0746

Notes: The results are based on DCC beta estimates for three shorter subsamples: the most recent 10-year, 5-year, and 2.5-year periods. See Table 4 for the full-sample results and explanatory notes.

Table 6 presents the Fama-MacBeth regression results for the DCC model estimated over shorter subsamples of the dataset. While the adjusted R^2 values remain fairly stable around 0.07 across all subsamples, no meaningful improvement in explanatory power is observed compared to the full 20-year sample (adjusted $R^2 =$ 0.0714; see Table 4). Notably, the intercept (λ_0) increases substantially in the 5-year sample (0.2217) relative to the full-sample estimate (0.0350; see Table 4), indicating a larger pricing error and a greater departure from the CAPM expectation of a zero intercept. The slope (λ_1), representing the market risk premium, remains statistically insignificant and even turns negative in the 10-year and 5-year samples, contradicting the CAPM prediction that higher beta should be associated with higher expected returns. Although the 2.5-year sample shows a positive slope, it is not statistically significant. Overall, these results suggest that shortening the estimation period does not resolve the DCC model's limited ability to capture cross-sectional variation in returns. This limitation highlights challenges in applying the DCC model for researchers and practitioners seeking reliable conditional beta estimates.

CONCLUSION

Accurate estimation of time-varying beta is essential for understanding systematic risk and for informing asset pricing, portfolio allocation, and performance evaluation. Two of the most common methods for modelling time-varying beta are the rolling-window OLS regression and the DCC model. This study compares them using daily returns of Turkish industry portfolios to assess how well each method reflects the core predictions of the CAPM, specifically whether Jensen's alpha is insignificant in time-series tests and whether the market risk premium is significant in cross-sectional tests.

The time-series analysis of Jensen's alpha shows that, in most industries, both estimation methods yield mean alphas close to zero, consistent with CAPM predictions. However, rolling betas more frequently produce smaller, insignificant alphas, suggesting that they capture systematic risk better than the DCC model. Cross-sectional Fama-MacBeth regressions further reveal that neither method produces a statistically significant market risk premium, with slope estimates well below the realized average excess return of the Turkish equity market. While both approaches explain a small fraction of cross-industry return variation, the rolling-window method generates a steeper slope coefficient, closer in magnitude to the observed average market return, preserving the CAPM's risk-return relation more faithfully than the DCC model. Using the 10-year, 5-year, and even 2.5-year subsamples did not improve, and in some cases worsened, the performance of the DCC model.

The findings suggest that, the rolling-window approach provides beta estimates more consistent with the CAPM framework when estimating time-varying betas for Turkish industry portfolios. Although the DCC model accounts for time variation in return volatility and its clustering, its additional complexity does not translate into improved performance according to the evaluation criteria used. The DCC model's reliance on multiple constant parameters imposes a rigid structure that may struggle to adapt when market dynamics shift, even in shorter samples. Furthermore, because the DCC model is designed to capture evolving conditional correlations through implied exponential weighting, its heightened responsiveness to recent return shocks can lead to over-adjustment in beta estimates, which may contribute to its lower performance. In contrast, the rolling-window method's periodic re-estimation over short windows, without rigid parametric constraints, allows it to better accommodate evolving market conditions, while the short estimation window limits the risk of misspecification when betas vary moderately.

The results highlight the importance of considering model flexibility and stability when selecting beta estimation methods, especially in emerging markets prone to structural changes or heightened volatility. Practitioners should recognize that rigid methods performing well in stable markets may yield unreliable risk assessments and misguided asset allocation decisions in more volatile or structurally changing markets. Future research could explore nonparametric models, such as those of Baillie, Calonaci and Kapetanios (2022), which generalize the rolling-window OLS regression without requiring a fixed window length. Additionally, comparing these generalized rolling-window approaches with the DCC model in a multifactor setting could be fruitful, for example using Engle (2016)'s extended DCC framework that estimates multiple betas simultaneously.

REFERENCES

Adrian, T. & Franzoni, F. (2009). Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM. *Journal of Empirical Finance, 16*(4), 537–556. https://doi.org/10.1016/j.jempfin.2009.02.003

Agrrawal, P., Gilbert, F. W. & Harkins, J. (2022). Time dependence of CAPM betas on the choice of interval frequency and return timeframes: Is there an optimum? *Journal of Risk and Financial Management, 15*(11), 520. https://doi.org/10.3390/jrfm15110520

Alexander, G. J. & Chervany, N. L. (1980). On the estimation and stability of beta. *The Journal of Financial and Quantitative Analysis, 15*(1), 123–137. https://doi.org/10.2307/2979022

Aloy, M., Laly, F., Laurent, S. & Lecourt, C. (2021). Modeling time-varying conditional betas: A comparison of methods with application for REITs. In G. Dufrénot & T. Matsuki (Eds.), *Recent Econometric Techniques for Macroeconomic and Financial Data* (pp. 229–264). Springer. https://doi.org/10.1007/978-3-030-54252-8_9

Baillie, R. T., Calonaci, F. & Kapetanios, G. (2022). Hierarchical time-varying estimation of asset pricing models. *Journal of Risk and Financial Management, 15*(1), 14. https://doi.org/10.3390/jrfm15010014

Bali, T. G., Engle, R. & Tang, Y. (2016). Dynamic conditional beta is alive and well in the cross-section of daily stock returns. *Management Science, 63*(11), 3760–3779. https://doi.org/10.1287/mnsc.2016.2536

Bauwens, L., Laurent, S. & Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey. *Journal of Applied Econometrics, 21*(1), 79–109. https://doi.org/10.1002/jae.842

Bollerslev, T., Engle, R. F. & Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. *Journal of Political Economy, 96*(1), 116–131. https://doi.org/10.1086/261527

Brooks, R. D., Faff, R. W. & Lee, J. H. H. (1992). The form of time variation of systematic risk: Some Australian evidence. *Applied Financial Economics, 2*(4), 191–198. https://doi.org/10.1080/758527100

Brooks, R. D., Faff R. W. & McKenzie, M. D. (1998). Time-varying beta risk of Australian industry portfolios: A comparison of modelling techniques. *Australian Journal of Management, 23*(1), 45–66. https://doi.org/10.1177/031289629802300101

Caporin, M. & McAleer, M. (2012). Do we really need both BEKK and DCC? A tale of two multivariate GARCH models. *Journal of Economic Surveys, 26*(4), 736–751. https://doi.org/10.1111/j.1467-6419.2011.00683.x

Choudhry, T. & Wu, H. (2008). Forecasting ability of GARCH vs Kalman filter method: Evidence from daily UK time-varying beta. *Journal of Forecasting, 27*(8), 670–689. https://doi.org/10.1002/for.1096

Çatık, A. N., Huyugüzel Kışla, G. & Akdeniz, C. (2020). Time-varying impact of oil prices on sectoral stock returns: Evidence from Turkey. *Resources Policy, 69*, 101845. https://doi.org/10.1016/j.resourpol.2020.101845

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics, 20*(3), 339–350. https://doi.org/10.1198/073500102288618487

Engle, R. F. (2016). Dynamic conditional beta. *Journal of Financial Econometrics*, *14*(4), 643–667. https://doi.org/10.1093/jjfinec/nbw006

Engle, R. F. & Sheppard, K. (2001). *Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH* (NBER Working Paper 8554). National Bureau of Economic Research. https://doi.org/10.3386/w8554

Esteban,M.V.&Orbe-Mandaluniz,S.(2010).Anonparametric approach for estimating betas: The smoothed rolling estimator. *Applied Economics, 42*(10), 1269–1279. https://doi.org/10.1080/00036840701721257

Fabozzi, F. J. & Francis, J. C. (1977). Stability tests for alphas and betas over bull and bear market conditions. *The Journal of Finance, 32*(4), 1093–1099. https://doi.org/10.2307/2326515

Fama, E. F. & French, K. R. (1992). The cross-section of expected stock returns. *The Journal of Finance, 47*(2), 427–465. https://dx.doi.org/10.2307/2329112

Fama, E. F. & French, K. R. (2004). The capital asset pricing model: Theory and evidence. *Journal of Economic Perspectives, 18*(3), 25–46. https://dx.doi.org/10.1257/0895330042162430

Fama, E. F. & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy, 81*(3), 607–636. https://dx.doi.org/10.1086/260061

Ferson, W. E. & Harvey, C. R. (1991). The variation of economic risk premiums. *Journal of Political Economy, 99*(2), 385–415. https://doi.org/10.1086/261755

Graham, J. R. & Harvey, C. R. (2001). The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics 60*(2–3), 187–243. https://doi.org/10.1016/s0304-405x(01)00044-7

Groenewold, N. & Fraser, P. (2000). Forecasting beta: How well does the 'five-year rule of thumb' do? *Journal of Business Finance & Accounting, 27*(7–8). https://doi.org/10.1111/1468-5957.00341

Hwang, S., & Valls Pereira, P. L. (2006). Small sample properties of GARCH estimates and persistence. *The European Journal of Finance, 12*(6–7), 473–494. https://doi.org/10.1080/13518470500039436

İlbasmış, M. (2024). A comparison of forecasting accuracy between two dynamic conditional correlation (DCC) models. *Journal of Economics Business and Political Researches, 9*(23), 1–11. https://doi.org/10.25204/iktisad.1388428

Jagannathan, R. & Wang, Z. (1996). The conditional CAPM and the cross-section of expected returns. *The Journal of Finance, 51*(1), 3–53. https://doi.org/10.2307/2329301

Korkmaz, T., Çevik, E. I., Birkan, E. & Özataç, N. (2010). Testing CAPM using Markov switching model: The case of coal firms. *Economic Research-Ekonomska Istraživanja, 23*(2), 44–59. https://doi.org/10.1080/1331677X.2010.11517411

Lewellen, J. & Nagel, S. (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics, 82*(2), 289–314. https://doi.org/10.1016/j.jfineco.2005.05.012

Lettau, M. & Ludvigson, S. (2001). Resurrecting the (C) CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy, 109*(6), 1238–1287. https://doi.org/10.1086/323282

Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. *The Journal of Finance, 20*(4), 587–615. https://doi.org/10.2307/2977249

Mergner, S. & Bulla, J. (2008). Time-varying beta risk of Pan-European industry portfolios: A comparison of alternative modeling techniques. *The European Journal of Finance, 14*(8), 771–802. https://doi.org/10.1080/13518470802173396

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica, 34*(4), 768–783. https://doi.org/10.2307/1910098

Nieto, B., Orbe, S. & Zarraga, A. (2014). Time-varying market beta: Does the estimation methodology matter? *Statistics and Operations Research Transactions, 38*(1), 13–42. http://hdl.handle.net/2117/88929

Shah, A. & Moonis, S. A. (2003). Testing for time-variation in beta in India. *Journal of Emerging Market Finance, 2*(2), 163–180. https://doi.org/10.1177/097265270300200202

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance, 19*(3), 425–442. https://doi.org/10.2307/2977928

Silvennoinen, A. & Teräsvirta, T. (2009). Multivariate GARCH models. In T. G. Andersen, R. A. Davis, J-P. Kreiß, T. Mikosch (Eds.), *Handbook of Financial Time Series* (pp. 201–229). Springer-Verlag. https://doi.org/10.1007/978-3-540-71297-8_9

Ustaoğlu, E. (2022). Time-varying beta coefficients of BIST sector indices. *Muhasebe ve Finansman Dergisi, 96*, 135–150. https://doi.org/10.25095/mufad.1143257