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Enhancing Geometry Education through Deep Learning Models: Addressing Challenges in Three-Dimensional Shape Visualization

Abdullah ŞENER[1*], Serdal POÇAN[2], Burhan ERGEN[3]

[1] Fırat University, Faculty of Economics and Administrative Sciences, Department of Management Information Systems, Elazığ, Türkiye
[2] Bingöl University, Genç Vocational School, Department of Computer Technologies, Bingöl, Türkiye
[3] Fırat University, Faculty of Engineering, Department of Computer Engineering, Elazığ, Türkiye
Abdullah ŞENER ORCID No: 0000-0002-8927-5638
Serdal POÇAN ORCID No: 0000-0001-6901-0889
Burhan ERGEN ORCID No: 0000-0003-3244-2615

**Corresponding author: asener@firat.edu.tr*

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Abstract: Integrating technology into mathematics education is crucial for enhancing the understanding of mathematical concepts and skills, as well as increasing motivation. This is particularly applicable in geometry classes, where technology can facilitate the detection of geometric shapes, impacting both the learning and teaching processes. In this context, emerging concepts of artificial intelligence and deep learning can be utilized as tools to overcome such limitations. This study addresses the challenges that teachers face when drawing three-dimensional geometric shapes in digital environments. Shapes drawn manually in digital environments can often be complex, making it difficult for teachers to create accurate and precise drawings. Deep learning models can assist teachers in correcting drawing errors, thereby providing students with clearer and more comprehensible visuals to facilitate the learning of geometric concepts. The study emphasizes the high accuracy rates achieved using various deep learning models, highlighting their impressive capabilities in accurately classifying geometric shapes.

Derin Öğrenme Modelleriyle Geometri Eğitiminin Geliştirilmesi: Üç Boyutlu Şekil Görselleştirmesindeki Zorlukların Ele Alınması

Anahtar Kelimeler
Yapay zaka,
Derin öğrenme,
Matematik eğitimi,
Üç boyutlu şekiller

Öz: Matematik eğitimine teknolojinin entegrasyonu, hem matematiksel kavram ve becerilerin anlaşılmasını geliştirmek hem de motivasyonu artırmak açısından büyük önem taşımaktadır. Bu durum özellikle geometri derslerinde geçerlidir; zira teknoloji, geometrik şekillerin algılanmasını kolaylaştırarak öğrenme ve öğretme süreçlerini olumlu yönde etkileyebilmektedir. Bu bağlamda, yapay zekâ ve derin öğrenme gibi gelişmekte olan kavramlar, mevcut sınırlılıkların aşılmasında birer araç olarak kullanılabilir. Bu çalışma, öğretmenlerin dijital ortamlarda üç boyutlu geometrik şekilleri çizerken karşılaştıkları zorlukları ele almaktadır. Dijital ortamlarda elle çizilen şekiller çoğu zaman karmaşık olabilmekte ve bu durum, öğretmenlerin doğru ve hassas çizimler yapmalarını güçleştirmektedir. Derin öğrenme modelleri, öğretmenlere çizim hatalarını düzeltmede yardımcı olarak, öğrencilerin geometrik kavramları daha açık ve anlaşılır görseller üzerinden öğrenmelerini sağlayabilir. Çalışma, çeşitli derin öğrenme modelleri kullanılarak elde edilen yüksek doğruluk oranlarına vurgu yaparak, bu modellerin geometrik şekilleri doğru şekilde sınıflandırmadaki etkileyici yeteneklerini ön plana çıkarmaktadır.

1. INTRODUCTION

Mathematics is a learning content that uses a symbolic language that refers to numbers, quantity, space and structure [1,2]. It is imperative to ensure that as many members of the society as possible have a solid grasp of the basics of mathematics and can learn and understand it effectively [3], but many students experience anxiety thinking that mathematics is difficult and that they are likely to fail, and consequently develop a negative attitude towards mathematics. The teaching methods that lead to this problem should be reconsidered [4], and it is important to provide suitable conditions for easy learning to occur [5].

When literature is considered, it is possible to see that technology, which is a part of our daily life, is heavily integrated with education. Many institutions supported adapting cognitive technologies to education, and stated the importance of teachers and students using technology in class actively [6]. Developments in technology also transform the perception of the classroom, and concepts such as flipped classrooms affect learning styles. In this context, virtual classroom environments created through online learning tools inevitably stretch [7]. Mathematics lessons and technology are getting more and more integrated, and the importance of digital technologies in improving learning experiences is being emphasized [1]. The integration of technology into education, especially mathematics learning, is considered important and is becoming a trend [8]. Recent studies (e.g., Chen et al [9]; Park & Kwon [10]) emphasize the increasing relevance of AI-enhanced learning platforms, highlighting the need for adaptive and intelligent systems in mathematics education.

Interest in how to use mobile devices to support learning and teaching, increases as technology becomes widespread [11]. Technology is integrated into daily life, and it plays an important role in education [12]. Technology, a part of daily life, affects knowledge-seeking behaviors, communication, and behaviors. Consequently, educational environments started to change, and digital culture in educational environments started to be cited in curricula [13]. These changes caused educational technologies with different interactions to become widespread, in addition to affecting education curricula. Students started using tablets instead of notebooks, and teachers are using different teaching tools, such as Google Classroom, Edmodo, PowerSchool, and Moodle. In addition, different online studying environments paved their way into education. The increasing number of active users of such courses show that distance learning methodologies are appreciated [13]. As a result, the role of technology is important in the development of mathematical concepts and skills in students [14].

Computer technologies can be integrated in every aspect of a classroom. Teaching specialized techniques become more interesting, and students can learn new concepts faster and easier [15]. Although there are differences in research questions or methodologies, we need to build bridges between different communities and learn from each other without having to reinvent the wheel [16]. At the same time, we think that in addition to different cultures learning from each other, the cooperation between different disciplines will contribute to each other in the context of teaching and learning.

Many studies in mathematics education show that artificial objects and especially technological objects are important in mediating mathematical issues [17]. It is stated that this contributes to the satisfaction levels of teachers since in-class participation and interest of students increase [18]. On the other hand, especially during online lessons, expending more energy to deliver a subject may cause teachers to be stressed, and students may need to work harder to understand the lesson [19]. This shows that more work should be done on what should be done to reduce stress for teachers in online courses and to facilitate learning for students [9].

Artificial intelligence (AI), which is a tool to support and even further develop human skills, is a hot topic in public debates as in many sciences [20]. AI, which is entrusted to futuristic societies, previously created in the imagination of science fiction writers and filmmakers, is now a reality of everyday life in our modern high-tech societies. There are many definitions of AI, and each of these definitions has been revised over time [21]. It is stated that the term AI was first used in 1956 at a conference held at Dartmouth University on how machines simulate the intelligent activities of humans [22]. Wang et al [23] define AI as "to make a computer work like a human mind" (p. 2), Rapaport [24] defines AI as "…a scientific study of what problems can be solved, what tasks can be accomplished, and what features of the world can be understood computationally (i.e., using the language of Turing Machines), and then to provide algorithms to show how this can be done efficiently, practically, physically, and ethically" (p. 54).

Important developments were experienced in artificial intelligence in education (AIED). Artificial intelligence technologies, one of the popular technology topics in some developed countries, have begun to be included in the training [10]. Two main questions are asked when thinking about the past and shaping the future: What are our strongest aspects, and what are the opportunities of the future [16]? Positive aspects of AI are expressed at the point of increasing the quality of education [25]. For example, AI can lift limitations such as time and place for teachers and offer unique learning environments such as collaborative learning environments to students [16]. AI, which offers innovative and creative opportunities in many science fields, became popular in mathematics education as well [26]. The relationship between mathematics and artificial intelligence is not one-sided. Artificial intelligence has an important place in the development of computer-based tools in the learning and teaching of mathematics, as well as the contribution of mathematics to artificial intelligence becoming a science field [27].

Geometry is a branch of mathematics that is difficult and feared by students [28]. In addition, the complexity of geometry compared to numerical operations and algebra causes major difficulties while trying to overcome problems experienced while learning [29]. Especially, representing a 3D shape as a 2D shape may affect students' reasoning [30] and this causes them to shy away from geometry. Spatial thinking, which is also expressed as the ability to establish the relationship between 2D and 3D representations, is considered important not only in mathematics but also in other branches of science [31]. The lattices formed with shapes in the geometry lessons may cause students to not be able to visualize the visual in their minds, and this causes the students to not be able to visualize spatial objects fully. These problems arising from the characteristics of geometric shapes appear as problems in distinguishing some shapes from each other [32].

Teaching geometric shapes is crucial in shaping children's thoughts. However, sufficient training material to do so is limited [33]. Although it is easier to draw on paper using drawing tools such as pencils in geometry, we have recently seen that many computer aided auxiliary tools are used in distance education [34]. The development of technology has also affected the teaching and learning process [35].

Online learning becomes more popular as the curricula in education institutes become digitized. The global pandemic, as well as the increasing number of students are stated as the causes, and consequently the need to use automated techniques in learning and teaching is highlighted [36]. Over a decade, advances in digital technologies have created new learning opportunities for rural and distance learners to provide distance education [37]. It has been incredibly challenging, especially in developing countries where teachers and organizations do not have the appropriate tools for effective teaching through distance learning [26]. Deficiencies or mistakes in the visualization of geometric concepts can cause students to fail learning knowledge they need to acquire about a concept [38]. Algorithms regarding machine learning in geometric shapes have become popular recently [39]. The emphasis on the combination of analytical and visual thinking, especially in animating a 3D shape in the mind or coping with the problems of plane and solid geometry [40], explains the reason for the emerging need. The visual and logical-structural foundation of a geometric concept are correlated. Logical-structural foundations refer to the correlations between properties and shape [38]. In addition, students' drawing skills, hence their spatial visualization and spatial orientation skills should be developed to draw and interpret shapes correctly [41]. Therefore, technological developments become popular in making geometric drawings easy, appealing to the eye, expressing ideas and understanding subjects more easily.

The spatial imagination plays a significant role in the cognitive development of individuals undergoing training within the educational system. Cognitive development, encompassing general intelligence, problem-solving skills, and 3D modeling, is influenced by various factors [42]. Distance learning has gained increasing popularity for various reasons, necessitating the exploration of challenges encountered in teaching mathematics within distance education environments. Specifically, limitations in drawing 3D shapes as 2D representations directly impact the process of learning and teaching. Furthermore, the imperative integration of Artificial Intelligence (AI) into education has become evident. The study underscores the growing trend of technology integration in mathematics education, with a particular emphasis on the pivotal role of digital technologies in enriching learning experiences. Within this framework, the objective is to predict and classify 3D drawings, a task that teachers often find challenging in digital environments. This endeavor, facilitated by deep learning models, is expected to directly influence the learning process.

Upon reviewing the literature, it becomes apparent that studies utilizing deep learning methods in mathematics as a component of AI are relatively limited. This scarcity may be attributed to the insufficient collaboration among different disciplines. Consequently, this study is deemed significant, and it is anticipated to serve as a pioneering effort for future research endeavors.

2. MATERIAL AND METHOD

2.1. Dataset

In this study, a custom dataset was constructed to evaluate the performance of the proposed deep learning models in classifying geometric shapes. The dataset comprises a total of 1,249 black-and-white images created manually in a digital environment. All images were drawn by twelve voluntary participants, including pre-service mathematics teachers and high school students, using drawing tablets and stylus pens. The drawings were created using basic digital drawing software, simulating a natural hand-drawing experience within a controlled digital setting.

To ensure standardization and consistency, participants were provided with predefined shape guidelines, including examples and basic dimensional constraints. Each drawing was evaluated based on visual clarity, geometric accuracy, and adherence to shape criteria before inclusion in the dataset. The finalized dataset includes three main geometric categories: spheres (142 images), pyramids (464 images: 120 square pyramids, 156 cones, and 188 triangular pyramids), and prisms (643 images: 175 rectangular prisms, 182 cubes, 142 cylinders, and 144 triangular prisms). All images vary in resolution and scale, then were normalized into grayscale format for compatibility with image processing algorithms. A sample of the image collection is presented in Figure 1, illustrating the diversity and structure of the dataset used in this research.

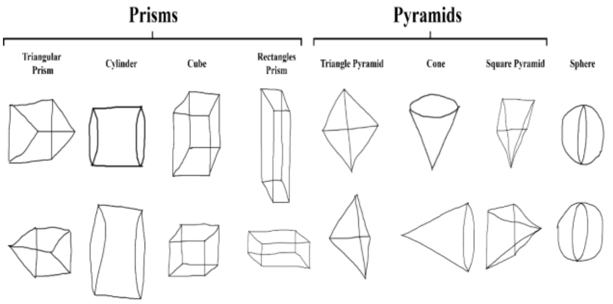

Figure 1. Example Images from the Dataset used in the Research

2.2. Deep Learning

Deep learning is a machine learning technique that predicts or generates results based on a dataset containing multiple layers. The primary objective of deep learning is to extract new information from the data it processes using the artificial neural network it employs. Among various deep learning methods, the Convolutional Neural Network (CNN) architecture is the most widely used [43]. CNNs recognize images with shared features and classify them by grouping them based on similarities, akin to the human brain's functioning [44]. The CNN architecture is composed of a series of layers, including the convolutional layer, non-linearity layer, pooling layer, flattening layer, and fully-connected layer [45]. The convolutional layer serves as the core of the CNN architecture, where the majority of computationally intensive operations occur. This layer detects both low-level and high-level features by applying filters to the input images. The non-linearity layer captures non-linear patterns by introducing transformations to the data. In the pooling layer, the size of the feature maps is reduced, which decreases the number of parameters and weights in the network. This reduction is typically achieved through max pooling, which selects the highest value within n×n regions, or average pooling, which computes the mean value of these regions. The flattening layer converts the data into a one-dimensional matrix, preparing it for the fully connected layer, which is the final stage of the CNN architecture. The fully connected layer establishes the relationship between the input and output layers [46, 47].

2.3. Visual Geometry Group-19

The Visual Geometry Group (VGG)-19 is a convolutional neural network with a depth of 24 layers. Its architecture consists of 16 convolutional layers, 5 pooling layers, and 3 fully connected layers. The network was pre-trained on more than one million images from the ImageNet database. VGG-19 accepts image inputs of 224 x 224 pixels and contains approximately 138 million parameters [48]. It utilizes 3x3 pixel filters in the convolutional layer to reduce the number of parameters in the architecture.

2.4. Xception

GoogLeNET (Inception V1), is created by Google engineers inspired by "Network-In-Network" [49]. Inception V2 and Inception V3 versions were developed later [50]. The Xception architecture is an extension of Inception architecture that replaces standard Inception modules with deeply separable convolution [51]. The Xception network was pre-trained on over one million images from the ImageNet database and processes image inputs of 224 x 224 pixels. Rather than dividing the input data into compressed chunks, the network independently maps spatial correlations for each output channel and employs 1 x 1 depthwise convolutions to capture inter-channel correlations.

2.5. The proposed approach

In the study, two new models are proposed based on VGG-19 and Xception architectures. A dataset containing geometric shapes created by drawing in a computer environment was used to evaluate the performances of the models. 70% of the images in the dataset were used for training and 30% for validation. The first step of the study involves adjusting the images in the dataset, which have varying resolutions and sizes, to a fixed resolution of 224 x 224 pixels, the input size required by the VGG-19 and Xception models. This step aims to optimize classification speed and minimize memory usage on the computer. During resizing, all images are scaled to this predetermined size. Care is taken to ensure that the dimensions are not excessively reduced, preserving as much image quality as possible. If the images are resized too much, it may become difficult to retain the necessary information for accurate image classification. In the next part of the study, the dataset images brought to fixed resolution and size were classified separately on the developed VGG-19 and Xception models. Classification processes were primarily carried out on prisms and pyramid classes. Then, classification was carried out by combining all classes. Based on the results obtained, the performances of the developed models were compared. Figure 2 illustrates the overall design of the VGG-19 model developed in the study.

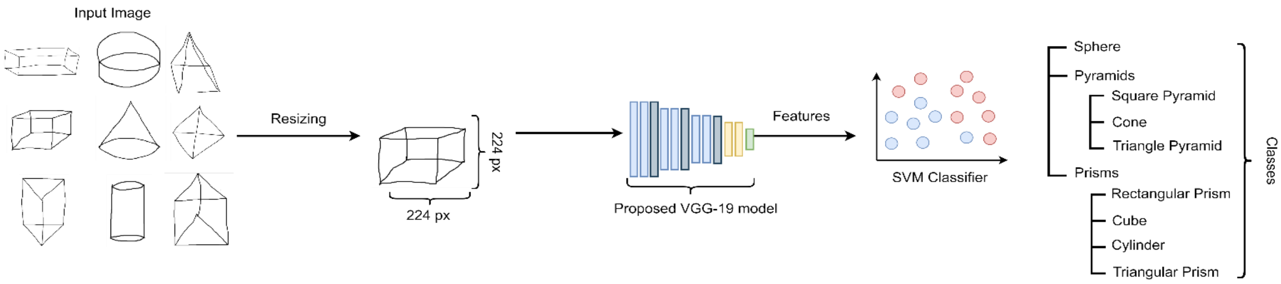

Figure 2. General Design of the Recommended VGG-19 Model

The model illustrated in Figure 2 comprises 25 layers in total. These include 13 convolutional layers, 5 max-pooling layers, 2 dropout layers, 2 batch normalization layers, 2 fully connected layers, and 1 softmax layer. In the developed VGG-19 model, 13 convolutional and 5 max-pooling layers are applied to the gray-scale computer-generated geometric shape images inputted into the model. After completing the convolution and pooling layers, the data is passed through the flattening and fully connected (FC1) layers, followed by the first batch normalization layer. A dropout operation is then applied with a rate between 0.3 and 0.5 to prevent overfitting by reducing the risk of the network memorizing the training data. Subsequently, the second fully connected layer and batch normalization layer are applied, followed by another dropout process with the same drop rate. The final classification output is generated using the softmax optimization algorithm. Additionally, the ReLU activation function is used in the convolutional layers of the model. Figure 3 illustrates the architecture of the Xception model, which is the second model developed in this study.

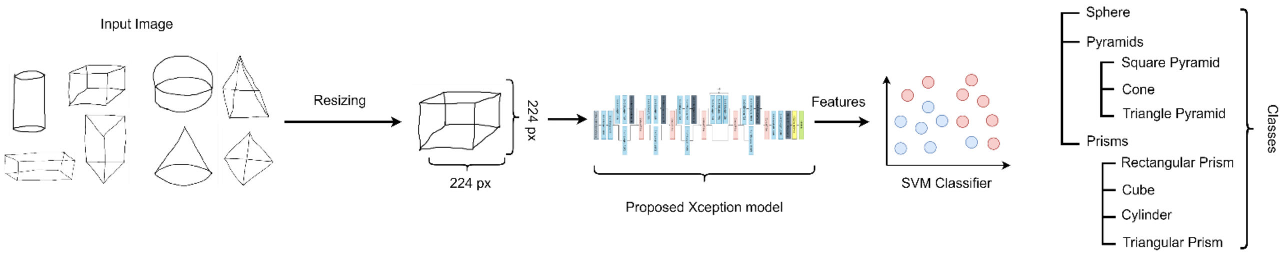

Figure 3. General Design of the Recommended Xception Model

The Xception model shown in Figure 3 consists of a total of 31 layers. These include 19 convolutional layers, 5 max-pooling layers, 2 dropout layers, 2 batch normalization layers, 2 fully connected layers, and 1 output (softmax) layer. The same parameters used in the VGG-19 model were also applied to the Xception model to enable a fair comparison between the models developed in the study.

2.6. Media and Libraries

The training and validation processes of the models developed based on VGG-19 and Xception are carried out using PyCharm 2021. Python 3.6 and OpenCV, Keras, Sklearn, Matplotlib, Imageio, NumPy, PIL and Os libraries were used to process the dataset used in models. The computer operating system used in the study is 64-bit Windows 11. The hardware specifications include an NVIDIA GeForce® RTX™ 3060 6GB graphics card, an 11th Gen Intel® Core™ i7 processor (2.3GHz, 24M Cache, up to 4.6GHz, 8 cores), and 16GB of memory.

3. RESULTS AND DISCUSSION

In the study, 70% of the dataset was used for training and 30% for validation processes in the evaluation of the performances of both models. A total of 464 images, 326 training and 138 validation images, were used in the classification of the pyramids. 643 images, 452 training and 191 validation, were used while classifying prisms. During general classification, 1249 images were used, of which 878 were for training and 371 for validation. Table 1 presents the training parameters of the developed models. The dropout rates between 0.3 and 0.5 were chosen to prevent overfitting while maintaining generalization. The number of epochs (10) was selected to ensure convergence without excessive computation. A mini-batch size of 16 was found optimal during preliminary testing to balance performance and training stability.

Table 1. Training Parameters of the Developed Model

Parameter	Value
Epoch	10
Mini Batch Size	16
Dropout	0,3-0,5
Activation Function	ReLu
Optimization Algorithm	Softmax

In the first stage of the study, the classification of pyramid images, which are divided into 3 subgroups, was performed using the VGG-19-based model. Figure 4 shows the accuracy and loss values obtained during the model's training and testing phases as a result of the classification. When the graphics are examined, it can be seen that the developed model learns quickly. It can be seen that the network continues to learn, as evident from the ups and downs until the 10th iteration. After the training of the model was completed, an accuracy rate of 81% was obtained.

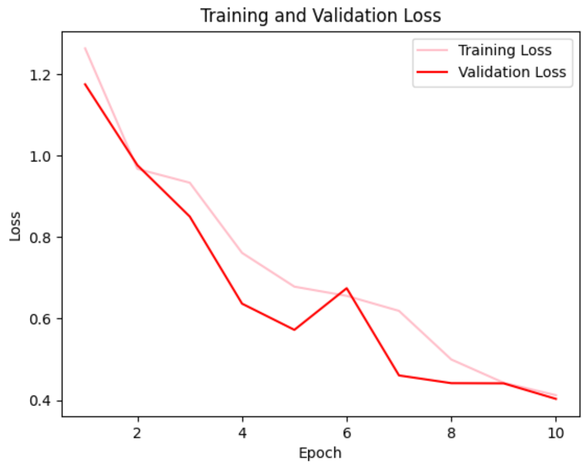

Figure 4. Accuracy and Loss Graphics of the VGG-19 Model Developed to Classify Pyramid Images

A confusion matrix is a table that summarizes the prediction results in a classification process. It is used to provide an idea about the error rate of a model developed through the classification process. Confusion matrix is frequently used in the literature to define the performance of models in classification problems. If we evaluate it for binary classification models, the confusion matrix is depicted as shown in Figure 5.

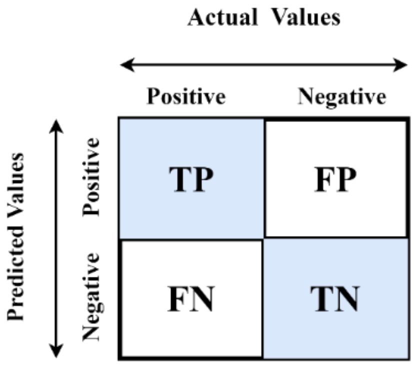

Figure 5. Confusion Matrix

The confusion matrix was used to evaluate the performance of the models whose classification operations were performed. Terms used while constructing the confusion matrix: TP: True Positive, FP: False Positive, TN: True Negative, and FN: False Negative. Precision, Recall, Accuracy and F1-Score values of the model can also be calculated using these values. The mathematical operations given in equations 1, 2, 3 and 4 are used to calculate these values.

$$Precision = \frac{TP}{TP + FP} \tag{1}$$

$$Recall = \frac{TP}{TP + FN} \tag{2}$$

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} \tag{3}$$

$$F1 - Score = \frac{2 * precision * recall}{precision + recall} \tag{4}$$

Table 2 presents the analysis results of the classification conducted using the test data from the proposed VGG-19 model.

Table 2. Analysis Results of Classification

Classes	Rec.%	Prc. %	F1-Scr. %	Acc. %
Square pyramid	100	68	81	81
Cone	100	75	86	
Triangle pyramid	64	100	78	

Figure 6 presents the confusion matrix of the VGG-19 model based on the results obtained from classification. This matrix provides a detailed view of the model's classification ability by showing the correct and incorrect predictions across the pyramid categories, offering insights into where misclassifications are most likely to occur.

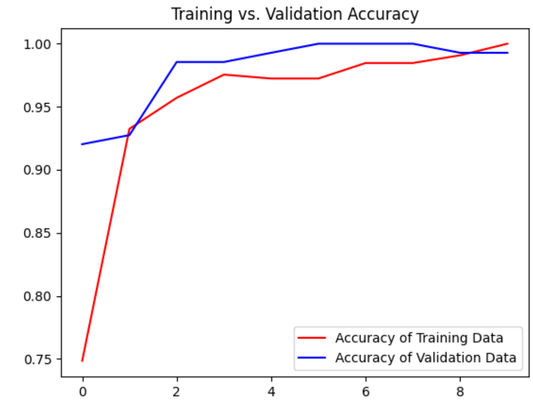

Figure 6. Confusion Matrix of VGG-19 Model Developed for Classifying Pyramid Images

In the second stage of the study, the classification of the pyramid images consisting of 3 subgroups was carried out using the Xception based model. Figure 7 presents the accuracy and loss values of the model after training and training after using test data.

Figure 7. Accuracy and Loss Graphics of Xception Model Developed for Classification of Pyramid Images

When the graphics are examined, it can be seen that the developed model learns quickly. It can be seen that the network continues to learn, as evident from the ups and downs until the 10th iteration. After the training of the model was completed, an accuracy rate of 100% was obtained. Table 3 presents the analysis results of the Xception model using test data.

Table 3. Analysis Results of Classification

Classes	Rec. %	Prc. %	F1-Scr. %	Acc. %
Square pyramid	100	100	100	100
Cone	100	100	100	
Triangle pyramid	100	100	100	

Figure 8 presents the confusion matrix of the recommended Xception model based on the results obtained after classification.

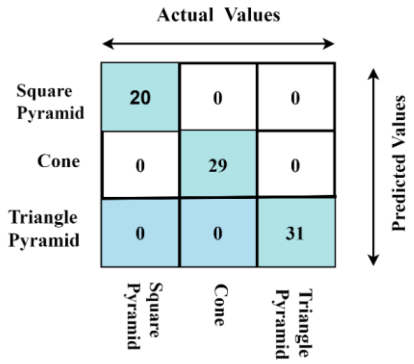

Figure 8. Confusion Matrix of Xception Model Developed to Classify Pyramid Images

In the third stage of the study, the classification of prism images, divided into four subgroups, was performed using the VGG-19-based model. Figure 9 presents the accuracy and loss rates as a result of training after classification and training using test dataset. When the graphics are examined, it can be seen that the developed model learns quickly. It can be seen that the network continues to learn, as evident from the ups and downs until the 10th iteration. After the training of the model was completed, an accuracy rate of 81% was obtained.

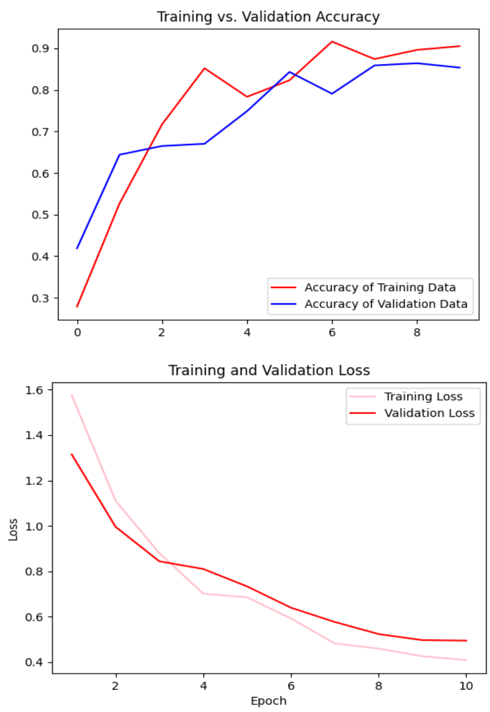

Figure 9. Accuracy and Loss Graphics of the VGG-19 Model Developed to Classify Prism Images

Table 4 shows the analysis results of the classification using the test data from the proposed VGG-19 model.

Table 4. Classification Analysis Results

Classes	Rec.%	Prc. %	F1-Scr. %	Acc. %
Rectangular prism	59	91	71	81
Cube	94	50	65	
Cylinder	90	100	95	
Triangular prism	100	94	97	

Figure 10 presents the confusion matrix of the recommended VGG-19 model based on the classification results.

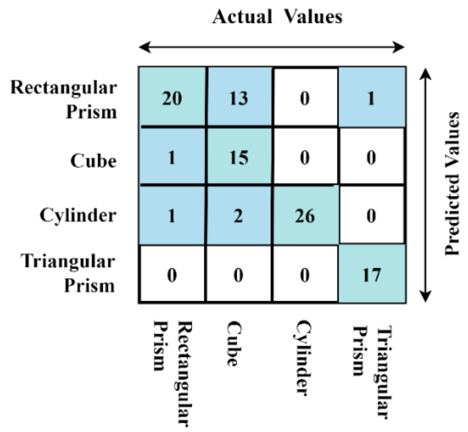

Figure 10. Confusion Matrix of the Developed VGG-19 Model to Classify Prism Images

In the fourth stage of the study, the classification of prism images consisting of 4 subgroups was carried out using the Xception based model. Figure 11 presents the accuracy and loss rates as a result of training after classification and training using test dataset. When the graphics
are examined, it can be seen that the developed model learns quickly. It can be seen that the network continues to learn, as evident from the ups and downs until the 10th iteration. After the training of the model was completed, an accuracy rate of 92% was obtained.

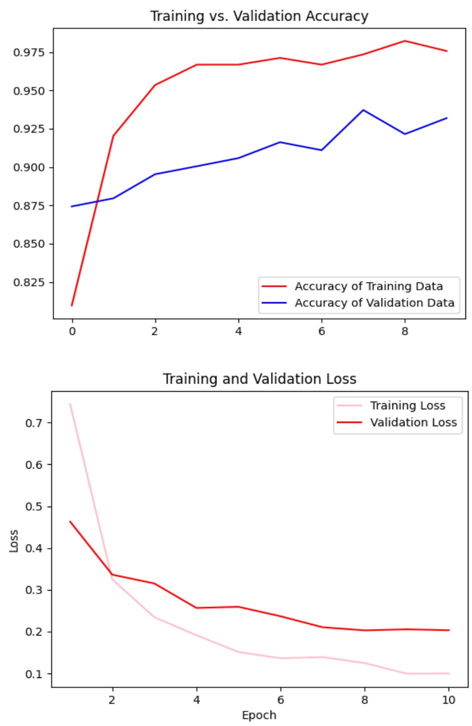

Figure 11. Accuracy and Loss Graphics of the VGG-19 Model Developed to Classify Prism Images

Table 5 shows the analysis results of the classification using the test data from the proposed Xception model.

Table 5. Analysis Results of Classification

Classes	Rec.%	Prec. %	F1-Scr.%	Acc.%
Rectangular prism	75	100	86	92
Cube	100	79	88	
Cylinder	100	100	100	
Triangle prism	100	91	95	

Figure 12 presents the confusion matrix of the recommended Xception model based on the classification results.

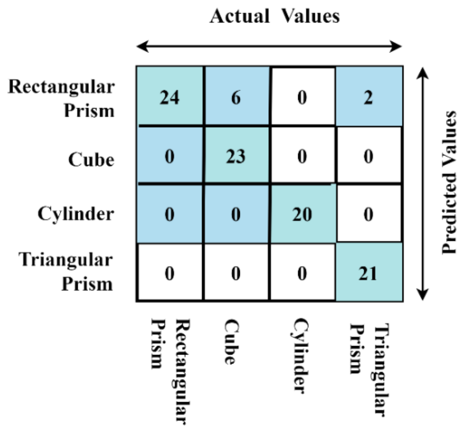

Figure 12. Confusion Matrix of the Recommended Xception Model Developed to Classify Prism Images

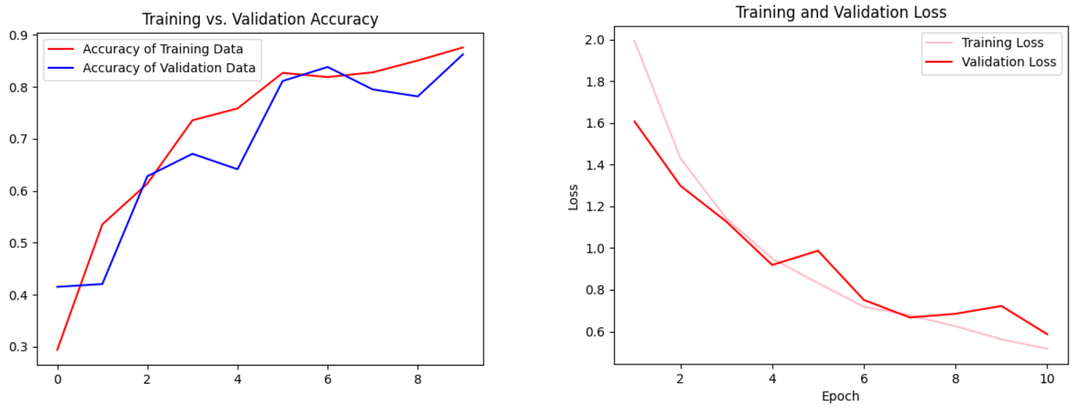

Figure 13. Accuracy and Loss Graphics of the Developed VGG-19 Model to Classify All Images in the Dataset

In the fifth stage of the study, the classification of images consisting of 8 subgroups was carried out using the VGG-19 based model. Figure 13 presents the accuracy and loss rates as a result of training after classification and training using test dataset. When the graphics are examined, it can be seen that the developed model learns quickly. It can be seen that the network continues to learn, as evident from the ups and downs until the 10th iteration. After the training of the model was completed, an accuracy rate of 84% was obtained.

Table 6 presents the classification results obtained using the test data from the proposed VGG-19 model.

Table 6. Classification Results Analysis

Classes	Rec.%	Prc. %	F1-Scr. %	Acc. %
Rectangular prism	71	89	79	84
Square pyramid	83	94	88	
Cone	96	100	98	
Cube	82	62	71	
Sphere	100	100	100	
Cylinder	95	91	93	
Triangle pyramid	100	55	71	
Triangular prism	64	95	76	

Figure 14 presents the confusion matrix of the recommended VGG-19 model according to the classification results.

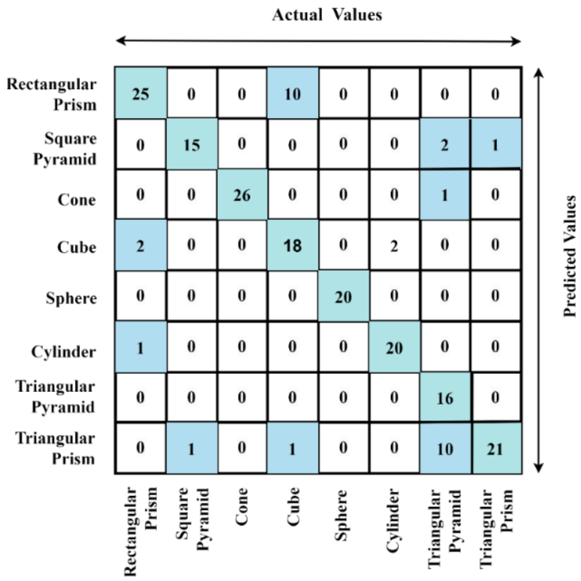

Figure 14. Confusion Matrix of VGG-19 Model Developed to Classify All Images in Dataset

In the last stage of the study, the classification of images consisting of 8 subgroups was carried out using the Xception based model. Figure 15 presents the accuracy and loss rates as a result of training after classification and training using test dataset. When the graphics are examined, it can be seen that the developed model learns quickly. It can be seen that the network continues to learn, as evident from the ups and downs until the 10th iteration. After the training of the model was completed, an accuracy rate of 95% was obtained.

Figure 15. Accuracy and Loss Graphics of Xception Model Developed to Classify All Images in Dataset

Table 7 presents the analysis results obtained from the test data of the proposed Xception model.

Table 7. Analysis results of classification

Classes	Rec. %	Prc. %	F1-Scr. %	Acc. %
Rectangular prism	76	100	86	95
Square pyramid	92	96	94	
Cone	100	100	100	
Cube	100	73	84	
Sphere	100	100	100	
Cylinder	100	100	100	
Triangle pyramid	100	100	100	
Triangular prism	94	89	92	

Figure 16 presents the confusion matrix of the recommended Xception model according to the classification results.

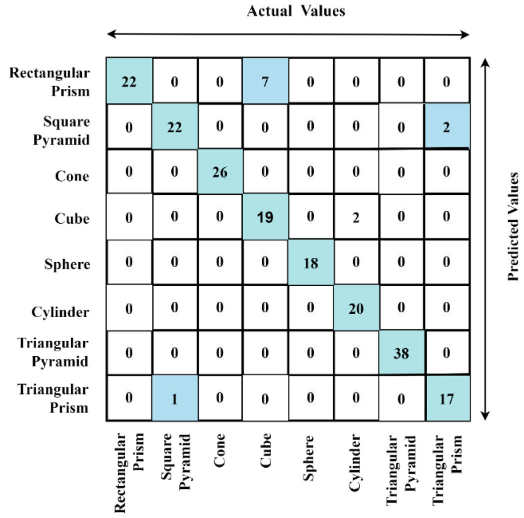

Figure 16. Confusion Matrix of Xception Model Developed to Classify All Images in Dataset

The performance of the VGG-19 and Xception models developed in this study was evaluated using a custom dataset of hand-drawn geometric figures. In the initial phase, classification was conducted on images categorized under pyramids. The VGG-19 model achieved an accuracy of 81%, whereas the Xception model reached a perfect accuracy of 100% in this category. In the second phase, classification was extended to prism images, where VGG-19 yielded 81% accuracy and Xception achieved 92%. Finally, when the models were tested across the complete dataset comprising spheres, pyramids, and prisms, VGG-19 recorded 84% accuracy, while Xception attained 95%. These results consistently highlight the superior performance of the Xception architecture.

When comparing these outcomes to existing studies in the field, varying conclusions can be observed. Patil and Golellu [52], for instance, found VGG-16 to outperform VGG-19 and Xception, while Özdemir and Arslan [53] reported higher performance with InceptionV3. Similarly, Humayun et al. [54] identified VGG-16 as the most effective model in their comparative analysis. These discrepancies underscore the influence of dataset characteristics and hardware configurations on model performance, highlighting the importance of contextual evaluation.

In terms of practical applications, the integration of these models into educational environments holds promising potential. Teachers can utilize simplified versions of such models through mobile applications or browser-based platforms to instantly assess and provide feedback on students' geometric drawings. These tools could be used during in-class activities by allowing students to sketch shapes digitally and receive automated evaluations, thus reinforcing spatial reasoning skills in real-time. In remote learning contexts, students can upload images of their drawings to receive instant classification and feedback, which may enhance engagement and comprehension outside traditional classroom settings. Moreover, these AI-powered systems can be integrated into digital whiteboards or geometry learning platforms, enabling teachers to demonstrate correct shape construction and offer error-correction suggestions interactively. By embedding deep learning into formative assessment practices, educators can create a more inclusive and adaptive learning experience tailored to individual student needs.

The utility of hand-drawn image datasets, such as MNIST, has been widely acknowledged in the literature for similar tasks. Studies by Garin & Tauzin [55], Grover & Toghi [56], Iyer [57], Kadam et al. [58], and Prabhu [59] illustrate the effectiveness of convolutional neural networks (CNNs) in classifying handwritten figures. Audibert and Maschio [60] introduced a CNN-based system named FINNger to identify mathematical hand signs in children, achieving an accuracy of 92%. They emphasized challenges related to background noise and spatial separation in image data but nonetheless demonstrated the pedagogical value of AI systems in mathematics education. Such findings further validate the current study's emphasis on the educational utility of AI-enhanced tools in geometry instruction. Table 8 presents a comparative analysis of the results obtained in this study with those reported in the literature using similar datasets.

Table 8. Comparative analysis of the results

Authors	**Method**	**Accuracy %**
Zhang L. [61]	Dual-channel CNN	%73
Hayat et al. [62]	Deep convolutional neural network-based (DCNN)	%94
Ali et al. [63]	Sketch-DeepNet	%95
This Study	**VGG19**	**%84**
This Study	**Xception**	**%95**

As shown in Table 7, the Xception model developed in this study achieved an accuracy rate comparable to the highest reported in the literature. While the VGG-19 model yielded slightly lower performance, it still demonstrated competitive results. These findings highlight the effectiveness of deep learning architectures, particularly Xception, in the classification of hand-drawn geometric figures, and underscore the potential of integrating such models into educational technologies.

The number of images in the dataset plays a crucial role in the training of deep learning models. To evaluate the performance of the models developed in this study, the researchers created a custom dataset. However, the relatively small number of images in the dataset is considered a limitation. Hand-drawn images by different people were used in the study to overcome this issue. In addition, it is thought that the study will contribute to the literature by sharing the datasets created from this and similar studies on different digital platforms. Due to the universal nature of geometry, it is possible to further improve the model developed by using the hand-drawn images by students and teachers in different countries, together. In addition, studies similar to this can be carried out in different learning areas apart from the geometry learning area. Similar studies can be conducted using different deep learning models and the results can be compared. In line with the results obtained, this study and similar studies can provide great benefits to students and learning in mathematics lessons in computer environments and mobile devices through applications.

4. CONCLUSION

Distance education is becoming widespread today. In this context, efforts to find solutions to problems encountered in mathematics lessons in distance education are increasing. In particular, drawing 3D objects in a computer environment creates problems related to spatial thinking for the learner. It is possible to come across different solutions in the literature to overcome these limitations. The study aimed to determine the geometric shapes using deep learning models by using the dataset containing the 3D hand-drawn geometric shapes in the computer environment. In this context, two models have been developed based on VGG-19 and Xception architectures, which are popular deep learning models today. The performances of the developed models were measured using the dataset in the study, and the two models were compared based on the obtained values, and it was determined that the Xception model provided better results. The Xception model classified geometric objects with 95% accuracy in general classification. Considering the results obtained, deep learning methods can be used to solve problems encountered in distance education.

Considering the obtained results, deep learning methods can be employed to address the challenges encountered in distance education. Consequently, the use of artificial intelligence, especially deep learning models, in geometry classes serves as a robust solution to the difficulties faced in drawing and visualizing geometric shapes in digital environments. As demonstrated in the study, the high accuracy rates achieved by these models underscore their potential to revolutionize mathematics education, making it more accessible, engaging, and effective for students. Teachers can leverage these technologies not only to enhance instructional methods but also to alleviate the stress associated with online courses and ultimately create a conducive environment for effective learning.

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