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NONDIFFERENTIABLE DESIRABILITY FUNCTIONS: DERIVATIVE FREE OPTIMIZATION WITH MATLAB/NOMAD

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ABSTRACT. Nondifferentiable desirability functions are one of the most preferred multi-response optimization methods in nonlinear robust parameter design. Their nondifferentiability makes the optimization problem hard to solve and researchers and scientists look for new softwares and new desirability function structures to overcome this problem. In this study, we suggest a new implementation of derivative free mash adaptive direct search algorithm (MADS) with MATLAB/NOMAD to nondifferentiable desirability functions. For doing this, we need to model the optimization problem of desirability functions as a mixed-integer nonlinear optimization program (MINLP) by introducing a new binary variable to the model. This integer shows the side of the two-sided desirability function which is active. Hence, the model of our problem becomes nondifferentiable nonconvex MINLP. We show our implementation on three well-known optimization problem from the multi-response optimization literature. We finally conclude with an outlook and future research projects.

1. INTRODUCTION

Taguchi studied quality improvement through robust design which made the field of robust design widespread among industrial quality engineers [32, 33]. Robust design aims at designing a product or process to which the effect of noise factors is minimum. Robust design is important in terms of minimizing variance of a product or process performance while keeping the difference between mean and the target of the responses (output variable) as small as possible which improves the quality during the design phase of a product or process. If there are more than one responses, we must solve a multi-response optimization problem to take into account all the characteristics of a product or process and to obtain an optimal factor setting reflecting the optimal properties of a product or process, simultaneously.

In the literature, there are many methods developed for multi-response optimization problems. Since these methods stem from multi objective optimization, they are classified according to articulation of preference information of a decision maker: no articulation,

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prior articulation, interactive articularion and posterior articularion [18, 19, 20, 21, 22, 23, 24, 25, 27, 30]. Prior articulation methods are the most popular methods where pitfalls needs to be studied and overcome. These methods collect the decision makers preferences and articulates before the optimization algorithm run.

The so called loss functions which is a prior articulation method in multi-response optimization ignore mean and variance information of responses to handle the different scales of responses in computations. Another important prior articulation method used for solving multi-response optimization is desirability functions. This method uses mean and variance information and has been improved a lot in the last decades. Desirability functions method overcomes the different scales of responses by assigning a desirability function, which takes value between 0 and 1, to each response and then combine them to an overall desirability function to be optimized.

Desirability functions developed by Derringer and Suich has a drawback of containing nondifferentiable points [13, 14] occuring at the target points of two-sided desirability functions [1]. They are obtained by the composition of nonsmooth piecewise functions with signomial response functions (depend on factor variables (input vector)). Before, optimization of desirability function was solved by either direct search techniques or by smoothing nondifferentible desirability functions with polynomial approximations or by changing the formula of desirability functions. Therefore new advances in numerical optimization made us suggest using these new methods for the optimization of desirability functions. In [1], we developed nonconvex model of desirability functions [7] and we obtained continuous optimization relaxations and convex relaxations of this model. We extend this nonconvex model to MIP relaxations [34] in an upcoming paper [5]. These relaxations are solved by GAMS/CPLEX [12], GAMS/BARON [6] and GAMS/CONOPT [11] in [1] and [5].

In [2], we made a topological generalization of desirability functions used in practice to provide the robust optimization [35] of the nondifferentiable desirability functions and solved it with generalized semi-definite programming and disjunctive optimization by using GAMS/BARON. In [4], we analyzed the topological structure of generalized desirability functions to explain the mechanism behind these functions that enables researchers and scientists to develop new desirability functions with better structural properties.

In this paper, we solve a mixed-integer nonlinear optimization model for the desirability functions first given in [1, 10]. We apply derivative-free optimization techniques (mesh adaptive direct search) [10] to desirability functions that is mentioned in Table 1 of [1]. For the researchers and scientists, who do not use GAMS environment [15] and its solvers, this method is available under MATLAB and NOMAD solver [26, 29]. This method is easy to use and have proven superiority in the literature for nonconvex MINLP [7]. In this paper, we show implementation of MATLAB/NOMAD solver on nonsmooth nonconvex MINLP [28] formulation of desirability functions of Derringer and Suich for wire-bonding process optimization problem [9] which includes quantitative variables [8], for tire-tread compound problem [13] and for a chemical process problem [17].

In this study, after introducing notation of desirability functions in Section 2. We will give numerical examples' statements and results in Section 3. We will finish with a conclusion and outlook to the future which will be given in Section 4. We present response models in Appendix A and MATLAB/NOMAD implementations in appendix B.

1.1. **Derringer and Suich Type of DFs.** An average or expected value of a response can be written as $Y_{ji} = f_j(x_1, x_2, ..., x_n) + \epsilon_{ji}$ $(i = 1, 2, ..., n)$, $(j = 1, 2, ..., m)$ where Y_{ji} is measured through design of experiment. These average value Y_{ji}s are related to factor variables by the polynomial expressions f_j with expected value of $\epsilon_{ji} = 0$ and covariance matrix $\alpha^2 I$. Polynomial expressions f_j are approximated through polynomial functions. Here, expected value of responses are estimated by $\hat{Y}_j$ using regression by second degree polynomials for better fit. In this study, we will show estimators of expected value of responses by Y_j where $Y_j(\mathbf{x}) = z(\mathbf{X})\beta_j$ with β_j is the vector of regression coefficient estimates and $z(\mathbf{x})$ is the vector of regression variables, i.e,
$(1, x_1, x_2, x_3, x_1x_2, x_1x_3, x_2x_3, x_1^2, x_2^2, x_3^2, x_1x_2x_3, ...)$. Here, β_j is the mean of the unique least squares estimator $\hat{\beta} = (X'X)^{-1}X'Y$ since $X'X$ is always a nonsingular matrix.

Since there are more than one-response in a multi-response optimization problem, desirability functions converts these response values to desirability values and combine them by geometic mean to obtain a single objective function. This objective function is optimized to find the best trade-off between responses. There are two types of desirability functions: one sided (for smaller-the-better responses type and larger-the-better type responses) and two-sided (for nominal-is-the-best type responses) [1]. The desirability functions considered in this study are of Derringer and Suich's type [13]. They can be linear or nonlinear; usually piecewise smooth including a finite number of nondifferentiable points at their target value, where the maximum desirability occurs. The optimization of overall desirability functions becomes a complicated task when there are two-sided individual desirability functions in the problem. Below, we give the optimization problem of overall desirability function as a nonsmooth MINLP problem:

$$
\begin{array}{l}
\text{maximize} \quad D(\mathbf{y}, \mathbf{z}) \\
\text{subject to} \\
\quad x_i \in [-1, 1] \quad (i = 1, 2, \ldots, n), \\
\quad 0 \leq d_j(y_j, z_j) \leq 1, \\
\quad 0 \leq d_j(y_j) \leq 1, \\
\quad z_j \in \{0, 1\} \quad (j = 1, 2, \ldots, m)
\end{array}
\tag{1.1}
$$

where $D(\mathbf{y}, \mathbf{z}) = (d_1(y_1, z_1)^{(w_1)} \cdot d_2(y_2, z_2)^{(w_2)} \cdot \ldots \cdot d_m(y_m, z_m)^{(w_m)})^{(\frac{1}{w_1+w_2+\ldots+w_m})}$. Here, $d_j(y_j, z_j) = z_j((y_j - l_j)/(t_j - l_j)) + (1 - z_j)((y_j - u_j)/(t_j - u_j))$ $(j = 1, 2, \ldots, m)$ for two-sided desirability functions, $d_j(y_j) = (y_j - l_j)/(t_j - l_j)$ for upper-the best one-sided desirability functions, $d_j(y_j) = (y_j - u_j)/(t_j - u_j)$ for lower-is-the- better one-sided desirability functions and $d_j(y_j) = (y_j - l_j)/(t_j - l_j)$ for upper-is-the-better one-sided desirability functions. Here, l_j,u_j,t_j corresponds to lower, upper and target of a response $y_j = Y_j(\mathbf{x})$ $(j = 1, 2, \ldots, m)$.

2. Examples and Results

In this Section, we solve three optimization problems with Derringer and Suich nondifferentiable desirability functions. We state the response models and necessary information in Appendix A. The problem given in Example 1 is solved by a modified desirability functions approach using Microsoft Excel GRG solver [9]. The problem given in Example 2 is solved by univariate direct search implemented under FORTRAN [13]. Lastly, Example 3 is solved by a hybrid genetic algorithm in combination with pattern search [17]. In this study, we obtained responses' models with better fits than those previously done by Design-Expert [1, 2] and solved the optimization problem of overall desirability by mesh adaptive direct search (MADS) implemented under MATLAB/NOMAD [26, 29].

2.1. **Numerical example: Wire Bonding Process Optimization.** The problem of wire bonding process optimization in semiconductor manufacturing has originally been presented in [9]. We use the 3 response models case given in [1]. In this problem, the overall desirability function $D^{\mathbf{Y}}(\mathbf{x},\mathbf{z}) = D(\mathbf{y},\mathbf{z}) = D(\mathbf{Y}(\mathbf{x}),\mathbf{z})$ with $y_j = Y_j(\mathbf{x})$ $(j = 1, 2, 3)$ is:

$$\begin{gathered}
D^{\mathbf{Y}}(\mathbf{x},\mathbf{z}) = ((((z_1(174.9333 + 23.3750x_2 + 3.6250x_3 - 19.0000x_2x_3 - 185)/(190 - 185))+ \\
((1 - z_1)(174.9333 + 23.3750x_2 + 3.6250x_3 - 19.0000x_2x_3 - 195)/(190 - 195)))\cdot \\
(z_2((154.8571 + 8.5000x_1 + 30.6250x_2 + 7.8750x_3 - 12.8571x_1^2 + 11.2500x_1x_2 - 185)/(190 - 185))+ \\
(1 - z_2)((154.8571 + 8.5000x_1 + 30.6250x_2 + 7.8750x_3 - 12.8571x_1^2 + 11.2500x_1x_2 - 195)/(190 - 195)))\cdot \\
(z_3((140.2333 + 5.3437x_1 + 18.2500x_2 + 19.5938x_3 - 170)/(185 - 170))+ \\
(1 - z_3)((140.2333 + 5.3437x_1 + 18.2500x_2 + 19.5938x_3 - 195)/(185 - 195))))^(1/3))
\end{gathered} \tag{2.1}$$

where the decision variables are $\mathbf{x} = (x_1, x_2, x_3)$ and $\mathbf{z} = (z_1, z_2, z_3)$ since all the desirability functions of the problem are two-sided. We added the nonlinear constraints of individual desirability functions being 0 and 1 to the model on which MATLAB/NOMAD is implemented.

Table 1. Optimal solutions of the Wire Bonding Process Optimization problem with 3 responses with MATLAB/NOMAD [29].

Method	$(x_1^0, x_2^0, x_3^0, z_1^0, z_2^0, z_3^0)$	$(x_1^*, x_2^*, x_3^*, z_1^*, z_2^*, z_3^*)$
1	$(0, 0, 0, 0, 0, 0)$	$(-0.4854, 0.9945, 1, 1, 1, 1)$
2	$(0.0920, 1.0000, 0.8170, 1, 1, 1)$	$(0.9999, 0.8317, 0.5932, 1, 1, 1)$
3	$(1.0000, 0.8630, 0.5880, 1, 0, 1)$	$(1.0000, 0.8630, 0.5880, 1, 0, 1)$

	$(d_1(y_1^*), d_2(y_2^*), d_3(y_3^*))$	D^*
1	$(-0.4182, -0.8795, 0.3589)$	infeasible
2	$(0.4301, 0.9999, 0.1585)$	0.4085
3	$(0.5223, 0.7492, 0.1880)$	0.4190

2.1.1. *Results.* We run MATLAB/NOMAD [29] to solve this nonsmooth MINLP problem. We use three different initial points to find the global optimal. In Table 1, on the first line, we give $(0, 0, 0, 0, 0, 0)$as the initial point and MATLAB/NOMAD finds an infeasible solution. In the second line, we give $(0.0920, 1.0000, 0.8170, 1, 1, 1)$ as the initial point which is the local solution produced by GAMS/CONOPT in combination with MSG (see [1]) and MATLAB/NOMAD converges with a deep local solution which is very close to global optimal. When we give the global optimal (that we know from the literature [1]) as the initial point $(1.0000, 0.8630, 0.5880, 1, 0, 1)$, MATLAB/NOMAD finds the global optimal given in the third line. We present the MATLAB/NOMAD implementation of this problem in Appendix B.

2.2. **Numerical example: Tire Tread Compound Optimization.** The problem of tire tread compund optimization has originally been presented in [13]. We use the 4 response models given in [1] (see Appendix A). In this problem, the overall desirability function $D^{\mathbf{Y}}(\mathbf{x},\mathbf{z}) = D(\mathbf{y},\mathbf{z}) = D(\mathbf{Y}(\mathbf{x}),\mathbf{z})$ with $y_j = Y_j(\mathbf{x})$ $(j = 1, 2, 3, 4)$ is:

$$D^{\mathbf{Y}}(\mathbf{x},\mathbf{z}) = ((((139.1192 + 16.4936*x_1 + 17.8808*x_2 + 10.9065*x_3 - 4.0096*x_1*x_1 - 3.4471*x_2*x_2- 1.5721*x_3*x_3 + 5.1250*x_1*x_2 + 7.1250*x_1*x_3 + 7.8750*x_2*x_3 - 120)/(170-120))* ((1261.1331 + 268.1511*x_1 + 246.5032*x_2 + 139.4845*x_3 - 83.5659*x_1*x_1 - 124.8155*x_2*x_2+ 199.1818*x_3*x_3 + 69.3750*x_1*x_2 + 94.1250*x_1*x_3 + 104.3750*x_2*x_3 - 1000)/(1300-1000))* (z_1*((400.3846 - 99.6664*x_1 - 31.3964*x_2 - 73.9190*x_3 + 7.9327*x_1*x_1 + 17.3076*x_2*x_2+ +0.4328*x_3*x_3 + 8.7500*x_1*x_2 + 6.250*x_1*x_3 + 1.2500*x_2*x_3 - 400)/(500-400))+ (1-z_1)*((400.3846 - 99.6664*x_1 - 31.3964*x_2 - 73.9190*x_3 + 7.9327*x_1*x_1 + 17.3076*x_2*x_2+ +0.4328*x_3*x_3 + 8.7500*x_1*x_2 + 6.250*x_1*x_3 + 1.2500*x_2*x_3 - 600)/(500-600)))* (z_2*((68.9096 - 1.4099*x_1 + 4.3197*x_2 + 1.6348*x_3 + 1.5577*x_1*x_1 + 0.0577*x_2*x_2- 0.3173*x_3*x_3 - 1.6250*x_1*x_2 + 0.1250*x_1*x_3 - 0.2500*x_2*x_3 - 60)/(67.5-60))+ (1-z_2)*((68.9096 - 1.4099*x_1 + 4.3197*x_2 + 1.6348*x_3 + 1.5577*x_1*x_1 + 0.0577*x_2*x_2- 0.3173*x_3*x_3 - 1.6250*x_1*x_2 + 0.1250*x_1*x_3 - 0.2500*x_2*x_3 - 75)/(67.5-75))))^(1/4)) \tag{2.2}$$

where the decision variables are $\mathbf{x} = (x_1, x_2, x_3)$ and $\mathbf{z} = (z_1, z_2)$ since there are two two-sided desirability function. We added the nonlinear constraints of individual desirability functions being 0 and 1 to the model on which MATLAB/NOMAD is implemented.

TABLE 2. Optimal solutions of the Wire Bonding Process Optimization problem with 3 responses with MATLAB/NOMAD [29].

Method	$(x_1^0, x_2^0, x_3^0, z_1^0, z_2^0)$	$(x_1^*, x_2^*, x_3^*, z_1^*, z_2^*)$
1	$(0,0,0,0,0)$	$(-0.0519, 0.1507, -0.8662, 1.0000, 0)$
2	$(0.0610, 0.0500, -0.8150, 1, 0)$	$(-0.0525, 0.1480, -0.8684, 1, 0)$
3	$(-0.0520, 0.1480, -0.8690, 1, 0)$	$(-0.0525, 0.1482, -0.8683, 1, 0)$

	$(d_1(y_1^*), d_2(y_2^*), d_3(y_3^*), d_4(y_4^*))$	D^*
1	$(0.1899, 1, 0.6564, 0.9285)$	0.5833
2	$(0.1886, 1, 0.6595, 0.9307)$	0.5833
3	$(0.1887, 1, 0.6593, 0.9305)$	0.5833

2.2.1. *Results.* We run MATLAB/NOMAD [29] to solve this nonsmooth MINLP problem. We use three different initial points to find the global optimal. In Table 2, on the first line, we give $(0,0,0,0,0,0)$as the initial point (an arbitrary point) and MATLAB/NOMAD finds the global optimal. In the second line, we give $(0.0610, 0.0500, -0.8150, 1, 0)$ as the initial point which is the local solution produced by GAMS/CONOPT in combination with MSG (see [1]) and MATLAB/NOMAD converges to global optimal. When we give the global optimal (that we know from the literature [1]) as the initial point $(-0.0520, 0.1480, -0.8690, 1, 0)$, MATLAB/NOMAD finds the global optimal given in the third line. We present the MATLAB/NOMAD implementation of this problem in Appendix C. Hence, in all three cases, MATLAB/NOMAD finds the global optima for this problem.

2.3. **Numerical example : A Chemical Process Optimization.** The problem of a chemical process optimization has originally been presented in [17]. We use the 3 response models case given in [1] (see appendix A). In this problem, the overall desirability function $D^{\mathbf{Y}}(\mathbf{x},\mathbf{z}) = D(\mathbf{y},\mathbf{z}) = D(\mathbf{Y}(\mathbf{x}),\mathbf{z})$ with $y_j = Y_j(\mathbf{x})$ $(j = 1, 2, 3)$ is:

$$D^{\mathbf{Y}}(\mathbf{x}, \mathbf{z}) = 0.7 * ((79.940 + 0.995 * x_1 + 0.515 * x_2 - 0.1376 * x_1 * x_1 - 1.001 * x_2 * x_2 + 0.250 * x_1 * x_2 - 78.5)/(80 - 78.5)) * (0.2 * z * ((69.552 - 0.948 * x_2 - 6.598 * x_2 * x_2 - 62)/(65 - 62)) + 0.2 * (1 - z) * ((69.552 - 0.948 * x_2 - 6.598 * x_2 * x_2 - 68)/(65 - 68))) * 0.1 * ((3386.2 + 205.10 * x_1 + 177.4 * x_2 - 3450)/(3100 - 3450)) \quad (2.3)$$

where the decision variables are $\mathbf{x} = (x_1, x_2)$ and $\mathbf{z} = (z)$ since there is only one two-sided desirability function. We added the nonlinear constraints of individual desirability functions being 0 and 1 to the model on which MATLAB/NOMAD is implemented.

2.3.1. *Results.* We run MATLAB/NOMAD [29] to solve this nonsmooth MINLP problem. We use two different initial points to find the global optimal. In Table 3, on the first line, we give $(0, 0, 0)$ as the initial point (an arbitrary point) and MATLAB/NOMAD converges. When we give the global optimal (that we know from the literature [2]) as the initial point $(0.1723, -0.8516, 0)$, MATLAB/NOMAD converges. We present the MATLAB/NOMAD implementation of this problem in Appendix D. Here, we note that although MATLAB/NOMAD converges, it gives an inferior solution than found in [17].

Table 3. Optimal solutions of the Chemical Process Optimization problem with 3 responses with MATLAB/NOMAD [29].

Method	(x_1^0, x_2^0, z^0)	(x_1^*, x_2^*, z^*)
1	$(0, 0, 0)$	$(-0.3774, -0.8865, 0)$
2	$(0.1723, -0.8516, 0)$	$(0.3774, -0.8865, 0)$

	$(d_1(y_1^*), d_2(y_2^*), d_3(y_3^*))$	D^*
1	$(0.2189, 0.1862, 0.0410)$	0.0017
2	$(0.2189, 0.1862, 0.0410)$	0.0017

3. Conclusion and Future Outlook

In this work, we investigate the derivative free optimization [10] to find out advantages of them over the global optimization approaches on wire bonding process optimization problem, tire tread compund problem and a chemical process problem. Although, MATLAB/NOMAD is a nonconvex MINLP solver, it highly depends on initial point selection. On wire bonding process optimization problem, we tried three different initial points to find out if it gives the global optimal however, it did not produce the global optimal unless the global optimal is the initial point itself. On tire tread compound problem, MATLAB/NOMAD succeed to find the global optimal whatever the initial point is. On chemical process optimization problem, MATLAB/NOMAD converges, however the optimal value is inferior than the results reported in the literature.

Another important issue which we faced with in our implementation is related with bound selection of decision variables, which effects the convergence of MATLAB/NOMAD. Anyway, MATLAB/NOMAD is a useful software when we know the global optima. This is important for the researchers and scientists who do not have the global optimizers available.

In this study, our work describes a new approach to model nondifferentiable functions via integer variables using a new tool. The methodology is new in the sense that we try different initial points one of which is global optimal. The solution process can be improved further by studying selections for bounds and initial values. We tested our computational approaches on different examples from the literature including one-sided and two-sided desirability functions.

In the future, it is possible to implement the desirability function which includes more than one nondifferentiable points [9, 3] since we have already tested global optimization [1], convex optimization [1], semi-infinite programming [2], mixed integer linear programming [5] and derivative free optimization on desirability functions including one nondifferentiable point. It is also possible to apply our derivative free approach to signomial [31] cases of desirability functions. This study is also connected in a broder sense to optimal control.

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Appendix A. Responses of Wire bonding process optimization problem

$$
\begin{aligned}
Y_1(\mathbf{x}) &= 174.9333 + 23.3750x_2 + 3.6250x_3 - 19.0000x_2x_3,\\
Y_2(\mathbf{x}) &= 154.8571 + 8.5000x_1 + 30.6250x_2 + 7.8750x_3 - 12.8571x_1^2 + 11.2500x_1x_2,\\
Y_3(\mathbf{x}) &= 140.2333 + 5.3437x_1 + 18.2500x_2 + 19.5938x_3.
\end{aligned}
\tag{A.1}
$$

Corresponding lower, target and upper values is given in 4

Table 4. Desirability Parameters of the responses for the Wire Bonding Problem [9].

	l_j	t_j	u_j	$d_j(l_j)$	$d_j(t_j)$	$d_j(u_j)$
y_1	185	190	195	0	1	0
y_2	185	190	195	0	1	0
y_3	170	185	195	0	1	0

Responses of Tire tread compound problem

$$
\begin{aligned}
Y_1(\mathbf{x}) = \quad & 139.1192 + 16.4936x_1 + 17.8808x_2 + 10.9065x_3 - 4.0096x_1x_1 - 3.4471x_2x_2\\
& -1.5721x_3x_3 + 5.1250x_1x_2 + 7.1250x_1x_3 + 7.8750x_2x_3,\\
Y_2(\mathbf{x}) = \quad & 1261.1331 + 268.1511x_1 + 246.5032x_2 + 139.4845x_3 - 83.5659x_1x_1 - 124.8155x_2x_2\\
& +199.1818x_3x_3 + 69.3750x_1x_2 + 94.1250x_1x_3 + 104.3750x_2x_3,\\
Y_3(\mathbf{x}) = \quad & 400.3846 - 99.6664x_1 - 31.3964x_2 - 73.9190x_3 + 7.9327x_1x_1 + 17.3076x_2x_2\\
& +0.4328x_3x_3 + 8.7500x_1x_2 + 6.250x_1x_3 + 1.2500x_2x_3,\\
Y_4(\mathbf{x}) = \quad & 68.9096 - 1.4099x_1 + 4.3197x_2 + 1.6348x_3 + 1.5577x_1x_1 + 0.0577x_2x_2\\
& -0.3173x_3x_3 - 1.6250x_1x_2 + 0.1250x_1x_3 - 0.2500x_2x_3.
\end{aligned}
\tag{A.2}
$$

Corresponding lower, target and upper values is given in 5

TABLE 5. Desirability Parameters of the responses for the Tire tread compound problem [9].

	l_j	t_j	u_j	$d_j(l_j)$	$d_j(t_j)$	$d_j(u_j)$
y_1	120	–	170	0	1	0
y_2	1000	–	1300	0	1	0
y_3	400	500	600	0	1	0
y_4	60	67.5	75	0	1	0

Responses of a Chemical process

$$\begin{aligned} Y_1(\mathbf{x}),\ y = &\ 79.940 + 0.995x_1 + 0.515x_2 - 1.376x_1x_1 \\ &-1.001x_2x_2 + 0.250x_1x_2, \\ Y_2(\mathbf{x}),\ y = &\ 69.522 - 0.948x_2 - 6.598x_2x_2, \\ Y_3(\mathbf{x}),\ y = &\ 3386.2 + 205.10x_1 + 177.4x_2. \end{aligned} \tag{A.3}$$

Corresponding lower, target and upper values is given in 6. The weights of the responses are $0.7, 0.2$ and 0.1, respectively.

TABLE 6. Desirability Parameters of the responses for the Tire tread compound problem [9].

	l_j	t_j	u_j	$d_j(l_j)$	$d_j(t_j)$	$d_j(u_j)$
y_1	78.5	–	80	0	1	0
y_2	62	65	68	0	1	0
y_3	3100	–	3450	0	1	0

APPENDIX B. MATLAB/NOMAD IMPLEMENTATION OF WIRE BONDING PROCESS OPTIMIZATION PROBLEM

```
clc
fun=@(x)-((((x(4)*(174.9333+23.3750*x(2)+3.6250*x(3)...
-19.0000*x(2)*x(3)-185)/(190-185))+...
((1-x(4))*(174.9333+23.3750*x(2)+3.6250*x(3)...
-19.0000*x(2)*x(3)-195)/(190-195)))*...
(x(5)*((154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)...
-12.8571*x(1)^2+11.2500*x(1)*x(2)-185)/(190-185))+...
(1-x(5))*((154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)...
-12.8571*x(1)^2+11.2500*x(1)*x(2)-195)/(190-195)))*...
(x(6)*((140.2333+ 5.3437*x(1)+18.2500*x(2)...
+19.5938*x(3)-170)/(185-170))+...
(1-x(6))*((140.2333+ 5.3437*x(1)+18.2500*x(2)...
+19.5938*x(3)-195)/(185-195))))^(1/3))

%x0 = [0.0920 1.0000 0.8170 1 1 1]';
x0 = [1.0000 0.8630 0.5880 1 0 1]';
```

```
%x0 = [0 0 0 0 0 0]';
lb = [-1;-1;-1;1;0;1];
ub = [1;1;1;1;0;1];

nlcon = @(x)[((x(4)*(174.9333+23.3750*x(2)+3.6250*x(3)...
-19.0000*x(2)*x(3)-185)/(190-185))+...
((1-x(4))*(174.9333+23.3750*x(2)+3.6250*x(3)...
-19.0000*x(2)*x(3)-195)/(190-195)))
(x(5)*((154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)...
-12.8571*x(1)^2+11.2500*x(1)*x(2)-185)/(190-185))+...
(1-x(5))*((154.8571+8.5000*x(1)+30.6250*x(2)...
+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)-195)/(190-195)))...
(x(6)*((140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)-170)/(185-170))+...
(1-x(6))*((140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)-195)/(185-195)))];
cl=[0 0 0]';
cu=[1 1 1]';

xtype='CCCBBB';
opts=optiset('solver','nomad','display','iter')
Opt=opti('fun',fun,'bounds',lb,ub,'nl',nlcon,cl,cu,'xtype',xtype,'options',opts)
[x,fval,exitflag,info] = solve(Opt,x0)
```

Appendix C. MATLAB/NOMAD Implementation of Tire tread compound optimization problem

```
clc

fun=@(x) -((((139.1192+16.4936*x(1)+17.8808*x(2)+10.9065*x(3)...
-4.0096*x(1)*x(1)-3.4471*x(2)*x(2)-1.5721*x(3)*x(3)+5.1250*x(1)*x(2)...
+7.1250*x(1)*x(3)+7.8750*x(2)*x(3)-120)/(170-120))*...
((1261.1331+268.1511*x(1)+246.5032*x(2)+139.4845*x(3)...
-83.5659*x(1)*x(1)-124.8155*x(2)*x(2)+...
199.1818*x(3)*x(3)+69.3750*x(1)*x(2)+...
94.1250*x(1)*x(3)+104.3750*x(2)*x(3)-1000)/(1300-1000))*...
(x(4)*((400.3846-99.6664*x(1)-31.3964*x(2)-73.9190*x(3)...
+7.9327*x(1)*x(1)+17.3076*x(2)*x(2)+...
+0.4328*x(3)*x(3)+8.7500*x(1)*x(2)+6.250*x(1)*x(3)+...
1.2500*x(2)*x(3)-400)/(500-400))+...
(1-x(4))*((400.3846-99.6664*x(1)-31.3964*x(2)-...
73.9190*x(3)+7.9327*x(1)*x(1)+17.3076*x(2)*x(2)+...
+0.4328*x(3)*x(3)+8.7500*x(1)*x(2)+6.250*x(1)*x(3)+...
1.2500*x(2)*x(3)-600)/(500-600)))*...
(x(5)*((68.9096-1.4099*x(1)+4.3197*x(2)+1.6348*x(3)+...
1.5577*x(1)*x(1)+0.0577*x(2)*x(2)-...
0.3173*x(3)*x(3)-1.6250*x(1)*x(2)+0.1250*x(1)*x(3)-...
```

```
0.2500*x(2)*x(3)-60)/(67.5-60))+...
(1-x(5))*((68.9096-1.4099*x(1)+4.3197*x(2)+...
1.6348*x(3)+1.5577*x(1)*x(1)+0.0577*x(2)*x(2)-...
0.3173*x(3)*x(3)-1.6250*x(1)*x(2)+0.1250*x(1)*x(3)-...
0.2500*x(2)*x(3)-75)/(67.5-75))))^(1/4))

%x0 = [-0.0520 0.1480 -0.8690 1 0]';
%x0 = [0.0610 0.0500 -0.8150 1 0 ]';
x0 = [0 0 0 0 0 ]';
lb = [-1;-1;-1;0;0];
ub = [1;1;1;1;1];

nlcon = @(x)[((139.1192+16.4936*x(1)+17.8808*x(2)...
+10.9065*x(3)-4.0096*x(1)*x(1)-3.4471*x(2)*x(2)-...
1.5721*x(3)*x(3)+5.1250*x(1)*x(2)+7.1250*x(1)*x(3)...
+7.8750*x(2)*x(3)-120)/(170-120))

((1261.1331+268.1511*x(1)+246.5032*x(2)...
+139.4845*x(3)-83.5659*x(1)*x(1)-124.8155*x(2)*x(2)+...
199.1818*x(3)*x(3)+69.3750*x(1)*x(2)...
+94.1250*x(1)*x(3)+104.3750*x(2)*x(3)-1000)/(1300-1000))

(x(4)*((400.3846-99.6664*x(1)-31.3964*x(2)...
-73.9190*x(3)+7.9327*x(1)*x(1)+17.3076*x(2)*x(2)+...
+0.4328*x(3)*x(3)+8.7500*x(1)*x(2)...
+6.250*x(1)*x(3)+1.2500*x(2)*x(3)-400)/(500-400))+...
(1-x(4))*((400.3846-99.6664*x(1)...
-31.3964*x(2)-73.9190*x(3)+7.9327*x(1)*x(1)+...
17.3076*x(2)*x(2)+...
+0.4328*x(3)*x(3)+8.7500*x(1)*x(2)+6.250*x(1)*x(3)+...
1.2500*x(2)*x(3)-600)/(500-600)))

(x(5)*((68.9096-1.4099*x(1)+4.3197*x(2)+...
1.6348*x(3)+1.5577*x(1)*x(1)+0.0577*x(2)*x(2)-...
0.3173*x(3)*x(3)-1.6250*x(1)*x(2)+0.1250*x(1)*x(3)-...
0.2500*x(2)*x(3)-60)/(67.5-60))+...
(1-x(5))*((68.9096-1.4099*x(1)+4.3197*x(2)+1.6348*x(3)+...
1.5577*x(1)*x(1)+0.0577*x(2)*x(2)-...
0.3173*x(3)*x(3)-1.6250*x(1)*x(2)+0.1250*x(1)*x(3)-...
0.2500*x(2)*x(3)-75)/(67.5-75)))];

cl=[0 0 0 0]';
cu=[1 1 1 1]';
xtype='CCCBB';

opts=optiset('solver','nomad','display','iter')
Opt=opti('fun',fun,'bounds',lb,ub,'nl',nlcon,cl,cu,'xtype',xtype,'options',opts)
[x,fval,exitflag,info] = solve(Opt,x0)
```

Appendix D. MATLAB/NOMAD Implementation of a Chemical process optimization problem

```
clc

fun=@(x)(-( 0.7*((79.940+0.995*x(1)+0.515*x(2)-0.1376*x(1)*x(1)-1.001*x(2)*x(2)+...
0.250*x(1)*x(2)-78.5)/(80-78.5))*...
(0.2*x(3)*((69.552-0.948*x(2)-6.598*x(2)*x(2)-62)/(65-62))+...
0.2*(1-x(3))*((69.552-0.948*x(2)-6.598*x(2)*x(2)-68)/(65-68)))*...
0.1*((3386.2+205.10*x(1)+177.4*x(2)-3450)/(3100-3450))))

%x0 = [0.1723 -0.8516 0]';
x0 = [0 0 0]';
lb = [-1;-1;0];
ub = [1;1;0];

nlcon = @(x) [ 0.7*((79.940+0.995*x(1)+0.515*x(2)-0.1376*x(1)*x(1)-1.001*x(2)*x(2)+...
0.250*x(1)*x(2)-78.5)/(80-78.5))
(0.2*x(3)*((69.552-0.948*x(2)-6.598*x(2)*x(2)-62)/(65-62))+...
0.2*(1-x(3))*((69.552-0.948*x(2)-6.598*x(2)*x(2)-68)/(65-68)))
0.1*((3386.2+205.10*x(1)+177.4*x(2)-3450)/(3100-3450))];

cl = [0 0 0]';
cu=[1 1 1]';

 xtype='CCB';
 opts=optiset('solver','nomad','display','iter')
 Opt=opti('fun',fun,'bounds',lb,ub,'nl',nlcon,cl,cu,'xtype',xtype,'options',opts)
 [x,fval,exitflag,info] = solve(Opt,x0)
```

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