

A Dynamic Analysis of Cosmic Strings in the $f(R, T)$ Gravity Model

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Abstract— This study investigates the dynamics of cosmic strings within the framework of the $f(R, T)$ gravity model, a modified theory of gravity proposed by Harko et al. in 2011. The $f(R, T)$ theory extends Einstein's general relativity by including functions of the Ricci scalar (R) and the trace of the energy-momentum tensor (T). Using the Friedman-Robertson-Walker (FRW) universe model, the study derives the field equations and explores the behavior of the key cosmological parameters, such as the scale factor, string energy density, cosmological term, deceleration parameter, and Hubble parameter. The analytical solutions show that the universe expands continuously with the increasing scale factor values. The study also demonstrates that energy density and the cosmological term decrease over time, while the Hubble parameter exhibits a similar declining trend. The results indicate that the model aligns well with the observed accelerated expansion of the current universe. These findings provide valuable insights into the role of cosmic strings and modified gravity theories in explaining large-scale cosmic evolution.

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1. Introduction

At the beginning of the universe, the four fundamental forces (gravity, electromagnetic, strong, and weak nuclear forces) coexisted together [1]. The energy that holds these four fundamental forces together is 10^{19} GeV. As the energy in the formation of the universe decreases over time, the four forces are separated from each other [1]. As a result of the work of scientists, Grand-Unification Theory (GUT) was introduced to investigate how and why the universe was formed. The purpose is to combine the four basic forces that differ from each other according to GUT [1]. According to the theory, symmetry breaks occurred when the universe passed from the high to the low-temperature phase [1]. As a result of symmetry breaks, some topological defects have occurred in the universe [1]. These topological defects are one-dimensional strings, two-dimensional domain walls, zero-dimensional monopoles, and textures [1, 2]. Strings are one-dimensional lines that are formed when cylindrical symmetry is broken. Monopoles are cube-shaped defects that have a magnetic charge in the form of north and south when spherical symmetry is broken. Domain walls are two-dimensional membranes formed during the phase transition [1, 2]. According to GUT, it is aimed to analyze topological defects

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caused by symmetry breaks while combining four fundamental forces.

In this study, one-dimensional strings will be discussed. In recent years, cosmic strings and the study of physical processes near such strings have attracted great attention, as they lead to density distortions that contribute to galaxy formation [3]. At the same time, strings serve as seeds for large structures such as galaxies and galaxy clusters in the universe [4]. Recent studies indicate that the universe is expanding by accelerating [5]. However, the reasons for the acceleration of the universe are still not fully explained. Although Einstein's theory of general relativity, proposed in 1916, explained the subjects of slight bending of light in the Sun's gravitational field, the deviation in Mercury's orbit, and gravitational redshift, it fell short in explaining the accelerating expansion of the universe. Einstein thought the universe to be stationary and added the cosmological constant to the field equations. However, in 1929, Hubble's observations of distant galaxies revealed a redshift in their light, indicating that these galaxies were receding from Earth [6]. Thus, the expanding universe model in Hubble's studies invalidated Einstein's model of the stationary universe. After Hubble demonstrated that the universe is expanding, scientists modified Einstein's general theory of relativity and introduced some alternative gravitation theories. Some of these theories include Lyra theory, Brans-Dicke theory, $f(R)$ theory, $f(G)$ theory, and $f(R, T)$ theory. Among these, one of the most recent is the $f(R, T)$ theory, introduced by Harko et al. in 2011 [7].

Many studies have been conducted on $f(R, T)$ theory and topological defects. Gad et al. [8] studied string clouds in the theory of general relativity in 2025. Ditta et al. [9] used a spherically symmetric space-time to derive field equations including quintessence and string cloud anisotropic matter fields in the $f(Q)$ theory. Lambat and Pund [10] investigated the cosmic string for the Bianchi-I universe within the framework of $f(R, T)$ theory. Islam et al. [11] considered the solution of Einstein field equations in a higher-dimensional spherically symmetric space-time using strange quark matter bound to a string cloud with a bulk viscous fluid. Gad and Al-Jedani [12] studied perfect fluid and cosmic string clouds in the Bianchi III universe. Sobhanbabu and Vijaya Shanthi [13] obtained the Kaluza-Klein cosmological model, which includes strange quark matter attached to a string cloud in the $f(R)$ gravitational theory. Vijaya Santhi and Chinnappalanaidu [14] researched $f(R)$ theory with strange quark matter attached to a string cloud in the Kantowski-Sachs universe. Kalkan et al. [15] studied magnetized strange quark matter for the inhomogeneous universe within the framework of $f(R, T)$ theory in 2022. Kalkan and Aktaş [16, 17] have researched for a solution for $f(R, T)$ theory using the deceleration parameter for different universe models. Amet Memet and Aktaş [18] studied Tsallis holographic dark energy for the $f(R, T)$ theory in 2023. Sahoo et al. [19] have investigated the Bianchi-III and $-VI_0$ cosmological models with string fluid origin in $f(R, T)$ gravity using a time-varying deceleration parameter. Kanakavalli and Ananda Rao [3] have studied the cosmological model of Locally Rotationally Symmetric (LRS) Bianchi-I in $f(R, T)$ gravitation theory. Sharma and Singh [20] have analyzed the Bianchi type II string cosmological model with a magnetic field in $f(R, T)$ gravity. Rani et al. [21] have researched the Bianchi type III magnetized string cosmological model for the perfect fluid distribution in $f(R, T)$ gravity. Naidu et al. [22] have investigated the bulk viscous string cosmological model for the Bianchi-V space-time in $f(R, T)$ gravitational theory. Katore and Hatkar [23] have studied the Bianchi type III and Kantowski-Sachs cosmological models with the domain wall in $f(R, T)$ theory. Yılmaz [24] has studied the Kaluza-Klein cosmological model for the string cloud and the quark matter attached to the domain wall in the context of general relativity theory. Agrawal and Pawar [25] have analyzed the Bianchi-V space-time using the magnetic domain wall in $f(R, T)$ theory. Dabre and Makod [26] have used the plane-symmetric LRS Bianchi type I metric to study a viscous bulk fluid coupled to a string cloud in the $f(R)$ gravity theory framework. Zhai and Zhang [27] have investigated the solutions of a

spherically symmetric Gauss-Bonnet black hole surrounded by a string fluid cloud with the cosmological constant in higher dimensions. In $f(R)$ theory, the distribution of strange quark matter containing string clouds was studied by Dixit et al. [28] for the Kantowski-Sachs universe.

The rest of the paper is organized as follows: In section 2, the field equations of the string matter distribution for the FRW metric in $f(R, T)$ theory are obtained and solutions are found. Section 3 analyzes the homogeneous and isotropic FRW metric within $f(R, T)$ theory, which aligns with the current universe model, and derives solutions for string matter arising from topological defects. The last section discusses the need for further research.

2. $f(R, T)$ Theory and String Solutions

In 2011, Harko et al. [7] obtained a new modification theory by taking the function $f(R, T)$ depending on R and T instead of the Ricci scalar R in the Einstein-Hilbert type action function. The action integral is defined as follows:

$$S = \frac{1}{16\pi} \int (f(R, T) + 2\Lambda)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \quad (2.1)$$

Here, T is the trace of the energy-momentum tensor (T_{ik}), L_m is the Lagrangian density, g is the determinant of the metric tensor g_{ik} , and Λ is the cosmological term. Moreover, T_{ik} is defined as [7]:

$$T_{ik} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ik}} \quad (2.2)$$

From (2.2), $T_{ik} = g_{ik}L_m - 2\frac{\partial L_m}{\partial g^{ik}}$. From the variation of the action S in (2.1),

$$f_R(R, T)R_{ik} - \frac{1}{2}f(R, T)g_{ik} + (g_{ik}\square - \nabla_i\nabla_k)f_R(R, T) = 8\pi T_{ik} - f_T(R, T)T_{ik} - f_T(R, T)\Theta_{ik} + \Lambda g_{ik} \quad (2.3)$$

Here, $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$, $\square = \nabla^i\nabla_i$, ∇_i covariant derivative, and $\Theta_{ik} = -T_{ik} + \frac{1}{2}Tg_{ik}$. The standard energy-momentum tensor for string matter is as follows [3]:

$$T_{ik} = \rho u_i u_k - \lambda x_i x_k \quad (2.4)$$

where ρ is the energy density, λ is the string tension, u^i is the four-velocity vector, and x^i represents the string direction. These vectors satisfy the conditions $u_i u^i = -1$, $x_i x^i = 1$, and $x_i u^i = 0$. From (2.4), the following equality is obtained [29]:

$$\Theta_{ik} = \rho g_{ik} + 2T_{ik} \quad (2.5)$$

In 2011, Harko et al. [7] proposed three different $f(R, T)$ functions for solutions. In this study, we analyze solutions using the function in the form of $f(R, T) = R + 2f(T)$, as proposed by Harko et al. [7], where $f(T)$ is an arbitrary function. Using (2.3) and (2.5), the field equations in $f(R, T)$ theory are obtained as follows:

$$R_{ik} - \frac{1}{2}Rg_{ik} = 8\pi T_{ik} + 2f'(T)T_{ik} + [f(T) + 2\rho f'(T)]g_{ik} + \Lambda g_{ik} \quad (2.6)$$

Here, $f'(T) = \frac{df(T)}{dT}$. If μ is constant, then $f(T) = \mu T$ in (2.6). In this case, (2.6) becomes as follows:

$$R_{ik} - \frac{1}{2}Rg_{ik} = T_{ik}(8\pi + 2\mu) + (\mu T + 2\rho\mu)g_{ik} + \Lambda g_{ik} \quad (2.7)$$

In this section, the homogeneous and isotropic FRW universe model that is largely consistent with today's universe is investigated. The space-time metric of the FRW universe model in the coordinates

(r, θ, ϕ, t) is defined as follows:

$$ds^2 = \frac{A^2}{1 - kr^2} dr^2 + A^2 r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) - dt^2 \quad (2.8)$$

where $k \in \{-1, 0, 1\}$ and A is a function of cosmic time. From (2.4), (2.7), and (2.8), the following field equations in $f(R, T)$ theory are obtained:

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{k}{A^2} = (8\pi + 3\mu)\lambda - \rho\mu + \Lambda \quad (2.9)$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{k}{A^2} = \lambda\mu - \rho\mu + \Lambda \quad (2.10)$$

and

$$\frac{3\dot{A}^2}{A^2} + \frac{3k}{A^2} = (8\pi + \mu)\rho - \lambda\mu + \Lambda \quad (2.11)$$

Here, the dot (\cdot) denotes the derivative with respect to cosmic time t . From (2.9)-(2.11), the string tension is obtained as follows:

$$\lambda = 0 \quad (2.12)$$

From (2.10) and (2.11), the energy density of the string and the cosmological term are found as follows:

$$\rho = -\frac{\ddot{A}}{(4\pi + \mu)A} + \frac{\dot{A}^2}{4\pi + \mu} + \frac{k}{(4\pi + \mu)A^2} \quad (2.13)$$

and

$$\Lambda = \frac{(8\pi + \mu)\ddot{A}}{(4\pi + \mu)A} + \frac{(4\pi + 2\mu)(\dot{A}^2 + k)}{(4\pi + \mu)A^2} \quad (2.14)$$

Since ρ and Λ depend on A , we can use an auxiliary equation to obtain the exact solution of the field equations. Moreover, (2.11)-(2.13) consist of three equations in four unknowns (A , λ , ρ , and Λ). As the auxiliary equation, we can take the Hubble parameter as in [30]:

$$H = \frac{\beta}{\sqrt{t + \alpha}} \quad (2.15)$$

Here, α and β are constant values. Since the Hubble parameter is always greater than zero, it must be $\beta > 0$. The deceleration parameter is defined with the help of the Hubble parameter as follows:

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1$$

From (2.15), the deceleration parameter is obtained as follows:

$$q = -1 + \frac{1}{2\beta\sqrt{t + \alpha}} \quad (2.16)$$

The value that makes the deceleration parameter zero is known as the transit time t_{tr} . From (2.16), it follows that $t_{tr} = \frac{1}{4\beta^2} - \alpha$. Since $t_{tr} > 0$, it must be $\beta\sqrt{\alpha} < \frac{1}{2}$, as shown in [30]. Since $H = \frac{\dot{A}}{A}$, the metric potential A is obtained as follows:

$$A = ce^{2\beta\sqrt{t + \alpha}} \quad (2.17)$$

where c is a constant. If (2.17) is substituted into (2.13) and (2.14), then the string density and the cosmological term are as follows:

$$\rho = \frac{1}{2} \frac{\beta}{(4\pi + \mu)(t + \alpha)^{\frac{3}{2}}} + \frac{k}{(4\pi + \mu)c^2(e^{4\beta\sqrt{t + \alpha}})}$$

and

$$\Lambda = -\frac{1}{2} \frac{(8\pi + \mu)\beta}{(4\pi + \mu)(t + \alpha)^{\frac{3}{2}}} + \frac{3\beta^2}{t + \alpha} + \frac{2(2\pi + \mu)k}{(4\pi + \mu)c^2 e^{(4\beta\sqrt{t+\alpha})}}$$

3. Results and Discussion

In this section, the homogeneous isotropic FRW metric, which is compatible with the present model of the universe in $f(R, T)$ theory, is investigated. Solutions for string matter from topological defects are derived. Finally, the solutions obtained in this section will be analyzed using various graphs. The values $\alpha = 0.11$, $c = 0.3$, $\mu = 33$, and $\beta = 0.93$ are used to plot the metric potential A , energy density ρ , and cosmological term Λ . The cases where k is 0, -1 and 1 are analyzed.

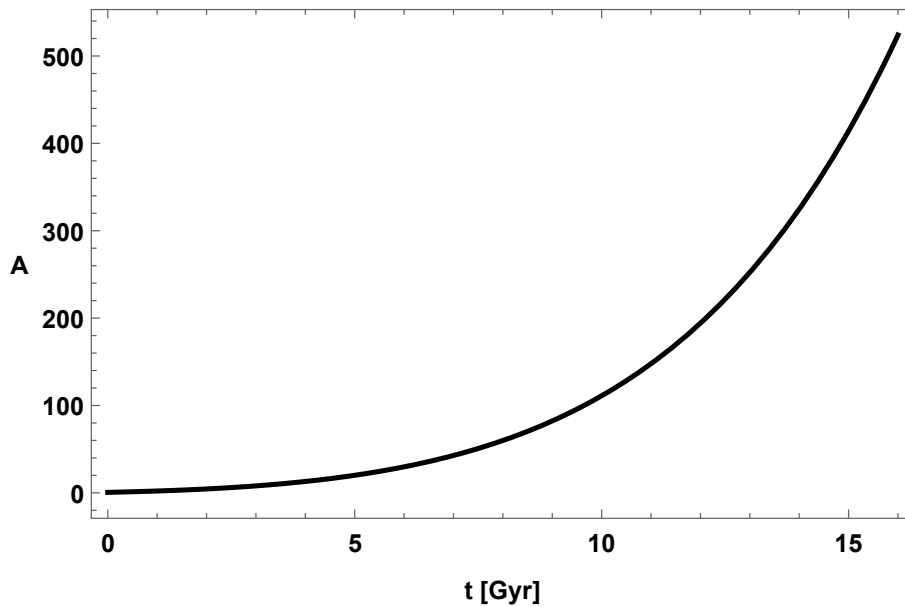


Figure 1. Time dependent change of the metric potential A

By examining the general trends in the graph in Figure 1, it can be observed that the metric potential A increases significantly with time. This increase suggests that the metric potential grows in parallel with the expansion of the universe in FRW space-time.

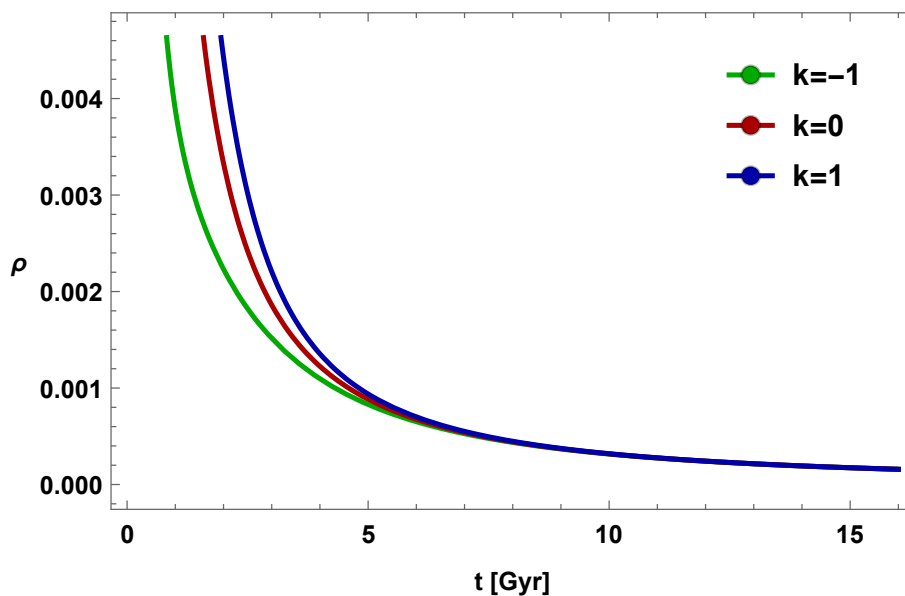


Figure 2. Time dependent change of energy density ρ

Figure 2 presents the graph showing the variation of string energy density over time. Examining the initial conditions, we observe that all curves have relatively high string energy density (ρ) values around $t = 0$. This may represent a period in the early universe when the energy density of string fields was dominant. As time progresses ($t > 5$ Gyr), a significant decrease in energy density is observed. This decrease may indicate that the energy density of string fields decreases in parallel with the expansion of the universe. For $k = -1$, the energy density decreases the slowest. In this scenario, since an open universe expands infinitely, the string energy density seems to decrease very slowly with this expansion, approaching a certain baseline value without ever reaching zero. For $k = 0$, there is a moderate decrease. The string energy density decreases to a certain level over time but at a faster rate than in the open universe. Since the flat universe is a model of a universe expanding at a critical density, this result can be interpreted as an expected situation. For $k = 1$, the energy density decreases the fastest. In this case, since the expansion of a closed universe is limited, the energy density of string fields also weakens rapidly. This model may suggest a scenario in which the universe could contract again at some point.

This graph illustrates the direct influence of the universe’s geometry on the temporal evolution of the string field energy density. The closed universe ($k = 1$) experiences rapid energy loss, while the open universe ($k = -1$) exhibits a slower decay due to the infinite nature of its expansion. The flat universe ($k = 0$) shows a balanced decrease between the two extremes. This temporal variation in the string energy density provides significant insights into how strings interact on cosmological scales and how they adapt to the expansion dynamics of the universe. The fact that energy density decreases at different rates in different universe geometries may offer a means of testing string-based cosmological models using observational data.

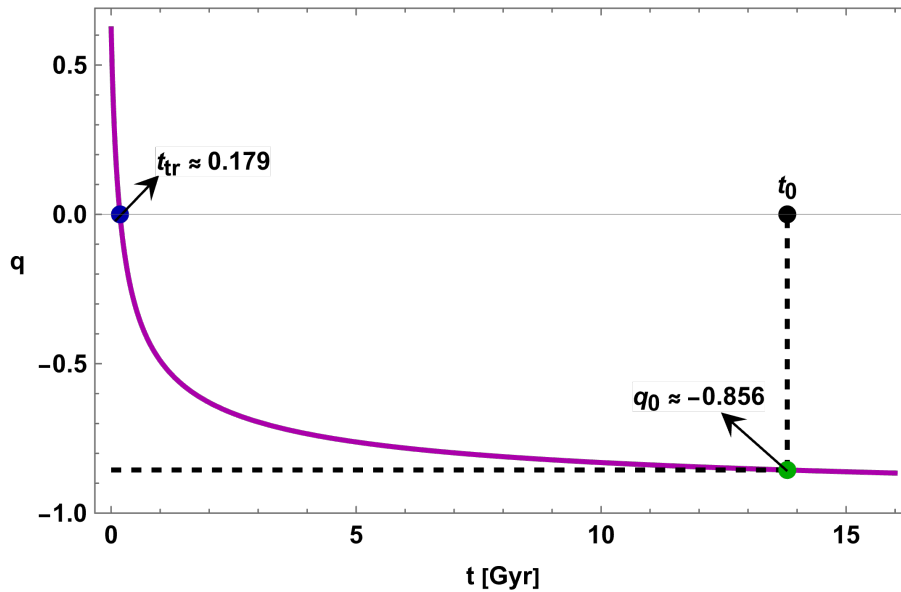


Figure 3. Time dependent change of the deceleration parameter q

It can be observed from the q values that the universe slows down when $q > 0$, expands at a constant speed when $q = 0$, and expands by accelerating when $q < 0$ (Figure 3). For $q = 0$, the phase transition occurs at $t_{tr} = 0.179$. When calculated using $t_0 = 13.8$, the value of t_{tr} is -0.856 .

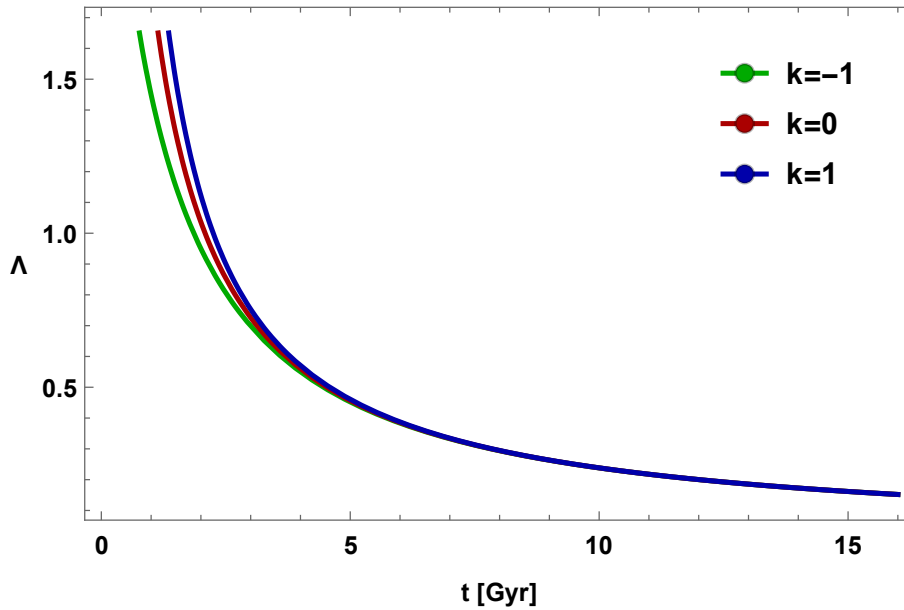


Figure 4. Time dependent change of the cosmological term

Figure 4 shows the plot of the cosmological term Λ with respect to time. Upon analyzing this graph, it can be observed that Λ starts at a high value at the beginning ($t = 0$). As time progresses, Λ gradually decreases. While all three curves exhibit a similar downward trend, the variation of Λ differs slightly across the different space curvatures.

4. Conclusion

This paper investigated the behavior of the string matter distribution, one of the topological defects in the homogeneous and isotropic FRW universe model that best describes the present-day universe within the framework of the $f(R, T)$ theory. In future studies, it is worth studying universe models with matter distributions, such as domain walls and textures, which are types of topological defects, in different space-time metrics within the framework of the $f(R, T)$ theory.

Author Contributions

All the authors equally contributed to this work. This paper is derived from the first author's master's thesis supervised by the second author. They all read and approved the final version of the paper.

Conflicts of Interest

All the authors declare no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

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