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A Spectral Taylor Polynomial Solution of Euler-Bernoulli Beam Equation By A Matrix Approach

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Abstract

The Euler-Bernoulli beam theory, widely applied in structural engineering, describes the relationship between applied loads and resulting deformations in beams. This study addresses the transverse vibration of beams with simply supported, cantilever, and fixed-fixed boundary conditions using the Taylor matrix method. The governing differential equation, which includes both boundary and initial conditions, is transformed into a matrix form through Taylor series expansion. This matrix approach simplifies the process of solving the Euler-Bernoulli equation, providing an efficient method for analyzing beam vibrations under various support conditions. The accuracy of the Taylor matrix method is validated by comparing it with exact solutions derived through the separation of variables. Numerical application examples illustrate that the method yields results closely aligning with the exact solution, with minimal discrepancies that decrease as the number of terms N in the Taylor series expansion increases. This method shows promise as an accessible and accurate approach for studying mechanical vibrations, especially in engineering applications requiring efficient computational techniques. The findings contribute to the literature on beam theory and demonstrate the applicability of the Taylor spectral matrix method in structural analysis.

Keywords: *Taylor matrix method, Euler-Bernoulli beam, Vibration, Spectral method*

1. Introduction

The Euler-Bernoulli beam theory, also known as the classical beam theory, is a fundamental engineering theory that describes the relationship between the applied loads on a beam and the resulting deflections, stresses, and strains. It is widely used in the analysis and design of structural elements subjected to bending.

The issue of a transversely vibrating beam is expressed in the context of a partial differential equation of motion, an external force function, as well as boundary and initial conditions. Considerable endeavors are dedicated to comprehending the solution to this initial-boundary-value problem with non-homogeneous characteristics. Four beam theories are employed for this purpose, with Euler-Bernoulli theory being the most widely used due to its simplicity and effectiveness. This theory offers a simplified mathematical representation of a beam's deformation and stresses, relying on specific assumptions to facilitate a more manageable analysis. It is particularly useful for analyzing slender beams subjected to lateral loads.

The transverse vibration of the Euler-Bernoulli beam has been widely studied by many researchers as Ahmed and Rifai [1], Özmen and Özhan [2], İnan and Oktav [3], Khatami et al. [4], Sivri and Temel [5], Craifaleanu et al. [6], Naz and Mahomed [7], Koç [8], Pakdemirli [9], Rahmani et al. [10], Karahan and Pakdemirli [11], Soltani and Asgarian [12], Sahin [13], Diyaroglu et al. [14], Baysal and Hasanov [15], Ike [16], Saraç [17], Ruiz et al. [18], and Haider et al. [19]. Detailed derivations of the Euler-Bernoulli beam model can be found in the books of Öchsner [20], Rao [21] and Inman [22]. Additionally, the vibration analyses of Euler-Bernoulli beams under different conditions and scenarios have been conducted using various mathematical methods by many researchers.

Everitt et al. [23] used the modified Adomian decomposition method for the free vibration problem of an Euler-Bernoulli beam with non-uniform cross-section under different conditions. Civalek and Demir [24] developed a non-local elasticity-based Euler-Bernoulli beam model to analyze the static bending and buckling behavior of cantilever carbon nanotubes, incorporating size effects using Eringen's non-local theory. Dong et al. [25] aimed to demonstrate the effectiveness of the classical Euler–Bernoulli beam theory in accurately predicting the bending behavior of single-walled BNNTs without the need for introducing scale parameters. Ishaquddin and Gopalakrishnan [26] developed a differential quadrature-based technique for the solution of the Euler-Bernoulli beam equation. Sınır et al. [27] studied the nonlinear free and forced vibrations of axially functionally graded Euler-Bernoulli beams with non-uniform cross-section. Liang et al. [28] developed a new Bernoulli-Euler beam model using simplified strain gradient elasticity theory, deriving equations from Hamilton's principle and solving for axial wave propagation, static bending, buckling, and free vibration of beams. Aslefallah et al. [29] employed the exponential approximation method, utilizing matrix representations of exponential functions and collocation points, to solve high-order nonlinear differential equations, highlighting its efficiency, accuracy, and reliability through comparisons with exact solutions and other approaches. Bassuony et al. [30] used a Galerkin method with Legendre and Laguerre polynomials to solve the Euler-Bernoulli beam equation, emphasizing the method's efficient matrix structure, which significantly reduces computational costs. Çayan et al. [31] presented Taylor-Matrix and Hermite-Matrix Collocation methods using Chebyshev–

Lobatto points and operational matrices to approximate the modal vibration behavior of a simply supported Euler–Bernoulli beam. Wu and Zheng [32] proposed using the Taylor series method combined with Padé approximants to solve a boundary value problem more efficiently, demonstrating improved accuracy and efficiency compared to the weighted residual method and validating their results with finite element method analysis. Demir et al. [33] analyzed the transverse vibration of a self-excited Euler–Bernoulli beam under distributed and singular loads using the differential transformation method.

Eq. (1) presents the Euler-Bernoulli beam vibration equation, including the axial load term.

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} + N_0\frac{\partial^2 u(x,t)}{\partial x^2} + \rho A\frac{\partial^2 u(x,t)}{\partial t^2} = f(x,t) \tag{1}$$

where, $u(x,t)$ is the transverse displacement with respect to position and time, E is the modulus of elasticity, I is the moment of inertia, N_0 is the axial load, ρ is the mass density, A is the cross-sectional area, and $f(x,t)$ is the external excitation force.

2. Spectral Taylor Matrix Method

2.1. Overview of the method

This method is a spectral technique employed to solve various differential equations by converting them into matrix form using Taylor polynomials. Differential equations for both specific and general solutions can be determined using this method. Taylor series has been utilized by many researchers in solving differential and integral equations. Karamete and Sezer [34] developed the Taylor collocation method, using a matrix approach to solve linear integro-differential equations by truncating the Taylor series, transforming the equation into a matrix equation with unknown Taylor coefficients. Kurt and Çevik [35] introduced a simple numerical method using Taylor polynomials in matrix form to solve single degree of freedom systems. They determined both particular and general solutions, demonstrated the method through a numerical application, and found that the results closely matched the exact solution. Gülsu and Sezer [36] examined the solution of m-th order and higher-order linear differential difference equations under variable coefficient uncertain conditions at any point using the Taylor series approach. Numerous studies and researches have been conducted on the Taylor matrix method, including those by Elmaci et al. [37], Bahşı and Çevik [38], Mukhtar [39], Wang et al. [40], Bayku and Sezer [41], Çevik [42], Çayan et al. [43] and Laib et al. [44]. Recently, Çevik et al. [45] provided a review on the development and applications of the collocation method, emphasizing its spectral formulation based on matrices and the incorporation of polynomial sequences, such as Taylor series.

In all these studies, this method was compared with other known techniques, and it was shown that the current approach is relatively easy and extremely accurate. In the first stage of the method, the given equations are transformed into a matrix equation, and then a new matrix equation is formed with the help of Taylor ordering points, where the unknown is only the Taylor coefficient matrix. From this, Taylor coefficients are found, resulting in finite Taylor series approximations.

2.2. Derivation of the Taylor matrix form of the governing equation

An arbitrary displacement function $u(x,t)$, dependent on both time and space variables, can be expressed through its Taylor expansion as follows.

$$\begin{gathered} u(x,t) = \sum_{p=0}^{N}\sum_{q=0}^{N} c_{p,q}(x-x_0)^p\,(t-t_0)^q \\ c_{p,q} = \frac{1}{p!\,q!}\;u^{(p,q)}(x_0\,,t_0) \end{gathered} \tag{2}$$

This Taylor series of order N is obtained by determining the unknown Taylor coefficient $c_{p,q}$. The displacement function can be represented in matrix form as follows:

$$u(x,t) = \mathbf{X}(x)\mathbf{C}\mathbf{T}(t) \tag{3}$$

where

$$\mathbf{X}(x) = [1 \quad (x-x_0) \quad (x-x_0)^2 \quad \dots \quad (x-x_0)^N] \tag{4}$$

$$\mathbf{T}(t) = [1 \quad (t-t_0) \quad (t-t_0)^2 \quad \dots \quad (t-t_0)^N]^T \tag{5}$$

$$\mathbf{C} = \begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0N} \\ c_{10} & c_{11} & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N0} & c_{N1} & \cdots & c_{NN} \end{bmatrix} \tag{6}$$

The first derivative with respect to space can be written as

$$\mathbf{X}'(x) = \mathbf{X}(x)\mathbf{D} \tag{7}$$

where

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 2 & & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & N \\ 0 & 0 & 0 & & 0 \end{bmatrix} \tag{8}$$

Therefore, the n^{th} space derivative becomes

$$\mathbf{X}^{(n)}(x) = \mathbf{X}(x)\mathbf{D}^n \tag{9}$$

Similarly, the n^{th} time derivative is expressed as

$$\mathbf{T}^{(n)}(t) = (\mathbf{D}^T)^n\mathbf{T}(t) \tag{10}$$

Using Eqs. (9) and (10), the space and time derivatives of (3) are

$$\frac{\partial^n u(x,t)}{\partial x^n} = \mathbf{X}(x)\mathbf{D}^n\mathbf{C}\mathbf{T}(t) \tag{11}$$

$$\frac{\partial^n u(x,t)}{\partial t^n} = \mathbf{X}(x)\mathbf{C}(\mathbf{D}^T)^n\mathbf{T}(t) \tag{12}$$

The external excitation function in Eq. (1) can be expressed in matrix form as

$$f(x,t) = \mathbf{X}(x)\mathbf{G}\mathbf{T}(t) \tag{13}$$

where

$$\mathbf{G} = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0N} \\ g_{10} & g_{11} & \cdots & g_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N0} & g_{N1} & \cdots & g_{NN} \end{bmatrix} \tag{14}$$

By substituting Eqs. (11-13) into Eq. (1), we obtain

$$EI\mathbf{X}(x)\mathbf{D}^4\mathbf{C}\mathbf{T}(t) + N_0\,\mathbf{X}(x)\mathbf{D}^2\mathbf{C}\mathbf{T}(t) + \rho A\mathbf{X}(x)\mathbf{C}(\mathbf{D}^T)^2\mathbf{T}(t) = \mathbf{X}(x)\mathbf{G}\mathbf{T}(t) \tag{15}$$

Simplifying Eq. (15) gives:

$$EI\mathbf{D}^4\mathbf{C} + N_0\,\mathbf{D}^2\mathbf{C} + \rho A\mathbf{C}(\mathbf{D}^T)^2 = \mathbf{G} \tag{16}$$

The left and right sides of Eq. (16) are matrices of size $(N+1)\times(N+1)$. To find the unknown $\mathbf{C}$ matrix with constant Taylor coefficients on the left side, a new matrix $\mathbf{S}_{(N+1)^2\times(N+1)^2}$ is defined, transforming the $\mathbf{C}$ and $\mathbf{G}$ matrices into column matrices of size $(N+1)^2\times 1$. In this case, Eq. (16) can be represented as follows:

$$\mathbf{S}_{(N+1)^2\times(N+1)^2}\bar{\mathbf{C}}_{(N+1)^2\times 1} = \bar{\mathbf{G}}_{(N+1)^2\times 1} \tag{17}$$

To find the matrix $\bar{\mathbf{C}}$ in Eq. (17), the following equation must be solved.

$$\bar{\mathbf{C}} = inv(\mathbf{S})\bar{\mathbf{G}} \tag{18}$$

2.3. Formulating boundary and intial condition matrices for the present method

2.3.1. Simple-simple supported beam

For a simple-simple supported beam, shown in Figure 1, the boundary conditions are

$$u(0,t) = 0;\ u''(0,t) = 0;\ u(L,t) = 0;\ u''(L,t) = 0 \tag{19}$$

and the initial conditions are assumed to be

$$u\left(\frac{L}{2},0\right) = u_0\ ;\ \dot{u}\left(\frac{L}{2},0\right) = v_0 \tag{20}$$

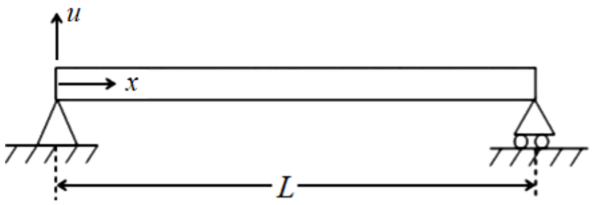

Figure 1. Simple-simple supported beam

By substituting the boundary conditions specified in Eq. (19) into Eq. (3), the matrix form of the boundary condition equations are obtained as follows.

$$\begin{aligned}
\mathbf{X}(0)\mathbf{C}\mathbf{T}(t) &= [0 \quad 0 \quad \dots \quad 0]\mathbf{T}(t) \Rightarrow \mathbf{X}(0)\mathbf{C} = [0 \quad 0 \quad \dots \quad 0] \\
\mathbf{X}(0)\mathbf{D}^2\mathbf{C}\mathbf{T}(t) &= [0 \quad 0 \quad \dots \quad 0]\mathbf{T}(t) \Rightarrow \mathbf{X}(0)\mathbf{D}^2\mathbf{C} = [0 \quad 0 \quad \dots \quad 0] \\
\mathbf{X}(L)\mathbf{C}\mathbf{T}(t) &= [0 \quad 0 \quad \dots \quad 0]\mathbf{T}(t) \Rightarrow \mathbf{X}(L)\mathbf{C} = [0 \quad 0 \quad \dots \quad 0] \\
\mathbf{X}(L)\mathbf{D}^2\mathbf{C}\mathbf{T}(t) &= [0 \quad 0 \quad \dots \quad 0]\mathbf{T}(t) \Rightarrow \mathbf{X}(L)\mathbf{D}^2\mathbf{C} = [0 \quad 0 \quad \dots \quad 0]
\end{aligned} \tag{21}$$

Each expression in Eq. (21) is applied individually to derive boundary condition matrices for each specific boundary condition.

$$\begin{aligned}
\mathbf{X}(0)\mathbf{C} = [1 \quad 0 \quad 0 \quad \dots \quad 0] &\begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0N} \\ c_{10} & c_{11} & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N0} & c_{N1} & \cdots & c_{NN} \end{bmatrix} \\
&= [c_{00} \quad c_{01} \quad c_{02} \quad \cdots \quad c_{0N}] \\
&= [0 \quad 0 \quad \dots \quad 0]
\end{aligned} \tag{22}$$

Rewriting Eq. (22) in terms of Eq. (17), the augmented **S** matrix is obtained as follows:

$$\mathbf{S}_{(N+1)\times(N+1)^2}\bar{\mathbf{C}}_{(N+1)^2\times1} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 \end{bmatrix} \bar{\mathbf{C}} = \mathbf{0}_{(N+1)\times1} \tag{23}$$

In Eq. (23), the first matrix on the left side is the boundary condition 1 matrix, denoted as **SS11**. Next,

$$\begin{aligned}
\mathbf{X}(0)\mathbf{D}^2\mathbf{C} = [1 \quad 0 \quad 0 \quad \dots \quad 0] &\begin{bmatrix} 0 & 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 6 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (\mathrm{N}-1)\mathrm{N} \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0N} \\ c_{10} & c_{11} & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N0} & c_{N1} & \cdots & c_{NN} \end{bmatrix} \\
= [2c_{20} \quad 2c_{21} \quad 2c_{22} \quad \dots \quad 2c_{2N}]&
\end{aligned} \tag{24}$$

Rewriting Eq. (24) in terms of Eq. (17), the augmented **S** matrix is obtained as follows:

$$\mathbf{S}_{(N+1)\times(N+1)^2}\bar{\mathbf{C}}_{(N+1)^2\times1} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 2 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 2 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 2 & & 0 & 0 & \cdots & 0 \end{bmatrix} \bar{\mathbf{C}} = \mathbf{0}_{(N+1)^2\times1} \tag{25}$$

In Eq. (25), the first matrix on the left side is the boundary condition 2 matrix, denoted as **SS12**. Next,

$$\begin{aligned}
\mathbf{X}(L)\mathbf{C} = [1 \quad L \quad L^2 \quad \dots \quad L^N] &\begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0N} \\ c_{10} & c_{11} & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N0} & c_{N1} & \cdots & c_{NN} \end{bmatrix} \\
&= [c_{00} + Lc_{10} + \cdots + L^N c_{N0} \quad \dots \quad c_{0N} + Lc_{1N} + \cdots + L^N c_{NN}] \\
&= [0 \quad 0 \quad 0 \quad \dots \quad 0]
\end{aligned} \tag{26}$$

Rewriting Eq. (26) in terms of Eq. (17), the augmented **S** matrix is obtained as follows:

$$\mathbf{S}_{(N+1)\times(N+1)^2}\bar{\mathbf{C}}_{(N+1)^2\times1} = \begin{bmatrix} 1 & 0 & \cdots & 0 & L & 0 & \cdots & 0 & L^2 & 0 & \cdots & 0 & & L^N & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & L & \cdots & 0 & 0 & L^2 & \cdots & 0 & \cdots & 0 & L^N & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & L & 0 & 0 & \cdots & L^2 & & 0 & 0 & \cdots & L^N \end{bmatrix} \bar{\mathbf{C}} = \mathbf{0}_{(N+1)\times1} \tag{27}$$

In Eq. (27), the first matrix on the left side is the boundary condition 3 matrix, denoted as **SS13**. Next,

$$
\begin{aligned}
\mathbf{X}(L)\mathbf{D}^2\mathbf{C} &= [1 \quad L \quad \dots \quad L^N]\begin{bmatrix} 0 & 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 6 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (N-1)N \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0N} \\ c_{10} & c_{11} & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N0} & c_{N1} & \cdots & c_{NN} \end{bmatrix} \\
&= [2c_{20} + 6Lc_{30} + \cdots + N(N-1)L^{N-2}c_{N0} \quad \dots \quad 2c_{2N} + 6Lc_{3N} + \cdots + N(N-1)L^{N-2}c_{NN}] \\
&= [0 \quad 0 \quad 0 \quad \dots \quad 0]
\end{aligned} \tag{28}
$$

Rewriting Eq. (28) in terms of Eq. (17), the augmented **S** matrix **SS14** is obtained.

Substituting the initial conditions specified in Eq. (20) into Eq. (3) yields the initial condition equations as shown below.

$$
\begin{aligned}
&\mathbf{BB11};\ \mathbf{X}\left(\frac{L}{2}\right)\mathbf{C}\mathbf{T}(0) = u_0 \\
&\mathbf{BB12};\ \mathbf{X}\left(\frac{L}{2}\right)\mathbf{C}\mathbf{D}^{\mathrm{T}}\mathbf{T}(0) = v_0
\end{aligned} \tag{29}
$$

Each expression in Eq. (29) is used to derive boundary condition matrices for each specific boundary condition.

$$
\begin{aligned}
\mathbf{X}\left(\frac{L}{2}\right)\mathbf{C}\mathbf{T}(0) &= \left[1 \quad \frac{L}{2} \quad \left(\frac{L}{2}\right)^2 \quad \dots \quad \left(\frac{L}{2}\right)^N\right]\begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0N} \\ c_{10} & c_{11} & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N0} & c_{N1} & \cdots & c_{NN} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\
&= \left[1 \quad \frac{L}{2} \quad \left(\frac{L}{2}\right)^2 \quad \dots \quad \left(\frac{L}{2}\right)^N\right]\begin{bmatrix} c_{00} \\ c_{10} \\ c_{20} \\ \vdots \\ c_{N0} \end{bmatrix} \\
&= c_{00} + \frac{L}{2}c_{10} + \left(\frac{L}{2}\right)^2 c_{20} + \cdots + \left(\frac{L}{2}\right)^N c_{N0} \\
&= u_0
\end{aligned} \tag{30}
$$

Rewriting Eq. (30) in terms of Eq. (17), the augmented **S** matrix is obtained as follows:

$$
\mathbf{S}_{1\times(N+1)^2}\bar{\mathbf{C}}_{(N+1)^2\times 1} = \left[1 \quad 0 \quad \cdots \quad 0 \quad \frac{L}{2} \quad 0 \quad \cdots \quad 0 \quad \dots \quad \left(\frac{L}{2}\right)^N \quad 0 \quad \dots \quad 0\right]\bar{\mathbf{C}} = u_0 \tag{31}
$$

In Eq. (31), the first matrix on the left side is the initial condition 1 matrix, denoted as **BB11**. Next,

$$
\mathbf{X}\left(\frac{L}{2}\right)\mathbf{C}\mathbf{D}^T\mathbf{T}(0) = c_{01} + \frac{L}{2}c_{11} + \left(\frac{L}{2}\right)^2 c_{21} + \cdots + \left(\frac{L}{2}\right)^N c_{N1} = v_0 \tag{32}
$$

Rewriting Eq. (32) in terms of Eq. (17), the augmented **S** matrix is obtained as follows:

$$
\mathbf{S}_{1\times(N+1)^2}\bar{\mathbf{C}}_{(N+1)^2\times 1} = \left[0 \quad 1 \quad 0 \quad \cdots \quad 0 \quad 0 \quad \frac{L}{2} \quad 0 \quad \cdots \quad 0 \quad \cdots \quad 0 \quad \left(\frac{L}{2}\right)^N \quad 0 \quad \dots \quad 0\right]\bar{\mathbf{C}} = v_0 \tag{33}
$$

In Eq. (33), the first matrix on the left side is the initial condition 2 matrix, denoted as **BB12**.

2.3.2. Cantilever beam

The boundary and initial conditions for the cantilever beam, shown in Figure 2, are

$$
u(0,t) = 0, \qquad u'(0,t) = 0, \qquad u''(L,t) = 0, \qquad u'''(L,t) = 0 \tag{34}
$$

$$
u(L,0) = u_0, \qquad \dot{u}(L,0) = v_0 \tag{35}
$$

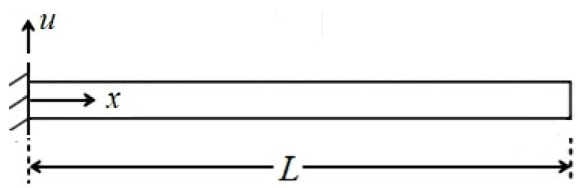

Figure 2. Cantilever beam

The matrix forms of these equations are derived in a manner similar to that described in the previous section.

2.3.3. Fixed-fixed beam

The boundary and initial conditions for the fixed-fixed beam are

$$u(0,t)=0,\qquad u'(0,t)=0,\qquad u(L,t)=0,\qquad u'(L,t)=0 \tag{36}$$

$$u\left(\frac{L}{2},0\right)=u_0,\qquad \dot{u}\left(\frac{L}{2},0\right)=v_0 \tag{37}$$

The matrix forms of these equations are derived in a manner similar to that described in Section 2.3.1.

2.4. Development of the general matrix equation for the system

The matrix form of the motion equation for the Euler-Bernoulli beam has been prepared in the previous sections. The matrix forms of the boundary and initial conditions obtained in Section (2.3) will be substituted into Eq. (15) by deleting its 30 rows. This allows for the creation of an augmented matrix equation that accommodates both the governing equation and the boundary-initial conditions. The resulting matrix is then solved using the Taylor matrix method to obtain the **S** matrix we seek. The inverse of this **S** matrix is calculated and multiplied by the **G** matrix to determine the unknown Taylor coefficients $c_{p,q}$.

3. Application Examples

In this section, we investigate the transverse vibration of the Euler-Bernoulli beam under axial load for three different support conditions. Since the first mode is generally the most critical, all numerical examples concentrate on the corresponding mode shape. The geometric and physical parameters considered for the beam in all numerical examples are as follows:

$$L=5m,\qquad A=0.01m^2,\qquad E=20\times10^{10}\frac{N}{m^2},\qquad \rho=8\times10^3\frac{kg}{m^3},$$
$$I=8.33\times10^{-6},\qquad f(x,t)=10^{-4}cos3t,\qquad N_0=100N$$

3.1. Application example 1: Simple-simple supported beam

Given the initial conditions $u_0=0.01m$ and $\dot{u}_0=0$, and for $N=6$ at the origin $(x_0,t_0)=(0,0)$, the unknown Taylor coefficients within the matrix series expansion are computed according to Eq. (18). These coefficients are then substituted into Eq. (3) to obtain the solution for the transverse forced vibration equation. The matrix forms corresponding to the boundary conditions for a simply supported beam, along with the initial condition matrices, are structured as follows.

The **SS11** matrix in Eq. (23) is calculated for $N=6$ as follows:

$$\mathbf{SS11}=\begin{bmatrix}1&0&\cdots&0&0&0&\cdots&0&&0&0&\cdots&0\\0&1&\cdots&0&0&0&\cdots&0&\cdots&0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&&\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&1&0&0&\cdots&0&&0&0&\cdots&0\end{bmatrix}_{7\times49} \tag{38}$$

Similarly, the **SS12** matrix in Eq. (25) is calculated for $N=6$ as follows:

$$\mathbf{SS12}=\begin{bmatrix}0&0&\cdots&0&0&0&\cdots&0&2&0&\cdots&0&&0&0&\cdots&0\\0&0&\cdots&0&0&0&\cdots&0&0&2&\cdots&0&\cdots&0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&&\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0&0&0&\cdots&0&0&0&\cdots&2&&0&0&\cdots&0\end{bmatrix}_{7\times49} \tag{39}$$

and the **SS13** matrix in Eq. (27) is calculated as follows:

$$\mathbf{SS13}=\begin{bmatrix}1&0&\cdots&0&5&0&\cdots&0&5^2&0&\cdots&0&&5^6&0&\cdots&0\\0&1&\cdots&0&0&5&\cdots&0&0&5^2&\cdots&0&\cdots&0&5^6&\cdots&0\\\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&&\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&1&0&0&\cdots&5&0&0&\cdots&5^2&&0&0&\cdots&5^6\end{bmatrix} \tag{40}$$

Finally, the **SS14** matrix is calculated as follows:

$$\mathbf{SS14}=\begin{bmatrix}0&0&\cdots&0&0&0&\cdots&0&2&0&\cdots&0&&305^4&0&\cdots&0\\0&0&\cdots&0&0&0&\cdots&0&0&2&\cdots&0&\cdots&0&305^4&\cdots&0\\\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&&\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0&0&0&\cdots&0&0&0&\cdots&2&&0&0&\cdots&305^4\end{bmatrix}_{7\times49} \tag{41}$$

The initial condition matrices for $N=6$ are computed using Eqs. (30-33) as follows:

$$\mathbf{X}\left(\frac{5}{2}\right)\mathbf{CT}(0)=0.01 \tag{42}$$

$$\mathbf{BB11} = \begin{bmatrix} 1 & 0 & \cdots & 0 & \frac{5}{2} & 0 & \cdots & 0 & \cdots & \left(\frac{5}{2}\right)^6 & 0 & \cdots & 0 \end{bmatrix}_{1\times 49} \tag{43}$$

$$\mathbf{X}\left(\frac{5}{2}\right)\mathbf{CD}^T\mathbf{T}(0) = 0 \tag{44}$$

$$\mathbf{BB12} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & \frac{5}{2} & 0 & \cdots & 0 & \cdots & 0 & \left(\frac{5}{2}\right)^6 & 0 & \cdots & 0 \end{bmatrix}_{1\times 49} \tag{45}$$

Substituting Eqs. (38-45) into Eq. (16) and rewriting in the form of Eq. (17) yields the $\bar{\mathbf{C}}_{49\times 1}$ and $\bar{\mathbf{G}}_{49\times 1}$ matrices.

$$\bar{\mathbf{C}} = \begin{bmatrix} c_{00} \\ c_{01} \\ \vdots \\ c_{06} \\ c_{10} \\ c_{11} \\ \vdots \\ c_{16} \\ \vdots \\ c_{60} \\ c_{61} \\ \vdots \\ c_{66} \end{bmatrix}_{49\times 1}, \quad \bar{\mathbf{G}} = \begin{bmatrix} g_{00} \\ g_{01} \\ \vdots \\ g_{06} \\ g_{10} \\ g_{11} \\ \vdots \\ g_{16} \\ \vdots \\ g_{60} \\ g_{61} \\ \vdots \\ g_{66} \end{bmatrix}_{49\times 1} \tag{46}$$

By substituting the column matrices above into Eq. (16) and isolating the **S** matrix, the element in the i^{th} row and j^{th} column of the resulting 49×49 **S** matrix is obtained as follows:

$$s_{i,j} = \begin{cases} (i \, mod \, (N+1)) \times \left((i \, mod \, (N+1)) + 1\right) \times 8 \times 10^3 \times 0.01 & ; \quad mod(i, N+1) < N \text{ and } j = i+2 \\ (((i-1) \, div \, (N+1)) + 1) \times (((i-1) \, div \, (N+1)) + 2) \times 100 & ; \quad j > 2(N+1) \text{ and } j = i + 2(N+1) \\ \dfrac{(((i-1) \, div \, (N+1)) + 4)!}{((i-1) \, div \, (N+1))!} \times 20 \times 10^{10} \times 8{,}33 \times 10^{-6} & ; \quad j > 4(N+1) \text{ and } j = i + (N+1) \\ 0 & ; \quad other \end{cases} \tag{47}$$

Here, $a \, mod \, b$ gives the remainder when the integer a is divided by the integer b, while $a \, div \, b$ gives the quotient obtained from dividing the integer a by the integer b.

In Eq. (47), following the insertion of rows **SS11**, **SS12**, **SS13**, **SS14**, **BB11**, and **BB12** into the **S** matrix, if the augmented $\bar{\mathbf{S}}$ matrix is singular, adjustments are made by modifying alternative rows until the singularity is resolved. The i^{th} row, j^{th} column element of the matrix is then computed as follows:

$$\bar{s}_{i,j} = \begin{cases} s_{i,j} & ; \quad i < (N+1) \times (N-3) - 1 \\ sb11_{1,j} & ; \quad i = (N+1) \times (N-3) - 1 \\ bb12_{1,j} & ; \quad i = (N+1) \times (N-3) \\ ss11_{(i \, mod \, (N+1))+1,j} & ; \quad (N+1) \times (N-3) < i \leq (N+1) \times (N-2) \\ ss12_{(i \, mod \, (N+1))+1,j} & ; \quad (N+1) \times (N-2) < i \leq (N+1) \times (N-1) \\ ss13_{(i \, mod \, (N+1))+1,j} & ; \quad (N+1) \times (N-1) < i \leq (N+1) \times N \\ ss14_{(i \, mod \, (N+1))+1,j} & ; \quad (N+1) \times N < i \leq (N+1)^2 \end{cases} \tag{48}$$

In Eq. (16), the expression on the right side represents the column matrix $\bar{\mathbf{G}}$. To determine the $\bar{\mathbf{G}}$ matrix, Eq. (13) can be alternatively expressed as follows:

$$f(x,t) = \mathbf{X}(x)\mathbf{G}\mathbf{T}(t) = \sum_{p=0}^{6}\sum_{q=0}^{6} \frac{1}{p!\,q!} f^{(p,q)}\left(x_p, t_q\right)(x-0)^p(t-0)^q \tag{49}$$

whereas

$$\begin{aligned} \mathbf{G} &= \sum_{p=0}^{6}\sum_{q=0}^{6} \frac{1}{p!\,q!} f^{(p,q)}\left(x_p, t_q\right)\mathbf{G} \\ &= f(x_0)\cos 3t_0 - 3f(x_0)\sin 3t_1 - \frac{9}{2}f(x_0)\cos 3t_2 + \frac{9}{2}f(x_0)\sin 3t_3 + \cdots \end{aligned}$$

Knowing that $f(x,t) = 10^{-4}cos3t$

$$\bar{\mathbf{G}} = \begin{bmatrix} f(x_0)\cos 3t_0 \\ -3f(x_0)\sin 3t_1 \\ -\frac{9}{2}f(x_0)\cos 3t_2 \\ \frac{9}{2}f(x_0)\sin 3t_3 \\ \frac{27}{8}f(x_0)\cos 3t_4 \\ -\frac{81}{40}f(x_0)\sin 3t_5 \\ -\frac{81}{80}f(x_0)\cos 3t_6 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{49\times 1} = \begin{bmatrix} 10^{-4} \\ 0 \\ -\frac{9}{2}10^{-4} \\ 0 \\ \frac{27}{8}10^{-4} \\ 0 \\ -\frac{81}{80}10^{-4} \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{49\times 1} \tag{50}$$

The values of the Taylor coefficients, $\bar{c}_{pq}$, are determined based on Eq. (18). Since the initial conditions have been inserted into the top two rows of the $\bar{\mathbf{G}}$, the corresponding values of 0.01 and 0 are likewise positioned in the first two rows of this column matrix.

$$\begin{bmatrix} c_{00} = 0 \\ c_{01} = 0 \\ c_{02} = 0 \\ c_{03} = 0 \\ c_{04} = 0 \\ c_{05} = 0 \\ c_{06} = 0 \\ c_{10} = 0.0064 \\ c_{11} = 0 \\ c_{12} = 1.4\times 10^{-9} \end{bmatrix} \begin{bmatrix} c_{13} = 0 \\ c_{14} = 1.05\times 10^{-9} \\ c_{15} = 0 \\ c_{16} = -3.16\times 10^{-10} \\ c_{20} = 0 \\ c_{21} = 0 \\ c_{22} = 0 \\ c_{23} = 0 \\ c_{24} = 0 \\ c_{25} = 0 \end{bmatrix} \begin{bmatrix} c_{26} = 0 \\ c_{30} = 5.00\times 10^{-4} \\ c_{31} = 0 \\ c_{32} = 1.13\times 10^{-10} \\ c_{33} = 0 \\ c_{34} = -8.44\times 10^{-11} \\ c_{35} = 0 \\ c_{36} = 2.53\times 10^{-11} \\ c_{40} = 5.00\times 10^{-5} \\ c_{41} = 0 \end{bmatrix} \begin{bmatrix} c_{42} = 1.13\times 10^{-11} \\ c_{43} = 0 \\ c_{44} = 8.44\times 10^{-12} \\ c_{45} = 0 \\ c_{46} = -2.53\times 10^{-12} \\ c_{50} = 1.54\times 10^{-9} \\ c_{51} = 0 \\ c_{52} = -3.38\times 10^{-16} \\ c_{53} = 0 \\ c_{54} = 2.53\times 10^{-16} \end{bmatrix} \begin{bmatrix} c_{55} = 0 \\ c_{56} = -7.60\times 10^{-17} \\ c_{60} = -1.02\times 10^{-10} \\ c_{61} = 0 \\ c_{62} = 2.25\times 10^{-17} \\ c_{63} = 0 \\ c_{64} = -1.69\times 10^{-17} \\ c_{65} = 0 \\ c_{66} = 5.06\times 10^{-18} \end{bmatrix} \tag{51}$$

By substituting the Taylor coefficients from Eq. (51) into Eq. (3), the displacement function is obtained as follows:

$$\begin{aligned} u(x,t) = {} & 0.0064 - 1.406\times 10^{-9}xt^2 + 1.055\times 10^{-9}xt^4 - 3.164\times 10^{-10}xt^6 - 5\times 10^{-4}x^3 + 1.125\times 10^{-10}x^3t^2 \\ & - 8.438\times 10^{-11}x^3t^4 + 2.531\times 10^{-11}x^3t^6 + 5\times 10^{-5}x^4 - 1.125\times 10^{-11}x^4t^2 + 8.438\times 10^{-12}x^4t^4 \\ & - 2.531\times 10^{-12}x^4t^6 + 1.536\times 10^{-9}x^5 - 3.375\times 10^{-16}x^5t^2 + 2.531\times 10^{-16}x^5t^4 - 7.595\times x^5t^6 \\ & - 1.024x^6 + 2.25\times 10^{-17}x^6t^2 - 1.688\times 10^{-17}x^6t^4 + 5.062\times x^6t^6 \end{aligned} \tag{52}$$

The solution function found in Eq. (52) is plotted for the time interval $t = 0 - 20s$, resulting in the graph depicted in Figure 3.

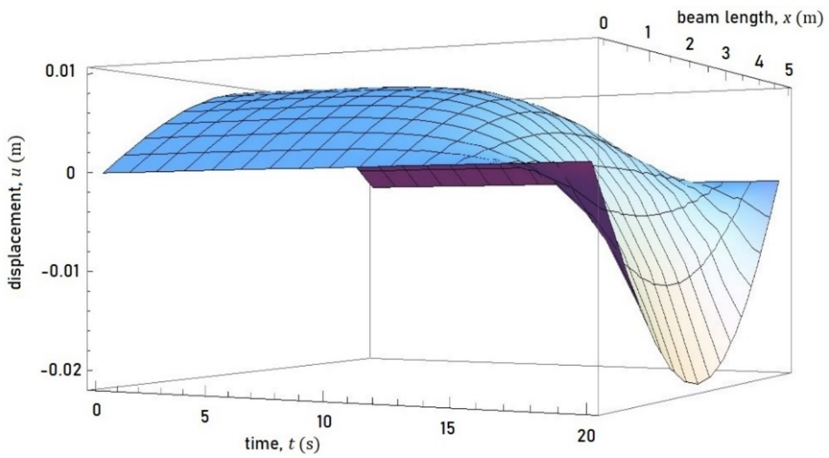

Figure 3. General solution using the Taylor matrix method for the first mode of the simple-simple supported beam in example 1

3.2. Application example 2: Cantilever beam

The initial conditions $u_0 = 0.01m$ and $u_0 = 0$ for $N = 6$ will be used. The unknown Taylor coefficients in the matrix series expanded at the origin $(x_0, t_0) = (0,0)$ will be calculated according to Eq. (18), then substituted into Eq. (3) to obtain the transverse forced vibration equation.

$$u(x,t) = 0.0008x^2 - 1.687x^2t^2 + 1.266\times10^{-9}x^2t^4 - 3.796\times10^{-10}x^2t^6 - 0.0001x^3 + 2.25\times10^{-10}x^3t^2 - 1.687\times10^{-10}x^3t^4 + 5.061\times10^{-11}x^3t^6 + 5.33\times10^{-6}x^4 - 1.12\times10^{-11}x^4t^2 + 8.43\times10^{-12}x^4t^4 - 2.53\times10^{-12}x^4t^6 + 3.2\times10^{-10}x^5 - 6.748\times10^{-16}x^5t^2 + 5.061\times10^{-16}x^5t^4 - 1.518\times10^{-17}x^5t^6 - 1.066\times10^{-11}x^6 + 2.248\times10^{-17}x^6t^2 - 1.686\times10^{-9}x^6t^4 + 5.059\times10^{-18}x^6t^6 \quad (53)$$

When Eq. (53) is plotted for the time interval $t = 0 - 20s$, the graph shown in Figure 4 is generated.

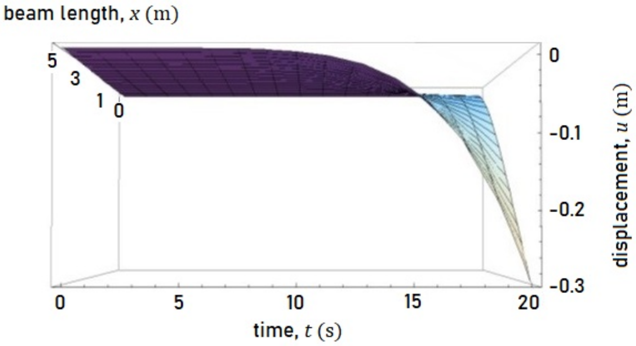

Figure 4. General solution using the Taylor matrix method for the first mode of the cantilever beam in example 2

3.3. Application example 3: Fixed-fixed beam

The initial conditions $u_0 = 0.01m$ and $u_0 = 0$ for $N = 6$ will be used. The unknown Taylor coefficients in the matrix series expanded at the origin $(x_0, t_0) = (0,0)$ will be calculated according to Eq. (18), then substituted into Eq. (3) to obtain the transverse forced vibration equation.

$$u(x,t) = 0.0064x^2 - 2.813\times10^{-10}x^2t^2 + 2.11\times10^{-10}x^2t^4 - 6.328\times10^{-11}x^2t^6 - 0.0026x^3 + 1.125\times10^{-10}x^3t^2 - 8.437\times10^{-11}x^3t^4 + 2.531\times10^{-11}x^3t^6 + 0.00026x^4 - 1.125\times10^{-11}x^4t^2 + 8{,}436\times10^{-12}x^4t^4 - 2.531\times10^{-12}x^4t^6 + 7.697\times10^{-9}x^5 - 3.375\times10^{-16}x^5t^2 + 2.531\times10^{-16}x^5t^4 - 7.593\times10^{-17}x^5t^6 - 5.12\times10^{-10}x^6 + 2.25\times10^{-17}x^6t^2 - 1.687\times10^{-17}x^6t^4 + 5.062\times10^{-18}x^6t^6 \quad (54)$$

When Eq. (54) is plotted for the time interval $\mathrm{t} = 0 - 20\mathrm{s}$, the graph shown in Figure 5 is generated.

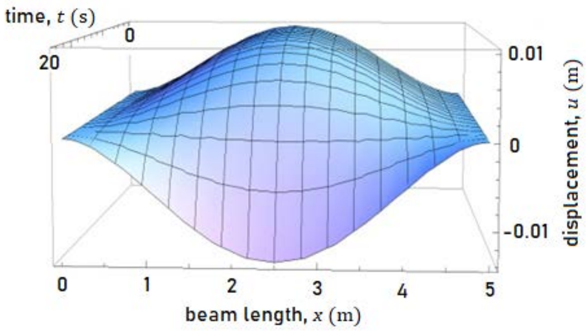

Figure 5. General solution using the Taylor matrix method for the first mode of the fixed-fixed beam in example 3

4. Validation of the Method

In this section, the separation of variables method is employed to derive the exact solution for the simply supported case. This solution is then compared with the results obtained through the Taylor matrix method. During this validation process, the beam and material properties applied in the numerical examples are also considered for both the Taylor matrix solution and the exact solution.

In applying the separation of variables method, we propose a solution of the form

$$u(x,t) = X(x)T(t) \quad (55)$$

where the displacement and time functions are defined as

$$T(t) = c_1e^{i\omega t} + c_2e^{-i\omega t} \quad (56)$$

$$X(x) = Asin\beta x + Bcos\beta x \quad (57)$$

By substituting the given values, boundary conditions, and initial conditions, and performing the necessary calculations, the homogeneous solution is obtained as follows:

$$u_h(x,t) = -0.005sin\left(\frac{n\pi}{L}x\right)\left(e^{i\omega t} + e^{-i\omega t}\right) \tag{58}$$

and the first mode's natural frequency is calculated for $n = 1$, using Eq. (59)

$$\omega = \sqrt{\frac{n^2\pi^2}{L^2\rho A}\left(\frac{n^2\pi^2}{L^2}EI - N_0\right)} \tag{59}$$

Similarly the particular solution will be

$$u_p = -\frac{10^{-4}}{9\rho A}cos(3t) \tag{60}$$

Thus the general solution is

$$\begin{aligned} u_g(x,t) &= -0.005sin\left(\frac{n\pi}{L}x\right)\left(e^{i\omega t} + e^{-i\omega t}\right) - \frac{10^{-4}}{9\rho A}cos(3t) \\ &= -0.005sin\left(\frac{n\pi}{L}x\right)\left(e^{i\sqrt{\frac{n^2\pi^2}{L^2\rho A}\left(\frac{n^2\pi^2}{L^2}EI - N_0\right)}t} + e^{-i\sqrt{\frac{n^2\pi^2}{L^2\rho A}\left(\frac{n^2\pi^2}{L^2}EI - N_0\right)}t}\right) - \frac{10^{-4}}{9\rho A}cos(3t) \end{aligned} \tag{61}$$

Substituting the given values, the general solution is obtained in its final form as follows:

$$u_g(x,t) = -0.005sin(0{,}6283x)\left(e^{56{,}97it} + e^{-56{,}97it}\right) - 0.0014 \times 10^{-4}cos(3t) \tag{62}$$

The exact solution for the first natural frequency mode, obtained from Eq. (62) and the solution derived using the Taylor matrix method are plotted for $t = 0.5$ and displayed in Figure 6. When the graphs of Eq. (62) and Eq. (52) are overlaid, a minimal difference between the two solutions is observed. This difference can be further reduced by increasing the value of N.

Figure 6. Comparison of the Taylor matrix method and the exact solution for the simple-simple supported case in the first natural frequency mode at $t = 0.5$ (- - - Taylor matrix method, — exact solution)

We have comprehensively evaluated the transverse vibration of Euler-Bernoulli beams, affirming the robustness of the Taylor matrix method in solving such problems. By comparing our results with existing literature, we have demonstrated that this method not only aligns closely with the exact solutions derived through the separation of variables but also offers considerable advantages in terms of computational efficiency and ease of application. The results show minimal discrepancies, which can be further reduced by increasing the number of terms in the Taylor series expansion. This validation underscores the method's effectiveness for beams under various support conditions, while its inherent versatility suggests promising applications for more complex scenarios, such as functionally graded material beams and higher vibrational modes. Our findings reinforce the practicality of the Taylor matrix method as a powerful tool for researchers and engineers in the field of structural analysis, contributing to the ongoing discourse on improvements in vibration analysis techniques.

5. Conclusion

In conclusion, this study demonstrates the effectiveness of the Taylor matrix method in solving the Euler-Bernoulli beam equation for various boundary conditions, including simply supported, cantilever, and fixed-fixed configurations. By transforming the governing differential equations into matrix form, the Taylor matrix approach provides a computationally efficient and accurate alternative to conventional methods, such as separation of variables. Through comparison with exact solutions, this method has proven to yield results with minimal discrepancies, which further decrease as the number of terms in the Taylor series expansion increases. The numerical examples validate the method's capability to handle complex vibration analyses for beams subjected to transverse loads, offering an accessible tool for engineers and researchers in structural analysis. This study highlights the Taylor matrix method as a valuable addition to the range of techniques available for solving differential equations in engineering, particularly in applications where efficiency and accuracy are critical. Future work may extend the method's application to more complex beam models and different load conditions, further broadening its utility in structural engineering.

Ethics committee approval and conflict of interest statement

This article does not require ethics committee approval.

This article has no conflicts of interest with any individual or institution.

Author Contribution Statement

MMB contributed to the analysis, methodology development, validation, and manuscript editing (34%); ÖT was responsible for the conceptualization, analysis, validation, and drafting of the manuscript (33%); MÇ contributed to the conceptualization and provided supervision, manuscript review, editing, and preparation of the final version (33%). All authors read and approved the final manuscript.

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