

Fermi-Walker Derivation of Magnetic Curves According to The Alternative Frame

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Abstract: Many studies have been carried out according to different frames in the theory of curves in Euclidean space. The alternative frame is one of these frames. In this paper, we investigate the Fermi-Walker derivative of magnetic curves according to the alternative frame. Firstly, we analyzed the magnetic curves in the alternative frame. We defined N-magnetic, C-magnetic and W-magnetic curves. Then, the Fermi-Walker derivation and theorems in this frame are given for N-magnetic, C-magnetic and W-magnetic curves.

Key words: Fermi-Walker derivations, alternative frame, magnetic curves.

Alternatif Çatıya Göre Manyetik Eğrilerin Fermi-Walker Türevi

Öz: Öklid uzayında eğriler teorisinde farklı çatılara göre birçok çalışma yapılmıştır. Bu çatılardan biri de alternatif çatıdır. Bu makalede alternatif çatıya göre manyetik eğrilerin Fermi-Walker türevini araştırılmıştır. İlk önce manyetik eğrileri alternatif çatıda incelenmiştir. Sonra, bu çatıda Fermi-Walker türevi ve teoremleri N-manyetik, C-manyetik ve W-manyetik eğrileri için verilmiştir.

Anahtar kelimeler: Fermi-Walker türev, alternatif çatı, manyetik eğri.

1. Introduction

The Frenet Frame in 3-dimensional Euclid space is the best known orthonormal frame. Many scientists use the Frenet Frame to calculate and characterize the properties of curves. Also, the shape of the space curve well local behavior of this curve is completely specific to its curvature and torsion. So, a curve must be at least 3rd order continuously differentiable to be completely examined. But Frenet frame is not defined for curves with zero 2nd derivative. Thus, we need a new frame. While the normal vector with respect to the alternative frame and the normal vector with respect to the Frenet frame are the same along the curve, C and W vectors are obtained by rotating the tangent and binormal vectors around the normal vector. This new frame is called alternative frame of the curve. There is the following relationship between alternative frame vectors and their derivative vectors:

$$\begin{bmatrix} N' \\ C' \\ W' \end{bmatrix} = \begin{bmatrix} 0 & \beta & 0 \\ -\beta & 0 & \gamma \\ 0 & -\gamma & 0 \end{bmatrix} \begin{bmatrix} N \\ C \\ W \end{bmatrix} \quad (1)$$

where $\beta = \kappa \sqrt{1 + \left(\frac{\tau}{\kappa}\right)^2}$, $\gamma = \frac{\kappa^2}{\kappa^2 + \tau^2} \left(\frac{\tau}{\kappa}\right)'$ [1]. Çakmak and Şahin examined the adjoint curves according to alternative frame [2]. Alıç and Yılmaz investigated the Smarandache curves according to alternative frame [3].

Magnetic curves have many applications in multidisciplinary science and we can say that they play a very important role in these scientific fields. The magnetic field is defined by the Lorentz force affecting a moving electric charge. When a charged particle enters the magnetic field V, the Serret-Frenet vectors of this particle are affected by this field. With this effect, the Lorentz force is generated. Therefore, the particle begins to trace a trajectory in this field. This trajectory is called a magnetic curve. In this subject, Barros and Romeo examined the trajectories of magnetic fields in 3-dimensional Riemannian manifolds [4,5]. Ling Xu and David Mould proposed that the continuous variation of the charge on the particle over time draws very complex trajectories with very different curves [6]. Körpınar and Demirkol gave magnetic curves in 3-dimensional Riemannian manifolds [7]. Z.Özdemir, İ.Gök, Y.Yaylı and N.Ekmekci also studied magnetic curves [8].

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Fermi-Walker derivative describes the transport of a vector along a curve. In other words, it describes how this vector is seen by a moving observer. Karakuş examined the Fermi-Walker derivations and geometric applications [9]. Maluf and Faria studied Fermi-Walker derivations [10]. Karakuş and Yaylı also investigated Fermi-Walker derivative of a curve with respect to the alternative frame [11].

In this paper, we investigate the magnetic curves according to alternative frame. We prove that the relationship between the Lorentz force and the alternative frame. Furthermore, we give Fermi-Walker derivative of the magnetic curves this frame.

2. Preliminaries

Definition 2.1: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be a unit speed curve with s arc parameters in Euclidean 3-space. The vectors T, N , and B , which are defined as follows, are called to as Frenet-Serret vectors:

$$T(s) = \alpha'(s), N(s) = \frac{T'(s)}{\|T'(s)\|}, B(s) = T(s) \times N(s) \quad (2)$$

where the vectors T, N , and B denote the tangent, normal, and binormal vectors, respectively. The derivative formulas for these vectors are defined as follows:

$$\begin{aligned} T'(s) &= \kappa N(s) \\ N'(s) &= -\kappa T(s) + \tau B(s) \\ B'(s) &= -\tau N(s) \end{aligned}$$

where κ and τ are the curvature and torsion of the curve α .

Definition 2.2: The Lorentz force, which governs the behaviour of a charged particle q moving at velocity v in an electric field E and a magnetic field B , can be expressed as follows:

$$F = q(E + v \times B) \quad (3)$$

It is possible to define a closed 2-form on a Riemannian manifold as a magnetic field. In the context of a Riemannian manifold, magnetic curves can be defined as orbits characterised by the movement of a charged particle under the influence of a magnetic field. The charged particles under scrutiny are subjected to a force known as the Lorentz force in a magnetic field. It can thus be deduced that each α orbit is

$$\nabla_{\alpha'} \alpha' = \Phi(\alpha'),$$

(where Φ is the Lorentz force corresponding to F and ∇ is the Levi-Civita connection).

Definition 2.3: Let X be any vector field along the α space curve with s arc parameters. The Fermi derivative of the X vector field along the α curve is defined as follows.

$$\tilde{\nabla}_T X = \nabla_T X - \langle T, X \rangle \nabla_T T + \langle \nabla_T T, X \rangle T \quad (4)$$

where $T = \frac{d\alpha}{ds}$, $\nabla_T T = \frac{dT}{ds}$. If $\tilde{\nabla}_T X = 0$, then X called Fermi-Walker parallel along the α curve.

3. Magnetic Curves according to the Alternative Frame

3.1 N-Magnetic Curves

Definition 3.1.1: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be a curve with s arc parameters in Euclidean 3-space. If the vector field N of the alternative frame satisfies the Lorentz force equation $\nabla_{\alpha'} N = V \times N$, then the curve α is called the N -magnetic curve.

Theorem 3.1.2: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be an the N -magnetic curve with unit velocity. Then, the Lorentz force according to the alternative frame is obtained as

$$\begin{bmatrix} \Phi(N) \\ \Phi(C) \\ \Phi(W) \end{bmatrix} = \begin{bmatrix} 0 & \beta & 0 \\ -\beta & 0 & \Omega \\ 0 & -\Omega & 0 \end{bmatrix} \begin{bmatrix} N \\ C \\ W \end{bmatrix}. \quad (5)$$

Proof: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be an the N -magnetic curve and let $\{N, C, W, \beta, \gamma\}$ be the alternative frame elements of this curve. Since $\Phi(N) \in sp\{N, C, W\}$,

$$\Phi(N) = aN + bC + cW \quad (6)$$

can be written. If both sides of Eq (6) are inner product by N and the necessary operations are performed, $a = 0$ is obtained. Thus,

$$\Phi(N) = bC + cW \quad (7)$$

can be written. Similarly, if both sides of Eq (7) are inner product by C , $b = \beta$ is obtained. If Eq (7) are inner product with W , $c = 0$ found. If the operations applied to $\Phi(N)$ are repeated for $\Phi(C)$, the following result is obtained:

$$\Phi(C) = -\beta N + \Omega W \quad (8)$$

Similarly,

$$\Phi(W) = -\Omega C \quad (9)$$

is obtained.

Theorem 3.1.3: Let α be an the N -magnetic curve with unit velocity. For α to be the trajectory of V magnetic field necessary and sufficient condition the magnetic field V along the curve α can be written as follows:

$$V = \Omega N + \beta W \quad (10)$$

Proof: Let α be defined as the trajectory of the magnetic field V . Since $V \in sp\{N, C, W\}$ according to the alternative frame, $V = a_1 N + a_2 C + a_3 W$ can be written. On the other hand,

$$\Phi(N) = V \times N \quad (11)$$

and from Eq(5), $\Phi(N) = \beta C$. Substituting the equation $\Phi(N) = \beta C$, into the Eq (11), allows the necessary calculations to be performed, thus resulting in a determination of a_2 equal to 0 and a_3 equal to β . Similarly,

$$\Phi(W) = V \times W \quad (12)$$

and from Eq (5), we can write $\Phi(W) = -\Omega C$. Substituting the equation $\Phi(W) = -\Omega C$ into Eq (12) and performing the necessary calculations, we obtain $a_1 = \Omega$ and , $a_2 = 0$.

3.2 C-Magnetic Curves

Definition 3.2.1: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be a curve in Euclidean 3-space. If the vector field C of the alternative frame satisfies the Lorentz force equation $\nabla_{\alpha'} C = \Phi(C) = V \times C$, then the curve α is called the C -magnetic curve.

Theorem 3.2.2: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be an the C -magnetic curve with unit velocity. Then, the Lorentz force according to the alternative frame is obtained as

$$\begin{bmatrix} \Phi(N) \\ \Phi(C) \\ \Phi(W) \end{bmatrix} = \begin{bmatrix} 0 & \beta & \lambda \\ -\beta & 0 & \gamma \\ -\lambda & -\gamma & 0 \end{bmatrix} \begin{bmatrix} N \\ C \\ W \end{bmatrix} \quad (13)$$

Proof: : Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be an the C -magnetic curve and let $\{N, C, W, \beta, \gamma\}$ be the alternative frame elements of this curve. Since $\Phi(N) \in sp\{N, C, W\}$,

$$\Phi(N) = b_1 N + b_2 C + b_3 W \quad (14)$$

can be written. If both sides of Eq (14) are inner product by N and the necessary operations are performed, $b_1 = 0$ is obtained. Similarly, if both sides of Eq (14) are inner product by C and the necessary operations are performed, then, we are given by $b_2 = \beta$. If similar operations are made for W , $b_3 = \lambda$ is obtained. If $b_1 = 0$, $b_2 = \beta$, $b_3 = \lambda$ is written at Eq (14), we obtained

$$\Phi(N) = \beta C + \lambda W$$

Similarly, if the operations applied to $\Phi(N)$ are repeated for $\Phi(C)$ and $\Phi(W)$, the following results are obtained:

$$\Phi(C) = -\beta N + \gamma W$$

$$\Phi(W) = -\lambda N - \gamma C$$

Theorem 3.2.3: Let α be an the C -magnetic curve with unit velocity. For α to be the trajectory of V magnetic field necessary and sufficient condition the magnetic field V along the curve α can be written as follows:

$$V = \gamma N - \lambda C + \beta W \quad (15)$$

Proof: Let α be defined as the trajectory of the magnetic field V . Since $V \in \text{span}\{N, C, W\}$ according to the alternative frame, $V = e_1 N + e_2 C + e_3 W$ can be written. On the other hand, $\Phi(N) = V \times N$ and from Eq (13), we given by $e_2 = -\lambda$ and $e_3 = \beta$. In a similar manner, by taking the equations

$$\Phi(C) = V \times C \text{ and } \Phi(W) = -\beta N + \gamma W,$$

executing the requisite computations, we obtain $e_1 = \gamma$. Therefore, the Eq(15) is arrived at.

3.3 W -Magnetic Curves

Definition 3.3.1: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be a curve in Euclidean 3-space. If the vector field W of the alternative frame satisfies the Lorentz force equation $\nabla_{\alpha'} W = \Phi(W) = V \times W$, then the curve α is called the W -magnetic curve.

Theorem 3.3.2: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be an the C -magnetic curve with unit velocity. Then, the Lorentz force according to the alternative frame is obtained as

$$\begin{bmatrix} \Phi(N) \\ \Phi(C) \\ \Phi(W) \end{bmatrix} = \begin{bmatrix} 0 & \Omega_2 & 0 \\ -\Omega_2 & 0 & \gamma \\ 0 & -\gamma & 0 \end{bmatrix} \begin{bmatrix} N \\ C \\ W \end{bmatrix} \quad (16)$$

Proof: Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{R}^3$ be an the W -magnetic curve and let $\{N, C, W, \beta, \gamma\}$ be the alternative frame elements of this curve. Since $\Phi(N) \in \text{span}\{N, C, W\}$,

$$\Phi(N) = f_1 N + f_2 C + f_3 W \quad (17)$$

can be written. If both sides of Eq (17) are inner product by N and the necessary operations are performed, $f_1 = 0$ is obtained. Similarly, If both sides of Eq (17) are inner product by C and the necessary operations are performed, then, we are given by $f_2 = \Omega_2$. If similar operations are made for W , $b_3 = 0$ is obtained. If f_1, f_2, f_3 is written at Eq (17), we obtained

$$\Phi(N) = \Omega_2 C$$

Similarly, if the operations applied to $\Phi(N)$ are repeated for $\Phi(C)$ and $\Phi(W)$, the following results are obtained:

$$\Phi(C) = -\Omega_2 N + \gamma W$$

$$\Phi(W) = -\gamma C$$

Theorem 3.3.3: Let α be an the W -magnetic curve with unit velocity. For α to be the trajectory of V magnetic field necessary and sufficient condition the magnetic field V along the curve α can be written as follows:

$$\mathbf{V} = \gamma \mathbf{N} + \Omega_2 \mathbf{W} \quad (18)$$

Proof: Let α be defined as the trajectory of the magnetic field $\mathbf{V}$. Since $\mathbf{V} \in \text{span}\{\mathbf{N}, \mathbf{C}, \mathbf{W}\}$ according to the alternative frame, $\mathbf{V} = k_1 \mathbf{N} + k_2 \mathbf{C} + k_3 \mathbf{W}$ can be written. On the other hand, $\Phi(\mathbf{N}) = \mathbf{V} \times \mathbf{N}$ and

from Eq (16), we given by $k_2 = 0$ and $k_3 = \Omega_2$. In a similar manner, by taking the equations

$$\Phi(\mathbf{C}) = \mathbf{V} \times \mathbf{C} \text{ and } \Phi(\mathbf{C}) = -\Omega_2 \mathbf{N} + \gamma \mathbf{W},$$

executing the requisite computations, we obtain $k_1 = \gamma$. Therefore, the Eq (18) is arrived at.

4. Fermi-Walker Derivations of Magnetic Curves According to The Alternative Frame

Theorem 4.1: Let α be a curve and $\mathbf{X} = \lambda_1 \mathbf{N} + \lambda_2 \mathbf{C} + \lambda_3 \mathbf{W}$ be any vector field along the curve α . Fermi-Walker derivations $\tilde{\nabla} \mathbf{X}$ according to the alternative frame is

$$\frac{\tilde{\nabla} \mathbf{X}}{\tilde{\nabla} s} = \frac{d\mathbf{X}}{ds} - \beta(\mathbf{W} \times \mathbf{X}). \quad (19)$$

Proof: If the Fermi Walker derivative is rewritten according to the alternative frame,

$$\frac{\tilde{\nabla} \mathbf{X}}{\tilde{\nabla} s} = \frac{d\mathbf{X}}{ds} - \langle \mathbf{N}, \mathbf{X} \rangle \frac{d\mathbf{N}}{ds} + \langle \frac{d\mathbf{N}}{ds}, \mathbf{X} \rangle \mathbf{N} \quad (18)$$

is obtained. From Eq (1) and Eq (18), we obtained Eq (19).

4.2 Fermi-Walker Derivations of N -Magnetic Curves

From theorem 4.1, we define Fermi-Walker derivations of N -magnetic curves as follows:

$$\tilde{\nabla}_N \Phi(\mathbf{N}) = \nabla_N \Phi(\mathbf{N}) - \beta(\mathbf{W} \times \Phi(\mathbf{N})) \quad (20)$$

Theorem 4.2.1: Let α be a N -magnetic curve according to the alternative frame. The magnetic field $\mathbf{V}$ and Lorentz forces $\Phi(\mathbf{N})$, $\Phi(\mathbf{C})$, $\Phi(\mathbf{W})$ with respect to this frame can write Fermi-Walker derivations as follows:

- i. $\tilde{\nabla}_N \Phi(\mathbf{N}) = \beta' \mathbf{C} + \beta \gamma \mathbf{W}$
- ii. $\tilde{\nabla}_N \Phi(\mathbf{C}) = -\beta' \mathbf{N} - \Omega \gamma \mathbf{C} + \Omega' \mathbf{W}$
- iii. $\tilde{\nabla}_N \Phi(\mathbf{W}) = -\Omega' \mathbf{C} - \Omega \gamma \mathbf{W}$
- iv. $\tilde{\nabla}_N \mathbf{V} = \Omega' \mathbf{N} - \beta \gamma \mathbf{C} + \beta' \mathbf{W}$.

Proof: i) : Let α be a N -magnetic curve. From Eq(20) and $\Phi(\mathbf{N}) = \beta \mathbf{C}$, we write

$$\tilde{\nabla}_N \Phi(\mathbf{N}) = \beta' \mathbf{C} + \beta \gamma \mathbf{W} \quad (21)$$

ii) From Eq(20) and $\Phi(\mathbf{C}) = -\beta \mathbf{N} + \Omega \mathbf{W}$ we obtain

$$\tilde{\nabla}_N \Phi(\mathbf{C}) = -\beta' \mathbf{N} - \Omega \gamma \mathbf{C} + \Omega' \mathbf{W} \quad (22)$$

We can prove iii and iv by using similar operations.

4.3 Fermi-Walker Derivations of C -Magnetic Curves

From theorem 4.1, we define Fermi-Walker derivations of C -magnetic curves as follows:

$$\tilde{\nabla}_N \Phi(\mathbf{C}) = \nabla_N \Phi(\mathbf{C}) - \beta(\mathbf{W} \times \Phi(\mathbf{C})). \quad (23)$$

Theorem 4.3.1: Let α be a C -magnetic curve according to the alternative frame. The magnetic field $\mathbf{V}$ and Lorentz forces $\Phi(\mathbf{N})$, $\Phi(\mathbf{C})$, $\Phi(\mathbf{W})$ with respect to this frame can write Fermi-Walker derivations as follows:

- i. $\tilde{\nabla}_N \Phi(\mathbf{N}) = (\beta' - \lambda \gamma) \mathbf{C} + (\beta \gamma + \lambda') \mathbf{W}$
- ii. $\tilde{\nabla}_N \Phi(\mathbf{C}) = -\beta' \mathbf{N} - \lambda^2 \mathbf{C} + \lambda' \mathbf{W}$
- iii. $\tilde{\nabla}_N \Phi(\mathbf{W}) = -\lambda' \mathbf{N} - \gamma' \mathbf{C} - \gamma^2 \mathbf{W}$

$$\text{iv. } \tilde{\nabla}_N V = \gamma' N + (-\lambda' - \beta\gamma)C + (\beta' - \lambda\gamma)W$$

Proof: i. : Let α be a C -magnetic curve. From Eq(23) and $\Phi(N) = \beta C + \lambda W$, we obtain

$$\tilde{\nabla}_N \Phi(N) = (\beta' - \lambda\gamma)C + (\beta\gamma + \lambda')W.$$

ii. From Eq(23) and $\Phi(C) = -\beta N + \lambda W$ we find

$$\tilde{\nabla}_N \Phi(C) = -\beta' N - \lambda^2 C + \lambda' W.$$

We can prove iii and iv by using similar operations.

4.4 Fermi-Walker Derivations of W-Magnetic Curves

From theorem4.1, we define Fermi-Walker derivations of W -magnetic curves as follows:

$$\tilde{\nabla}_N \Phi(W) = \nabla_N \Phi(W) - \beta(W \times \Phi(W)) \quad (24)$$

Theorem 4.4.1: Let α be a C -magnetic curve according to the alternative frame. The magnetic field V and Lorentz forces $\Phi(N)$, $\Phi(C)$, $\Phi(W)$ with respect to this frame can write Fermi-Walker derivations as follows:

$$\text{i. } \tilde{\nabla}_N \Phi(N) = \Omega'_2 C + \Omega_2 \gamma W$$

$$\text{ii. } \tilde{\nabla}_N \Phi(C) = -\Omega'_2 N - \gamma^2 C + \gamma' W$$

$$\text{iii. } \tilde{\nabla}_N \Phi(W) = -\gamma' C - \gamma^2 W$$

$$\text{iv. } \tilde{\nabla}_N V = \gamma' N - \Omega_2 \gamma C + \Omega'_2 W$$

Proof: i. Let α be a W -magnetic curve. From Eq(24) and $\Phi(N) = \Omega_2 C$, we obtain

$$\tilde{\nabla}_N \Phi(N) = \Omega'_2 C + \Omega_2 \gamma W.$$

ii. From Eq(24) and $\Phi(N) = \Omega_2 C$, we obtain

$$\tilde{\nabla}_N \Phi(C) = -\Omega'_2 N - \gamma^2 C + \gamma' W.$$

We can prove iii and iv by using similar operations

5. Conclusion

In this study, magnetic curves are defined according to the alternative frame in 3-Euclidean space. Some characterizations related to these curves are obtained. In conclusion, a congruent methodology can be utilised to delineate magnetic curves in three-dimensional dual space according to the alternative frame. This approach facilitates the investigation of the relationships between these curves.

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