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Rényi Entropy Based Dark Energy Models in Bianchi I Cosmology

Mutlu Battaloğlu[1], Can Aktaş[2]

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Abstract— In this study, the Rényi holographic dark energy (RHDE) model is analyzed in the framework of the $f(R,T)$ theory for the homogeneous anisotropic Bianchi metric. For this purpose, field equations are obtained in $f(R,T)$ theory and Hubble parameter and energy density of Rényi holographic dark energy are utilized to obtain exact solutions of these equations. The obtained solutions were plotted using four different data sets and the models were analyzed in detail with the help of these plots.

Keywords — *Renyi holographic dark energy, Bianchi I metric, $f(R,T)$ theory*

1. Introduction

The fact that the universe is expanding at an accelerating rate was first confirmed by supernova observations in the late 1990s, and this phenomenon has raised profound questions about the fundamental nature of cosmic evolution [1]. This expansion contradicts predictions based on Newtonian and General Relativity theories, suggesting that the expansion should slow down due to matter's gravitational effects. Observations show that the force behind the accelerated expansion of the universe is dark energy. This component is not directly observed but makes up much of the universe's energy density.

Dark energy, proposed as the source of this acceleration, constitutes approximately 68% of the total energy density of the universe and remains one of the greatest puzzles of modern cosmology [2]. Among the theories of dark energy, the most common is the Λ-Cold Dark Matter (ΛCDM) model, which assumes that dark energy is a cosmological constant (Λ) [3]. However, this model is under theoretical scrutiny due to its inability to explain why the cosmological constant has a very small but nonzero value [4]. In addition, the idea that dark energy could vary over time has led to the development of alternative models to explain the expansion rates in the history of the universe [5].

Among these models, one of the notable approaches is the holographic dark energy (HDE) concept. The holographic energy model is based on the holographic principle, which suggests that the energy density and entropy are directly related to the boundary conditions of the universe [6]. The theoretical foundation of HDE is based on Bekenstein-Hawking entropy and the universal energy bounds of the holographic principle [7]. The formulations of HDE that express the energy density in the universe are generally associated with the concept of entropy. Different definitions of entropy have led to the

[1]mutlu@comu.edu.tr; [2]canaktas@comu.edu.tr (Corresponding Author)
[1]Business Administration, Çanakkale Vocational School of Social Sciences, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye
[2]Department of Mathematics, Faculty Sciences, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye

diversification of HDE models [8]. Among them, Rényi entropy plays an important role in understanding the dynamic nature of dark energy. Rényi entropy is a generalized form of the standard Bekenstein-Hawking entropy, which allows for the dark energy density to have an evolving structure over time [9]. HDE models structured with Rényi entropy provide a powerful framework for understanding the dynamics of dark energy in anisotropic and complex cosmic environments [10].

Many cosmologists have developed Rényi Holographic Dark Energy (RHDE) models in modified gravity theories in recent studies. Bhardwaj and others examined the RHDE model within the *f(T)* modified gravity framework [11]. Saha et al. have presented cosmological models of dark energy defined for RHDE in the DGP brane-world cosmology with and without interactions between cosmic fluids in their studies [12]. Alam et al. have studied RHDE and its behavior in an anisotropic and homogeneous Bianchi type-I Universe within the $f(G)$ gravity framework [13]. Bharali and Das have investigated the dynamics of the Bianchi type VI_0 space-time within the framework of RHDE $f(R,T)$ gravity theory [14]. Saleem et al. [15] have studied the Rényi, Sharma-Mittal, and generalised holographic dark energy distributions in the LRS Bianchi I metric in $f(Q)$ theory. Sardar et al. [16] have Tsallis, Rény,i and Sharma-Mittal analysed holographic dark energy models for the non-flat metric in Hořava-Lifshitz gravitation theory. Rényi's holographic dark energy model was analyzed using the framework of $F(Q,C)$ theory in the FRW universe by Sarddar et al. The Rényi holographic dark-geometry model was studied in the FRW universe in the framework of $f(Q,C)$ theory by Samaddar et al. [17]. Aktaş and Aygün [18] Barrow investigated the holographic dark energy model for Bianchi I space-time in Lyra and General relativity theories. Moreover, the RHDE solution for the FRW metric was obtained by Aktaş and Yılmaz [19] in General relativity theory.

Einstein's General Theory of Relativity has successfully explained cosmological phenomena, but it has some shortcomings in explaining components such as dark energy and dark matter. Therefore, various alternative gravity theories have been developed to extend Einstein's equations. One such theory is the $f(R,T)$ gravity theory, which provides a generalization based on the Ricci scalar (R) and the trace of the energy-momentum tensor (T) [20]. This theory provides a flexible structure to study the dynamic interactions between matter and geometry and goes beyond the standard gravity theories used in cosmology.

Studies under the $f(R,T)$ theory to model dark energy are mostly based on isotropic geometries in the literature. However, examining anisotropic models to understand the conditions of the early universe and explain the expansion dynamics is also important. In this context, the $f(R,T)$ theory provides a strong framework to examine the dark energy density and the expansion rate of the universe under anisotropic scenarios. Anisotropic models play a critical role in understanding the expansion dynamics that arose in the early universe and reconstructing the physical conditions of that era. The Bianchi I metric, as a homogeneous but anisotropic model of the universe, allows for different expansion rates along the three orthogonal axes of space. The Bianchi I metric has important advantages in explaining the possible anisotropic effects and energy distributions, especially in cases where isotropic models fall short, particularly in the early universe. When holographic dark energy models are studied in the context of an anisotropic universe, one can understand how the dark energy density responds to the expansion rates and how this response affects the level of anisotropy.

This study analyzes the Rényi entropy-based holographic dark energy model in the context of $f(R,T)$ gravity theory, using the anisotropic Bianchi I metric as a model. Our aim is to understand the effects of RHDE on the dynamics of an anisotropic universe, explain cosmic acceleration, and contribute to the expansion of entropy-related holographic dark energy models. The rest of the paper is organised as follows: Section 2 presents the field equations in $f(R,T)$ theory for the Bianchi I metric. In section 3, solutions of the field equations are obtained, and the solutions are discussed in detail with the help of graphs. In the last section, future work is mentioned.

2. Filed Equations in $f(R,T)$ Theory

$f(R,T)$ theory was put forward by Harko et al. in 2011 [20]. In this study, the authors made 3 suggestions for the $f(R,T)$ function. If $f(R,T) = \lambda_1 R + \lambda_2 T$ is taken as suggested by Harko et al. in this study, the field equations are obtained as follows [16]. Here λ_1 and λ_2 are constants. If $\lambda_1 = 1$ and $\lambda_2 = 0$ are taken, the GRT field equations are obtained.

$$G_{ik} = \left(\frac{\lambda_2 + 8\pi}{\lambda_1}\right) T_{ik} + \left(P + \frac{T}{2}\right) \frac{\lambda_2}{\lambda_1} g_{ik} \tag{2.1}$$

The homogeneous anisotropic Bianchi I metric is expressed as follows.

$$ds^2 = -A^2dx^2 - B^2dy^2 - C^2dz^2 + dt^2 \tag{2.2}$$

where A, B, and C are functions of cosmic time. From (2.2), we obtain the Hubble parameter, expansion scalar, and spatial volume as follows,

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)$$

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}$$

$$V = a^3 = ABC$$

Here, the dot is the derivative of cosmic time concerning t. As is well known, the universe is filled with a hypothetical fluid known as holographic dark energy. Its energy-momentum tensor,

$$T_{ij}^m = \rho_m u_i u_j \tag{2.3}$$

where u_i is the 4-velocity and ρ_m is the energy density of matter. Moreover, energy momentum tensor for RHDE T_{ij}^R,

$$T_{ij}^R = (\rho_R + P)\, u_i u_j - P g_{ij} \tag{2.4}$$

Here ρ_R is the energy density of RHDE and P is the pressure. In this study, it is assumed that the universe is filled with both holographic dark energy and matter. From (2.3) and (2.4), the total energy-momentum tensor can be written as follows:

$$T_{ij} = T_{ij}^m + T_{ij}^R = (\rho_m + \rho_R + P)\, u_i u_j - p g_{ij} \tag{2.5}$$

From (2.1), (2.2) and (2.5), the field equations in the $f(R,T)$ theory are obtained as follows.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \frac{8\pi P}{\lambda_1} + \frac{3P\lambda_2}{2\lambda_1} - \frac{\lambda_2\rho_m}{2\lambda_1} - \frac{\lambda_2\rho_R}{2\lambda_1} \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \frac{8\pi P}{\lambda_1} + \frac{3P\lambda_2}{2\lambda_1} - \frac{\lambda_2\rho_m}{2\lambda_1} - \frac{\lambda_2\rho_R}{2\lambda_1} \tag{2.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \frac{8\pi P}{\lambda_1} + \frac{3P\lambda_2}{2\lambda_1} - \frac{\lambda_2\rho_m}{2\lambda_1} - \frac{\lambda_2\rho_R}{2\lambda_1} \tag{2.8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \frac{P\lambda_2}{2\lambda_1} - \frac{8\pi\rho_m}{\lambda_1} - \frac{3\lambda_2\rho_m}{2\lambda_1} - \frac{8\pi\rho_R}{\lambda_1} - \frac{3\lambda_2\rho_R}{2\lambda_1} \tag{2.9}$$

3. Results and Discussions

The system of equations (2.6) - (2.9) consists of four equations in six unknowns (A, B, C, ρ_m, ρ_R, P). In order to obtain an exact solution of this equation, we need two additional equations.

Firstly, by estimating the Hubble horizon as an IR cut-off, we can get the energy density of the RHDE as follows [21]

$$\rho_R = \frac{3c^2 \cdot H^2}{1 + \pi \cdot \xi \cdot H^{-2}} \tag{3.1}$$

Here, c^2 is a numerical constant and ξ is a free parameter known as the real non-extensive parameter.

Secondly, the Hubble parameter can be used. The Hubble parameter, which shows how fast the universe is expanding, is critical in cosmology, and this information can be used to calculate the age of the universe. This study takes the preferred Hubble parameter in the following form [22].

$$H=\frac{\beta}{\sqrt{t+\alpha}} \tag{3.2}$$

Here, α and β are constants.

Energy density of Renyi holographic dark energy from (3.1) and (3.2), we get

$$\rho_R=\frac{3c^2\beta^4}{\left(\pi\left(t+\alpha\right)\xi+\beta^2\right)\left(t+\alpha\right)} \tag{3.3}$$

From (3.3), (2.6) - (2.9), the metric potentials A, B and C are obtained as follows, respectively.

$$A=c_1 e^{-\frac{\left(\frac{\sqrt{t+\alpha}}{6}+\beta(t+\alpha)\right)(c_1+c_3)e^{-6\beta\sqrt{t+\alpha}}+\left(-2\beta^2-\frac{c_2}{6}-\frac{c_4}{6}\right)\sqrt{t+\alpha}-6\beta^3(t+\alpha)}{3\sqrt{t+\alpha}\beta^2}} \tag{3.4}$$

$$B=e^{-\frac{\left(\frac{\sqrt{t+\alpha}}{6}+\beta(t+\alpha)\right)c_3 e^{-6\beta\sqrt{t+\alpha}}+\left(-\beta^2-\frac{c_4}{6}\right)\sqrt{t+\alpha}+6\beta^3(t+\alpha)}{3\sqrt{t+\alpha}\beta^2}} \tag{3.5}$$

and

$$C = e^{-\frac{\left(\frac{\sqrt{t+\alpha}}{6}+\beta(t+\alpha)\right)c_1 e^{-6\beta\sqrt{t+\alpha}}+\left(-\beta^2-\frac{c_2}{6}\right)\sqrt{t+\alpha}+6\beta^3(t+\alpha)}{3\sqrt{t+\alpha}\beta^2}} \tag{3.6}$$

Here, $c_1 - c_4$ are arbitrary constants. Energy density of matter is,

$$\begin{aligned}\rho_m = \quad & -\frac{3c^2\beta^4}{(t+\alpha)\left(\pi\left(t+\alpha\right)\xi+\beta^2\right)} - \frac{\left(c_1^2+c_3c_1+c_3^2\right)\lambda_1 e^{-12\beta\sqrt{t+\alpha}}}{8\pi+\lambda_2} \\ & +\frac{\lambda_1\lambda_2\beta}{(t+\alpha)^{\frac{3}{2}}\left(32\pi+4\lambda_2\right)\left(4\pi+\lambda_2\right)} + \frac{6\lambda_1\beta^2\left(8\pi+\lambda 2\right)}{(t+\alpha)\left(32\pi+4\lambda_2\right)\left(4\pi+\lambda_2\right)}\end{aligned} \tag{3.7}$$

Moreover, pressure is

$$P=\frac{3\lambda_1\beta^2}{2\left(4\pi+\lambda_2\right)\left(t+\alpha\right)}-\frac{\lambda_1\left(16\pi+3\lambda_2\right)\beta}{4\left(8\pi+\lambda_2\right)\left(4\pi+\lambda_2\right)\left(t+\alpha\right)^{3/2}}+\frac{\left(c_1^2+c_3c_1+c_3^2\right)e^{-12\beta\sqrt{t+\alpha}}\lambda_1}{8\pi+\lambda_2} \tag{3.8}$$

The equation of the state parameter is denoted as w. From (3.3) and (3.8), we obtain the equation of the state parameter for the universe model we have analyzed as follows.

$$w=\frac{P}{\rho_R}=\frac{\frac{3\lambda_1\beta^2}{2(4\pi+\lambda_2)(t+\alpha)}-\frac{\lambda_1(16\pi+3\lambda_2)\beta}{4(8\pi+\lambda_2)(4\pi+\lambda_2)(t+\alpha)^{3/2}}+\frac{\left(c_1^2+c_3c_1+c_3^2\right)e^{-12\beta\sqrt{t+\alpha}}\lambda_1}{8\pi+\lambda_2}}{\frac{3c^2\beta^4}{(\pi(t+\alpha)\xi+\beta^2)(t+\alpha)}}$$

In order to draw the graphs of our solutions, the values of the constants must be known. We can use some observational data to determine the values of the constants in solutions. In this study, four different observational values in Table 1 below were used to obtain the graphs.

Table 1. Values of the constants

Data Set	α	β	c	ξ	λ_1	λ_2	c_1	c_2	c_3
SN Ia [23]	1	0.4814	5	1.4	1	$-8\pi+1$	1	1	1
SN Ia+H(z)+ BAO/CMB [24]	1.6	0.2770	12	1.4	1	$-8\pi+1$	1	1	1
SN Ia + BAO + H(z) [25]	1.3	0.3762	8	1.4	1	$-8\pi+1$	1	1	1
CC+SN Ia + BAO + R18 [26]	1.6	0.2963	10	1.4	1	$-8\pi+1$	1	1	1

The graphs of the metric potentials with the help of equations (3.4)-(3.7) and the values of the constants in Table 1 are as follows:

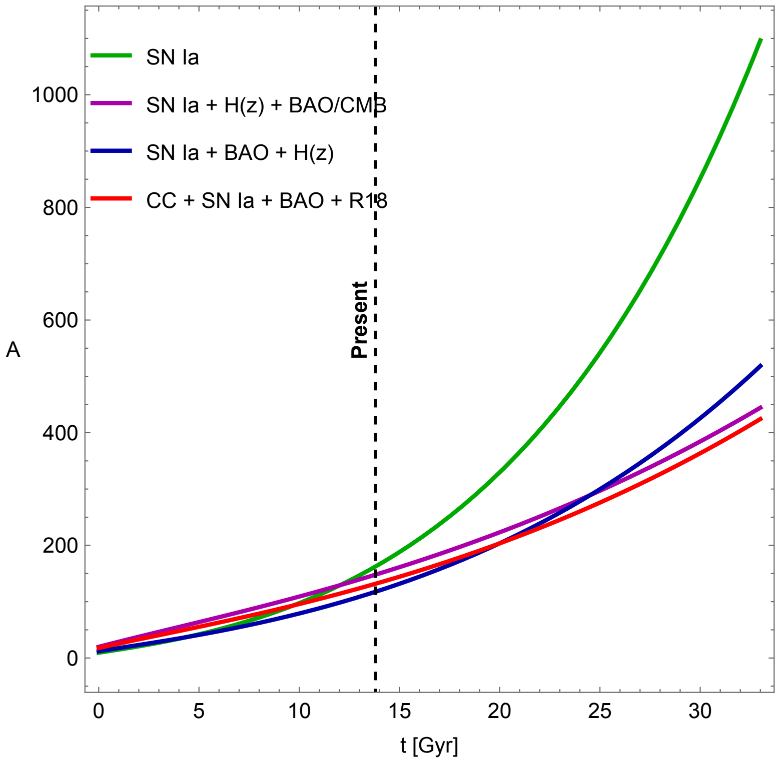

Figure 1. Variation of A against cosmic time

The metric potential A shows a similar trend in all data sets. When Figure 1 is examined, it is seen that A shows acceleration in the range of 5-10 Gyr and a linear increase after 20 Gyr. SN Ia data shows a slightly faster acceleration and a higher potential when used alone. CC + SN Ia + BAO + R18 data present the earliest acceleration and the highest late period value. SN Ia + H(z) + BAO/CMB and SN Ia + BAO + H(z) sets stand out with more balanced and soft increases.

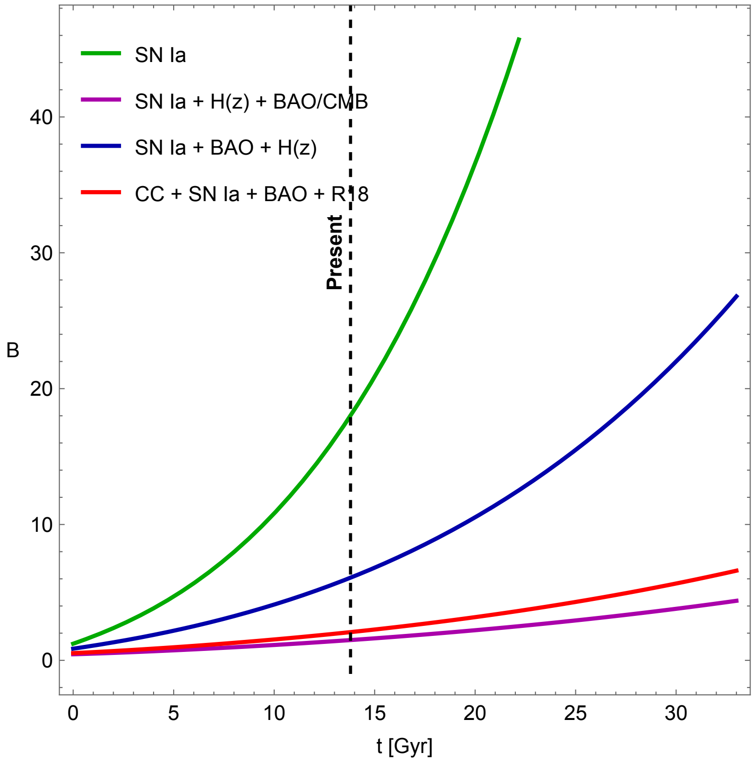

Figure 2. Variation of B against cosmic time

The metric potential B starts from very low levels in all data sets. Differences between data sets gradually become apparent between 5-10 Gyr. When Figure 2 is examined, it is understood that the acceleration is earlier and stronger in SN Ia and CC + SN Ia + BAO + R18 data in this range. As a result, although all data sets show a similar growth trend, CC + SN Ia + BAO + R18 shows a faster and higher increase.

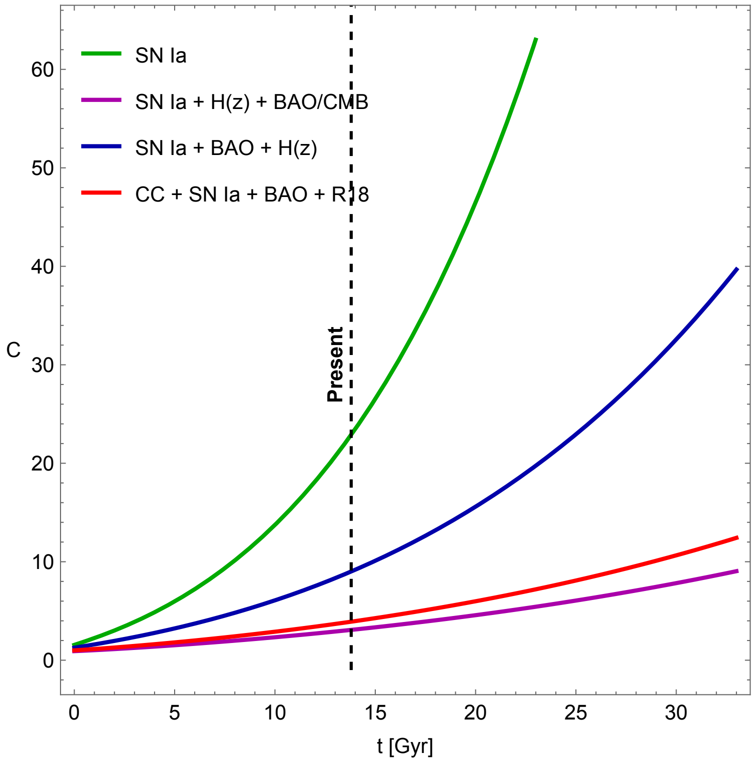

Figure 3. Variation of C against cosmic time

Although the change of C potential in the Bianchi I metric with time shows a similar trend in general in four different data sets, significant differences are noted in the acceleration timing and late period values. The CC + SN Ia + BAO + R18 data set showed an earlier acceleration than the other sets. SN Ia data, when used alone, showed a rapid increasing trend. This situation indicates that Type Ia supernovae are a strong cosmological indicator. SN Ia + H(z) + BAO/CMB and SN Ia + BAO + H(z) data sets have a more balanced and stable growth. When 5-10 Gyr in Figure 3 is examined, the separation in the acceleration curves begins, and CC + SN Ia + BAO + R18 and SN Ia data show a more pronounced acceleration in this period. The 20-30 Gyr interval is when the growth rate between different data sets becomes clear. CC + SN Ia + BAO + R18 stand out as reaching the highest potential. This comparison clearly shows the influence of different observational data on cosmological models and how the anisotropic expansion in the Bianchi I metric evolves at different speeds.

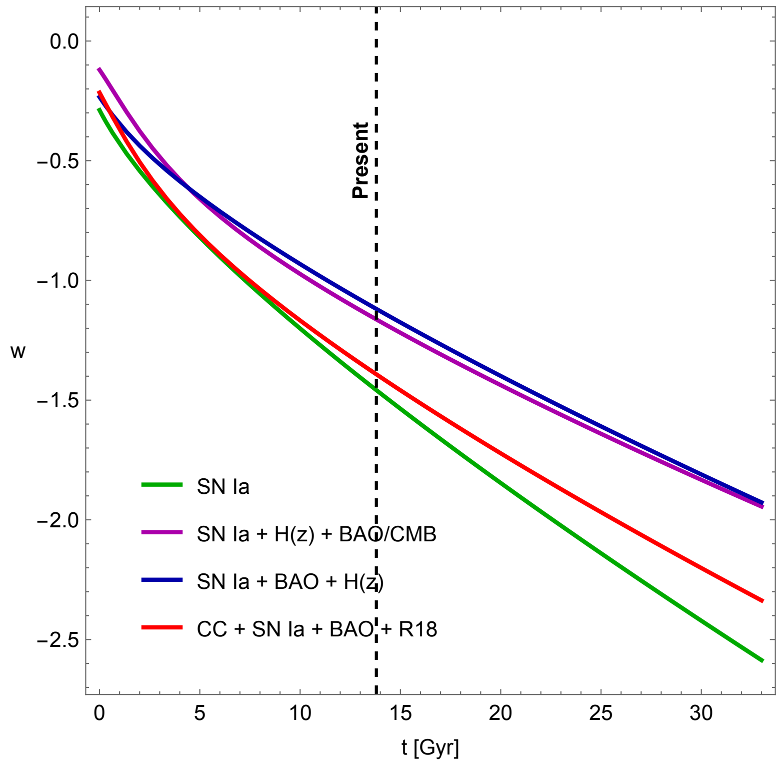

Figure 4. Variation of w against cosmic time

Figure 4 shows the variation of the equation of state parameter w with time. Since the equation of state parameter is used to determine the dynamical properties of dark energy, this parameter plays a critical role in cosmological models. The values of the EosS parameter for Planck + Pantheon + BAO, WMAP + eCMB + BAO + H0 + SNe and 68% Planck TT,TE,EE+lowE+lensing+SNe+BAO are -1.03 ±0.03 [27], -1.084 ±0.063 [28] and -1.028 ±0.031 [27], respectively. Furthermore, if the value of the EoS parameter of our model is calculated for today, it is -1.45827, -1.16293, -1.11914, -1.39217 for SN Ia, SN Ia + H(z) + BAO/CMB, SN Ia + BAO + H(z) and CC + SN Ia + BAO + R18 models respectively. Accordingly, it can be observed that our SN Ia + H(z) + BAO/CMB and SN Ia + BAO + H(z)models are compatible with the observational data. When the graph is analysed, it is seen that w takes negative values. This shows that the universe model we obtained is compatible with dark energy studies. In particular, $w = -1$ corresponds to the cosmological constant model (ΛCDM). If w is more negative than this value in the graph (e.g., $w < -1$), this indicates a 'phantom energy' scenario. If $w > -1$ but negative, this indicates 'quintessence' models.

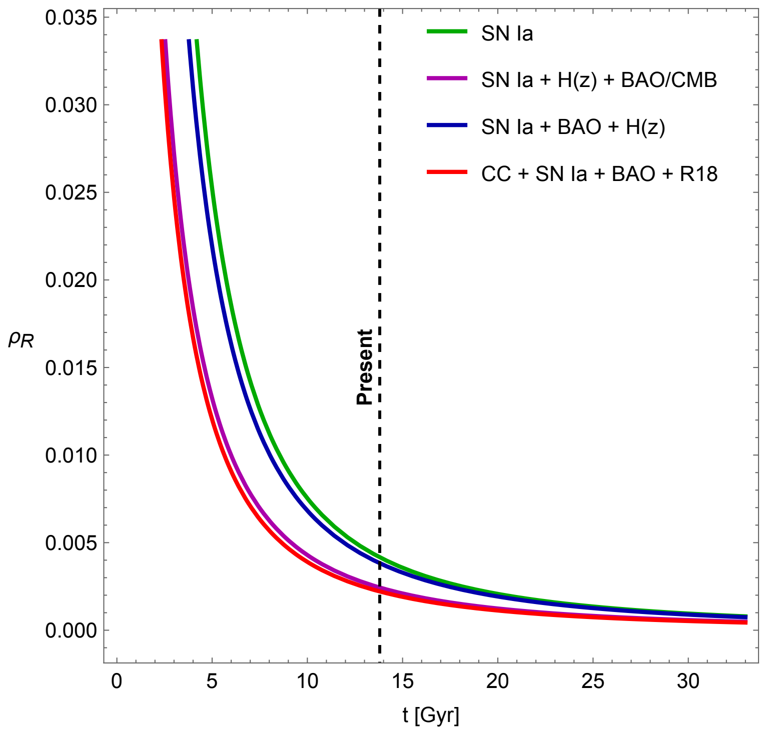

Figure 5. Variation of ρ_R against cosmic time

As can be seen from Figure 5, ρ_R follows a markedly decreasing trend with time. This is an expected behaviour in cosmological models. Because as the universe expands, the dark energy density is expected to change. However, the unique aspect of the RHDE model is that it describes this energy density in terms of entropy and space-time relation. The curves obtained with different observational data (such as SN Ia, H(z), BAO/CMB) are very close. This shows that the model agrees quite well with the observations. The fact that ρ_R approaches zero, especially in the later stages, indicates that the universe will have an increasingly sparse energy density, and dark energy will dominate in the long run. For holographic models based on Rényi entropy, this decay pattern can provide essential clues about the parameters of the model. This graph is very valuable for analyzing the dynamical structure of RHDE and its consistency with observations. In particular, paying attention to how the energy density decreases in which period and the agreement of observational data with the model strongly support this study.

Figure 6 shows that the pressure is negative. This is an expected feature in dark energy models. Negative pressure appears to be the main reason for the accelerating expansion of the universe. This is consistent with the cosmological constant (Λ) or dynamical dark energy models. In the graph, the curves obtained with different observational data (such as SN Ia, H(z), BAO/CMB) overlap greatly. This suggests that the model is in good agreement with observational data. The fact that the pressure approaches less negative values with time indicates that the expansion rate of the universe may settle down to a certain level while the dark energy becomes dominant. This negativity in the pressure of RHDE supports the cosmological effect of entropy-based models. How the pressure changes with time provides important information about the dynamical nature of dark energy and provides a critical observation for testing energy-momentum relations in the framework of Rényi entropy.

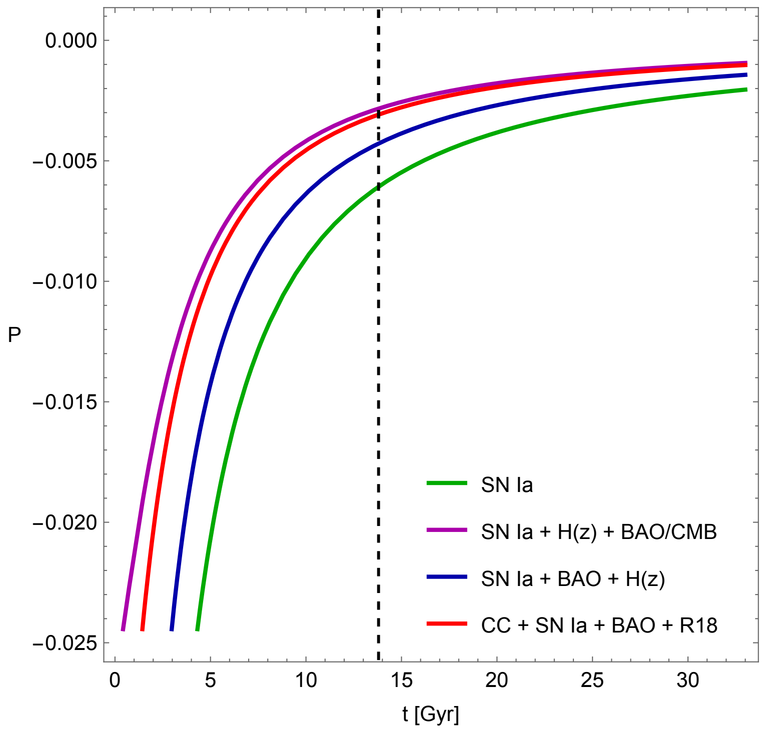

Figure 6. Variation of P against cosmic time

4. Conclusion

This study investigates the behaviour of RHDE matter distribution in the homogeneous and anisotropic Bianchi I universe model within the framework of $f(R,T)$ theory. This study analyses the dynamics of RHDE in an anisotropic universe and investigates its effect on the expansion rates. The solutions obtained using the Hubble parameter and the energy density of the RHDE are compared with four different data sets and supported by graphs. The results show that the accelerating expansion of the Universe is consistent with the RHDE and that the equation of state parameter w takes negative values. In particular, some data sets w values are close to the ΛCDM model, while the others support the 'phantom energy' or 'quintessence' scenarios. Furthermore, it is observed that the RHDE exhibits a decreasing energy density with time and that negative pressure explains the acceleration of the universe. These findings emphasise that anisotropic models play an important role in understanding the conditions of the early universe. In future studies, it will be useful to investigate other holographic dark energy models in different space-time metrics within the framework of $f(R,T)$ theory. This work can be presented as a master's or doctoral thesis using a different holographic dark energy.

Author Contributions

All the authors contributed equally to this work. This paper is derived from the first author's doctoral dissertation supervised by the second author. They all read and approved the final version of the paper.

Conflicts of Interest

All the authors declare no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

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