Mathematical Sciences And Applications E-Notes
Volume 3 No. 2 pp. 45–53 (2015) © MSAEN

CERTAIN SEQUENCE OF FUNCTIONS INVOLVING GENERALIZED HYPERGEOMETRIC FUNCTIONS

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(Communicated by Ali ÇEVİK)

Abstract. A remarkably large number of operational techniques have drawn the attention of several researchers in the study of sequence of functions and polynomials. In this sequel, here, we aim to introduce a new sequence of functions involving the generalized Gauss hypergeometric function by using operational techniques. Some generating relations and finite summation formula of the sequence presented here are also considered.

1. Introduction

In recent years, operational techniques have been attract the attention of many researchers due to their importance and applications in various sub-fields of analysis (see [5], [9],[8], [17],Srivastava and Singh[20], Mittal[11, 12, 13], Chandal[6, 7], Srivastava[16], Joshi and Parjapat[10],Patil and Thakare[15] and Srivastava and Singh[19]). In the sequels a remarkably large number of sequence of functions involving a variety of special functions have been developed by many authors (see, for example, [19]; for a very recent work, see also [18]). In the present study we aim to develop a new sequence of functions involving the $F_p^{(\alpha,\beta)}(.)$ by using operational techniques, which are expressed in terms of the generalized Gauss hypergeometric function. Furthermore, some generating relations and finite summation formula are also obtained.

For our purpose, we begin by recalling some known functions and earlier works.

In 1971, Mittal [11] gives the Rodrigues formula for the generalized Lagurre polynomials defined as:

$$T_{kn}^{(\alpha)}(x) = \frac{1}{n!}x^{-\alpha}\exp\left(p_k(x)\right)D^n\left[x^{\alpha+n}\exp\left(-p_k(x)\right)\right] \tag{1.1}$$

where $p_k(x)$ is a polynomial in x of degree k.

Mittal [12] also proved the following relation for (1.1) defined as:

$$T_{kn}^{(\alpha+s-1)}(x) = \frac{1}{n!}x^{-\alpha-n}\exp\left(p_k(x)\right)T_s^n\left[x^{\alpha}\exp\left(-p_k(x)\right)\right] \tag{1.2}$$

Date: Received: August 13, 2015; Accepted: October 9, 2015.
2010 *Mathematics Subject Classification.* 33E10, 33E12, 33E99, 44A45.
Key words and phrases. Special function, generating relations , generalized Gauss hypergeomtric functions, Sequence of function,finite summation formula.

where s is constant and $T_s \equiv x\,(s+xD)$.

In this sequel, in 1979, Srivastava and Singh [19] studied a sequence of functions $V_n^{(\alpha)}\,(x;a,k,s)$ defined as:

$$V_n^{(\alpha)}\,(x;a,k,s) = \frac{x^{-\alpha}}{n!}\exp\{p_k\,(x)\}\,\theta^n\,[x^{\alpha}\exp\{-p_k\,(x)\}] \tag{1.3}$$

By employing the operator $\theta \equiv x^a\,(s+xD)$, where s is constant and $p_k\,(x)$is a polynomial in x of degree k.

A new sequence of function $\left\{V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}\,(x;a,k,s)\right\}_{n=0}^{\infty}$ is introduced in this paper as:

$$\begin{aligned} V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}\,(x;a,k,s) &= \frac{1}{n!}x^{-\alpha}F_p^{(\rho,\sigma)}\,[\lambda,\mu;\nu;p_k\,(x)] \\ &\quad\times (T_x^{a,s})^n\left\{x^{\alpha}F_p^{(\rho,\sigma)}\,[\lambda,\mu;\nu;-p_k\,(x)]\right\}, \end{aligned} \tag{1.4}$$

where $T_x^{a,s} \equiv x^a\,(s+xD)\,, D \equiv \dfrac{d}{dx}$, a and s are constants, k is finite and non-negative integer, $p_k\,(x)$ is a polynomial in x of degree k and $F_p^{(\rho,\sigma)}\,[\lambda,\mu;\nu;x]$ is a generalized Gauss hypergeometric functions of one variables. For the sake of completeness, we define this function here (for more detail see [14]):

$$F_p^{(\rho,\sigma)}\,(\lambda,\mu;\nu;x) = \sum_{n=0}^{\infty}(\lambda)_n\frac{B_p^{(\rho,\sigma)}\,(\mu+n,\nu-\mu)}{B\,(\mu,\nu-\mu)}\frac{x^n}{n!};\qquad (|x|<1), \tag{1.5}$$

where $\min(\Re(\rho),\,\Re(\sigma)) > 0; \Re(\nu) > \Re(\mu) > 0$ and $\Re(p) \geq 0$ and $B_p^{(\rho,\sigma)}(x,y)$ is generalized Beta type function, which is introduced and studied by Özergin *et al.* [23] in their paper and defined by (see, *e.g.*, [23, p. 4602, Eq.(4)]; see also, [22, p.32, Chapter 4.]):

$$B_p^{(\rho,\sigma)}\,(x,y) := \int_0^1 t^{x-1}\,(1-t)^{y-1}\,{}_1F_1\left(\rho;\sigma;\frac{-p}{t\,(1-t)}\right)dt, \tag{1.6}$$

$$(\Re(p) \geq 0; \min(\Re(x),\Re(y),\Re(\rho),\Re(\sigma)) > 0 \text{ and } B_0^{(\rho,\sigma)}\,(x,y) = B\,(x,y)),$$

where $B(x,y)$ is a well known Euler's Beta function defined by:

$$B(x,y) := \int_0^1 t^{x-1}(1-t)^{y-1}dt\ (\Re(x) > 0, \Re(y) > 0). \tag{1.7}$$

Some generating relations and finite summation formula of class of polynomials or sequence of function have been obtained by using the properties of the differential operators. $T_x^{a,s} \equiv x^a\,(s+xD)$, where $D \equiv \dfrac{d}{dx}$, is based on the work of Mittal[13], Patil and Thakare[15], Srivastava and Singh [19].

Some useful operational techniques are given below:

$$\exp\left(tT_x^{a,s}\right)\left(x^{\beta}f\,(x)\right) = x^{\beta}\,(1-ax^at)^{-\left(\frac{\beta+s}{a}\right)}\,f\left(x\,(1-ax^at)^{-1/a}\right), \tag{1.8}$$

$$\exp\left(tT_x^{a,s}\right)\left(x^{\alpha-an}f\,(x)\right) = x^{\alpha}\,(1+at)^{-1+\left(\frac{\alpha+s}{a}\right)}\,f\left(x\,(1+at)^{1/a}\right), \tag{1.9}$$

$$\left(T_x^{a,s}\right)^n(xuv) = x\sum_{m=0}^{\infty}\binom{n}{m}\left(T_x^{a,s}\right)^{n-m}(v)\left(T_x^{a,1}\right)^m(u)\,, \tag{1.10}$$

$$(1.11)\qquad (1+xD)\,(1+a+xD)\ldots(1+(m-1)a+xD)\,x^{\beta-1}=a^m\left(\frac{\beta}{a}\right)_m x^{\beta-1},$$

$$(1.12)\qquad (1-at)^{\frac{-\alpha}{a}}=(1-at)^{\frac{-\beta}{a}}\sum_{m=0}^{\infty}\left(\frac{\alpha-\beta}{a}\right)_m\frac{(at)^m}{m!}.$$

2. Generating Relations

In this section, we present some generating relations involving the $F_p^{(\alpha,\beta)}(.)$.
First generating relation:

$$(2.1)\qquad \sum_{n=0}^{\infty}V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,x^{-an}t^n=(1-at)^{-\left(\frac{\alpha+s}{a}\right)}$$
$$\times F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k\left(x\,(1-at)^{-1/a}\right)\right].$$

Second generating relation:

$$(2.2)\qquad \sum_{n=0}^{\infty}V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha-an)}(x;a,k,s)\,x^{-an}t^n=(1+at)^{-1+\left(\frac{\alpha+s}{a}\right)}$$
$$\times F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k\left(x\,(1+at)^{1/a}\right)\right].$$

Third generating relation:

$$(2.3)\qquad \sum_{m=0}^{\infty}\binom{m+n}{n}V_{m+n}^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,x^{-am}t^m=(1-at)^{-\left(\frac{\alpha+s}{a}\right)}$$
$$\times\frac{F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]}{F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k\left(x\,(1-at)^{-1/a}\right)\right]}$$
$$\times V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}\left(x\,(1-at)^{-1/a};a,k,s\right).$$

Proof of first generating relation
From (1.4), we obtain

$$(2.4)\qquad \sum_{n=0}^{\infty}V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,t^n=x^{-\alpha}F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]$$
$$\times\exp(tT_x^{a,s})\left\{x^{\alpha}F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k(x)\right]\right\}.$$

Using operational technique (1.6), above equation (2.4) is reduces to

$$(2.5)\qquad \sum_{n=0}^{\infty}V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,t^n=(1-ax^at)^{-\left(\frac{\alpha+s}{a}\right)}$$
$$\times F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k\left(x(1-ax^at)^{-1/a}\right)\right].$$

Replacing t by tx^{-a}, (2.1) is obtained.

Proof of second generating relation
Again from (1.4), we have

$$\sum_{n=0}^{\infty} x^{-an} V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha-an)}(x;a,k,s)\, t^n = x^{-\alpha} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \times \exp(tT_x^{a,s})\left\{x^{\alpha-an} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\}. \tag{2.6}$$

Applying the operational technique (1.7), we get

$$\sum_{n=0}^{\infty} x^{-an} V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha-an)}(x;a,k,s)\, t^n = (1+at)^{\frac{\alpha+s}{a}-1} \times F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)]\, F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k\left(x(1+at)^{1/a}\right)\right], \tag{2.7}$$

which is desired.
Proof of third generating relation
We can write (1.4) as

$$(T_x^{a,s})^n \left[x^{\alpha} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right] = n! x^{\alpha} \frac{V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)}{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)]}. \tag{2.8}$$

Thus we obtain

$$\exp\left(t\left(T_x^{a,s}\right)\right)\left\{(T_x^{a,s})^n \left[x^{\alpha} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right]\right\} = n!\exp\left(tT_x^{a,s}\right) \times \left[x^{\alpha} \frac{V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)}{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)]}\right] \tag{2.9}$$

$$\sum_{m=0}^{\infty} \frac{t^m}{m!} (T_x^{a,s})^{m+n} \left\{x^{\alpha} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\} = n!\exp\left(tT_x^{a,s}\right) \times \left\{x^{\alpha} \frac{V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)}{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)]}\right\}. \tag{2.10}$$

Using the operational technique (1.6), above equation can be written as

$$\sum_{m=0}^{\infty} \frac{t^m}{m!} (T_x^{a,s})^{m+n} \left[x^{\alpha} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right] = n! x^{\alpha} \times (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}\left(x(1-ax^a t)^{-1/a};a,k,s\right)}{F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k\left(x(1-ax^a t)^{-1/a}\right)\right]}. \tag{2.11}$$

Using (2.9), above equation gives

$$\sum_{m=0}^{\infty} \frac{t^m (m+n)!}{m!n!} x^{\alpha} \frac{V_{m+n}^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)}{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]} = x^{\alpha} \times (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}\left(x(1-ax^a t)^{-1/a};a,k,s\right)}{F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k\left(x(1-ax^a t)^{-1/a}\right)\right]}. \tag{2.12}$$

Therefore

$$\sum_{m=0}^{\infty}\binom{m+n}{n}V_{m+n}^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,t^{m}=(1-ax^{a}t)^{-\left(\frac{\alpha+s}{a}\right)}$$
$$\times\frac{F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}\left(x(1-ax^{a}t)^{-1/a};a,k,s\right)}{F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k\left(x(1-ax^{a}t)^{-1/a}\right)\right]}. \tag{2.13}$$

Replacing t by tx^{-a}, this gives the result (2.3).

Remark 2.1. If we give some suitable parametric replacement in (2.1), (2.2) and (2.3) respectively, then we can see the known results (see [5, 6, 7, 8, 9, 10, 11, 12, 15, 18, 17, 19, 20]).

3. Finite Summation Formulas

Here, we study some finite summation formulas involving the $F_p^{(\alpha,\beta)}(.)$

First finite summation formula:

$$V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)=\sum_{m=0}^{n}\frac{1}{m!}(ax^{a})^{m}\left(\frac{\alpha}{a}\right)_m V_{n-m}^{(\lambda,\mu;\nu;\rho,\sigma;p;0)}(x;a,k,s). \tag{3.1}$$

Second finite summation formula:

$$V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)=\sum_{m=0}^{n}\frac{1}{m!}(ax^{a})^{m}\left(\frac{\alpha-\beta}{a}\right)_m \times V_{n-m}^{(\lambda,\mu;\nu;\rho,\sigma;p;\beta)}(x;a,k,s). \tag{3.2}$$

Proof of first finite summation formula

From equation (1.4), we have

$$V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)=\frac{1}{n!}x^{-\alpha}F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right] \times\left(T_x^{a,s}\right)^{n}\left\{xx^{\alpha-1}F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k(x)\right]\right\}. \tag{3.3}$$

Using the operational technique (1.8), we have

$$\begin{aligned}V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)&=\frac{1}{n!}x^{-\alpha}F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]x\\&\times\sum_{m=0}^{\infty}\binom{n}{m}\left(T_x^{a,s}\right)^{n-m}\left\{F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k(x)\right]\right\}\left(T_x^{a,1}\right)^{m}\left(x^{\alpha-1}\right)\\&=\frac{1}{n!}x^{-\alpha}F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;p_k(x)\right]x\sum_{m=0}^{\infty}\frac{n!}{m!\,(n-m)!}x^{a(n-m)}\\&\times\left[(s+xD)(s+a+xD)(s+2a+xD)\dots(s+(n-m-1)a+xD)\right]\\&\times F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k(x)\right]x^{am}\\&\times\left[(1+xD)(1+a+xD)(1+2a+xD)\dots(1+(m-1)a+xD)\right]\left(x^{\alpha-1}\right).\end{aligned} \tag{3.4}$$

Using the result (1.9), we have

$$V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s) = F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \sum_{m=0}^{n} \frac{1}{m!\,(n-m)!} x^{an} \times \prod_{i=0}^{n-m-1}(s+ia+xD)\left\{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\} a^m \left(\frac{\alpha}{a}\right)_m. \tag{3.5}$$

Putting $\alpha = 0$ and replacing n by $n-m$ in (3.3), we get

$$V_{n-m}^{(\lambda,\mu;\nu;\rho,\sigma;p;0)}(x;a,k,s) = \frac{1}{(n-m)!} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \times (T_x^{a,s})^{n-m}\left\{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\} \tag{3.6}$$

$$\Rightarrow \frac{1}{(n-m)!}(T_x^{a,s})^{n-m}\left\{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\} = \frac{V_{n-m}^{(\lambda,\mu;\nu;\rho,\sigma;p;0)}(x;a,k,s)}{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)]}. \tag{3.7}$$

This gives

$$\frac{1}{(n-m)!}\prod_{i=0}^{n-m-1}(s+ia+xD)\left\{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\} = x^{a(m-n)} \frac{V_{n-m}^{(\lambda,\mu;\nu;\rho,\sigma;p;0)}(x;a,k,s)}{F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)]}. \tag{3.8}$$

From equations (3.5) and (3.8) we have the main result.

Proof of second finite summation formula

Equation (1.4) can be written as

$$\sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,t^n = x^{-\alpha} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \times \exp\left(tT_x^{(a,s)}\right)\left\{x^{\alpha} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\}. \tag{3.9}$$

Applying the equation (1.6) in the equation (3.9), we have

$$\sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,t^n = (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \times F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k\left(x\,(1-ax^a t)^{-1/a}\right)\right]. \tag{3.10}$$

Applying the result from the equation (1.12) the equation (3.10) is reduced to

$$\begin{aligned}
&\sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s)\,t^n = (1-ax^a t)^{-\left(\frac{\beta+s}{a}\right)} \\
&\quad\times \sum_{m=0}^{\infty}\left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \\
&\quad\times F_p^{(\rho,\sigma)}\left[\lambda,\mu;\nu;-p_k\left(x(1-ax^a t)^{-1/a}\right)\right] \\
&= \sum_{m=0}^{\infty}\left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} x^{-\beta} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \exp\left(tT_x^{a,s}\right) \\
&\quad\times \left\{x^{\beta} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\} \\
&= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m\, t^{n+m}}{m!n!} x^{-\beta} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \left(T_x^{a,s}\right)^n \\
&\quad\times \left\{x^{\beta} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\} \\
&= \sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m\, t^{n}}{m!\,(n-m)!} x^{-\beta} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \left(T_x^{a,s}\right)^{n-m} \\
&\quad\times \left\{x^{\beta} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\}.
\end{aligned} \tag{3.11}$$

Now equating the coefficient of t^n, we get

$$\begin{aligned}
V_n^{(\lambda,\mu;\nu;\rho,\sigma;p;\alpha)}(x;a,k,s) &= \sum_{m=0}^{n}\left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m}{m!\,(n-m)!} x^{-\beta} \\
&\quad\times F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;p_k(x)] \left(T_x^{a,s}\right)^{n-m} \left\{x^{\beta} F_p^{(\rho,\sigma)}[\lambda,\mu;\nu;-p_k(x)]\right\}.
\end{aligned} \tag{3.12}$$

Using the equation (1.4) in (3.12), we have the result (3.2).

4. Special Cases and Conclusion

(I) All the results established in equation (2.1), (2.2), (2.3), (3.1) and (3.2) can be reduced in to the known works (see, [5, 2, 3, 4]) by assigning suitable value to the parameters in generalized Gauss hypergeometric function $F_p^{(\alpha,\beta)}(.)$.
(II) If we apply the Wright function $W(\alpha,\delta;z)$ very special case of hypergeomtric function ${}_pF_q$; all the results established in equation (2.1), (2.2), (2.3), (3.1) and (3.2) reduced to the work of Joshi and Prajapati [On New Sequence of Functions and Their MATLAB Computation].

Now, we conclude present investigate by remarking that by using our main sequence formula we presented some generating relations and finite summation formula of the sequence here. All results of this paper are important due to presence of $F_p^{(\alpha,\beta)}(.)$. On account of the most general nature of the $F_p^{(\alpha,\beta)}(.)$ a large number of sequences and polynomials involving simpler functions can be easily obtained as their special cases but due to lack of space we can not mention here.

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